The First Japan-Taiwan Joint Conference on Differential Geometry

Titles and Abstracts

Professor Chih-Wei Chen (National Taiwan University, Taiwan)

Title: Curvature estimates of the Ricci flow based on Ricci curvature

Abstract: We construct a uniform local bound of curvature operator from local bounds of Ricci curvature and injectivity radius among all n-dimensional Ricci flows. Thus new compactness theorems for the Ricci flow and Ricci solitons are derived. In particular, we show that every Ricci flow with |Ric|\leq K must satisfy |Rm| \leq Ct^{-1} for all t in (0,T], where C depends only on the dimension n and T depends on K and the injectivity radius inj_g(t).

Professor Hajime Fujita (Japan Women's University, Japan)

Title: Localization of index via torus fibrations and a Danilov-type formula for toric origami manifolds

Abstract: In our joint work with M. Furuta and T. Yoshida, we developed an analytic index theory on open Riemannian manifolds which leads to a localization formula of the index. Our localization formula does not rely on the group action but uses a family of torus fibrations and Dirac-type operators along the fibers. In this talk we will explain an application of the localization formula to the equivariant index of Dirac-type operators for a class of manifolds called "toric origami manifolds". The formula is a generalization of Danilov's classical theorem for symplectic toric manifolds, which asserts that the space of holomorphic sections of a torus equivariant ample line bundle can be identified with the representation space of the torus defined by the integral lattice points in the momentum polytope (Delzant polytope). The formula itself can be obtained as a consequence of the symplectic cobordism theorem of Cannas da Silva-Guillemin-Pires or the multi fan description of toric origami manifolds by Masuda-Park. In contrast to them, our proof is direct and geometric.

Professor Keizo Hasegawa (Niigata University, Japan)

Title: Homogeneous locally conformally Kaehler and Sasaki manifolds

Abstract: We discuss recent developments in the study of locally conformally Kaehler and Sasaki homogeneous spaces. In our previous works we have shown a holomorphic structure theorem and a metric structure theorem for a compact homogeneous locally conformally Kaehler manifold, asserting that it is a holomorphic fiber bundle over a flag manifold with fiber a 1-dimensional complex torus and is always of Vaisman type, that is, its associated Lee form is parallel with respect to the Hermitian metric. For non-compact cases we have given a certain sufficient condition to be of Vaisman type. Recently, applying a powerful technique of "modifications", we have been attacking the problem of classifying homogeneous Vaisman spaces of unimodular Lie groups. For the case of Vaisman unimodular Lie groups, we have shown that their Lie algebras are divided into three types: R x h_2n-1, R x sl(2), R x u(2), up to modifications, where h_2n-1 is a Heisenberg Lie algebra of dimension 2n-1. Note that we have shown at the same time that a Sasakian unimodular Lie algebra is one of the three types: h_2n-1, sl(2), su(2), up to modifications. We also discuss non-unimodular cases and non-Vaisman cases. This talk is based on joint works with D. Alekseevsky, V. Cortes and Y. Kamishima.

Professor Kota Hattori (Keio University, Japan)

Title: The nonuniqueness of tangent cones at infinity of Ricci flat manifolds

Abstract: Colding and Minicozzi proved the uniqueness of the tangent cones at infinity of Ricci-flat manifolds with Euclidean volume growth which has a tangent cone at infinity with a smooth cross section. In this talk I explain about an example of the Ricci-flat manifold implying that the assumption for the volume growth in the above result is essential. More precisely, I will show that some of the hyperKaehler manifolds constructed by Anderson, Kronheimer and LeBrun have infinitely many tangent cones at infinity and one of them has a smooth cross section.

Professor Chung-I Ho (National Kaohsiung Normal University, Taiwan)

Title: Minimal genus problem

Abstract: The minimal genus of embeded surfaces within a homology class in 4-manifolds has been a study topic for a long time. It involves many important technique in 4-dimensional topology. There has a big breakthrough since the invention of Seiberg-Witten theory. In this talk, I will survey the progress in the past as well as some recent work in this problem.

