# Research Report (April, 2023- March, 2024)

| Enrollment from April 2020 | Department of Pure and Applied Mathematics | Tadashi Udagawa |
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## I. List of Papers

## II. List of Talks

[1] Tadashi Udagawa, 'tt*-Toda equation and fusion algebra'', Koriyama Geometry and Physics Days 2023 "tt*-Toda equations and infinite-dimensional Lie algebras", May 2023, Nihon University.

[2] Tadashi Udagawa, ''Solutions of the tt*-equations constructed from the SU(2)k-fusion ring, And Smyth potentials'', Geometry seminar, October 2023, Waseda University.

[3] Tadashi Udagawa, ''Solutions of the tt*-equations constructed from the SU(2)k-fusion ring, And Smyth potentials'', The 4th Taiwan-Japan Joint Conference on Differential Geometry, November 2023, National Taiwan University.

[4] Tadashi Udagawa, ''Globality of the DPW construction for Smyth potentials'', Geometry colloquium, November 2023, Hokkaido University.

[5] Tadashi Udagawa, ''Solutions of the tt*-equations constructed from the SU(2)k-fusion ring, And Smyth potentials'', Research meeting on Integrable Systems, Harmonic Maps and Surface Geometry, December 2023, Kobe University.

## III. Research Results in AY2023

We constructed the topological anti-topological fusion(tt*)-structure from the SU(2)k-fusion ring by using the DPW method. The tt*-structure is related to conformal field theory. We investigated the relationship between the tt*-structure and irreducible representations. The tt*-structure is a special case of flat bundles and DUbrovin showed that the connection form gives a harmonic map into the symmetric space.

In my study, we show that a finite number of solutions to the sinh-Gordon equation gives a Hermitiam metric on our tt*-structure. There is a one-to-one correspondence between the Hermitian metric and representation of SU(2). From a physical standpoint, the tt*-structure constructed from the SU(2)k-fusion ring is related to the supersymmetric minimal model.

## IV. Research Plan for AY2024

We try to give a global description of constant mean curvature(CMC)-surface from the tt*-structure with rank two over a Riemann surface. We also investigate the isomonodromic aspects of tt*-structure over a Riemann surface. It is characterized by an upper triangular matrix S and for the case that $S+S^t$ is positive-definite, the tt*-structures wer classified by Cecotti and Vafa from the viewpoint of Physics. We try to give ``mathematical interpretation'' of this classification.