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スーパーグローバル大学創成支援 早稲田大学 数物系科学拠点
Mathematics and Physics Unit "Multiscale Analysis, Modelling and Simulation"
Top Global University Project, Waseda University

Lecture series on Mathematical Fluid Dynamics in Waseda “Maximal Regularity Theorem and Mathematical Fluid Dynamics”

Waseda University, Tokyo, Japan   March 9–12, 2021

Venue: Online via ZOOM

Timetable (in Japan Time = GMT+9)

	Mar. 9 (Tue)	Mar. 10 (Wed)	Mar. 11 (Thu)	Mar. 12(Fri)
17:00 | 18:00	Yoshihiro Shibata ①	Senjo Shimizu ①	Yoshihiro Shibata ②	Senjo Shimizu ②
18:15 | 19:15	Robert Denk ①	Patrick Tolksdorf ①	Mads Kyed ②	Patrick Tolksdorf ②
19:30 | 20:30	Mads Kyed ①	Robert Denk ②	Robert Denk ③	Mads Kyed ③

Lecturer	Title
Robert Denk (Universität Konstanz)	Maximal regularity and the Newton polygon approach
Mads Kyed (Hochschule Flensburg)	Fluid-structure interaction under periodic forcing
Yoshihiro Shibata (Waseda Univ.)	R-Bounded Solution Operators and Mathematical Fluid Dynamics
Senjo Shimizu (Kyoto Univ.)	Maximal L^1-regularity and a free boundary problem for the incompressible Navier-Stokes equations
Patrick Tolksdorf (Johannes Gutenberg-Universität Mainz)	Free Boundary Problems via Da Prato - Grisvard Theory

Maximal regularity and the Newton polygon approach

Robert Denk

University of Konstanz, Germany

robert.denk@uni-konstanz.de

Lecture 1: Maximal regularity for parabolic boundary value problems

Maximal regularity is one of the standard approaches to investigate semilinear and quasilinear parabolic evolution equations. The basic idea of maximal regularity is to find appropriate spaces for the right-hand sides and for the solution, in which the operator associated to the linearized equation induces an isomorphism. We consider a linearized boundary value problem of the form

$$\begin{aligned} \partial_t u - Au &= f \quad \text{in } (0,T)\times G,\\ B_j u &= g_j \quad \text{on } (0,T)\times \partial G, \quad (j=1,\dots,m),\\ u|_{t=0} &= u_0. \end{aligned}$$

For maximal L^p-regularity, one considers L^p-Sobolev spaces in time t and space x with $p \in (1,\infty)$ for the data and the solution. Here the trace spaces for $t=0$ or on the boundary will Sobolev spaces of non-integer order in the sense of Besov spaces.

To analyze the above boundary value problem, one can study the related model problem in the half-space $\mathbb{R}^n_+ := \{x \in \mathbb{R}^n : x_n > 0\}$. We assume the boundary value problem to be parabolic which means, in particular, that the Shapiro-Lopatinskii condition has to be satisfied. Under this condition, the solution operators to the model problem can be constructed.

Lecture 2: The Newton polygon approach

In contrast to classical parabolic boundary value problems, there are several examples where the equations have an inhomogeneous structure, which implies that the principal part of the symbol cannot be defined. In particular, this appears in equations with a free boundary or with some dynamics on the boundary like the Stefan problem. In these examples, uniform a priori estimates and maximal regularity cannot be obtained by the classical results.

One possible approach to show maximal regularity makes use of the Newton polygon related to the boundary value problem. One obtains a generalized definition of parameter-ellipticity and uniform a priori estimates. The Sobolev spaces for the data and the right-hand sides are defined with the help of the Newton polygon, which typically leads to intersection spaces with dominating mixed smoothness.

Lecture 3: Applications

Typical applications for the Newton polygon method are given by boundary value problems with free boundary which appear in crystallization theory or in fluid mechanic. We discuss an example of a fluid-structure interaction problem, where the Navier Stokes equation for the fluid is coupled with a damped plate equation for the elastic boundary. One obtains maximal L^p-regularity in Sobolev spaces which are defined with the help of the Newton polygon. The maximal regularity approach leads to a unique strong solution for small times (or small data) of the nonlinear boundary value problem.

Date: I. Tuesday, March 9, 18:15–19:15
II. Wednesday, March 10, 19:30–20:30
III. Thursday, March 11, 19:30–20:30

Fluid-structure interaction under periodic forcing

MADS KYED
HOCHSCHULE FLENSBURG, GERMANY
mads.kyed@hs-flensburg.de

A periodic forcing of an undamped elastic structure can lead to onset of resonance. If the elastic structure is in contact with a viscous fluid, the interaction with the fluid provides a damping mechanism. Depending on the specific nature of the interaction, one can typically argue from a purely physical point of view that the damping via energy dissipation in the viscous fluid prevents the onset of resonance in the structure. The main objective in my talks is to rigorously confirm this behavior from a mathematical point of view. I will discuss the possibility of quantifying the damping effect of the fluid interaction. In order to mathematically investigate the fluid-structure system, I will introduce a general technique and function analytic framework to obtain suitable a priori estimates of maximal regularity type in the periodic setting.

