Research Report (April, 2021 - March, 2023)

In the SGU course of Mathematical Physical Science: April 2021-March 2023

Conferring university	Degree name (by completing a course / by thesis only)	Date of conferment
Waseda University	Doctor of Science (course)	March 15, 2023

Enrollment from April 2021 | Department of Pure and Applied Mathematics | Taiki TAKEUCHI

I. List of Papers

01. T. Takeuchi, "The Keller-Segel system of parabolic-parabolic type in homogeneous Besov spaces framework", J. Differential Equations **298** (2021), 609—640.
02. T. Takeuchi, "Maximal Lorentz regularity for the Keller-Segel system of parabolic-elliptic type", J. Evol. Equ. **21** (2021), no. 4, 4619—4640.
03. T. Takeuchi, "Space-time analytic smoothing effect of the heat semigroup defined on homogeneous Besov spaces", Partial Differential Equations in Applied Mathematics **4** (2021), 100174.
04. T. Takeuchi, "Various regularity estimates for the Keller-Segel-Navier-Stokes system in Besov spaces", J. Differential Equations **343** (2023), 606—658.

II. Record of Awards

FY 2020 Azusa Ono Memorial Award (Award of academic work)

III. List of Talks

01. T. Takeuchi, "The Keller-Segel system of parabolic-parabolic type in homogeneous Besov spaces framework", Young Researchers Workshop on Nonlinear PDE, online (Zoom), Mar. 6, 2021.
02. T. Takeuchi, "On the Keller-Segel system of parabolic-parabolic type with initial data in homogeneous Besov spaces", 42nd Young Researchers Seminar on Evolution Equations, online (Zoom), Aug. 31, 2021.
03. T. Takeuchi, "On the Keller-Segel system of parabolic-parabolic type in homogeneous Besov spaces framework", MSJ Autumn Meeting 2021, online (Zoom), Sep. 15, 2021.
04. T. Takeuchi, "On the maximal Lorentz regularity theorem for the Keller-Segel system of parabolic-parabolic type", International Workshop on Multi-Phase Flows: Analysis, Modelling and Numerics, online (Zoom), Dec. 3, 2021.
05. T. Takeuchi, "Inviscid limit problem for the Keller-Segel-Navier-Stokes system of parabolic-elliptic type", Young Researchers' Forum on Mathematical Fluid Mechanics, online (Zoom), Jun. 21, 2022.
06. T. Takeuchi, "On the inviscid limits for the Keller-Segel-Navier-Stokes system of parabolic-elliptic type", OS Special Seminar in Tohoku University, Tohoku University, Jun. 24, 2022.
07. T. Takeuchi, "Vanishing viscosity limit of solutions of the Keller-Segel-Navier-Stokes system", International Workshop on "Fundamental Problems in Mathematical and Theoretical Physics", Waseda University, Jul. 26, 2022.
08. T. Takeuchi, "On the vanishing viscosity method for the Keller-Segel-Navier-Stokes system", 43rd

Young Researchers Seminar on Evolution Equations, online (Zoom), Sep. 7, 2022.

09. T. Takeuchi, "On the local well-posedness and inviscid limits for the Keller-Segel-Navier-Stokes system", MSJ Autumn Meeting 2022, Hokkaido University, Sep. 9, 2022.
10. T. Takeuchi, "Inviscid limits of solutions to the Keller-Segel-Navier-Stokes system", Memorial research meeting for Prof. Masaki Kurokiba "Nonlinear PDE and Chemotaxis", Kitakyushu International Conference Center, Dec. 1, 2022.
11. T. Takeuchi, "Inviscid limits for the Keller-Segel-Navier-Stokes system of parabolic-elliptic type", International Workshop on Multi-Phase Flows: Analysis, Modelling and Numerics, Waseda University, Dec. 6, 2022.
12. T. Takeuchi, "On the inviscid limit problem for the Keller-Segel-Navier-Stokes system", Workshop on fundamental equations of fluid mechanics by young researchers, Nagoya University, Jan. 6, 2023.
13. T. Takeuchi, "Vanishing viscosity method for the Keller-Segel-Navier-Stokes system", the 6th International Workshop on the Mathematical Analysis of Chemotaxis, Leibniz University Hannover, Feb. 13, 2023.

IV. Research Results in AY 2022

Concerning my previous paper on the smoothing effects of the heat semigroup, I conducted joint research with Prof. Tohru Ozawa, and we obtained a refined result on the decay estimate of the heat semigroup defined on homogeneous Besov spaces.

For the Chemin-Lerner space, a new function space was introduced by considering the Lorentz spaces in time direction. Some fundamental properties were shown.

The smoothing effects of solutions to the Fujita type equation were considered. It is expected that the solution is not smooth whenever the corresponding initial data is given by a suitable sign-changing function. This research is still a work in progress.

The Keller-Segel-Navier-Stokes system with nonlinear boundary conditions was considered. The global solution was obtained under the suitable smallness assumption of the given data. It is expected that the solution is real analytic in space and time provided that the given data has sufficient regularity. This research is still a work in progress joint with Dr. Keiichi Watanabe.

V. Summary (From April 2021 to May 2023)

In this course, I studied the fundamental theory of the heat equation and the regularity theory of solutions to the nonlinear parabolic problems.

I attended the lecture “Special Lecture on Mathematical Fluid Mechanics” by Prof. Yoshihiro Shibata, and I learned a lot of methods for the exterior problems and free boundary problems of the Navier-Stokes system. In addition, I could obtain opportunities such that I gave presentations in “Special Lecture on Quantum Physics” by Prof. Tohru Ozawa and in the research conference in honor of Prof. Shibata’s 70th birthday. It was a pleasure to give my presentations in these great conferences.

Moreover, I also visited Paderborn Univ. and Hannover Univ. and gave a presentation in the research conference from the end of Jan. 2023 to the middle of Mar. Under the support of this course, I could meet Prof. Michael Winker and his research group and get the opportunity to conduct joint research. Regarding this research visit, I am deeply grateful for Ms. Yukari Ishizaki’s help.

Finally, I also appreciate Prof. Kenji Takizawa and Prof. Yoshihiro Shibata.