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スーパーグローバル大学創成支援 早稲田大学 数物系科学拠点
Mathematics and Physics Unit "Multiscale Analysis, Modelling and Simulation"
Top Global University Project, Waseda University

International Workshop on Multiphase Flows: Analysis, Modelling and Numerics

Waseda University, Tokyo, Japan November 30 – December 3, 2021

Venue: Online via Zoom

Program (in Japan Time)

	Tue., Nov. 30	Wed., Dec. 1		Thu., Dec. 2	Fri., Dec. 3
14:30 | 14:40	**Opening**		14:40 | 15:30	**Murata Miho**	**Yusuke Ishigaki**
14:40 | 15:30	**Yoshihiro Shibata**	**Kubo Takayuki**	15:45 | 16:10	**Kenta Tomioka**	**Taiki Takeuchi**
15:50 | 16:40	**Shota Sakamoto**	**Liutang Xue**	16:20 | 16:45	**Tomoki Takahashi**	**Tatsuki Yamamoto**
17:00 | 17:50	**Kai Koike**	**Wang Chao**	17:00 | 17:50	**Oishi Kenta**	**Keiichi Watanabe**
17:50 | 18:30	**Break & Disscussion**				
18:30 | 19:20	**Jürgen Saal**	**Martin Saal**	18:30 | 19:20	**Bogdan-Vasile Matioc**	**Yoshiyuki Kagei**
19:40 | 20:30	**Piotr Mucha**	**Jan Burczak**	19:40 | 20:30	**Vladimir Kozlov**	**Mariana Haragus**

●…50-minutes Talk ●…25-minutes Talk

Organizers Yoshihiro Shibata, Takayuki Kubo, Hirokazu Saito, Keiichi Watanabe, Kenta Oishi

Tuesday, November 30th, 2021

Global well posedness for two phase problem of Navier-Stokes equations in unbounded domains

Yoshihiro Shibata
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In this talk, I would like to give a formulation of incompressible-incompressible, incompressible-compressible, compressible-compressible, two phase problem. And then, in the incompressible-incompressible case, I will explain how to prove the global well-posedness by using the maximal regularity and decay properties for linearized equations. My methods are different in the case of with surface tension and without surface tension.

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The well-posedness of the Boltzmann equation without angular cutoff in some Banach-algebra spaces

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In this talk we will study a Cauchy problem of the Boltzmann equation near an equilibrium without angular cutoff in the whole space $\mathbb{R}^3$ and on the torus $\mathbb{T}^3$. For this problem, well-posedness is proved in Sobolev, Besov, and the Wiener spaces. The key property of these spaces is that they are Banach algebra, i.e., $\|fg\| \leq C\|f\|\|g\|$ for any f, g belonging to one of these spaces (if their indices satisfy certain conditions). We will first summarize these results, and then introduce a recent result of well-posedness in $\mathbb{R}^3$ over L^1_k, a Fourier-Lebesgue space (k stands for the variable on the Fourier side). Our proof is based on the Banach-algebra property of the space to control a quadratic term, and the macro-micro decomposition to obtain sufficient dissipation estimate to close energy estimates. Compared to the case on $\mathbb{T}^3$, we will also employ a detailed calculation of algebraically weighted estimates. This talk is based on a joint work with Renjun Duan (Chinese University of Hong Kong) and Yoshihiro Ueda (Kobe University).

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Some results on the motion of point particles in a 1D viscous compressible fluid

Kai Koike
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In this talk, I present some results on the long-time behavior of point particles in a 1D viscous compressible fluid. I shall first explain that the velocity $V(t)$ of a point particle obeys a power law $t^{-3/2}$ and that this can be proved by an application of pointwise estimates of Green's function (https://www.sciencedirect.com/science/article/pii/S0022039620304666 and https://arxiv.org/abs/2010.06578). Then I also explain a recent development which extends this result to several point particles.

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Is the Helmholtz decomposition necessary for well-posedness of the Stokes equations?