Professor Naoyuki Koike (Tokyo University of Science, Japan)

Title: Volume-preserving mean curvature flow for tubes with boundary in symmetric spaces

Abstract: In this talk, I first state the evolution equations for the radius function and its gradient along the volume-preserving mean curvature flow starting from a tube (of nonconstant radius) over a closed geodesic ball of a reflective submanifold in a symmetric space under certain condition for the radius function. Next, we prove that the tubeness is preserved along the flow in the case where the ambient space is a rank one symmetric space of non-compact type and the reflective submanifold is an invariant submanifold. Furthermore, in this case, we prove that the flow reaches to the invariant submanifold in finite time or it exists in infinite time and converges to another tube of constant mean curvature over the invariant submanifold in the C^infinity-topology in infinite time.

Professor Kwok-Kun Kwong (National Cheng Kung University, Taiwan)

Title: Weighted Hsiung-Minkowski formulas and rigidity of hypersurfaces

Abstract: The well-known Alexandrov theorem states that the only closed embedded surfaces with constant mean curvature in Euclidean space are the round spheres. There are many generalizations, commonly known as rigidity theorems. In this talk I am going to illustrate how we can use the weighted Hsiung-Minkowski formulas to obtain simple proofs of these kinds of rigidity results. More precisely, I will give Alexandrov type results for closed embedded hypersurfaces with radially symmetric higher order mean curvature in a large class of warped product manifolds, including space forms. I will also show the rigidity of closed immersed self-expanding solitons to the weighted generalized inverse curvature flow. Part of it is joint work with Hojoo Lee and Juncheol Pyo.

Professor Kuo-Wei Lee (National Taiwan University, Taiwan)

Title: Constant mean curvature foliations in the extended Schwarzschild spacetime

Abstract: In this talk, we first give an introduction to the constant mean curvature (CMC) foliations and the CMC time functions. Then we summarize some CMC foliations results in cosmological spacetimes. For spatially noncompact cases, Schwarzschild

spacetimes for example, CMC foliation properties were conjectured by Malec and ´O Murchadha in [1, 2]. These conjectures are completely proved in [3, 4, 5] and we will show the ideas of these proofs in this talk as well.

References

[1] Malec, E.; ´O Murchadha, N.: Constant mean curvature slices in the extended Schwarzschild solution and the collapse of the lapse, Phys. Rev. D (3) 68 (2003), no. 12, 124019, 16 pp.
[2] Malec, E.; ´O Murchadha, N.: General spherically symmetric constant mean curvature foliations of the Schwarzschild solution, Phys. Rev. D 80, (2009), 024017, 8 pp.
[3] Lee, Kuo-Wei; Lee, Yng-Ing: Spacelike spherically symmetric CMC foliation in the extended Schwarzschild spacetime. Ann. Henri Poincar´e 17 (2016), no. 6, 1477–1503.
[4] Lee, Kuo-Wei: Dirichlet problem for the constant mean curvature equation and CMC foliation in the extended Schwarzschild spacetime. Classical Quantum Gravity 33, (2016) 185001.
[5] Lee, Kuo-Wei: Existence of a constant mean curvature foliation in the extended Schwarzschild spacetime. Phys. Rev. D 94, (2016) 084038.

Professor Chun-Chi Lin (National Taiwan Normal University, Taiwan)

Title: On the gradient flow of elastic open curves with fixed length and clamped ends

Abstract: In this talk, I will discuss the regular open curves in R^n with clamped ends subject to a fixed length constraint and moving according to the L^2-gradient flow of the elastic energy, which is a 4th-order parabolic equation. A standard procedure in deriving the long-time existence of smooth solutions is applying interpolation inequalities to obtain integral estimates of various order. For this equation, the main difficulty under such approach comes from the constraint of fixed total length and the clamped ends. We focus on how to overcome the difficulty and then prove the long time existence and unique convergence to critical points. The result in this talk is based on a joint work with Anna Dall'Acqua and Paola Pozzi.

