Mathematics and Physics Unit “Multiscale Analysis, Modelling and Simulation”
Top Global University Project, Waseda University

International Workshop on “Fundamental Problems in Mathematical and Theoretical Physics”

Date: July 22 - July 26, 2019
Venue: Large Conference Room, 1st Floor, 55N Bldg., Waseda University, Nishi-Waseda Campus
早稲田大学 西早稲田キャンパス　55 号館 N 棟 1 階 大会議室
Access: https://www.waseda.jp/fsci/en/access/

Part II. Mathematical Physics

Mini Courses

◆ Luis Vega (Basque Center for Applied Mathematics, Spain) ◆

◆ Mini Course I	July 24, Wednesday	13:00 - 14:00
◆ Mini Course II	July 24, Wednesday	14:10 - 15:10
◆ Mini Course III	July 25, Thursday	16:00 - 17:00
◆ Mini Course IV	July 26, Friday	15:00 - 16:00

The vortex filament equation, the Talbot effect and non-circular jets

1.- Vortex filament equation: the localized induction approximation of Biot-Savart Integral and Hasimoto transformation.

2.- Self-similar solutions: characterization and their connection with the case of regular polygons. The turbulence of non-circular jets. Non-linear Talbot effect.

3.- Skew polygonal lines: self-similar solutions have finite energy. Formation of singularities and continuation of the solutions beyond the blow up time.

◆ Vladimir Georgiev (University of Pisa, Italy) ◆

◆ Mini Course I	July 24, Wednesday	16:50 - 17:50
◆ Mini Course II	July 25, Thursday	17:10 - 18:10
◆ Mini Course III	July 26, Friday	10:00 - 11:00
◆ Mini Course IV	July 26, Friday	11:10 - 12:10

Fractional NLS and semilinear half - wave equations

Lecture I: Semi - relativistic NLS and Half Wave equation with gauge invariant noninearities. The case of mass and energy conservation laws.

We consider the Cauchy problems associated with semi - relativistic NLS (sNLS) and half wave (HW). In particular we focus on the following two main questions: local/global Cauchy theory; existence and stability/instability of ground states. In between other results, we prove the existence and stability of ground states for sNLS in the L^2 supercritical

regime. This is in sharp contrast with the instability of ground states for the corresponding HW, which is also established, by showing an inflation of norms phenomenon. Concerning the Cauchy theory we show, under radial symmetry assumption the following results: a local existence result in H^1 for energy subcritical nonlinearity and a global existence result in the L^2 subcritical regime.

Lecture II: Half-wave equation with Landau - Ginzbug gauge invariant power type nonlinearity.

We discuss the half-wave equation with Landau - Ginzbug power type nonlinearity on Euclidean spaces. The model manifests diffusion phenomena, therefore mass and the energy are not conserved. In two dimensional H^s scaling subcritical case with $1 \leq s \leq 2$, the local well-posedness follows from a Strichartz estimate. In higher dimensional H^1 scaling subcritical case, the local well-posedness for radial solutions follows from a weighted Strichartz estimate. Moreover, in three dimensional H^1 scaling critical case, the local well-posedness for radial solutions follows from a uniform bound of solutions which may be derived by the corresponding one dimensional problem. Local solutions may be extended by a priori estimates.

Lecture III: On Traveling Solitary Waves and Absence of Small Data Scattering for Nonlinear Half-Wave Equations

We consider nonlinear half-wave equations with focusing power-type nonlinearity of power $1 < p < \infty$ for $d = 1$ and $1 < p < (d+1)/(d-1)$ for $d \geq 2$. We study traveling solitary waves with frequency $\omega \in \mathbb{R}$, velocity $v \in \mathbb{R}^d$, and some finite-energy profile $Q_v \in H^{1/2}(\mathbb{R}^d)$, $Q_v \not\equiv 0$. We prove that traveling solitary waves for speeds $|v| \geq 1$ do not exist. Furthermore, we generalize the non-existence result to the square root Klein–Gordon operator $\sqrt{-\Delta + m^2}$ and other nonlinearities.As a second main result, we show that small data scattering fails to hold for the focusing half-wave equation in any space dimension. The proof is based on the existence and properties of traveling solitary waves for speeds $|v| < 1$. Finally, we discuss the energy-critical case when $p = (d+1)/(d-1)$ in dimensions $d \geq 2$.

Lecture IV: Blow-up for self-interacting fractional Ginzburg-Landau equation

The blow-up of solutions for the Cauchy problem of fractional Ginzburg-Landau equation with non-positive nonlinearity is shown by an ODE argument. Moreover, the optimal lifespan estimate for size of initial data is obtained.

