Young Researchers' Forum on Mathematical Fluid Mechanics

Dates: June 20 – 21, 2022
Location: Online (Zoom)

Tokyo Time JST (UTC+9)	Monday June 20	Tuesday June 21	Berlin Time CEST (UTC +2)
3:25 p.m. – 3:30 p.m.	Opening		8:25 a.m. – 8:30 a.m.
3:30 p.m. – 4:10 p.m.	Ken Furukawa	Antonio Agresti	8:30 a.m. – 9:10 a.m.
4:20 p.m. – 5:00 p.m.	Hiroyuki Tsurumi	Tomoki Takahashi	9:20 a.m. – 10:00 a.m.
5:10 p.m. – 5:30 p.m.	Break	Break	10:10 a.m. – 10:30 a.m.
5:30 p.m. – 5:45 p.m.	Motofumi Aoki	Taiki Takeuchi	10:30 a.m. – 10:45 a.m.
5:45 p.m. – 6:00 p.m.	Dáithí Ó hAodha	Christian Gesse	10:45 a.m. – 11:00 a.m.
6:00 p.m. – 6:15 p.m.	Daniele Barbera	Christiane Bui	11:00 a.m. – 11:15 a.m.
6:15 p.m. – 6:30 p.m.	Tomonori Tsuruhashi		11:15 a.m. – 11:30 a.m.
6:40 p.m. – 7:00 p.m.	Break	Break	11:40 a.m. – 12:00 p.m.
7:00 p.m. – 8:00 p.m.	Miho Murata	Matthias Köhne	12:00 p.m. – 1:00 p.m.
8:10 p.m. – 8:15 p.m.		Closing	1:10 p.m. – 1:15 p.m.

June 20, 2022

Singular Limit Problem of the Primitive Equations from the Anisotropic Navier-Stokes Equations in $L^\infty_{x'}L^q_{x_3}(\mathbb{T}^3)$ Spaces

Ken Furukawa

RIKEN, Japan

The primitive equations are a system to describe the motion of fluid in thin domains, e.g., ocean and atmosphere. The primitive equations can be derived from the Navier-Stokes equations with anisotropic (ANS) viscosity, i.e., horizontal $O(1)$ and vertically $O(\varepsilon)$ for small $\varepsilon > 0$. Applying a scaling to ANS and taking a formal limit ($\varepsilon > 0$), we obtain the primitive equation. These procedures are equivalent to the so-called hydrostatic approximation. In this talk, we consider the mathematical aspect of the hydrostatic approx- imation in the primitive equations to justify the convergence ($\varepsilon > 0$) mathematically rigorously. The function anisotropic spaces $L^\infty_{x'}L^1_{x_3}(\mathbb{T}^3)$ is a one of critical function space of the primtive equations. It is natural to consider the convergence in $L^\infty_{x'}L^q_{x_3}(\mathbb{T}^3)$ for $q \geq 1$. We present the $O(\varepsilon)$-convergence result in $L^\infty_{x'}L^q_{x_3}(\mathbb{T}^3)$. This is joint work with Prof. Kashiwabara of the University of Tokyo.

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The 2D stationary Navier-Stokes flow on the whole plane around a radial flow

Hiroyuki Tsurumi

Kyoto University, Japan

This talk is based on a joint work with Yasunori Maekawa (Kyoto University).

We consider the two-dimensional stationary Navier-Stokes equations

$$\begin{cases} -\Delta u + u \cdot \nabla u + \nabla p = F, & x \in \mathbb{R}^2, \\ \nabla \cdot u = 0 & x \in \mathbb{R}^2. \end{cases} \tag{1}$$

Here $u = (u_1(x), u_2(x))$ and $p = p(x)$ denote the unknown velocity vector and the unknown pressure of the fluid at the point $x = (x_1, x_2) \in \mathbb{R}^2$, respectively, while $F = (F_1(x), F_2(x))$ is the given external force. Here $u \cdot \nabla u := \sum_{j=1,2} u_j \partial_{x_j} u$.

In this talk, we suppose that a external force F is divergence-free and can be expressed as $F = \nabla^\perp \phi := (\partial_{x_2}\phi, -\partial_{x_1}\phi)$ with some flow ϕ. Actually, by the Helmholtz decomposition, we can write F as a sum of the curl-free term and divergence-free one. Then the curl-free term is absorbed in the pressure term ∇p, and the divergence-free one is written as $\nabla^\perp \phi$ if F decays sufficiently. Under such a situation, we will show that for every small compact supported radial flow ϕ_* having non-zero integral and its smaller perturbation φ decaying faster than $|x|^{-2}$, there exists a solution u of (1) for $F = \nabla^\perp(\phi_* + \varphi)$. In our result, we also impose the smoothness of ϕ_* and φ so that we may obtain u as a classical solution in $C^2(\mathbb{R}^2; \mathbb{R})$, while it is not necessary to impose any symmetric conditions on φ.

