Research Report (September, 2021- March, 2022)

Enrollment from September 2021 Department of Pure and Applied Mathematics Tatsuki YAMAMOTO

I. List of Papers

[1] R. Kanamaru and T. Yamamoto, Logarithmically Improved Extension Criteria Involving the Pressure for the Navier-Stokes Equations in R^n", Mathematische Nachrichten, (in press).

II. List of Talks

[1] R. Kanamaru and T. Yamamoto, "Logarithmically Improved Extension Criteria Involving the Pressure for the Navier-Stokes Equations in R^3", MSJ Autumn Meeting, Chiba Univ., Online (zoom), Sep. 17, 2021.

[2] T. Yamamoto, "Logarithmically Improved Extension Criteria Involving the Pressure for the Navier-Stokes Equations in R^3", International Workshop on Multiphase Flows: Analysis, Modelling, and Numerics, Waseda Univ., Online (zoom), Dec. 3, 2021.

[3] T. Yamamoto, "Logarithmically Improved Extension Criteria Involving the Pressure for the Navier-Stokes Equations in R^3", The 2nd Young Researchers Workshop on Nonlinear Partial Differential Equations, Online (zoom), Mar. 15, 2022.

III. Research Results in AY2021

I considered the stationary magnetohydrodynamic equations (MHD equations) in two- and three-dimensional bounded domains with multiple boundary components. In view of the incompressibility condition of the fluid, the total flux of the fluid over the boundary is required to be zero. In 2021, I considered the solvability of the problem under stronger conditions that the flux of the velocity vector across each connected component of the boundary vanishes. The existence of a weak solution is readily proved by Leray-Schauder principle, provided we could establish the suitable a priori estimate.

One way of attempting to obtain the desired estimate is to construct a special extension of the boundary value for the fluid, which satisfies the so-called Leray-Hopf inequality. In the MHD problem, it seems difficult to apply the classical argument by Hopf directly because the tangential component of the magnetic field on the boundary is not controlled. This difficulty is already pointed out by Yanagisawa (2015).

Another approach of proving an a priori estimate is to use a contradiction argument. If we assume that the a priori estimate does not hold, then we can take certain normalized sequences of solutions. In the Navier-Stokes problem, it is well known that the limits (in appropriate topology) of the sequences satisfy the Euler system and the pressure for the system takes constant values on each connected components of the boundary. I applied same argument to the MHD problem and found that it was difficult to obtain the information of the pressure on the boundary because of the same reason as in the first approach.

IV. Research Plan for AY2022

I will study the stationary Navier-Stokes system with nonhomogeneous slip boundary conditions of Navier type in 2D bounded domains and 3D bounded axially symmetric domains.