２０２６年度 早稲田大学大学院教育学研究科
博士後期課程　一般・外国学生入学試験問題　資料解読
【教科教育学専攻（数学科教育学・数学科内容学）】

解答上の注意

１．問題は１問のみである。

２．解答用紙の所定欄に、研究指導名・指導教員名・受験番号・氏名を必ず記入すること。

３．解答用紙が複数枚配付された場合、ホッチキスははずさないこと。また、無解答の解答用紙でも提出すること。

４．問題用紙は「２枚」（本ページ含む）、解答用紙は「１枚」です。必ず枚数を確認すること。

以　上

２０２６年度　早稲田大学大学院　教育学研究科
博士後期課程　入学試験問題

科目名　資料解読（数学科教育学・数学科内容学）

問題

次の文章のうち，「The situation is much」から始まる段落の内容を日本語でわかりやすく説明せよ。他の段落も踏まえて記号・用語の説明なども補足すること。

Type-I Berkovich Points. Each point a in the standard unit disk $\bar{D}(0,1)$ of $\mathbb{C}_p$ is associated to a point of the Berkovich disk, which we denote by

$$\xi_{a,0} \in \bar{D}^{\mathcal{B}}.$$

Type-II Berkovich Points. Each closed disk $\bar{D}(a,r)$ contained in $\bar{D}(0,1)$ with radius $r \in |\mathbb{C}_p^*| = p^{\mathbb{Q}}$ is associated to a point of the Berkovich disk, which we denote by

$$\xi_{a,r} \in \bar{D}^{\mathcal{B}}.$$

Type-III Berkovich Points. Similarly, each closed disk $\bar{D}(a,r) \subseteq \bar{D}(0,1)$ with positive radius $r \notin |\mathbb{C}_p^*| = p^{\mathbb{Q}}$ is associated to a point of the Berkovich disk, which we naturally also denote by $\xi_{a,r}$.

中略

In order to visualize the Berkovich disk, we place the Gauss point $\xi_{0,1}$ at the top of the page and observe that there is a line segment running from any point $\xi \neq \xi_{0,1}$ up to the Gauss point. If $\xi = \xi_{a,r}$ is of Type-I, II, or III, then this line segment is the set of points

$$L_{a,r} = \{\xi_{a,t} : r \le t \le 1\}.$$

Notice that any two line segments $L_{a,r}$ and $L_{b,s}$ merge with one another at the point $\xi_{a,t} = \xi_{b,t}$ determined by

$$t = \max\{r, s, |b-a|\}.$$

This is the smallest allowable value of t for which the disks $\bar{D}(a,t)$ and $\bar{D}(b,t)$ acquire a common point, hence for which they coincide. Thus one can imagine the various line segments continually merging as they run upward toward the Gauss point at the top of the tree.

中略

The picture for points of Type I is clear. In order to understand Types II and III, suppose that we fix a point $\xi_{a,r}$ of Type II or III. Each $b \in \bar{D}(a,r)$ gives a line segment $L_{b,0}$ that runs up from the Type-I point $\xi_{b,0}$ and through the point $\xi_{r,a}$. Two such line segments $L_{b,0}$ and $L_{b',0}$ merge before reaching $\xi_{a,r}$ if and only if $|b-b'| < r$, so it is really each open disk $D(b,r)$ inside the closed disk $\bar{D}(a,r)$ that gives a line segment running up to $\xi_{a,r}$.

If $\xi_{a,r}$ is of Type III, then $D(b,r) = \bar{D}(a,r)$ for any $b \in \bar{D}(a,r)$, so there is only one segment running downward from $\xi_{a,r}$.

The situation is much more interesting, and complicated, if $\xi_{a,r}$ is of Type II. In this case $\bar{D}(a,r)$ is covered by a countable union of open disks $D(b,r)$, so there is a countable set of branches downward from $\xi_{a,r}$. A convenient, although noncanonical, way to describe the branches is as follows. Let $\mathfrak{P} = \{z \in \mathbb{C}_p : |z| < 1\}$ denote the maximal ideal in the ring of integers of $\mathbb{C}_p$ and fix some $c \in \mathbb{C}_p$ with $|c| = r$. Then the open disks of radius r are in one-to-one correspondence with the residue field $\bar{\mathbb{F}}_p$ via the map

$$\{\text{disks } D(b,r) \text{ inside } \bar{D}(a,r)\} \longrightarrow \bar{\mathbb{F}}_p, \qquad D(b,r) \longmapsto (b/c) \bmod \mathfrak{P}.$$

The surjectivity of this map is clear, and the injectivity follows from the fact that $D(b,r) = D(b',r)$ if and only if $|b-b'| < r = |c|$.

出典 Joseph H. Silverman, The Arithmetic of Dynamical Systems, Graduate Texts in Mathematics (2007), vol. 241, Springer, pp. 295―299.

研究指導	
教員名（	）
受験番号	
氏　名	

早稲田大学大学院教育学研究科
博士後期課程　一般・外国学生入学試験解答用紙
科目名：資料解読
【教科教育学専攻（数学科教育学・数学科内容学）

大学記入欄

問題番号

▼　　　裏面に続く　▼

▼　裏面を使用する場合はここから記入すること ▼

▲　ここまで ▲