WINPEC Working Paper Series No. E2522
October 2025

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Masashige Hamano Yuki Murakami

Waseda INstitute of Political EConomy
Waseda University
Tokyo, Japan

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Masashige Hamano[†] Yuki Murakami[‡]

First version: September 2025.
This version: October 13, 2025.

Abstract

This paper highlights the potential for decoupling economic growth from CO_2 emissions under strong policy, while providing a tractable framework for analyzing the long-run global green transition. We develop a dynamic stochastic general equilibrium model with heterogeneous firms: green firms abate emissions at higher costs, while brown firms do not. Emissions reduce aggregate productivity but are not internalized in competitive equilibrium. Using global data from 1981 to 2022, we calibrate the model to match observed trends in GDP and emissions. The analysis delivers three main findings. First, while emissions continue to rise, the share of green firms grows over time. Second, faster technological progress amplifies the growth–emissions trade-off, whereas slower progress attenuates it. Third, welfare analysis shows that the optimal emission tax must be substantially higher than current levels, though its role is moderated when combined with abatement innovation. Together, these results underscore the importance of policy in sustaining growth while mitigating environmental externalities.

Keywords: Climate change, Green transition, Heterogeneous firms, Economic growth, DSGE models.
JEL Classifications: Q54, Q58, E32, F44.

*We thank Toshihide Arimura, Rainer Klump, and seminar and conference participants at the University of Frankfurt, Waseda Brussels Office, Bank of Thailand, and University of Stellenbosch for their valuable comments. This research was supported by JSPS KAKENHI Grant Number 19KK0338 and 24KJ2083. All remaining errors are our own.

[†]Waseda University, School of Political Science and Economics, 1-6-1 Nishiwaseda Shinjuku-ku, Tokyo 169-8050, JP, email: masashige.hamano@waseda.jp

[‡]Waseda University, Graduate School of Economics, 1-6-1 Nishiwaseda Shinjuku-ku, Tokyo 169-8050, JP, email: yuki.murakami.ym1@gmail.com

1 Introduction

Climate change and its negative impact on economic activity have become increasingly evident in recent decades. A seemingly apparent trade-off exists between economic growth and CO_2 emissions. Figure 1 shows the growth of world GDP and the stock of CO_2 from 1981 to 2022. While world GDP increased by a factor of 3.4, the stock of CO_2 also rose, becoming 1.2 times higher than its 1981 level.

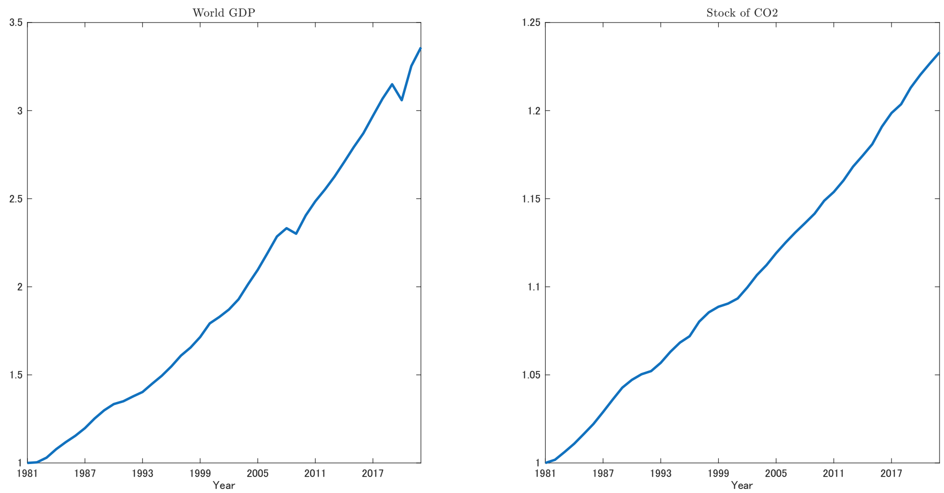


Figure 1: World GDP (level) and CO_2 Emissions (stock), 1981–2022

Note: CO_2 is expressed in GtC (billion metric tons of carbon), reflecting the absolute amount of carbon in the atmosphere. Source: World Bank World Development Indicators and Global Carbon Budget.

