Last update: August 2, 2021

Germany-Japan Workshop on Problems Related to Free Boundaries and Moving Contact Lines

Dates: August 18 – 20, 2021, Location: Online (Zoom)

Timetable (in Japan Time)

August 18 (Wed)

16:00 – 17:00 Chao Wang (Peking University)
The local well-posedness of water wave equations

17:00 – 17:20 Break & Discussion (Breakout rooms are available)

17:20 – 18:20 Maximilian Moser (University of Regensburg)
Sharp Interface Limits with Contact Angle

18:20 – 18:40 Break & Discussion (Breakout rooms are available)

18:40 – 19:40 Helmut Abels (University of Regensburg)
On the sharp interface limit of a Navier-Stokes/Allen-Cahn system

19:40 – 20:00 Discussion (Breakout rooms are available)

August 19 (Thu)

16:00 – 17:00 Kenta Oishi (Waseda University)
On the global well-posedness and decay for a free boundary problem of the Navier-Stokes equation in unbounded domains

17:00 – 17:20 Break & Discussion (Breakout rooms are available)

17:20 – 18:20 Mathias Wilke (Martin-Luther-Universität Halle-Wittenberg)
Well-Posedness and qualitative behaviour of the Mullins-Sekerka problem with ninety-degree angle boundary contact.

18:20 – 18:40 Break & Discussion (Breakout rooms are available)

18:40 – 19:40 Irina Vladimirovna Denisova (Russian Academy of Sciences)
The Problem on the Rotation of a Two-Phase Drop

19:40 – 20:00 Discussion (Breakout rooms are available)

August 20 (Fri)

16:00 – 17:00 Hajime Koba (Osaka University)
Mathematical modeling and analysis of diffusion equations on evolving surfaces

17:00 – 17:20 Break & Discussion (Breakout rooms are available)

17:20 – 18:20 Manuel Gnann (Delft University of Technology)
Tanner's law for the apparent contact angle in viscous thin films

18:20 – 18:40 Break & Discussion (Breakout rooms are available)

18:40 – 19:40 Jonas Sauer (Delft University of Technology)
On the Linear Analysis of the 2D Stokes Problem With a Moving Contact Line

19:40 – 20:00 Discussion (Breakout rooms are available)

Organized by Matthias Köhne, Jürgen Saal, Yoshihiro Shibata, Keiichi Watanabe.

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Wednesday, August 18th 2021

The local well-posedness of water wave equations

Chao Wang

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In this talk, I will talk about the water wave equations. First, I will present the local well-posedness in the low regularity space. Second, I will talk about our results on the water wave equations in the corner domain.

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Sharp Interface Limits with Contact Angle

Maximilian Moser

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We consider the sharp interface limit for the Allen-Cahn equation with a nonlinear Robin boundary condition in a two-dimensional domain, in the situation that a diffuse interface has developed and intersects the boundary of the domain. The boundary condition is designed such that in the limit one obtains mean curvature flow with constant contact angle. For times when a smooth solution to this limit problem exists and for angles close to 90° , we show convergence in strong norms using the method of de Mottoni and Schatzman: in the first step, one constructs an approximate solution for the Allen-Cahn equation via asymptotic expansions based on the solution of the limit problem. Therefore suitable curvilinear coordinates defined on a trapeze are constructed and an interesting model problem on the half space appears. In the second step one estimates the difference of the exact and approximate solution with a Gronwall argument. Here a spectral estimate for the corresponding Allen-Cahn operator (linearized at the approximate solution) enters. – Joint work with Helmut Abels (Regensburg).

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On the sharp interface limit of a Navier-Stokes/Allen-Cahn system

Helmut Abels

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We consider the sharp interface limit of a Navier-Stokes/Allen-Cahn system, when a parameter $\varepsilon > 0$ that is proportional to the thickness of the diffuse interface tends to zero, in a two dimensional bounded domain. We present a recent result in which we prove convergence for sufficiently small times of the solutions of the Stokes/Allen-Cahn system to solutions of a sharp interface model, where the interface evolution is given by the mean curvature equation with an additional convection term coupled to a two-phase Navier-Stokes system with an additional contribution to the stress tensor, which describes the capillary stress. To this end we construct a suitable approximation of the solution of the Navier-Stokes/Allen-Cahn system. To this end a new Ansatz based on partial linearization in highest order is used, which simplifies the analysis compared to previous results. Then the difference of approximate and exact solution is estimated with the aid of a suitable spectral estimate of the linearized Allen-Cahn operator. This is a joint work with Mingwen Fei from Anhui Normal University, China.