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(Incomressible) Fluid flow in a domain is described by the fundamental Stokes (linear) and Navier-Stokes (nonlinear) equations. The Helmholtz decomposition into solenoidal and gradient fields serves as a helpful tool to analyze these systems. Apparently it has been an open question for some time, whether the existence of the Helmholtz decomposition (which is equivalent to weak well-posedness of the Neumann problem) is necessary for well-posedness of Stokes and Navier-Stokes equations in the L^q-setting for $1 < q < \infty$. Note that by a classical result of Bogovskiĭ and Maslennikova there are (uniformly) smooth domains, so-called non-Helmholtz domains, such that the Helmholtz decomposition does not exist. In my talk, I intent to present positive and negative results on well-posedness of the Stokes and Navier-Stokes equations in L^q for a large class of uniform $C^{2,1}$-domains. In particular, classes of non-Helmholtz domains are addressed. This will include a comprehensive answer to the open question for the case of partial slip type boundary conditions. The project is a joint work with Pascal Hobus.

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Compressible Navier-Stokes with rough density

Piotr Mucha

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We prove the all-time propagation of the Sobolev regularity for the velocity field solution of the two-dimensional compressible Navier-Stokes equations, provided the volume (bulk) viscosity coefficient is large enough. The initial velocity can be arbitrarily large and the initial density is just required to be bounded. In particular, one can consider a characteristic function of a set as an initial density. Uniqueness of the solutions to the equations is shown, in the case of a perfect gas. As a by-product of our results, we give a rigorous justification of the convergence to the inhomogeneous incompressible Navier-Stokes equations when the volume viscosity tends to infinity. Similar results are proved in the three-dimensional case, under some scaling invariant smallness condition on the velocity field.

The talk is based on joint results with Raphael Danchin (Paris12).

See: R Danchin, PB Mucha, Compressible Navier-Stokes equations with ripped density arXiv:1903.09396

Wednesday, December 1st, 2021

On the Evolution of Compressible and Incompressible viscous fluids with a sharp interface

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In this talk, we consider some two phase problems of compressible and incompressible viscous fluids' flow without surface tension under the assumption that the initial domain is a uniform $W_q^{2-1/q}$ domain in $\mathbb{R}^N$ $(N \geq 2)$. We prove the local in the time unique existence theorem for our problem in the L_p in time and L_q in space framework with $2 < p < \infty$ and $N < q < \infty$ under our assumption. In our proof, we first transform an unknown time-dependent domain into the initial domain by using the Lagrangian transformation. Secondly, we solve the problem by the contraction mapping theorem with the maximal L_p-L_q regularity of the generalized Stokes operator for the compressible and incompressible viscous fluids' flow with the free boundary condition. The key step of our proof is to prove the existence of a $\mathcal{R}$-bounded solution operator to resolve the corresponding linearized problem. The Weis operator-valued Fourier multiplier theorem with $\mathcal{R}$-boundedness implies the generation of an analytic semigroup and the maximal L_p-L_q regularity theorem.

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Revisit the patch problems of the 2D Boussinesq system and 2D inhomogeneous Navier-Stokes system

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Recently the density patch problem of 2D inhomogeneous Navier-Stokes (INS) system and the temperature patch problem of 2D Boussinesq system have attracted a lot of interest. Due to that the density/temperature solves the transport equation and the patch data is of piecewise characteristic functions, the patch problem is about whether the initial regularity of the patch boundary can be preserved for all the time. Some noticeable global regularity persistence results of the patch boundary have already been established. In this talk we revisit the patch problem of 2D INS and 2D Boussinesq with initial $C^{k,\gamma}(k \geq 3, 0 < \gamma < 1)$ boundary regularity, which seems not addressed in the previous works. By introducing a good unknown and using the striated estimates method initialized by J.-Y. Chemin, we proved that the $C^{k,\gamma}$-regularity of temperature patch for 2D Boussinesq will be globally persisted. By applying the striated estimates method, we also showed that, under the assumption that initial density is sufficiently close to constant 1, the boundary regularity of the density patch for 2D INS will globally belong to $C^{k,\gamma'}$ with any $0 < \gamma' < \gamma$.