Professor Chih-Chung Liu (National Cheng Kung University, Taiwan)

Title: Bubble Tree for Vortex Moduli Spaces

Abstract: We study the asymptotic behaviours of vortex moduli spaces near the boundary. Formation of bubbles follows from arguments in Gromov compactness and removal of

singularities require careful choices of gauges and addition of mild singularities to the metric on the surfaces.

Professor Ziming Ma (National Taiwan University, Taiwan)

Title: From Witten-Morse theory to SYZ mirror symmetry

Abstract: Wedge product on deRham complex of a Riemannian manifold M can be pulled back to H*(M) via explicit homotopy, constructed using Green's operator, to give higher product structures. Fukaya conjectures the Witten deformation of these higher product structures have semi-classical limits as operators defined by counting gradient flow trees with respect to Morse functions, which generalizes the remarkable Witten deformation of deRham differential. We will describe briefly the proof of Fukaya's conjecture, and an application to Mirror symmetry which realizes the scattering diagram as semi-classical limit of solution to the Maurer-Cartan equation.

Professor Shinichiro Matsuo (Nagoya University, Japan)

Title: Yamabe invariants and the twisted Seiberg-Witten equations

Abstract: We compute the Yamabe invariants for a new infinite class of closed four-dimensional manifolds by using a "twisted" version of the Seiberg-Witten equations.

Professor Akane Nakamura (Josai University, Japan)

Title: Isospectral limit of the Painleve-type equations and degeneration of curves

Abstract: Recently, a classification of the four-dimensional analogs of the Painleve equations have been accomplished using isomonodromic deformation (Sakai, Kawakami-N-Sakai, Kawakami). In order to characterize the 40 types of the 4-dimensional Painleve-type equations in their list, we consider their abelianization (i.e. isospectral limit, autonomous limit) leading to the Hitchin systems, which are algebraically completely integrable. We study the degenerations of the theta divisors appearing as the spectral curves and the Painleve divisors of the Liouville tori.

Professor Shin-ichi Ohta (Kyoto University, Japan)

Title: Nonlinear geometric analysis on Finsler manifolds

Abstract: A Finsler manifold is a manifold equipped with a (Minkowski) norm on each tangent space. Although the natural Laplacian is nonlinear for Finsler manifolds, one can establish the Bochner formula (O-Sturm, 2014) and use it to develop the nonlinear analogue of the Gamma-calculus (of Bakry et al). In this talk, we review the basic notions in this "nonlinear geometric analysis" on Finsler manifolds, and give some applications in isoperimetric and functional inequalities.

Professor Hajime Ono (Saitama University, Japan)

Title: Volume minimization principle for conformally Kaehler Einstein-Maxwell metrics

Abstract: Apostolov and Maschler introduced the notion of conformally Kaehler Einstein-Maxwell metrics (cKEM metrics for short), which is a generalization of strongly Hermitian solutions of the Einstein-Maxwell equations on complex surfaces. Moreover they defined an obstruction to the existence of cKEM metrics, cKEM-Futaki invariants. In this talk, I would like to explain that cKEM-Futaki invariants are considered as the first variation of some “volume function”. We also give some examples of non-Kaehler cKEM metrics in any dimensions. This is joint work with A. Futaki.

Professor Takashi Sakai (Tokyo Metropolitan University, Japan)

Title: The intersection of two real flag manifolds in a complex flag manifold

Abstract: In 1988, Chen-Nagano introduced the notion of antipodal set of a compact symmetric space, and defined a geometric invariant, so-called 2-number, as the maximal possible cardinality of antipodal sets. An orbit of the adjoint representation of a compact connected Lie group G admits a G-invariant Kaehler structure, and is called a complex flag manifold. In this talk, first we give a definition of an antipodal set of a complex flag manifold using a torus action, and show that a maximal antipodal set is characterized as an orbit of the Weyl group of G. An orbit of the linear isotropy representation of the compact symmetric space G/K is embedded in a complex flag manifold as a real form, and called a real flag manifold. We give a necessary and sufficient condition for two real flag manifolds, which are not necessarily congruent to each other, in a complex flag manifold to intersect transversally in terms of symmetric triads. Then we show the antipodal structure of the intersection of two real flag manifolds. Furthermore, as an application, we study the Floer homology for a pair of real forms in a Hermitian symmetric space

of compact type. This talk is based on a joint work with Osamu Ikawa, Hiroshi Iriyeh, Takayuki Okuda, and Hiroyuki Tasaki.

