Recent Topics on the Mathematical Fluid Mechanics

Dates: December 7 – 8, 2023
Venue: Waseda University, Green Computing Systems Research and Development Center, (**Access**) Wasedamachi-27, Shinjuku-ku, Tokyo
Website: https://www.waseda.jp/fsci/mathphys/news-en/17623
Registration: https://forms.gle/728Wp29Vdurpju7F7 (Before November 30, 2023)

Timetable

Dec. 7 (Thu)		Dec. 8 (Fri)	
9:50〜9:55	Opening		
10:00〜10:50	Matthias Hieber	10:00〜10:50	Alexey Cheskidov
11:00〜11:50	Tatsu-hiko Miura	11:00〜11:50	Tim Binz
Lunch Break		Lunch Break	
13:30〜14:20	Ryosuke Nakasato	13:20〜14:10	Masahiro Suzuki
14:30〜15:20	Keiichi Watanabe	14:20〜14:40	Yoshiki Iida
Coffee Break		14:45〜15:05	Kei Noda
15:40〜16:00	Taichi Eguchi	Coffee Break	
16:05〜16:25	Naoto Deguchi	15:25〜16:15	Senjo Shimizu
16:35〜17:25	Yoshiyuki Kagei	16:15〜16:20	Closing
Research Exchanges			

Organizers:
Hideo Kozono (Waseda University)
Keiichi Watanabe (Suwa University of Science)

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Thursday, December 7th, 2023

The Primitive Equations subject to Transport Noise and Stochastic Boundary Conditions

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In this talk we discuss first the primitive equations of geophysical flows subject to transport noise. The associated equations are deduced from the anisotropic compressible Navier-Stokes equations and the stochastic variants of the Boussinesq and hydrostatic approximations. We then show that this set of equations admits a strong, global solution under certain assumptions on the noise.

Secondly, we consider related stochastic boundary value problems and show that they admit a unique, local pathwise solution in anisotropic Sobolev spaces of negative order This is joint wirk with A. Agresti, T. Binz, A. Hussein and M. Saal.

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Nonlinear stability of the two-jet Kolmogorov type flow on the unit sphere under a perturbation with nondissipative part

Tatsu-Hiko Miura

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We consider the vorticity form of the Navier-Stokes equations on the 2D unit sphere and study the nonlinear stability of the two-jet Kolmogorov type flow which is a stationary solution given by the zonal spherical harmonic function of degree two. Due to the structure of the viscous term, a perturbation contains a nondissipative part given by a linear combination of the spherical harmonics of degree one. Our interest is in studying the effect of the nondissipative part on the evolution of the whole perturbation. We show that the nondissipative part is preserved in time for all initial perturbation. In fact, this corresponds to conservation of angular momentum. Moreover, we prove that the dissipative part of the perturbation converges exponentially in time towards a nontrivial equilibrium. We further express the equilibrium explicitly in terms of the nondissipative part of the initial perturbation. To prove these results, we make use of some properties of Killing vector fields on a manifold.

On the multi-dimensional diffusion waves for the compressible Navier-Stokes-Korteweg system in scaling critical spaces

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We consider the initial-value problem for the compressible Navier-Stokes-Korteweg system in the d-dimensional Euclidean space $\mathbb{R}^d$ $(d \geq 2)$. The system is well-known as the Diffuse Interface model describing the motion of a vaper-liquid mixture in a compressible viscous fluid. The purposes of this talk are to obtain the global-in-time solution around the constant equilibrium states $(\rho_*, 0)$ $(\rho_* > 0)$ and investigate the L^p-L^1 type time-decay estimates in scaling critical function spaces based on the Fourier-Herz spaces $\widehat{L^p}(\mathbb{R}^d)$. In addition, we consider the time decay with the *diffusion wave property* came from the structure of the linearized system similar to the wave equation with strong damping in the critical Besov spaces. The key idea is the derivation of the time-decay for the Besov norm with higher derivatives by using L^1-maximal regularity for the linearized system. This talk is based on the joint work with Prof. Takayuki Kobayashi (Osaka University).

