２０２２年度 早稲田大学大学院教育学研究科
博士後期課程 一般・外国学生入学試験問題 資料解読
【教科教育学専攻（数学科教育学・数学科内容学）】

解答上の注意

１． 教科教育学専攻（数学科教育学・数学科内容学）の入学試験問題は、出願時に届け出た指導教員の欄に従い、下記の表の解答すべき問題を解答しなさい。

志願票に記入した研究指導名	志願票に記入した指導教員名	解答すべき問題・ページ
数学科教育学研究指導	宮川 健	問題 Ⅰ（P.２）または問題 Ⅱ（P.３）のいずれかを選択
数学科教育学研究指導	高木 悟	
数学科内容学研究指導	新井 仁之	
数学科内容学研究指導	梁 松	
数学科内容学研究指導	戸松 玲治	
数学科内容学研究指導	村井 聡	
数学科内容学研究指導	小森 洋平	
数学科内容学研究指導	小柴 健史	
数学科内容学研究指導	髙島 克幸	
数学科内容学研究指導	谷山 公規	

２． 解答用紙の所定欄に、「問題番号」（例：「Ⅰ」・「Ⅱ」など）を必ず記入すること。また、全ての解答用紙の所定欄に研究指導名・指導教員名・受験番号・氏名を必ず記入すること。

３． 解答すべき問題以外を解答した場合、当該解答は「０点」となります。

４． 解答用紙が複数枚配付された場合、ホッチキスははずさないこと。また、無解答の解答用紙でも提出すること。

５． 問題用紙は「３枚」（本ページ含む）、解答用紙は「１枚」です。必ず枚数を確認すること。

以 上

科目名　**資料解読（数学科教育学・数学科内容学）**

問題 I　次の文章にある subset sum problem および grid shading problem とはどのような問題か日本語でわかりやすく説明せよ。

(i) *Pick six positive real numbers—any six positive real numbers. If you chose*

$$1, e, \pi, 4, \sqrt{67}, 98.6,$$

you're in trouble, because the object of this game is to have as many subsets as possible adding up to the same sum. For example, choosing

$$5, 6, 8, 9, 13, 14$$

yields three subsets with the same sum:

$$14 = 5 + 9 = 6 + 8.$$

The set

$$3, 4, 5, 6, 7, 8$$

has four subsets adding up to the same number:

$$7 + 8 = 3 + 4 + 8 = 3 + 5 + 7 = 4 + 5 + 6.$$

Choosing

$$1, 2, 3, 4, 5, 6$$

yields five subsets with the same sum:

$$4 + 6 = 1 + 3 + 6 = 1 + 4 + 5 = 2 + 3 + 5 = 1 + 2 + 3 + 4.$$

Is this the best possible? This problem can be more generally posed for n positive real numbers: Then it seems that no collection does better than

$$1, 2, 3, \ldots, n - 1, n.$$

Call this problem the **subset sum problem**.

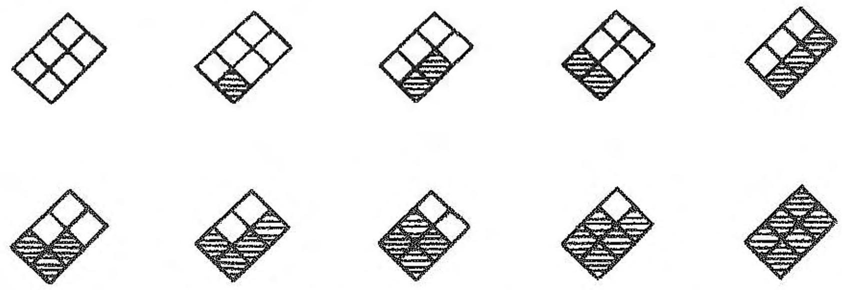

Fig. 1.