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On the motion of interfaces of compressible and incompressible fluids with surface tension– A priori estimates

Chao Wang

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In this talk, I will talk about two phase flow separated by a free boundary with surface tension. The upper flow is described by compressible Euler equations and the lower flow satisfies the incompressible Euler equations. In this talk, we obtain a priori estimate for the system.

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The Stochastic Primitive Equations with Transport Noise and Turbulent Pressure

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We consider the stochastic primitive equation for geophysical flows subject to transport noise and turbulent pressure. Admitting very rough noise terms, the global existence and uniqueness of solutions to this stochastic partial differential equation are proven using stochastic maximal L^2-regularity, the theory of critical spaces for stochastic evolution equations, and global a priori bounds. Compared to other results in this direction, we do not need any smallness assumption on the transport noise which acts directly on the velocity field and we also allow rougher noise terms. The aptations to Stratonovich type noise are discussed as well.

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From ketchup to concentration-driven convex integration

Jan Burczak

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It is much easier to make hair gel or shaving foam flow after applying some force to it. Such fluid is called non-Newtonian: it changes its viscosity under applied force. This behaviour is abundant in nature: ketchup, ice, concrete, molten lava, blood, certain polymers, porridge, not forgetting the eponymous ketchup are all non-Newtonian. A simple model of such fluid (a power-law model) is known to be well-posed in the 'subcritical' regime and to have energy solutions above the 'compactness threshold'. A recent result obtained with S. Modena and L. Székelyhidi shows that also a dual picture holds. Namely, the power-law model is ill posed below the 'compactness threshold' and it has many (very) weak solutions in the 'supercritical regime'. The last result is of consequence to the classical Navier-Stokes equations.

Thursday, December 2nd, 2021

Global well-posedness for a Q-tensor model of nematic liquid crystals

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We consider the model for a viscous incompressible liquid crystal flow proposed by Beris and Edwards in 1994. This model is coupled system by the Navier-Stokes equations with a parabolic-type equation describing the evolution of the director fields $\mathbb{Q}$, which is called $\mathbb{Q}$-tensor. In this talk, we prove the global well posedness and the decay estimates for a $\mathbb{Q}$-tensor model of nematic liquid crystals in $\mathbb{R}^N$, $N \geq 3$. The proof is based on the maximal L_p -L_q regularity and the L_p -L_q decay estimates to the linearized problem. This is a joint work with Professor Yoshihiro Shibata from Waseda University.

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The Cauchy problem of nonlinear partial differential equations in plasma physics

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We study the Cauchy problem for the Zakharov system and the Schrödinger-improved Boussinesq system in a two dimensional domain. Under natural assumption on the initial data, we prove the existence and uniqueness of global solutions. We show the difference between the Zakharov system and the Schrödinger-improved Boussinesq system. Moreover, we consider the vanishing "improvement" limit of global solutions of the Schrödinger-improved Boussinesq system as the coefficient of the linear term of the highest order in the equation of ion sound wave tends to zero. These results are based on a joint work with Professor Tohru Ozawa.

References

[1] T. Ozawa, K. Tomioka, Schrödinger-improved Boussnesq system in two space dimensions, submitted.

[2] T. Ozawa, K. Tomioka, Zakharov system in two space dimensions, Nonlinear Analysis, 214 (2022) 112532.

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Anisotropic weighted L^q-L^r estimates of the Oseen semigroup in exterior domains, with application to the Navier-Stokes flow past a rigid body

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We consider the Navier-Stokes flow past a three dimensional rigid body. We develop analysis in Lebesgue spaces with anisotropic weights $(1+|x|)^{\gamma}(1+|x|-x_1)^{\delta}$, which naturally arise in the asymptotic structure of fluid when the translational velocity of the body is parallel to the x_1-direction. We first provide necessarily and sufficiently conditions so that anisotropic weights belong to the Muckenhoupt class. We next derive anisotropic weighted L^q-L^r estimates for the Oseen semigroup in exterior domains. As an application, we prove the stability of a stationary solution in anisotropic weighted Lebesgue spaces.

