

“Multiscale Analysis, Modelling and Simulation”

Top Global University Project, Waseda University

REPORT ON STUDY ABROAD

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Date: February 5, 2023

- 1. Destination:** University of Pittsburgh
- 2. Dates of stay:** August 29, 2022 – December 11, 2022 (105 days)
- 3. Purpose:** To visit Prof. Giovanni Paolo Galdi and study the Navier-Stokes equations and related topics.
- 4. Host Professor:** Prof. Giovanni Paolo Galdi (University of Pittsburgh)
- 5. Research Activity:**

I) Seminars:

- Prof. Dallas Albritton (IAS Princeton): Non-uniqueness of Leray solutions to the forced Navier-Stokes equations, CNA Seminar, Carnegie Mellon University, November 8, 2022

II) Presentations:

- Logarithmically Improved Extension Criteria Involving the Pressure for the Navier-Stokes Equations in $\mathbb{R}^3$, PDE and Analysis Seminar, University of Pittsburgh, November 7, 2022

III) Research Results:

We considered the steady Navier-Stokes equations with nonhomogeneous slip boundary conditions of Navier-type in a two-dimensional bounded domain. The existence of a weak solution to the problem is readily proved by the Leray-Schauder theorem, provided we can establish a suitable a priori estimate. The constructive proofs have so far been given by several authors. When the domain is simply connected, we could derive the desired estimate by a different method, which is based on a contradiction argument. In the proof, the Bernoulli law for a weak solution to the Euler equations plays an important role. We hope that our method can be generalized to the case when the boundary has more than one connected component.

6. Other comments

Firstly, I would like to express my sincere gratitude to Prof. Giovanni Paolo

Galdi for his warm hospitality and encouragement. During my stay in Pittsburgh, I could discuss various problems related to the Navier-Stokes equations with him so many times. Through the discussions, I could learn his approach to the problems and realized that many fundamental problems are still open. I am also grateful to Prof. Armin Schikorra for giving me a precious chance to talk in PDE and Analysis Seminar and for fruitful discussions on the local regularity theory for the non-stationary Navier-Stokes equations.

My deep thanks go as well to Prof. Yoshihiro Shibata and Top Global University Project (Waseda University) for giving me such a wonderful opportunity to visit Pittsburgh. Lastly, I would like to thank Ms. Ishizaki for making travel arrangements for me.



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