早稲田大学理工学術院　英語プログラム特別入試

2 次選考：「数理科学的思考力を問う総合試験」

サンプル問題

2017 年 4 月公開

早稲田大学理工学術院

英語プログラム特別入試（4 月入学）
2 次選考 :「数理科学的思考力を問う総合試験」サンプル問題の公開につきまして

早稲田大学理工学術院では、英語学位取得プログラムのための〝英語プログラム特別入試〞を 2018 年 4 月入学者より新たに導入します。
英語プログラム特別入試では「1 次選考（書類審査）」合格者に対し「2 次選考（総合試験および面接試験）」を通じて評価いたします。

このたび、「2 次選考（総合試験および面接試験）」において重要な評価項目となる「数理科学的思考力を問う総合試験」のサンプルを公開しましたので、出願を検討される上での材料としてご活用下さい。

・公開されるサンプルは〝英語プログラム特別入試〞における「2 次選考 : 数理科学的思考力を問う総合試験」の出題の概要や狙いをお伝えすることを目的として作成されたものであり、実際の出題とは形式・分量等において異なることがありますことをご了承ください。
・当日の試験時間は 90 分を予定しており、今回のサンプル問題においても 1 問あたり 30 分程度の解答時間を想定しております。
・全て「英語」でご解答いただきます。

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1. A triangle $\triangle$ABC is such that its angles satisfy the condition

$$\sin \mathrm{A} = 2 \cos \mathrm{B} \sin \mathrm{C}.$$

What can you say about this triangle?

2. Find all positive integers x, y which satisfy the condition

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}.$$

3. Find all third-order polynomials $f(x)$ and real numbers a which satisfy the following three conditions:

(i) The coefficient of x^3 is 1. (ii) The graph of $y = f(x)$ is tangent to the x axis at $x = a$. (iii) $y = f(x)$ takes its local maximum value $\frac{1}{2}$ at $x = -\frac{a}{3}$.

4. Consider the points $\mathrm{A} = (a, 0, 0)$, $\mathrm{B} = (0, b, 0)$, $\mathrm{C} = (0, 0, c)$ and the origin $\mathrm{O} = (0, 0, 0)$ in three-dimensional space. Assume that $a, b, c > 0$. Answer the following questions.

   (i) Show that the vector $\overrightarrow{\mathrm{OH}} = \left(\frac{k}{a}, \frac{k}{b}, \frac{k}{c}\right)$ is perpendicular to the vectors $\overrightarrow{\mathrm{AB}}$, $\overrightarrow{\mathrm{BC}}$, $\overrightarrow{\mathrm{CA}}$.

   (ii) Find k if the point H lies within the triangle $\triangle$ABC.

   (iii) Find the area of the triangle $\triangle$ABC.

5. Consider the sequence of numbers $\{a_n\}$ with initial terms and recurrence relation given by

$$a_1 = 1, \quad a_2 = e, \quad a_n (a_{n+2})^3 = (a_{n+1})^4 \qquad (n = 1, 2, 3, \dots)$$

Here e denotes the base of the natural logarithm. Answer the following questions.

   (i) Let $b_n = \log a_{n+1} - \log a_n$. Find the general term of $\{b_n\}$.

   (ii) Find the general term of $\{a_n\}$.

   (iii) Find the limit $\lim_{n \to \infty} a_n$.

6. Consider a point $\mathrm{P}(a, e^a)$ of the plane curve C given by the equation $y = e^x$. Let C' be the plane curve given by the equation $y = \log x$. Assume that the tangent line to C at P touches C' at the point Q. Answer the following questions.

(i) Show that a is a solution of the equation

$$(*) \quad te^t - e^t - t - 1 = 0.$$

(ii) If a_0 is a solution of $(*)$, show that $-a_0$ is also a solution of $(*)$.

(iii) Show that $(*)$ has exactly one solution in the range $t > 0$.

7. Let P be a point in the interior of an equilateral triangle $\triangle$ABC which has sides of length 1. Denote by A′, B′, C′ the intersection points of the lines through AP, BP, CP with the opposite edges of the triangle. (For example, the line from A passing through P meets the edge BC in the point A′.) Denote the lengths of the segments BA′. CB′, AC′ by x, y, z. Answer the following questions.

   (i) Express z in terms of x, y.

   (ii) If the areas of the triangles $\triangle$ABC, $\triangle$A′B′C′ are denoted by S, S', show that $S' = 2xyzS$.

8. Denote by $\mathrm{P}_1, \mathrm{P}_2, \ldots, \mathrm{P}_6$ the vertices of a regular hexagon whose sides have length 1. Denote by F the figure given by connecting the vertices $\mathrm{P}_{\mathrm{i}_1}, \mathrm{P}_{\mathrm{i}_2}, \mathrm{P}_{\mathrm{i}_3}, \mathrm{P}_{\mathrm{i}_4}, \mathrm{P}_{\mathrm{i}_1}$ in that order by line segments, where $\mathrm{i}_1, \mathrm{i}_2, \mathrm{i}_3, \mathrm{i}_4$ are obtained by rolling a dice 4 times. Answer the following questions.

   (i) Find the probability that F is a convex quadrilateral.

   (ii) Find the probability that the area of F is maximal.

9. Let z be a point of the complex plane which satisfies the condition

$$|z - 4| = 5 - \frac{2}{5}(z + \overline{z})$$

   Answer the following questions.

   (i) Find the quadratic equation satisfied by the real numbers x, y where $z = x + yi$.

   (ii) Sketch the curve C traced out by the points z.

   (iii) Find the volume of the solid body which is obtained when the region enclosed by C is rotated once around the line $y = -3$.