Research Report（April, 2021- March, 2022）

Enrollment from April 2020 | Department of Pure and Applied Mathematics | Jumpei INOUE

I. List of Papers

None.

II. List of Talks

1. Jumpei Inoue, “拡散の種類を変えての拡散ロジスティック方程式における最適棲息分布の考察”, 第 42 回発展方程式若手セミナー, August 30th, online.

III. Research Results in AY2021

The research theme is an optimization problem for the stationary diffusive logistic equation. The equation is elliptic type differential equation over a bounded domain: $d\Delta u + u(m(x) - u) = 0$, imposing the Neumann boundary condition. I studied the supremum of the ratio of the integral of $u(x)$ and $m(x)$ by varying the diffusion rates d and the profiles of resources $m(x)$. In the ecological sense, the integral of $u(x)$ and that of $m(x)$ are the total population of species and the total amount of feeds, respectively. A conjecture that the supremum of that ratio is finite for any dimension was proposed by Wei-Ming Ni. In particular, the supremum was to be 3 in the 1-dimensional case. Bai-He-Li (Proc AMS, 2015) showed the validity of that. However, I.-Kuto (DCDS-B, 2021) proved that the supremum is infinity in the higher-dimensional case. This is a big contrast to the 1-dimensional case. In the 2-dimensional cases, let the diffusive coefficient be fixed and the resource function be the Dirac delta function, then the integral of the solution goes to infinity. This mechanism comes from the quadratic reaction term $u(m(x)-u)$ respect to u. Because the maximum of this term seems to be the square of the delta function, the solution $u(x)$ is much larger than the resource function $m(x)$. Consequently, one of the natural proposal is $d\Delta u+h(u)(m(x)-u)=0$, where K is a positive constant and $h(u)=u(u<K)$; $=K(u>K)$. This modification makes the reaction term be linear with respect to u when u is large, and the integral of the solution is expected to be finite. More precisely, let the domain be the 2-dimensional disk, and the resource function be an approximation sequence of the Dirac delta function. It seemed that the radial solution of the modified diffusive logistic equation was close to the special solution $4d/r^2$ of $d\Delta u-u^2=0$. However, recently, the solution has appeared to be close to that special solution. This problem is being addressed on now.

IV. Research Plan for AY2022

Next plan is to study the modified diffusive logistic equation mentioned above. There is a plan for study abroad, that is to visit Prof. Xueli Bai (Northwestern Polytechnical University, China) who is one of the best researcher for reaction-diffusion systems.