Our empirical window (1981–2022) is not arbitrary. The early 1980s mark both the beginning of consistent global data coverage on GDP and CO_2 emissions and the emergence of international climate governance. The *First World Climate Conference* (1979) called for coordinated international climate research and led to the World Climate Programme. This was followed by the *Vienna Convention for the Protection of the Ozone Layer* (1985) and the *Montreal Protocol* (1987), which introduced binding commitments to phase out ozone-depleting substances and became a blueprint for later climate agreements (United Nations Environment Programme, 1985, 1987). In 1987, the *Brundtland Report* popularized the concept of "sustainable development" and firmly linked growth and environmental protection in policy debates (World Commission on Environment and Development, 1987). The establishment of the *Intergovernmental Panel on Climate Change* (IPCC) in 1988 institutionalized regular scientific assessments of climate change, providing the foundation for subsequent international negotiations. These developments culminated in the 1992 Earth Summit in Rio de Janeiro, which created the *United Nations Framework Convention on Climate Change* (UNFCCC) (United Nations, 1992). The UNFCCC later underpinned the *Kyoto Protocol* (1997) (United Nations, 1997) and the *Paris Agreement* (2015) (United Nations, 2015).

These agreements signaled growing international consensus that climate change poses systemic risks to both the economy and the environment. Alongside official initiatives, the private sector has played a central role in developing and adopting environmentally friendly technologies. The environmental economics literature emphasizes the importance of technology adoption and policy incentives—such as carbon pricing and subsidies—in accelerating the "green transition" (Popp, 2002; Acemoglu et al., 2012; Aghion et al., 2019). Empirical studies further show how renewable energy innovation, energy efficiency, and clean production processes have been stimulated by international policy pressure and market forces (Johnstone et al., 2010; Calel and Dechezleprtre, 2016).

This paper develops a DSGE framework with heterogeneous firms to capture the endogenous green transition of the global economy. Firms are of two types: green firms, which abate greenhouse gas emissions using more costly technologies, and brown firms, which do not. Similar to the integrated assessment framework of Nordhaus (1992, 1994, 2017), emissions reduce productivity but are not internalized in competitive equilibrium, appearing instead as negative externalities. Governments impose emission taxes to encourage abatement.

Using global data, we calibrate the growth rates of exogenous productivity, abatement innovation, and the emission tax to match the observed evolution of world GDP and the stock of CO_2, along with a damage function that captures negative externalities. The model delivers three main findings. First, while emissions continue to rise, the world economy is undergoing a green transition, characterized by a growing share of green firms. Second, counterfactual analysis shows that faster technological progress amplifies the trade-off between growth and emissions, whereas slower progress attenuates it, while the share of green firms remains broadly unchanged. Third, welfare analysis indicates that the optimal emission tax must rise substantially relative to current levels. Such a policy induces a significant green transition, reduces the stock of CO_2, and simultaneously sustains higher economic growth by mitigating the negative externalities of emissions. A similar outcome can be achieved by combining abatement innovation with the optimal emission tax, although the role of the latter is less pronounced.