The approach to (1) is to analyze the vorticity-streamfunction system as

$$\begin{cases} \Delta \psi = -\omega, & x \in \mathbb{R}^2, \\ \Delta \omega = \nabla \times (\nabla^\perp \psi \cdot \nabla(\nabla^\perp \psi)) + \Delta \phi, & x \in \mathbb{R}^2. \end{cases} \tag{2}$$

Here $\psi = \psi(x)$ is the stream function, which generates the divergence-free flow as $u = \nabla^\perp \psi$, while $\omega := \nabla \times u := \partial_{x_1} u_2 - \partial_{x_2} u_1$ is the vorticity field. We will analyze this system (2) in the polar coordinates. Actually, some key structures are found by decomposing the system (2) into the Fourier mode with respect to the angular valuable. For this method, we especially refer to Hillairet-Wittwer (2013), which showed the existence of solutions around $\mu x^{\perp}/|x|^2$ (with a sufficiently large constant μ) in the exterior of the disk centered at the origin.

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Global well posedness for a Q-tensor model of nematic liquid crystals

Miho Murata

Shizuoka University, Japan

We consider the model for a viscous incompressible liquid crystal flow proposed by Beris and Edwards in 1994. The model is coupled system by the Navier-Stokes equations with a parabolic-type equation describing the evolution of the director fields $\mathbb{Q}$, which is called $\mathbb{Q}$-tensor. The aim of this talk is to prove the existence and uniqueness of the strong solution to a $\mathbb{Q}$-tensor model of nematic liquid crystals in $\mathbb{R}^N$, $N \geq 3$ with the help of the maximal L_p -L_q regularity and the L_p -L_q decay estimates to the linearized equations. In this talk, we discuss mathematical analysis of the linearized problem and a general framework to prove the global well posedness for small initial data of quasilinear parabolic equations in unbounded domains. This is a joint work with Professor Yoshihiro Shibata from Waseda University.

June 21, 2022

The primitive equations for turbulent flows

Antonio Agresti

Institute of Science and Technology, Austria

Primitive equations are one of the fundamental models for geophysical flows and are used to describe oceanic and atmospheric dynamics. They are derived from the Navier-Stokes equations by the small aspect ratio limit in domains where the vertical scale is much smaller than the horizontal one. Starting from the celebrated result by C. Cao, E.S. Titi and R.M. Kobelkov on global well-posedness in three dimensions, primitive equations have received an increasing interests in the mathematical community. In this talk, after briefly reviewing the known results, we investigate primitive equations with stochastic perturbations of transport type. Such noise is widely used in fluid mechanics to model turbulent flows. The presence of the transport noise gives rise to several mathematical problems that naturally lead to the use of stochastic maximal regularity techniques, as well as to the use of theory of critical spaces for stochastic evolution equations. Among others, we will discuss the global well-posedness of the stochastic primitive equations. Finally we will present some open problems.

This talk is based on a joint work with M. Hieber (TU Darmstadt), A. Hussein (TU Kaiserslautern) and M. Saal (TU Darmstadt).

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Anisotropic weighted L^q-L^r estimates of the Oseen semigroup in exterior domains, with applications to the Navier-Stokes flow past a rigid body

Tomoki Takahashi

Nagoya University, Japan

We consider the space-time behavior of the Navier-Stokes flow past a three dimensional rigid body. In particular, we develop analysis in Lebesgue spaces with anisotropic weights $(1+|x|)^\gamma(1+|x|-x_1)^\delta$, which naturally arise in the asymptotic structure of fluid when the translational velocity of the body is parallel to the x_1-direction. Our first aim is to derive anisotropic weighted L^q-L^r estimates for the Oseen semigroup in exterior domains. We apply those estimates to the stability of a stationary solution and to Finn's starting problem.

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Fluid Flow in a Wedge Type Domain subject to Partial Slip Boundary Conditions and the Contact Line Problem

Matthias Köhne

Heinrich-Heine-Universität Düsseldorf, Germany

We consider two Problems, which are naturally related to each other: Fluid Flow in Wedge type domains subject to partial slip boundary conditions and two-phase flows with moving contact line subject to reasonable boundary and transmission conditions. We examine the relations, present recent results and discuss open problems.

Short Presentations

Motofumi Aoki (Tohoku University)
On the energy conservation law for the compressible Navier–Stokes equations

Daniele Barbera (Pisa University)
On Standing waves and Gradient-Flow for the Landau-De Gennes Model of Nematic Liquid Crystals

Christiane Bui (Heinrich-Heine-Universität Düsseldorf)
Stability Analysis for Living Fluids in the periodic setting

Christian Gesse (Heinrich-Heine-University Düsseldorf)
Stability for a class of heterogeneous catalysis models

Dáithí Ó hAodha (Tohoku University)
Large-Time Behaviour of Solutions to the Surface Quasi-Geostrophic Equations

Taiki Takeuchi (Waseda University)
Inviscid limit problem for the Keller-Segel-Navier-Stokes system of parabolic-elliptic type

Tomonori Tsuruhashi (The University of Tokyo)
Kinetic formulation for irregular pairs in a divergence equation