◆ Luca Fanelli (Sapienza University of Roma, Italy) ◆

◆ Mini Course I	July 24, Wednesday	15:40 - 16:40
◆ Mini Course II	July 25, Thursday	10:00 - 11:00
◆ Mini Course III	July 25, Thursday	11:10 - 12:10
◆ Mini Course IV	July 26, Friday	13:30 - 14:30

Uniform Resolvent Estimates and the point spectra of non self-adjoint relativistic Hamiltonians

- Lecture 1: Introduction, non self-adjoint Hamiltonians and eigenvalue problems. The Birman-Schwinger Principle and absence of eigenvalues, as a consequence of uniform resolvent estimates for the free Hamiltonian. An overview of recent results in the non-relativistic case (Frank, Seiringer, Simon et al).

- Lecture 2: A uniform Sobolev estimate by multiplier techniques.

- Lecture 3: The uniform Sobolev Estimate for the resolvent of the free Schrödinger Hamiltonian by Kenig-Ruiz-Sogge. Proof, and failure of the estimate for the Dirac Hamiltonian.

- Lecture 4: A new Agmon-Hörmander-type estimate for the free Dirac operators. As a consequence, absence of eigenvalues for Dirac operators with almost scaling invariant complex potentials. The results are obtained in collaboration with P. D'Ancona, and N. Schiavone (Roma Sapienza).

Invited Talks

◆ Kazumasa Fujiwara (Tohoku University, Sendai) ◆

July 25, Thursday 13:20 - 14:00

An estimate for commutator of fractional Laplacian with rough metric

We show an estimate for commutator of weight functions and fractional Laplacian with rough metric. Commutators of fractional Laplacian has been studied mainly by Fourier analysis. On the other hand, Fourier analysis is not sufficient to consider the commutator estimate with rough metric. In this talk, we consider the commutator estimate by combining spectrum analysis and Fourier analysis. This talk is based on a joint work with Luigi Forcella, Professors Vladimir Georgiev and Tohru Ozawa (arXiv:1804.02524).

◆ Hiroyuki Hirayama (Miyazaki University, Miyazaki) ◆

July 25, Thursday 14:05 - 14:45

Well-posedness for a system of quadratic derivative nonlinear Schrödinger equations with radial initial data

We consider the Cauchy problem of the system of quadratic derivative nonlinear Schrödinger equations introduced by M.Colin and T.Colin as a model of laser-plasma interaction. The well-posedness of this system was obtained in our previous works. In this talk, we improve the well-posedness results for radial initial data. In particular, we prove that the results for 2D radial data is better than that for 1D initial data.
To construct the non-trivial radial solution, we rewrite the system radial form.
This talk is based on the joint work with Shinya Kinoshita (Universitat Bielefeld) and Mamoru Okamoto (Shinshu University).

◆ Jun-ichi Segata (Kyusyu University, Fukuoka) ◆

July 25, Thursday 14:50 - 15:30

Modified scattering for the complex valued nonlinear Klein-Gordon equation

We consider the long time behavior of solutions to the initial value problem for the "complex valued" cubic nonlinear Klein-Gordon equation (NLKG) in one space dimension. The complex valued nonlinear Klein-Gordon equation arises in various fields of physics. For example, the nonlinear Dirac equation in the relativistic quantum fields can be reduced to the system of the complex valued nonlinear Klein-Gordon equations. In this talk, we give a large time asymptotic profile of solutions to (NLKG). We also consider the final state problem for the gauge invariant quadratic nonlinear Klein-Gordon equation in two space dimension.

◆ Haruya Mizutani (Osaka University, Osaka) ◆

July 26, Friday 16:20 - 17:00

Wave operator on Sobolev space

Motivated by applications to scattering theory in the energy space for nonlinear Schr̈odinger equations with linear potentials, in this talk we consider linear scattering theory in the scale of Sobolev spaces. We introduce a simple sufficient condition in an abstract framework to deduce the existence of wave operators on Sobolev spaces from the existence of the ordinary wave operators. In case of the Sobolev space of order one, this condition is general enough to accommodate short-range potentials with subcritical singularities, inverse-square potentials and the 1D point interaction. The case of homogeneous Sobolev spaces is also discussed.

◆ Takahisa Inui (Osaka University, Osaka) ◆

July 26, Friday 17:10 - 17:50

The Strichartz estimates for the damped wave equation and its application to a nonlinear problem

We give the space-time estimates of the solution to the damped wave equation, which is called Strichartz estimates. Moreover, we apply them to a nonlinear equation.