This paper contributes to several strands of the literature. First, it relates to the integrated assessment literature linking economic growth and climate change, pioneered by Nordhaus (1992, 1994, 2017), with subsequent refinements in damage functions (e.g. Dietz and Stern, 2015). Unlike these studies, we focus on the long-run green transition of the world economy under negative externalities. Second, it connects to research on business cycles and environmental policy. Heutel (2012) shows that optimal environmental taxes may be countercyclical, while Fischer and Springborn (2011) and Pizer (2002) emphasize how uncertainty and fluctuations affect policy design. Although our analysis is conducted under the assumption of perfect foresight and a long-run transition, short-run fluctuations can also be analyzed within our theoretical framework as a natural extension. Third, it builds on the literature on directed technical change and the green transition. Acemoglu et al. (2012) highlight how policy can redirect innovation toward clean technologies, and Aghion et al. (2019) stress the role of sustained policy in breaking technological lock-in. Our work shares the same interest and complements these studies by providing an alternative theoretical framework. Fourth, our framework relates to the macroeconomic literature on heterogeneous firms (Melitz, 2003; Ghironi and Melitz, 2005), extended to environmental questions. Finally, our calibration speaks to the "decoupling" debate: while some advanced economies have stabilized or reduced emissions relative to GDP, emerging economies continue to exhibit strong positive links between growth and emissions (Stern, 2004; Grunewald et al., 2017; OECD and United Nations Development Programme, 2025). Our analysis points to the possibility of decoupling through the introduction of strong policy initiatives.

The rest of the paper is structured as follows. Section 2 develops the theoretical model. Section 3 presents the calibration strategy. Section 4 compares model simulations with the data. Section 5 conducts counterfactual experiments. Section 6 derives the optimal emission tax. Section 7 concludes.

2 The Model

We propose a theoretical model of the world economy in which households supply labor and consume by maximizing expected utility. The key feature of the model is its ability to capture the green transformation of firms. By green transformation, we mean the adoption of environmentally friendly technologies that reduce carbon emissions.

Firms are heterogeneous with respect to their idiosyncratic productivities and decide whether to engage in costly abatement. However, they fail to internalize the negative impact of emissions on aggregate productivity. To address this externality, the government imposes a tax on emissions and redistributes the proceeds to households in a lump-sum manner.

2.1 Households

The representative household maximizes the following expected discounted sum of utility $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_s$, where $0 < \beta < 1$ is the discount factor and

$$U_t = \ln C_s - \chi \frac{L_s^{1+\psi}}{1+\psi}.$$

In this expression, C_t denotes consumption at time t and L_t represents labor supply. The parameter $\chi > 0$ captures the disutility from supplying labor, while $\psi > 0$ is the inverse of the Frisch elasticity of labor supply.

The consumption basket is composed of two types of goods:

$$C_t = \left(\int_{\omega \in \Omega_g} c_{g,t}^{1-\frac{1}{\sigma}} (\omega)\, d\omega + \int_{\omega \in \Omega_b} c_{b,t}^{1-\frac{1}{\sigma}} (\omega)\, d\omega \right)^{\frac{1}{1-\frac{1}{\sigma}}}.$$

Here, $c_{g,t}(\omega)$ denotes consumption of "green" goods, produced by firms adopting environmentally friendly technology, while $c_{b,t}(\omega)$ represents consumption of "brown" goods, produced by firms that do not adopt such technology. Both technologies are defined in detail in the following section. The parameter $\sigma > 1$ is the elasticity of substitution among varieties.

Minimizing total expenditure yields the optimal demand for each type of good:

$$c_{\nu,t}(\omega) = \left(\frac{p_{\nu,t}(\omega)}{P_t}\right)^{-\sigma} C_t, \quad \nu = g, b, \tag{1}$$

with the corresponding price index:

$$P_t = \left[\int_{\omega \in \Omega_g} p_{g,t}(\omega)^{1-\sigma} d\omega + \int_{\omega \in \Omega_b} p_{b,t}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}},$$

which we take as the numeraire in the following analysis.

2.2 Production and Pricing

Firms are heterogeneous with respect to their productivity levels, which are drawn upon entry from a distribution $G(z)$. Firms emit CO_2 in proportion to their production. Emissions are harmful for aggregate economic activity, but these damages are not internalized and instead appear as negative externalities.

Production is conducted by two types of firms with different abatement technologies. "Green" firms invest in abatement, while "brown" firms do not. The choice of technology for a firm with productivity z depends on the relative profitability of the two options. Each firm adopts the technology that delivers higher profits (dividends). Thus, the green transformation in the economy is endogenous.

