# Research Report（September, 2018- September, 2019）

Enrollment from September 2017 | Department of Pure and Applied Mathematics | Keiichi WATANABE

**I. Publications**

(in preparation)

**II. Talks**

01. K. Watanabe, “Free boundary problem of compressible and incompressible two-phase flows with surface tensions and phase transitions in bounded domains”, Nagoya University, January 21.

02. K. Watanabe, “Global solvability of compressible-incompressible two-phase flows with phase transitions and surface tensions in bounded domains”, Tokyo Institute of Technology, March 20.

03. K. Watanabe, “The free boundary problem of compressible-incompressible two-phase viscous fluids with phase transitions”, Tohoku University, May 15.

04. P. Tolksdorf and K. Watanabe, “Navier-Stokes equations in exterior Lipschitz domains”, Kanazawa University, September 20.

**III. Scientific visits**

01. United Kingdom, Imperial College London, September 09, 2018--December 16, 2018, Join the Ph.D. program “EPSRC Center for Doctoral Training in the Mathematics of Planet Earth.”

02. United States, University of Pittsburgh, May 25, 2019--September 02, 2019.

**IV. Grants and Fellowships**

01. JSPS Research Fellowship for Young Scientists, "Free boundary problem of compressible-incompressible viscous two-phase flows with phase transitions in unbounded domains", #19J10168, April 2019--March 2021, JPY 1,900,000.

**V. Research Results in 2nd year**

01. I showed the existence of the Helmholtz decomposition of $\mathrm{L}^p(\Omega)$, where $\Omega \subset \mathbb{R}^{\mathrm{n}}\ (\mathrm{n} \geq 3)$ is an exterior Lipschitz domain, assuming that $p$ satisfies $|1/p - 1/2| < 1/6 + \varepsilon$ with some constant $\varepsilon$. Compared with the result obtained by Lang-Méndez (2006), this result relaxes their condition on $p$ and includes the case $p = 3$. Furthermore, I proved that the Stokes operator admits maximal regularity and the $\mathrm{L}^p - \mathrm{L}^q$ mapping properties for the Stokes semigroup, which make us to construct mild solutions to the three-dimensional Navier-Stokes equations in the scale-critical function space $\mathrm{L}^{\infty}\big(0, T; \mathrm{L}^3_{\sigma}(\Omega)\big)$. This result is based on joint work with Dr. Patrick Tolksdorf (Univ. Mainz).

02. I proved the existence of a unique global strong solution to the Navier-Sokes-Korteweg equations in $\mathbb{R}^{\mathrm{n}}\ (\mathrm{n} \geq 2)$ in the $\mathrm{L}^{\mathrm{p}}$-in-time and $\mathrm{L}^{\mathrm{q}}$-in-space framework which includes the $\mathrm{L}^2$-in-time and $\mathrm{L}^2$-in-space framework results. The proof relies on the maximal regularity property of the negative of Laplace operator and the Mikhlin-type Fourier multiplier theorem. In the proof, a standard energy method is not required.

**VI. Research Plan for 3rd year**

01. I will study a free boundary problem of the Navier-Stokes equations with contact angles.

02. I will study the Navier-Stokes equations with Robin boundary conditions in a domain with non-compact boundaries.