“Multiscale Analysis, Modelling and Simulation”
Top Global University Project, Waseda University

Report on Study Abroad

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Date: March 21, 2023

1. Destination

“Paderborn University” and “Leibniz University Hannover” (Germany).

2. Dates of Stay

From January 30, 2023 to March 15, 2023 (44 days).

3. Purpose

- Paderborn University
  To visit Prof. Michael Winkler and his research group for studying the mathematical analysis of the chemotaxis system and obtaining the opportunity to conduct joint research.

- Leibniz University Hannover
  To participate in the international research conference “The 6th International Workshop on Mathematical Analysis of Chemotaxis (iWMAC6)” and give a presentation on my research results.
  (URL: https://www.ifam.uni-hannover.de/en/iwmac6)

4. Host Professor

Prof. Michael Winkler (Paderborn University).

5. Research Activity

Firstly, I participated in the international research conference “iWMAC6” and gave a presentation:

T. Takeuchi, “Vanishing viscosity method for the Keller-Segel-Navier-Stokes system”, the 6th International Workshop on the Mathematical Analysis of

Chemotaxis, Leibniz University Hannover, Feb. 13, 2023 (Invited).

Secondly, I studied the Keller-Segel-Navier-Stokes system with nonlinear boundary conditions. Precisely, for the usual Keller-Segel-Navier-Stokes system, the cell density n and the concentration of the chemo-attractant c satisfy the Neumann boundary conditions $\nabla n \cdot \nu = \nabla c \cdot \nu = 0$. However, recently, Xue and Othmer (SIAM J. Appl. Math. **70** (2009), no. 1, 133—167) derived a new model such that the boundary condition of the cell density n is given by the nonlinear terms $\nabla n \cdot \nu = nS\nabla c \cdot \nu$. For this model, Prof. Winkler has given some mathematical results from the viewpoint of the functional equation (J. Evol. Equ. **18** (2018), no. 3, 1267—1289 and Int. Math. Res. Not. IMRN (2021), no. 11, 8106—8152). On the other hand, I showed the existence and uniqueness of global strong solutions of the system based on the maximal regularity theorem. Here, the most important point is how to deal with the nonlinear boundary condition $\nabla n \cdot \nu = nS\nabla c \cdot \nu$. In contrast to the previous results, I obtained the solutions directly without considering the approximation system.

Finally, by the above result, I obtained the opportunity to conduct joint research with Dr. Frederic Heihoff, who is a young researcher in Prof. Winkler's group. Here, the aim of our joint research is to establish a new blow-up criterion of the system. Because I obtained the solutions with the initial data in Besov spaces, we expect that another blow-up criterion will be shown in the Besov spaces framework. This is still work in progress, so I will continue to consider the problems.

6. Other comments

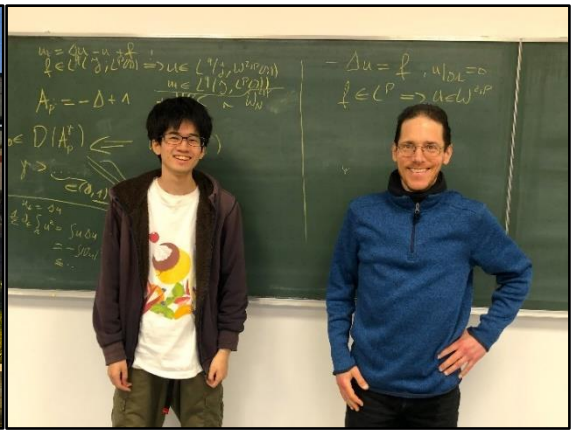
I would like to express my appreciation for Prof. Michael Winkler for accepting my research visit. In addition, I am truly honored to be able to see and discuss him face-to-face. By the discussion, I could obtain the opportunity to conduct joint research with a young researcher in Paderborn. During my stay in Paderborn, young researchers kindly helped me a lot, mainly around university procedures and life in Germany. Thanks to them, I could really enjoy spending time in Paderborn until the end.

Concerning the international research conference "iWMAC6" in Hannover, I would like to thank the organizers; Dr. Johannes Lankeit, Dr. Sachiko Ishida, Dr. Masaaki Mizukami, and Dr. Mario Fuest. By virtue of the invitation, I could go through the first experience of giving a presentation in the international conference held overseas. During the conference, I could talk to a lot of foreign

researchers from not only Germany but also Austria, China, Poland, Italy, Korea, Poland, and the United Kingdom. And then I could enjoy many kinds of nice German foods with them during dinner time in Hannover, so I could have a wonderful experience in various aspects.

Lastly, I am deeply indebted to Prof. Hideo Kozono and Top Global University Project in Waseda University for giving me the important opportunity to visit Paderborn and Hannover. I am also thankful to Ms. Yukari Ishizaki for a permission request and an office procedure around the research visit.

▲ Paderborn University



▲ With Prof. Michael Winkler

▲ Leibniz University Hannover

▲ During “iWMAC6” in Hannover