Mathematics and Physics Unit “ Multiscale Analysis, Modelling and Simulation ”
Top Global University Project, Waseda University

International Workshop on “ Fundamental Problems in Mathematical and Theoretical Physics ”

Mathematical Physics

Date: July 25 – July 26, 2022
Venue: Large Conference Room, 1st Floor, 55N Bldg., Waseda University, Nishi-Waseda Campus
早稲田大学 西早稲田キャンパス 55 号館 N 棟 1 階 大会議室

Abstract of MINI Courses

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On global dynamics of NLS and wave equation

◆ Mini Course I July 25, Monday 10:00 – 11:00
◆ Mini Course II July 25, Monday 13:30 – 14:30
◆ Mini Course III July 26, Tuessday 10:00 – 11:00

I. **Stability/instability results for NLS with singularity in 2D**

We study a class of two-dimensional non-linear Schrödinger equations with point-like singular perturbation and Hartree or local type non-linearity. Our analysis has the following key points: we establish existence, symmetry, and regularity of ground states, and we demonstrate the well-posedness of the associated Cauchy problem in the singular perturbed energy space. Stability/instability properties for the local nonlinearities are discussed too. The talk is based on collaborations with Masahiro Ikeda, Noriyoshi Fukaya, Alessandro Michelangeli and Raffaele Scandone.

II. **Scattering and global dynamics for Chern-Simons-Schrödinger system**

In this talk, we are concerned with solutions to the Cauchy problem for Chern-Simons-Schrödinger equations in the mass supercritical case. First we establish the local well-posedness of solutions in the radial space. Then we consider scattering versus blow-up dichotomy for radial data below the ground state threshold. The talk is based on collaboration with Tianxiang Gou.

III. **On Strauss conjecture for wave equation with inverse square potential**

In this talk we shall discuss the global existence for 3d semilinear wave equation with non-negative potential satisfying generic decay assumptions.

In the supercritical case $p > 1 + \sqrt{2}$ we establish the small data global existence result. The approach is based on appropriate conformal energy estimate in combination with Hardy inequality for conformal energy on space - like surfaces. Moreover, it is obtained in collaboration with Hideo Kubo.

♢ Fumio Hiroshima ♢

Faculty of Mathematics, Kyushu University

Renormalization, ground states, and localization by path measures

◆ Mini Course I July 25, Monday 11:10 – 12:10
◆ Mini Course II July 26, Tuessday 11:10 – 12:10
◆ Mini Course III July 26, Tuessday 13:30 – 14:30

I. We provide the tools to study spectral analysis of quantum field theory. We give definitions of Fock space $\mathcal{F}$, its Schrödinger representation $L^2(Q)$, second quantization $d\Gamma$, Gaussian random variable $\phi(f)$ indexed by functions f, and Euclidean quantum fields, etc. Define the Nelson model H as a self-adjoint operator in $L^2(\mathbb{R}^3) \otimes L^2(Q)$, which is a toy model of the Yukawa model. This model describes a linear interaction between a scalar quantum field and quantum particles, and is of the form

$$H_\Lambda = \left(-\frac{1}{2m}\Delta + V\right) \otimes 1 + 1 \otimes d\Gamma(\omega) + \phi_\Lambda(x).$$

Physically it describes the interaction between pions and nucleon in an atomic core. Here $\phi_\Lambda(x)$ is the field operator with an ultraviolet cutoff Λ. We give a functional integral representation of its heat semi-group e^{-tH} by

$$(F, e^{-tH_\Lambda}G) = \int_{\mathbb{R}^3} \mathbb{E}^x \left[(F(B_0), e^{K_\Lambda}G(B_t))_{L^2(Q)}\right] \mathrm{d}x.$$

Here (B_t) denotes Brownian motion and $\mathbb{E}^x$ describes the expectation of Brownian motion starting from x at time $t = 0$. We deeply study the integral kernel K. The references are [9, 4].

