Multiscale Analysis, Modeling and Simulation
-Top Global University Project, Waseda University-
REPORT ON STUDY ABROAD
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Date: November 14, 2015

1. Study Abroad Destination: University of Pisa, Italy

2. Dates of Stay: October 7, 2015 - November 14, 2015 (39days)

3. Purpose:
To prove the fractional Leibnitz rule for high order derivative.

4. Host Professor: Prof. Vladimir Georgiev (University of Pisa)

5. Education and Research Activity in the Destination

Research Results:

The purpose of our study is to show the fractional Leibnitz rule for order $s \geq 1$. Professor Vladimir Georgiev, professor Tohru Ozawa, and I have studied the estimate since March, 2015. The fractional Leibnitz rule is an estimate for the remainder of classical Leibnitz rule, which is an expression of the derivative of a product of functions, with pseudo differential operators of or order s. Kenig, Ponce, and Vega(1993) obtained the fractional Leibnitz rule for $0 < s < 1$ and they estimated the remainder by derivatives of order s_1 and s_2, where $s = s_1 + s_2$. The fractional Leibnitz rule is proposed by many authors and sometimes referred as fractional Leibniz rule estimates, commutator estimates, Kato-Ponce estimates, and so on. The fractional Leibnitz rule is of interest on their own in Harmonic Analysis as well as in applications to nonlinear partial differential equations. Professor Ozawa and I showed that the fractional Leibnitz rule holds even for $s = 1$ in one space dimension. This means that we could expect in a general setting. Moreover, for $s = 2$, the fractional Leibnitz rule is obtained by the classical Leibnitz rule. This suggests that correction terms might be necessary for $s > 1$.

During this stay, we obtained the fractional Leibnitz rule for $0 \leq s_1, s_2 \leq 2$. Especially, we showed that, when $0 \leq s_1, s_2 \leq 1$, the fractional Leibnitz rule holds without any correction terms and when $s \geq 2$, the fractional Leibnitz rule holds with natural correction terms from the view point of the classical Leibnitz rule. The serious difficulty to show the fractional Leibnitz rule is to redistribute the derivative of high-frequency components to low-frequency components. We overcame this difficulty by the improved method of Bourgain and Li(2014). More precisely, we showed that the Jacobian, which comes from the fundamental principle of calculus of the difference of low-frequency components, decreases the order of derivative of higher exponents. Roughly speaking, the difference between our method and that of Bourgain and Li is that ours is for Triebel-Lizorkin spaces and theirs is for Besov spaces, and we need more careful calculations to treat Triebel-Lizorkin spaces. We can apply this method not only when $0 \leq s_1, s_2 \leq 1$ but also $s \geq 2$, and it also explains why the correction terms are required when $s \geq 2$. We also remark that our method can be applied in the case when $s \geq 4$.

Talking about future works, we are interested in the application of the fractional Leibnitz rule for $s \geq 1$ and that of our method. We found that we can apply the fractional Leibnitz rule to obtain a priori estimates for some nonlinear partial differential equations, and we shall study the time-global solutions for them. Moreover, we are interested in the fractional calculus with potentials and we shall study the fractional Leibnitz rules with potentials by our method.

6. Other Comments:

At first, I would like to express my deepest gratitude to professor Georgiev for his magnificent hospitality. I spent really wonderful days in Pisa thanks to him. Even thought his schedule was tight, he made time to discuss with me almost every day and he gave his interesting ideas and helpful advices. These discussions improved our study very well. Moreover, in Pisa, I had some

troubles because this is my first time to stay a foreign country but every time, he tried his best to solve my problems. He also introduced me his students and colleagues and I found a possibility to start some joint works with them.

I also would like to thank professor Ozawa for his kind supports. He kindly helped me to start this joint work and gave me many helpful advices. He also visited Pisa for 4 days during my stay so that we could discuss, and there he gave nice comments for our future works with his work.

Lastly, I'm deeply grateful to professor Yoshihiro Shibata and "Top Global University Project" for giving me such a great opportunity to study abroad. For me, it is almost impossible to stay abroad to study there by myself, but to study abroad is definitely important to join international academic societies. I truly hope that Top Global University Project keeps to help many Japanese students to join international projects.