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Navier–Stokes flow past a moving rigid body with a rough surface

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Consider the three-dimensional Navier–Stokes flow past a rigid body $\mathscr{O} \subset \mathbb{R}^3$. Let Ω be a bounded Lipschitz domain, which is assumed to be simply connected for simplicity, and let $\Omega := \mathbb{R}^3 \setminus \mathscr{O}$ be the exterior of the rigid body. Assume that the rigid body is rotating about y_3-axis with an angular velocity $\omega = (0, 0, a)^\top$, $a \in \mathbb{R}$, as well as moving with a translational velocity $\eta = (\eta_1, \eta_2, \eta_3)^\top \in \mathbb{R}^3$. Here, a and η_ℓ, $\ell \in \{1, 2, 3\}$, are constants. The aim of this talk is to show the L^p-L^q-estimates of the corresponding Stokes semigroup, which extends the results due to Geissert, Heck, and Hieber (2006) as well as Hishida and Shibata (2009) to the case that the boundary is rough, i.e., Lipschitz. The proof used here is strongly inspired from the idea employed in Hishida (2018, 2020) but further careful analyses are required since the Helmholtz decomposition of $L^p(\Omega)$ holds for *some* $p \in (1, \infty)$ due to the roughness of the boundary. As an application of the L^p-L^q-estimates of the corresponding Stokes semigroup, the global existence of a unique solution to the Navier–Stokes equations are shown. This talk is based on a joint work with Dr. Tomoki Takahashi (Tokyo Institute of Technology).

The energy equality of the magnetohydrodynamic system in the framework of Lorentz-Besov spaces

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We find a new criterion of the Lorentz–Besov spaces to establish the energy equality of the incompressible magnetohydrodynamic (MHD) system in $\mathbb{R}^3$. If the magnetic field is equal to zero, i.e., the Navier–Stokes equations, Cheskidov–Constantin–Friedlander–Shvydkoy (2008) proved the energy equality if the weak solution to the Navier–Stokes equations belongs to $L^3(0,T;B^{1/3}_{3,\infty})$. Recently, Cheskidov–Luo (2020) obtained the class of the Lorentz–Besov spaces $L^{p,\infty}(0,T;B^s_{q,\infty})$ that are not contained in $L^3(0,T;B^{1/3}_{3,\infty})$. In this talk, we obtain a new criterion to establish the energy equality of the MHD system. We may regard our criterion on the MHD system as a generalization of the criterion of Cheskidov–Luo (2020). Moreover, our class of magnetic field is larger than that of velocity field even though the nonlinear structure of the magnetic field is almost the same as that of the velocity field.

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On the stability of stationary compressible Navier-Stokes flows

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We consider the stability of the stationary solution of the compressible Navier-Stokes equation in the 3D whole space with an external force which decays at spatial infinity. The stationary solution is known to be asymptotically stable if the external force is small enough. In our work, the time decay rates of the L^p norms of the perturbations are derived under the smallness assumption on the initial perturbations. It is also showed that the decay rates are optimal. The proof is based on the combination of the spectral analysis and energy method in Besov spaces. The time-space integral estimates for the linearized semigroup around the constant state in some Besov spaces play a crucial role in the proof.

Eckhaus instability of the compressible Taylor vortices

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This talk is concerned with the bifurcation and stability of the compressible Taylor vortex. Consider the compressible Navier-Stokes equations in a domain between two concentric infinite cylinders. If the outer cylinder is at rest and the inner one rotates with sufficiently small angular velocity, a laminar flow, called the Couette flow, is stable. When the angular velocity of the inner cylinder increases, beyond a certain value of the angular velocity, the Couette flow becomes unstable and a vortex pattern, called the Taylor vortex, bifurcates and is observed stably. This phenomenon is mathematically formulated as a bifurcation and stability problem. In this talk, the compressible Taylor vortices are shown to bifurcate near the criticality for the incompressible problem when the Mach number is sufficiently small. The localized stability of the compressible Taylor vortices is considered under axisymmetric perturbations and it is shown that the Eckhaus instability of compressible Taylor vortices occurs as in the case of the incompressible ones.

Thursday, December 8th, 2023

Dissipation anomaly and anomalous dissipation in incompressible fluid flows

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Dissipation anomaly, a phenomenon predicted by Kolmogorov's theory of turbulence, is the persistence of a non-vanishing energy dissipation for solutions of the Navier-Stokes equations as the viscosity goes to zero. This law has been extensively verified experimentally for turbulent fluid flows as well as numerically for solutions of the 3D Navier-Stokes equations at high Reynolds numbers. Anomalous dissipation, predicted by Onsager, is a failure of solutions of the limiting Euler equations to preserve the energy balance, necessary for the validity of the inviscid equations in turbulent regimes. The precise regularity threshold conjectured by Onsager for inviscid solutions to sustain the energy cascade has been a focus of intensive mathematical investigations for decades. In turbulent flows, the energy injected at forced low modes (large scales) cascades to small scales through the inertial range where viscous effects are negligible, and only dissipates above Kolmogorov's dissipation wavenumber that goes to infinity in the limit of vanishing viscosity. The persistence of the energy flux through the inertial range is what constitutes dissipation anomaly for viscous fluid flows as well as anomalous dissipation for the limiting inviscid flows.