II. The Nelson model can be realized as a self-adjoint operator in a Hilbert space. An ultraviolet cutoff Λ is introduced in H_Λ to be defined as a self-adjoint operator. Let $\sigma(H_\Lambda)$ be the spectrum of H_Λ. The infimum of $\sigma(H_\Lambda)$ is called the ground state energy. Let $\inf \sigma(H_\Lambda) = e_\Lambda$. The eigenvector Φ such that

$$H_\Lambda \Phi = e_\Lambda \Phi$$

is called the ground state. The existence and absence of the ground state is not trivial. We discuss the existence and absence of the ground state of H_Λ. Mathematically, it can be viewed as a question of the existence and absence of embedded eigenvalues of the continuous spectrum.

Next, we discuss the renormalization theory of H_Λ. Let E_Λ be a renormalization term and $\lim_{\Lambda\to\infty} E_\Lambda = -\infty$. We discuss the existence of the linit of $H_\Lambda - E_\Lambda$ as $\Lambda \to \infty$. Edward Nelson himself succeeded in renormalizing in 1964 [8], but we give an alternative proof using functional integrals. I.e., there exists H_∞ such that

$$\lim_{\Lambda\to\infty} e^{-t(H_\Lambda - E_\Lambda)} = e^{-tH_\infty}$$

We also furthermore show the existence and absence of the ground state of the renormalized Nelson model H_∞. The references are [1, 2, 5, 3, 7].

III. The localization of the ground state Φ will be discussed for both cases H_Λ and H_∞. $\Phi = \Phi(x, n, \phi)$ includes three variables x, n, ϕ. Here $x \in \mathbb{R}^3$ denotes the spatial variable, n the number of bosons, and ϕ the field variable. This is where the functional integral representation is most powerful. Non-perturbative results are derived: (1) the spatial decay from above and below,

$$ce^{-c|x|} \le \|\Phi(x)\|_{L^2(Q)} \le ce^{-C|x|},$$

(2) the super-exponential decay of the number of bosons,

$$\|e^{+\beta N}\Phi\| < \infty \quad \forall \beta > 0,$$

where N denotes the number operator, (3) Gaussian domination with respect to field operators

$$\|e^{\beta\phi(f)^2}\Phi\| < \infty \Longleftrightarrow \beta < \frac{1}{2\|f\|}.$$

These results are derived from applications of the functional integral representation of $e^{-tH_\#}$ for $\# = \Lambda, \infty$. The references are [5, 6].

References

[1] M. Gubinelli, F. Hiroshima, and J. Lörinczi. Ultraviolet renormalization of the Nelson Hamiltonian through functional integration. *J. Funct. Anal.*, 267:3125–3153, 2014.

[2] F. Hiroshima. Functional integral approach to semi-relativistic Pauli-Fierz models. *Adv. Math.*, 259:784–840, 2014.

[3] F. Hiroshima. *Ground states in quantum field models.* Springer, 2019.

[4] F. Hiroshima and J. Lőrinczi. *Feynman-Kac type theorems and its applications, volume 2 (2nd ed).* De Gruyter, 2020.

[5] F. Hiroshima and O. Matte. Ground states and their associated Gibbs measures in the renormalized nelson model. *Rev. Math.Phys.*, 33:2250002 (84 pages), 2021.

[6] F. Hiroshima and O. Matte. Point-wise spatial decay of eigenvectors in the nelson model. *in preparation*, 2022.

[7] O. Matte and J. Møller. Feynman-Kac formulas for the ultra-violet renormalized Nelson model. *arXiv:1701.02600*, preprint, 2017.

[8] E. Nelson. Interaction of nonrelativistic particles with a quantized scalar field. *J. Math. Phys.*, 5:1990–1997, 1964.

[9] H. Spohn. *Dynamics of Charged Particles and their Radiation Field.* Cambridge University Press, 2004.

♢ Satoshi Masaki ♢

Graduate School of Engineering Science, Osaka university

Global dynamics below first excited solitons for NLS equation with potential

In this talk, we consider the focusing cubic nonlinear Schrodinger (NLS) equation with a linear potential in three space dimensions. When the corresponding Schrodinger operator possesses one negative eigenvalue, the NLS equation has a small stable ground state that is close to an eigenfunction of the eigenvalue. Under the small mass constraint, there also exist first excited solitons which is a perturbation of the ground state of the potential-less NLS equation. In this setting, we will show the global behavior of a solution that has a small mass and energy less than that of the first excited soliton with the same mass is classified into two types. One is scattering to the family of the ground states for both time directions and the other is blowup (or grow-up) for both time directions. This is previously known under the radial assumption (Nakanishi 2017). We deal with the non-radial case. This talk is based on joint work with Jason Murphy (Missouri S&T) and Jun-ichi Segata (Kyushu).