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On the global well-posedness and decay for a free boundary problem of the Navier-Stokes equation in unbounded domains

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We develop the global well-posedness and some decay properties for a free boundary problem of the incompressible Navier-Stokes equations in unbounded domains. Our assumptions are maximal L_p-L_q regularity for the Stokes equations and L_q-L_r decay for the Stokes semigroup. The novelty is that the compactness of the boundary is not assumed while it is essentially used when a similar result has been obtained in exterior domains by Shibata. Owing to this improvement, we obtain the global well-posedness and decay in the half space. This is a joint work with Yoshihiro Shibata (Waseda University).

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The two-phase Stokes flow by capillarity in the plane

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We discuss a two-phase moving boundary problem that describes the two-dimensional Stokes flow of two fluids with different densities and viscosities that occupy the entire plane in the regime where surface tension effects are taken into account at the interface that separates the fluids. In this setting the classical methods of potential theory can be used to transform the model into a nonlinear and nonlocal evolution problem for the function that parameterizes the interface between the fluids, the nonlinearities being expressed by singular integral operators. This problem is of parabolic type, well-posed in all Sobolev spaces up to critical regularity, and it features some parabolic smoothing properties. Joint work with Georg Prokert.

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Modeling of fluid flow in a flexible pipe with laminar wall

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We consider a three dimensional model of fluid flow in an elastic pipe with laminate wall surrounded by another elastic material. It can serve as a model of blood flow in a vessel in the blood circulatory system, where the vessel's wall consists of several layers (adventitia, media and intima) reenforced by collagen fibres and surrounded material is the muscle tissue. An asymptotic approximation of this problem by a problem for fluid flow in the same pipe but with two-dimensional boundary is presented in [1], [2] and [3]. The two-dimensional model of the laminate wall together with a new boundary condition, which, in particular, takes into account the interaction of the wall with surrounding elastic material of muscles and the fluid flow, is obtained via a dimension reduction procedure.

In this talk a study of time periodic flows in an infinite cylinder with new "elastic" boundary conditions is presented. The main result is that solutions of this problem do not depend on the period and they are nothing else but the time independent Poiseuille flow. Similar solutions of the Stokes equations for the rigid wall (the no-slip boundary condition) depend on the period and their profile depends on time.

Compared with the classical work of J.R. Womersley our formulation of problem has much in common, both of them involve momentless shell theory for modeling the elastic wall. In Wormerley's work, axisymmetric pulsative blood flow in a vessel with circular isotropic elastic wall is found as a perturbation of the steady Poisseulle flow. Apart from inessential generalizations like arbitrary shape of vessel's cross-section and orthotropic wall, there is a certain difference in our models and results and this will be discussed in the talk.

Next step is derivation and analysis of one dimensional model, which is obtained from the above model by using dimensional reductions procedure, is treated in our papers [4]–[8].

This is a joint work with Sergei Nazarov and German Zavorokhin (Russia).

References

[1] Kozlov V., Nazarov S., *Dokl. Phys.*, **56**(11):560–566, 2011.

[2] Kozlov V., Nazarov S., *J. Math. Sci.*, **213**(4):561–581, 2016.

[3] Ghosh A., Kozlov V., Nazarov S., and Rule D., Q. J. Mech. Appl. Math, Vol. 71, Issue 3, 349-367, 2018.

[4] Kozlov V., Nazarov S., *J. Math. Sci.*, **207**(2):249–269, 2015.

[5] V. Kozlov, S. Nazarov, Sbornik: Mathematics 208 (8), 1138, 2017.

[6] F. Berntsson, M. Karlsson, V. Kozlov, S. Nazarov, Appl. Math. Comp. 274, 125-132, 2016.

[7] F. Berntsson, A. Ghosh, V. Kozlov, S. Nazarov, AMM 63, 633-643, 2018.

[8] Kozlov V., Nazarov S., Zavorokhin G., *EJMCA*, 7:3, 33–55, 2019.