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Thursday, August 19th 2021

On the global well-posedness and decay for a free boundary problem of the Navier-Stokes equation in unbounded domains

Kenta Oishi

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We develop the global well-posedness and some decay properties for a free boundary problem of the incompressible Navier-Stokes equations in unbounded domains. Our assumptions are maximal L_p-L_q regularity for the Stokes equations and L_q-L_r decay for the Stokes semigroup. The novelty is that the compactness of the boundary is not assumed while it is essentially used when a similar result has been obtained in exterior domains by Shibata. Owing to this improvement, we obtain the global well-posedness and decay in the half space. This is a joint work with Yoshihiro Shibata (Waseda University).

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Well-Posedness and qualitative behaviour of the Mullins-Sekerka problem with ninety-degree angle boundary contact

Mathias Wilke

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In the first part of the talk, we prove local-in-time well-posedness for the Mullins-Sekerka system with ninety degree angle boundary contact with the help of maximal L_p-L_q-regularity. The motion of the free boundary is described by a height function over a fixed reference surface. In the second part we show that solutions starting close to certain equilibria exist globally in time, are stable, and converge to an equilibrium at an exponential rate. This is a joint work with Helmut Abels and Maximilian Rauchecker.

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The Problem on the Rotation of a Two-Phase Drop

Irina Vladimirovna Denisova[1]
and
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The stability of an axially symmetric liquid drop consisting of two viscous incompressible capillary fluids and rotating about the x_3-axis with the angular velocity ω is considered. We study a free boundary problem for the perturbation of velocity vector field $\boldsymbol{v} = (v_1, v_2, v_3)$ and pressure function p of the two-phase fluid written in the coordinate system rotating with ω. We obtain global unique solvability of the problem provided that the initial data and the rotation speed are small, as well as the unknown surfaces are close to certain axially symmetric equilibrium figures. It is proved that if the second variation of energy functional is positive, the perturbation of an axially symmetric equilibrium figure tends to zero exponentially as $t \to \infty$, the motion of the drop going over to the rotation of the liquid mass as a rigid body. We develop the technique used by V. A. Solonnikov to prove the stability of a solution near the slow rotation of an axisymmetric equilibrium figure for a single fluid of finite volume. The existence of a two-phase axisymmetric equilibrium figure close to a ball was demonstrated earlier.

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Friday, August 20th 2021

Mathematical modeling and analysis of diffusion equations on evolving surfaces

Hajime Koba

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We are interested in diffusion processes such as concentration and thermal diffusions on evolving surfaces. An evolving surface means that the surface is moving or the shape of the surface is changing along with the time. This talk deals with mathematical modeling and analysis of diffusion equations on evolving surfaces.

In the first half of this talk, we derive and study diffusion equations on an evolving surface from an energetic point of view. We employ an energetic variational approach to make a mathematical model for a diffusion process on an evolving double bubble. Moreover, we investigate the boundary conditions for our equations to study the conservation and energy laws of them.

In the second half of this talk, we show the existence of local and global-in-time strong solutions to the diffusion equation on an evolving surface with a boundary. We apply both the maximal L^p-in-time regularity for Hilbert space-valued functions and the semigroup theory to construct strong solutions to the surface diffusion system.

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Tanner's law for the apparent contact angle in viscous thin films

Manuel Gnann

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We rigorously investigate the asymptotics of traveling-wave solutions relevant for Tanner's law describing the apparent contact angle. It is proved that the velocity-dependent apparent contact angle is given by a universal law that only weakly depends on the modeling at the contact line and the liquid-solid interface. More precisely, we prove that the apparent contact angle depends continuously differentiably on the microscopic Young angle and on the mobility exponent describing the form of slip at the liquid-solid interface. We expect the same behavior to be valid also for the Stokes or Navier-Stokes equations of a liquid droplet of which the thin-film equation is the lubrication approximation. This indicates that there is no need to model contact-angle dynamics at least if one considers intermediate asymptotics.

The talk is based on joint works with Lorenzo Giacomelli (Rome), Felix Otto (Leipzig), and Anouk Wisse (Delft).

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On the Linear Analysis of the 2D Stokes Problem With a Moving Contact Line

Jonas Sauer

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In my talk I am interested in well-posedness and regularity results for a free boundary problem for the two-dimensional Stokes equations, modelling a droplet on a substrate close to its moving front. The free boundary is the liquid-gas interface whose evolution is driven by surface tension. The considered model is subject to a linear Navier-slip condition and to a fixed microscopic contact angle.

It is well-known that this relatively parsimonious model leads to a singularity in the pressure close to the triple junction which excludes the existence of smooth (or even C^2) solutions. This has sparked new proposed physical models seeking to remove this singularity.

I want to advocate for keeping the simple model, accepting that solutions will develop singularities and building a suitable mathematical framework that can capture this behaviour. In particular, the presented (L^2-based) analysis gives precise information on the singular expansion of the velocity, the pressure and the profile in the vicinity of the triple junction, and aims at providing global well-posedness of solutions for initial data close to a linear equilibrium profile.

The talk is based on ongoing work with Manuel Gnann, Hans Knüpfer and Nader Masmoudi.