Motivated by a recent dissipation anomaly construction for the 3D Navier-Stokes equations by Brue and De Lellis (2023), we prove the existence of various scenarios in the limit of vanishing viscosity: the total and partial loss of the energy due to dissipation anomaly, dissipation anomaly and anomalous dissipation with a continuous energy limit, anomalous dissipation without dissipation anomaly, and the existence of infinitely many limiting solutions of the Euler equations in the limit of vanishing viscosity. We also discover a relation between dissipation anomaly and the discontinuity of the energy of the limit solution. Finally, expanding on the obtained total dissipation anomaly construction, we show the existence of dissipation anomaly for long time averages, relevant for turbulent flows, proving that the Doering-Foias (2002) upper bound is sharp.

Global strong well-posedness of the coupled atmosphere-ocean model

TIM BINZ
(JOINT WORK WITH F. BRANDT, M. HIEBER AND T. ZÖCHLING)
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In their seminal article [1] from 1993 Lions, Temam and Wang introduced the primitive equations of the coupled atmosphere-ocean. It describes the dynamics of the atmosphere and the ocean on a large scale. This model consists of two primitive equations coupled by non-linear wind-driven boundary conditions or non-linear traction conditions at the interface.

Lions, Teman and Wang proved global existence of weak solutions of the primitive equations of the coupled atmosphere-ocean. Uniqueness of weak solutions remains an open problem.

In this talk we show the existence and uniqueness of global strong solutions to the coupled atmosphere-ocean model.

Our proof rely on a new maximal L^p-regularity result for the hydrostatic Stokes operator with inhomogeneous boundary conditions, a Kato-Ponce type para-product inequality in Triebel-Lizorkin spaces due to Chae, and the splitting into barotropic and baroclinic modes for primitive equations discovered by Cao and Titi, as well as a careful analysis of the boundary coupling terms in each step.

References

[1] J.L. Lions, R. Temam, Sh. H. Wang, Models for the coupled atmosphere and ocean. (CAO I,II). Comput. Mech. Adv. 1 (1993), 3 – 119

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Solitary Waves of the Vlasov–Poisson System

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We consider the Vlasov–Poisson system describing a two-species plasma with spatial dimension 1 and the velocity variable in $\mathbb{R}^n$. We find the necessary and sufficient conditions for the existence of solitary waves of the system. To this end, we need to investigate the distribution of ions trapped by the electrostatic potential. Furthermore, we classify completely in all possible cases whether or not the solitary wave is unique, when we exclude the variant caused by translation. There are both cases that it is unique and nonunique. This talk is based on a joint work with Professor M. Takayama (Keio Univ.) and Professor K. Z. Zhang (Northeastern Univ.).

Uniqueness of weak solutions to the primitive equations in some anisotropic spaces

Yoshiki Iida

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We consider the three dimensional primitive equations for oceans and atmosphere in the isothermal setting. The seminal work Cao–Titi (2007) proved the global well-posedness of this system for arbitrary large initial data in H^1, unlike the three dimensional Navier–Stokes equations. However, the uniqueness of weak solutions to the primitive equations is still open. In this talk, based on regularity in terms of Besov spaces, we give a new class in which the uniqueness holds. Our class is different from z-weak solutions, which was introduced by several authors in previous uniqueness results. In this context, the difference between the class of z-weak solution and ours is clarified. The proof of our result is basically relied on Littlewood–Paley theory argument. Such a technique of Littlewood–Paley theory enables us to obtain a slightly larger class which guarantees the energy equality. This talk is based on the joint work with Dr. Tim Binz (TU Darmstadt), arXiv:2309.03443.

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Analyticity in space-time of solutions to evolution equations with multilinear operators based on maximal regulatrity

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Space-time analyticity for evolution equations with nonlinear term represented by multilinear operators is considered via the parameter trick. By extending the case of bilinear operators in the previous work, we obtain space-time analyticity for the equations of Fujita type. In this poster session, we will introduce previous studies in the bilinear case and outline the proof of the main result by extending the result to the multilinear case.

Free boundary problems of the Navier–Stokes equations based on maximal L^1-regularity

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In this talk, we consider end-point maximal L^1-regularity upon the homogeneous Besov space for the Stokes equations with inhomogeneous stress boundary condition. We decompose the Fourier symbol of the Stokes equations in time and space regions. Utilizing the almost orthogonal properties between the boundary potential and the Littlewood–Paley decomposition, we show maximal L^1-regularity in the Besov and the Lizorkin–Triebel spaces. We further discuss an application to free boundary problems of the Navier–Stokes equations. This is a joint work with Takayoshi Ogawa (Tohoku University).