(ii) *Fix two positive integers m and n. Draw an $m \times n$ grid of squares "on tilt" as in Fig. 1. Now shade in some of the squares so that there are no unshaded squares below shaded squares—i.e., so that if the shaded squares were blocks in a rectangular frame, none would slide down. Call such a shading a* **proper shading**. *As an index k runs from 0 to mn, count how many proper shadings there are with k shaded squares. For example, if $m = 2$ and $n = 3$, then the count is*

$$1, 1, 2, 2, 2, 1, 1$$

as k runs from 0 to 6. For $m = 2$ and $n = 4$, the count is

$$1, 1, 2, 2, 3, 2, 2, 1, 1.$$

And for $m = 3$ and $n = 4$ one counts

$$1, 1, 2, 3, 4, 4, 5, 4, 4, 3, 2, 1, 1.$$

It seems that the count always weakly increases until half of the squares have been shaded, and then weakly decreases until all of the squares have been shaded. In other words, there seem to be no "dips" in the count. It gets bigger, then smaller. Is this always true for any size grid? Call this the **grid shading problem**.

出典 R.A. Proctor, Solution of Two Difficult Combinatorial Problems with Linear Algebra, The American Mathematical Monthly (1982), vol. 89, pp. 721-734.

２０２２年度

早稲田大学大学院教育学研究科博士後期課程入学試験問題

科目名　資料解読（数学科教育学・数学科内容学）

問題 II　(1) 次の英文を和訳せよ

(2) 文中にある point of accumulation（集積点）とはどのような点であると考えられるか、日本語で説明せよ。

A sequence $\{x_n\}$ is said to be a **Cauchy sequence** if given ϵ there exists N such that for all $m, n \geqq N$ we have

$$|x_m - x_n| < \epsilon.$$

Intuitively we see that the terms of a Cauchy sequence come closer and closer together. *We observe that if a sequence converges, then it is a Cauchy sequence.* The proof for this is easy, for if the sequence $\{x_n\}$ converges to the limit a, given ϵ there exists N such that for all $n \geqq N$ we have

$$|x_n - a| < \frac{\epsilon}{2}.$$

Also for all $m \geqq N$ we have

$$|x_m - a| < \frac{\epsilon}{2}.$$

Hence for $m, n \geqq N$, we have

$$|x_m - x_n| \leqq |x_m - a| + |a - x_n| < \epsilon,$$

thus proving that our sequence is a Cauchy sequence. The converse of the statement we just proved is also true, but we need the completeness axiom to prove it, via the Weierstrass–Bolzano theorem.

Theorem 1.5. *Let $\{x_n\}$ be a Cauchy sequence of numbers. Then $\{x_n\}$ converges, i.e. it has a limit.*

Proof. First we need a lemma.

Lemma 1.6. *If $\{x_n\}$ is a Cauchy sequence, then it is bounded.*

Proof. Given 1 there exists N such that if $n \geqq N$ then

$$|x_n - x_N| < 1.$$

From this it follows that $|x_n| \leqq |x_N| + 1$ for all $n \geqq N$. We let B be the maximum of $|x_1|, \dots, |x_N|, |x_N| + 1$. Then B is a bound for the sequence.

From the lemma, we conclude that $-B \leqq x_n \leqq B$ for all n. By the Weierstrass–Bolzano theorem, the sequence $\{x_n\}$ has a point of accumulation c. We shall prove that c is a limit of the sequence. Given ϵ, there exists N such that if $m, n \geqq N$ we have

$$|x_n - x_m| < \frac{\epsilon}{2}.$$

Since c is a point of accumulation, we can select m such that $m \geqq N$ and

$$|x_m - c| < \frac{\epsilon}{2}.$$

Then for all $n \geqq N$, we have

$$|x_n - c| \leqq |x_n - x_m| + |x_m - c| < \epsilon,$$

as was to be shown.

出典　S. Lang, Undergraduate Analysis, Second edition, Springer-Verlag, New York, 1997.