Mathematical Analysis of the Navier-Stokes Equations: Foundations and Overview of Basic Open Problems

-Top Global University Project, Waseda University-

REPORT ON STUDY ABROAD

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1. Study Abroad Destination: Hotel S. Michele Cetraro, Cosenza, Italy

2. Dates of Stay: September 3, 2017 - September 10, 2017 (8days)

3. Purpose:

To study the fundamental knowledge of the latest works on the Navier-Stokes equations.

4. Host Professor: Prof. Giovanni P. Galdi (University of Pittsburgh)

5. Education and Research Activity in the Destination

I) Seminars, Lectures, Conferences, etc:

- G.P. Galdi (University of Pittsburgh): Interaction of a Navier-Stokes Liquid with a Rigid Body
- Matthias Hieber (TU Darmstadt): Analysis of Incompressible Viscous Fluid Flow: an Approach by Maximal L^p-Regularity
- Hideo Kozono (Waseda University): Method of the Besov space and its applications to the strong solutions of the Navier-Stokes equations
- James Robinson (University of Warwick): Partial regularity for the 3D Navier Stokes equations and applications

II) Abstract of the lectures:

- Professor Galdi presented a lecture on physical applications of the Navier-Stokes equations. He considered mainly two types of physical phenomena regarding the interaction of a Navier-Stokes liquid with a rigid body, the coupled motion of a rigid body with a cavity filled with a liquid, and the viscous flow of a liquid past a rigid obstacle. He proved that these different phenomena have the common mathematical property from a view point of the spectrum analysis of linear operators. He also showed us the result of experiments by pictures, which helped us to understand these theories visually.
- Professor Hieber explained the methods of approaching the analysis of fluid dynamics, such as nematic liquid crystal flow and two phase free boundary problems. As the foundation of the approach, he considered the maximal L^p-regularity property of the equation. More precisely, he introduced and explained the notion of H^∞-calculus to show the $\mathcal{R}$-boundedness of the resolvent of the generator associated with the equations.
- Professor Kozono considered the Navier-Stokes equations in the homogeneous Bseov spaces. He first introduced the Besov spaces, with the notion of Littlewood-Paley decomposition of distributions. Then he precisely showed the bilinear estimate of nonlinear terms and the $L^p - L^q$ type estimate of the heat semi-group in these spaces, which are critical for the proof of the well-posedness of the stationary and non-stationaly Navier-Stokes equations.

- Professor Robinson investigate the regularity of weak solutions to the Navier-Stokes equations. He introduced the notion of the box-counting dimension, which is used to measure the time interval where the weak solutions blow up. He deeply considered the condition of the solution and the pressure to shorten such singular time, and presented the bound of that dimension. He also explained that his method is meaningful for considering the uniqueness problem of the weak solutions.

6. Other Comments:

In this summer school, four experts gave lectures on their own fields associated with the Navier-Stokes equations. Although it is rather hard for me to understand their lectures, this event was a good opportunity to know the frontiers of the studies. Moreover, I had a chance to talk with foreign students whose study fields are similar to mine. On the other hand, the hotel where the event hold was comfortable so that I could make good progress in my study.

During the event, Mr. Watanabe, who is my colleague and studies the similar field to Professor Hieber's work, explained the basic knowledge associated with the professor's lecture to me. Therefore, I would like to appreciate his kindness. Furthermore, Professor Saitoh kindly supported our travel in Italy. He also taught us mental preparation for studying abroad. Hence, I am thankful to him for giving me advice. Finally, I am deeply grateful to the staffs in the top global university project of Waseda university and CIME for giving this beneficial opportunity.