Professor Chiung-Jue Sung (National Tsing Hua University, Taiwan)

Title: Poisson equation and its applications

Abstract: In this talk, we will explain our recent joint work with Ovidiu Munteanu and Jiaping Wang concerning the Poisson equation on smooth metric measure spaces, emphasizing sharp estimates of the solutions. Applications will also be discussed.

Professor Asuka Takatsu (Tokyo Metropolitan University, Japan)

Title: Riemannian Wasserstein geometry on the space of Gaussian measures over the Wiener space

Abstract: An abstract Wiener space is a triple (H,E, mu), where mu is a standard Gaussian measure on a separable Hilbert space H and E is a separable Banach space on which mu is supported. It is known that the space of Gaussian measures on E being equivalent to mu becomes a Hilbert manifold, and the manifold admits a non-positive Riemannian metric derived from the information geometry. In this talk, we consider another geometry on the manifold, called the Wasserstein geometry, which is a metric geometry on the space of probability measures. We construct a compatible Riemannian metric on the manifold with the Wasserstein geometry and show the non-negativity of the Riemannian manifold, which provides the difference from the information geometry. This is based on a joint work with Hiroshi Kawabi (Okayama University).

Professor Kazunaga Tanaka (Waseda University, Japan)

Title: Domain shape and multiplicity of positive solutions for nonlinear elliptic equations

Abstract: We consider the existence of multiple positive solutions of the following nonlinear elliptic equation equations:

-Delta u= u^p and u>0 on Omega, u=0 on the boundary of Omega

where Omega is a bounded domain in R^N with smooth boundary, and 1 < p < (N+2)/(N-2) if N>2, 1 < p < infinity if N=2.

Domain shape significantly affects the multiplicity of positive solutions of this equation. We study singularly perturbed domains and we give new multiplicity results via our variational approach, which is different from the classical Lyapunov-Schmidt reduction method. This talk is partially based joint works with Jaeyoung Byeon (KAIST, Korea).

Professor Chung-Jun Tsai (National Taiwan University, Taiwan)

Title: Calibrated submanifolds in bundle manifolds of special holonomy

Abstract: In manifolds with special holonomy, it is interesting to study calibrated submanifolds, which are volume minimizers in their homology classes. We study the calibrated submanifolds in several famous local models of manifolds with special holonomy. These model spaces are all total spaces of some vector bundles, and the zero section is a calibrated submanifold. We show that the zero section is the only compact minimal submanifold, and is dynamically stable under the mean curvature flow. This is a joint work with Mu-Tao Wang.

Professor Mu-Tao Wang (Columbia University, USA)

Title: Surface geometry and general relativity

Abstract: Many classical results regarding surfaces in 3-dimensional Euclidean space such as Weyl's isometric embedding problem and the Minkowski inequality have their counterparts for surfaces in spacetime. These generalization are not only of mathematical interest, but also of physically relevant importance. They are closely related to fundamental concepts such as gravitational energy and Cosmic censorship. In my talk, I shall discuss some recent developments in these directions.

Professor Chin-Tung Wu (National Pingtung University, Taiwan)

Title: CR Reilly formula and its applications

Abstract: In this talk, we derive the CR Reilly's formula and its applications to studying of the first eigenvalue estimate for the CR Dirichlet and the Neumann eigenvalue problem, and embedded p-minimal hypersurfaces. In particular, we prove the CR Aleksandrov Theorem for compact oriented embedded hypersurfaces of nonvanishing constant p-mean curvature in the n-dimensional Heisenberg group. This is a joint work with Shu-Cheng Chang.