Date: I. Tuesday, March 9, 19:30–20:30
II. Thursday, March 11, 18:15–19:15
III. Friday, March 12, 19:30–20:30

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R-bounded solutions operators and Mathematical Fluid Dynamics

YOSHIHIRO SHIBATA
WASEDA UNIVERSITY, JAPAN
yshibata325@gmail.com

I would like to explain a systematic method of obtaining the maximal regularity of solutions for a system of a linear parabolic equations with non-homogeneous boundary conditions based on R-solution operators for the resolvent problem with non-homogeneous boundary conditions. In fact, combination of R-bounded solution operators with the Weis operator valued Fourier multiplier theorem and extension of de Leeuv transference theorem to the operator valued Fourier multiplier yield the maximal regularity theorem for the initial boundary value problem for linear parabolic systems with non-homogeneous boundary conditions and high frequency part of periodic solutions for linear parabolic system with non-homogeneous boundary conditions.

As application of our approach based on R-bounded solution operators, I discuss the local and global well posedness of a free boundary problem for the Navier-Stokes equations in an exterior domain, and the unique existence theorem of periodic solutions of the Navier-Stokes equations in a periodically moving three dimensional domain.

Date: I. Tuesday, March 9, 17:00–18:00
II. Thursday, March 11, 17:00–18:00

Maximal L^1-regularity and a free boundary problem for the incompressible Navier-Stokes equations

SHIMIZU SENJO
KYOTO UNIVERSITY, JAPAN
shimizu.senjo.5s@kyoto-u.ac.jp

Time-dependent free surface problem for the incompressible Navier-Stokes equations which describes the motion of viscous incompressible fluid nearly half-space is considered. We obtain a global well-posedness of the problem for a small initial data in scale invariant critical Besov spaces. Our proof is based on maximal L^1-regularity of the corresponding Stokes problem in the half-space. This is a joint work with Takayoshi Ogawa (Tohoku University).

Date: I. Wednesday, March 10, 17:00–18:00
II. Friday, March 12, 17:00–18:00

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Free Boundary Problems via Da Prato - Grisvard Theory

PATRICK TOLKSDORF
JOHANNES GUTENBERG-UNIVERSITÄT MAINZ, GERMANY
tolksdorf@uni-mainz.de

A common way to prove global well-posedness of free boundary problems for incompressible viscous fluids is to transform the equations governing the fluid motion to a fixed domain with respect to the time variable. An elegant and physically reasonable way to do this is to introduce Lagrangian coordinates. These coordinates are given by the transformation rule

$$x(t) = \xi + \int_0^t u(\tau, \xi) \, \mathrm{d}\tau,$$

where $u(\tau, \xi)$ is the velocity vector of the fluid particle at time τ that initially started at position ξ. The variable $x(t)$ is then the so-called Eulerian variable and belongs to the coordinate frame where the domain that is occupied by the fluid moves with time. The variable ξ is the Lagrangian variable that belongs to time fixed variables. In these coordinates the fluid only occupies the domain Ω_0 that is occupied at initial time $t = 0$.

To prove a global existence result for such a problem, it is important to guarantee the invertibility of this coordinate transform globally in time. By virtue of the inverse function theorem, this is the case if

$$\nabla_\xi x(t) = \mathrm{Id} + \int_0^t \nabla_\xi u(\tau, \xi) \, \mathrm{d}\tau$$

is invertible. By using a Neumann series argument, this is invertible, if the integral term on the right-hand side is small in $\mathrm{L}^\infty(\Omega_0)$. Thus, it is important to have a global control of this L^1-time integral with values in $\mathrm{L}^\infty(\Omega_0)$. If the domain is bounded, this can be controlled by decay properties of the corresponding semigroup operators that describe the motion of the linearized fluid equation. On certain unbounded domains, however, these decay properties are not true anymore. While there are technical possibilities to fix these problems if the boundary is compact, these fixes cease to work if the boundary is non-compact.

As a model problem, we consider the case where Ω_0 is the upper half-space. To obtain estimates of the L^1-time integral we establish a homogeneous version of the celebrated theorem of Da Prato and Grisvard (1975) about maximal regularity in real interpolation spaces. In these lectures, we will describe this homogeneous Da Prato–Grisvard theorem in detail and show how it can be applied to solve problems from fluid mechanics involving a free non-compact boundary.

This is a joint work with Raphaël Danchin, Matthias Hieber, and Piotr Mucha.

Date: I. Wednesday, March 10, 18:15–19:15
II. Friday, March 12, 18:15–19:15