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Friday, December 3rd, 2021

Diffusion wave phenomena and L^p estimates of solutions of compressible viscoelastic system

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This talk is concerned with the system of equations describing motion of compressible viscoelastic fluids in a three dimensional whole space. We investigate the large time behavior of solutions around a motionless state, and obtain the L^p estimates of solutions for $1 \leq p \leq \infty$, provided that the initial data is sufficiently close to the motionless state. In addition, we clarify the diffusion wave phenomena caused by sound wave, viscous diffusion and elastic wave.

On the maximal Lorentz regularity theorem for the Keller-Segel system of parabolic-parabolic type

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In this talk, we show the maximal Lorentz regularity theorem for the Keller-Segel system of parabolic-parabolic type, i.e, the existence and uniqueness of local strong solutions for arbitrary initial data in the scaling invariant homogeneous Besov space, where the solutions belong to the Lorentz space in time direction. We also construct global strong solutions for small initial data. The method is based on the maximal Lorentz regularity theorem of heat equations established by Kozono-Shimizu (2019).

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Logarithmically Improved Extension Criteria Involving the Pressure for the Navier-Stokes Equations in $\mathbb{R}^3$

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This talk is concerned with the extension criterion involving the pressure for the Navier-Stokes equations in $\mathbb{R}^3$. We prove that if a strong solution u on $[0,T)$ and the pressure π associated with u satisfy the condition

$$\int_0^T \frac{\|\nabla\pi(\tau)\|^r_{\dot{B}^{-3/p}_{\infty,\infty}}}{\log(e+\|u(\tau)\|_{H^s})}d\tau < \infty \quad \text{for } \frac{2}{r}+\frac{3}{p}=3 \text{ with } \frac{3}{2} \leq p < \infty,$$

then there is $T' > T$ such that u can be continued to the strong solution on $[0,T')$. Our method is based on the interpolation inequality due to Gérard-Meyer-Oru (1997) and the trilinear estimate due to Guo-Kučera-Skalák (2018). This is a joint work with Dr. Ryo Kanamaru (Photon Sansu Club).

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Stability of uniformly rotating liquid

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This talk is concerned with the stability of a stationary solution of free boundary problems of the incompressible Navier–Stokes equations in a three-dimensional bounded domain with surface tension. More precisely, it is shown that if the initial angular momentum is sufficiently small and if the initial configuration is sufficiently close to equilibrium, then there exists a global strong solution that converges exponentially fast to a uniform rigid rotation of the liquid as $t \to \infty$ with respect to a certain axis, where the moving interface may be understood in the classical sense. The proof of the existence of a stationary solution is also given.

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Stability of the compressible Taylor vortices under axisymmetric perturbations

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Bifurcation problem is considered for the compressible Navier-Stokes equations in a domain between two concentric infinite cylinders. If the outer cylinder is at rest and the inner one rotates with sufficiently small angular velocity, a laminar flow, called the Couette flow, is stable. When the angular velocity of the inner cylinder increases, beyond a certain value of the angular velocity, the Couette flow becomes unstable and a vortex pattern, called the Taylor vortex, bifurcates and is observed stably. This phenomena is mathematically formulated as a bifurcation and stability problem. In this talk, the bifurcation problem of the Taylor vortex and its stability are considered for the compressible Navier-Stokes equations. It is shown that the compressible Taylor vortex bifurcates near the criticality for the incompressible problem when the Mach number is sufficiently small. The localized stability of the compressible Taylor vortex is shown under sufficiently small axisymmetric perturbations.

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A spectral approach to transverse linear instability of line periodic water waves

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Line periodic waves are solutions of the three-dimensional water-wave problem which are periodic in one horizontal coordinate and do not depend on a second, transverse, horizontal coordinate. The transverse stability question is concerned with their stability with respect to three-dimensional perturbations, hence also depending on the horizontal coordinate in which the line periodic waves are constant. Relying upon an abstract, rather simple, linear instability criterion, we show that capillary-gravity periodic water waves are transversely unstable in several parameter regimes.