♢ Shuji Machihara ♢

Graduate School of Science and Engineering, Saitama University

Rellich inequalities in the framework of equalities

We observe some kinds of Rellich inequalities from corresponding equalities. We split the functions into the spherical harmonics and its reminder to express our equalities and inequalities. Since we treat with smooth functions whose support are away from the origin, we will consider the inequalities in spatial dimensions greater than or equal to 2. By using those equalities, we also investigate the relation of two operators between the Laplacian and the radial Laplacian. We also investigate the Rellich inequalities of the radial Laplacian, we will call it radial Rellich inequality in this talk. If we have enough time to spare, I will talk about the non-existence of non-trivial maximisers for some inequalities. We give an extensions of those inequalities for general Sobolev space functions by showing the density lemma. This talk is based on joint research with Neal Bez from Saitama university and Tohru Ozawa from Waseda university.

♢ Takuya Sato ♢

Graduate School of Science, Tohoku University

Optimal mass decay of solutions to nonlinear Schrödinger equations with a critical dissipative nonlinearity

We consider the optimality of mass decay of solutions to the Cauchy problem of nonlinear Schrödinger equations with a long-range dissipative nonlinearity. We show that the L^2-norms (mass) of any global solutions do not decay more rapidly than $(\log t)^{-1/2}$, and we also prove that there exists a solution decaying just at the rate of $(\log t)^{-1/2}$ in L^2. This talk is based on the joint work with Professor Naoyasu Kita (Kumamoto University).

♢ Taiki Takeuchi ♢

Department of Mathematics, Faculty of Science and Engineering Waseda University

Vanishing viscosity limit of solutions of the Keller-Segel-Navier-Stokes system

This talk is concerned with the Cauchy problem for the Keller-Segel-Navier-Stokes system of parabolic-elliptic type in $\mathbb{R}^N$, $N \geq 3$. We show the local well-posedness of this system for arbitrary initial data in Sobolev spaces, where the existence time interval of the solution may be chosen independently of the viscosity. In addition, we obtain inviscid limits of the above system. The proof is based on a priori estimates in the Sobolev spaces combined with the commutator estimate established by Kato-Ponce (1988). We also used the method of Kato-Lai (1984) who showed the continuous dependence of solutions of the Euler system with respect to the initial data. The method can be applied to obtain inviscid limits and their property of the convergence.

♢ Ryunosuke Kusaba ♢

Department of Pure and Applied Physics, Faculty of Science and Engineering, Waseda University

Nonexistence of global solutions to Nakao's problem via a test function method

We consider the following Cauchy problem for a weakly coupled system of a semilinear damped wave equation and a semilinear wave equation:

$$\begin{cases} \partial_t^2 u - \Delta u + \partial_t u = |v|^p\,, & t > 0,\ x \in \mathbb{R}^n, \\ \partial_t^2 v - \Delta v = |u|^q\,, & t > 0,\ x \in \mathbb{R}^n, \\ (u, \partial_t u, v, \partial_t v)\,(0, \cdot\,) = (u_0, u_1, v_0, v_1)\,, & x \in \mathbb{R}^n. \end{cases}$$

The problem to determine the critical exponent which gives a threshold for the existence of global solutions of the above system was proposed by Professor Mitsuhiro Nakao, Emeritus of Kyushu University, Japan. The purpose of this problem is to investigate the difference in structure between damped wave equations and wave equations. There are two results on this problem (Wakasugi (2017), Chen-Reissig (2021)), which discuss the blowing up of solutions in finite time, but there is a gap between their statements. In this talk, we bridge this gap with a test function method while providing more concise proof. This talk is based on a joint work with Kosuke Kita (Waseda University).