"Multiscale Analysis, Modeling and Simulation"
Top Global University Project, Waseda University
REPORT ON STUDY ABROAD

Date: September 10, 2017

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1. **Study abroad destination:** Hotel S. Michele, Cetraro, Cosenza, Italy
2. **Dates of stay:** September 3, 2017 – September 10, 2017 (8days)
3. **Purpose:** To participate "CIME International Mathematical Summer Center: Mathematical Analysis of the Navier-Stokes Equations: Foundations and Overview of Basic Open Problems."
4. **Education and research activity in the destination**
   I. **Lectures:**
      (1) G. P. Galdi (Univ. Pittsburgh), "Analysis of basic problems in liquid-solid interaction."
      (2) M. Hieber (TU Darmstadt), "Analysis of Incompressible Viscous Fluid Flow: an approach by Maximal Lp-Regularity."
      (3) H. Kozono (Waseda Univ.), "Method of the Besov space and its applications to the strong solutions of the Navier-Stokes equations."
      (4) J. Robinson (Univ. Warwick), "Partial regularity for the 3D Navier Stokes equations and applications."

   II. **Research Results:**

      In this stay, I was able to learn various approach to the Navier-Stokes equation. For example, the spectrum theory, the maximal Lp regularity, the inhomogeneous Besov space, the partial regularity, and so on. Let me explain summaries of lectures below.

      Prof. Galdi gave a lecture on an interaction of a liquid with a rigid body. In particular, Prof. Galdi gave a lecture on the stability of its rotation. In the lecture, there was not only an explanation of mathematical theory but also animations of experiments. It was very helpful in understanding the lecture. The lecture consisted of two parts: one was the coupled motion of a rigid body with a cavity entirely filled with a Navier-Stokes liquid while the other was the classical phenomena of self-oscillation observed in a viscous flow past a rigid obstacle. In the lecture, there were explanations of mathematical interpretations of physical phenomenon

and rigorous mathematic explanations using the spectral theory.

Prof. Hieber gave a lecture on the maximal Lp regularity and its applications. The lecture was started from the introduction of the maximal Lp regularity and H-infinity calculus. Although I use the maximal regularity theorem in my study, but I did not know much about an approach from H-infinity calculus. The lecture may help my research. In the lecture, Prof. Hieber also treated the Navier-Stokes equations and the Ericksen-Leslie model as an application of the maximal Lp regularity and proved the local and global existence results for strong solutions under several assumptions.

Prof. Kozono gave a lecture on the homogeneous Besov space. First, Prof. Kozono introduced the homogeneous Besov space and inhomogeneous Besov space, and then he explained the embedding theorem and proved the bilinear estimate. Then he explained its application to the stationary and non-stationary Navier-Stokes equations. Part of the lecture was difficult to understand. I think, however, that it was very good just under understanding the basic theory of the Besov space.

Prof. Robinson gave a lecture on partial regularity for the 3D Navier-Stokes equations. In the lecture, the equations were considered in not the whole space but the three-dimensional torus because of some mathematical reasons. The main point of the lecture was that considering the set of singular time and discussing the measure of its set. To define the measure, the Minkowski dimension was used. It was interesting that the weak solution which satisfies the strong energy inequality yields the weak-strong uniqueness and the partial regularity results.

5. **Other comments:**

I was glad to receive various lectures on mathematical analysis of the Navier-Stokes equations. I would like to make use of what I learned in lecture for my research. The content of this lecture was very exciting because there were lots of content I did not know before. In addition, it was very interesting to discuss the contents of the lecture with other participants.

Finally, I am grateful to Prof. Yoshihiro Shibata and "Top Global University Project, Waseda University" for giving me such a great opportunity to take lectures abroad.