Research Report（April, 2021- March, 2022）

Enrollment from April 2021 | Department of Department of Pure and Applied Mathematics | Taiki TAKEUCHI

I. List of Papers

01. T. Takeuchi, “The Keller-Segel system of parabolic-parabolic type in homogeneous Besov spaces framework”, J. Differential Equations **298** (2021), 609—640.
02. T. Takeuchi, “Maximal Lorentz regularity for the Keller-Segel system of parabolic-elliptic type”, J. Evol. Equ. **21** (2021), 4619—4640.
03. T. Takeuchi, “Space-time analytic smoothing effect of the heat semigroup defined on homogeneous Besov spaces”, Partial Differential Equations in Applied Mathematics **4** (2021), 100174.

II. List of Talks

01. T. Takeuchi, “On the Keller-Segel system of parabolic-parabolic type with initial data in homogeneous Besov spaces”, 42nd Young Researchers Seminar on Evolution Equations, online (Zoom), Aug. 31.
02. T. Takeuchi, “On the Keller-Segel system of parabolic-parabolic type in homogeneous Besov spaces framework”, MSJ Autumn Meeting, online (Zoom), Sep. 15.
03. T. Takeuchi, “On the maximal Lorentz regularity theorem for the Keller-Segel system of parabolic-parabolic type”, International Workshop on Multi-Phase Flows: Analysis, Modelling and Numerics, online (Zoom), Dec. 3.

III. Research Results in AY2021

The smoothing effect of the heat semigroup defined on homogeneous Besov spaces in the whole space was considered. It was shown that even if the initial data is a singular data such as the Dirac measure, the solution of the linear heat equation is analytic jointly space and time. In fact, a radius of convergence of the Taylor expansion is given explicitly.

The Keller-Segel-Navier-Stokes (KSNS) system of parabolic-elliptic type in the whole space was considered. The local well-posedness for the system with initial data in the Sobolev spaces was obtained. In addition, it was shown that the solution is uniformly bounded with respect to the viscosity in the sense of the norm of the initial data. The existence of inviscid limits was also given.

On the other hand, the local well-posedness for the KSNS system with initial data in the scaling invariant Besov spaces was shown. Moreover, it was proven that the solution belongs to the Lorentz spaces in time direction with revealing the relation between the regularity of the initial data and the regularity of the solution. The global existence of solutions was also given for small initial data.

IV. Research Plan for AY2022

To show that inviscid limits of the 2D KSNS system exist globally in time even if the initial velocity of the fluid is arbitrarily large. This result corresponds to that of the global existence of solutions to the 2D Euler system.

To establish the maximal regularity theorem for the Laplacian by introducing the Lorentz space of Chemin-Lerner type. For its applications, construct solutions to the Navier-Stokes system.