# Research Report（April, 2017 - March, 2023）

## In the SGU course of Mathematical Physical Science: April 2017-March 2023

| Conferring university | Degree name (by completing a course / by thesis only) | Date of conferment |
|---|---|---|
| Waseda University | Doctor of Science | 3, 23, 2023 |

Enrollment from April 2017    Department of Pure and Applied Mathematics    Yoshiki KANEKO

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**I. List of Papers**

**Yoshiki Kaneko**, (2022.6). SOLUTIONS OF THE tt\*-TODA EQUATIONS AND QUANTUM COHOMOLOGY OF MINUSCULE FLAG MANIFOLDS. Nagoya Mathematical Journal, 1-15. doi:10.1017/nmj.2022.17

H. Yudai and **Y. Kaneko**, "On Some Lie-theoretic Solutions of the tt\*-Toda Equations with Ingeter Stokes Data" preprint.

**II. Record of Awards**

**III. List of Talks**

International Presentations:

**Yoshiki Kaneko**, "Solutions of the tt\*-Toda Equations from Minuscule Flag Manifolds", The international workshop "Geometry of submanifolds and integrable systems", Osaka city University (zoom), 2022/2.

**Yoshiki Kaneko**, "Solutions of the tt\*-Toda equations and quantum cohomology of minuscule flag manifolds", Short course on Nonabelian Hodge theory, Waseda University(zoom), 2021/7.

**Yoshiki Kaneko**, "Solutions of the tt\*-Toda Equations and Quantum Cohomology of Flag Manifolds", International Workshop on Multiphase Flows: Analysis, Modelling and Numerics, Oxford-Waseda(zoom), 2020/12.

**Yoshiki Kaneko**, "Solutions of The tt\*-Toda Equations Corresponding to Quantum Cohomology of Flag Manifolds", UK-Japan Winter School 2019, Leeds University, 2019/1.

**Yoshiki Kaneko**, "Introduction to orbifolds", Korimaya Geometry and Physics Days 2018, Nihon University, 2018/2.

Domestic Presentations:

**Yoshiki Kaneko**, "Solutions of the tt\*-Toda Equations with integer Stokes data", Poisson geometry and related topics 22, Tokyo University of Science, 2022/12.

**Yoshiki Kaneko**, "Deployment of the tt\* geometry", Koriyama Geometry and Physics Days, Nihon University, 2021/11.

**Yoshiki Kaneko**, "Quantum Drinfeld- Sokolov reduction", Koriyama Geometry and Physics Days, Nihon University, 2021/11.

**Yoshiki Kaneko**, "Local solutions of the tt*-Toda equations and quantum cohomology of minuscule flag manifolds", geometry symposium, Hokkaido University (zoom), 2021/8.

**Yoshiki Kaneko**, "Solutions of the tt*-Toda equations and quantum cohomology of flag manifolds", Mathematical Society of Japan Spring Meeting 2021, Keio University, 2021/3.

**Yoshiki Kaneko**, "Virasoro algebra and coadjoint action", Koriyama Geometry and Physics Days, Nihon University、2020/2.

**Yoshiki Kaneko**, "Pseudodifferential symbol and Hamiltonian equations", Koriyama Geometry and Physics Days, Nihon University、2020/2.

**Yoshiki Kaneko**, "Solutions of tt*-Toda equations from flag manifolds", Ibunya Igyousyu, Tokyo University, 2019/10.

**Yoshiki Kaneko**, "Local solutions of tt*-Toda equations from generalized flag manifolds", Kanto Wakate geometry seminar, Tokyo Metropolitan University, 2019/6.

**Yoshiki Kaneko**, "Local solutions of the tt*-Toda equations from flag manifolds", Waseda Wakate Ibunya, Waseda University, 2019/3.


## IV. Research Results in AY 2022

It is known that solutions of the tt*-Toda equations correspond to holomorphic data, asymptotic data, and Stokes data by one-to-one. In particular, integer Stokes data is related to physics models and mathematical geometric object. Some examples of them are known. In this year, we obtain some properties of polynomials corresponding to integer Stokes data and the formula to calculate the number of integer Stokes data. When we focus on $\rho$-line, we have precisely 4 integer Stokes data on it. We compare the Guest-Its-Lin classification (the GIL classification) with the Cecotti and Vafa classification (The CV classification). The CV classification is the orbits of the upper triangular matrix by the semidirect product of the braid group and the sign group. The GIL classification is the classifying solutions in which we identify the same solutions up to permutations. We can define a map from the set of the GIL classification to the set of the CV classification. We found this map is injective. Thus we conclude that the GIL classification is the same of the CV classification. These are joint works with Y. Hateruma.

My first paper is accepted in Nagoya. Math. J. in June, 2022.


## V. Summary (From April 2017 to May 2023)

I focused on integer Stokes data of the tt*-Toda equations. We have some properties of polynomials corresponding to Integer Stokes data. Then we can calculate the number of integer Stokes data. We determined the four asymptotic data with integer Stokes data on $\rho$-line. Especially when we consider $m=-\rho$, the quantum cohomology of complex projective space corresponds to a global solution of the tt*-Toda equation. I found the quantum cohomology of minuscule flag manifolds correspond to solutions of the tt*-Toda equations. By this result, we obtain new interpretations of solutions of the tt*-Toda equations which correspond to quantum cohomology. Finally, we compare the GIL classification with the CV classification. We conclude that the GIL classification coincides with the CV classification.