Multiscale Analysis, Modeling and Simulation
-Top Global University Project, Waseda University-
REPORT ON STUDY ABROAD
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1. **Study Abroad Destination:** University of Pisa, Italy.
2. **Dates of Stay:** October 3rd, 2016 - February 4th, 2017 (125days).
3. **Purpose:**

    To study the global well-posedness of semirelativistic equations.
4. **Host Professor:** Vladimir Georgiev (University of Pisa).
5. **Education and Research Activity in the Destination**

    Presentation:

    Kazumasa Fujiwara, Masahiro Ikeda, Yuta Wakasugi, "A direct proof of blow-up for a damped wave equation with a non-gauge invariant power type nonlinearity,"
    University of Pisa, December 15th, 2016.

    Research Results:

    The purpose of our study is to show the global well-posedness of semirelativistic equations with a power type nonlinearity and an imaginary coefficient on Euclidean spaces. During the last stay (from 7th October to 14th November in 2015), professors Georgiev and Ozawa, and I obtained a priori estimate for the solutions. In particular, the charge and energy of solutions are decreasing with respect to time in the case where spacial dimension is less than 4. Since the charge and energy are decreasing, we expected the global well-posedness in positive time in the energy space and finite time blow-up in negative time in the frame work of the charge. In three dimensional case, however, we could not obtain the local well-posedness of the corresponding regularity and therefore the global well-posedness was also unclear. In addition, because of the fractional derivative and non-positivity of nonlinearity, we could not show the blow-up in negative time.

    In this stay, we obtained both of the global well-posedness in positive time in three dimensional case under the radial assumption and blow-up in negative time in one dimensional case. In particular, in the scaling subcritical case, we applied the Strichartz estimate of Professors Bellazzini, Georgiev, and Visciglia (arXive:1611.04823) and obtained the global well-posedness in the energy space. Moreover, in the scaling critical case, we also obtained the global well-posedness by transforming semirelativistic equations of three dimension into the corresponding equations of one dimension. The article of these argument was submitted to arXive with arXive number:1611.09674. We also obtained the finite time blow-up in negative time by the corresponding ordinary differential inequality. The ordinary differential inequality is obtained by the ideas of Virial identity and modified test function method of professor Ozawa and me

(J. Math. Phys. 57 (2016)). Especially, a commutator estimate plays a critical role in order to derive the corresponding ordinary differential inequality without the positivity of nonlinearity.

6. Other Comments:

In the beginning, I would like to express my sincere gratitude to professor Georgiev for his warm hospitality. Thanks to his help, I had no serious problem during my stay.

In this stay, I could discuss with professor Georgiev for many times. This was my second time to visit him and this time was longer and more challenging than the last time. Therefore, I could experience his powerful approach much better than the last time. Specifically, for instance, I learned how to simplify problems with symmetry and how to modify our problems by commutators from him. I am sure that this experience gives a new basis of my future works.

Lastly, I am deeply grateful to professors Shibata and Ozawa and "Top Global University Project" for their various supports.