2.2.1 Brown Firms

We first characterize "brown" firms. The emissions of a brown firm with productivity z are given by

$$e_{b,t}(z) = \frac{y_{b,t}(z)}{z},$$

where $e_{b,t}(z)$ denotes CO_2 emissions and $y_{b,t}(z)$ denotes output. Emissions are assumed to be proportional to production, given technology z.

The labor demand of a brown firm is

$$l_{b,t}(z) = \frac{y_{b,t}(z)}{A_t z} + \tau_t \frac{e_{b,t}(z)}{A_t},$$

where A_t is aggregate labor productivity. The term $\tau_t \frac{e_{b,t}(z)}{A_t}$ captures the cost of emissions in terms of effective labor. Here, $\tau_t \geq 0$ is interpreted as an emissions tax. A higher τ_t directly increases marginal production costs.

Profits of the brown firm, $d_{b,t}(z)$, are

$$d_{b,t}(z) = \rho_{b,t}(z)\, y_{b,t}(z) - w_t l_{b,t}(z),$$

where $\rho_{b,t}(z)$ is the firm's output price and w_t is the real wage.

Under monopolistic competition, each firm maximizes profits taking into account the demand (1) for its products. This yields the following optimal price:

$$\rho_{b,t}(z) = \frac{\sigma}{\sigma - 1} \frac{w_t}{A_t z} (1 + \tau_t).$$

Note that profits can also be expressed as

$$d_{b,t}(z) = \frac{1}{\sigma} \rho_{b,t}(z)^{1-\sigma} C_t.$$

Thus, a higher τ_t raises the firm's price but reduces its profits.

2.2.2 Green Firms

Green firms also emit CO_2, but abate a fraction Ω_t of emissions:

$$e_{g,t}(z) = (1 - \Omega_t) \frac{y_{g,t}(z)}{z}.$$

Abatement, however, requires additional effective labor. The labor demand of a green firm is

$$l_{g,t}(z) = \frac{y_{g,t}(z)}{A_t z} (1 + g(\Omega_t)) + \frac{\tau_t e_{g,t}(z)}{A_t} + \frac{f_{a,t}}{A_t}.$$

Here, the first term corresponds to production and abatement, the second to emissions, and the third to fixed operational abatement costs. The abatement cost function is denoted $g(\Omega_t)$, where

$$g(\Omega_t) = \theta_1 \Omega_t^{\theta_2}, \quad \theta_1 > 0,\ \theta_2 > 0,$$

following Nordhaus (2008) and Heutel (2012). The parameter $f_{a,t} > 0$ captures fixed operational abatement costs, implying that the green technology always requires non-negative overhead costs.

Profits of the green firm are

$$d_{g,t}(z) = \rho_{g,t}(z) y_{g,t}(z) - w_t l_{g,t}(z).$$

which is rewritten as

$$d_{g,t}(z) = \frac{1}{\sigma} \rho_{g,t}(z)^{1-\sigma} C_t - w_t \frac{f_{a,t}}{A_t}.$$

As in the case of brown firms, green firms maximize profits, which yields the following optimal price:

$$\rho_{g,t}(z) = \frac{\sigma}{\sigma - 1} \frac{w_t}{A_t z} \left[1 + \tau_t (1 - \Omega_t) + \theta_1 \Omega_t^{\theta_2}\right].$$

The optimal abatement share Ω_t is chosen as

$$\Omega_t = \left(\frac{\tau_t}{\theta_1\theta_2}\right)^{\frac{1}{\theta_2-1}}. \tag{2}$$

Note that all firms choose the same abatement level regardless of their productivity z. The extent of abatement depends on the emission tax and abatement cost parameters: a higher τ_t induces a higher abatement rate through a larger Ω_t.

2.2.3 Cutoff Productivity for Abatement

Having described the two technologies, we can establish the following proposition.

Proposition 1 (Existence of a cutoff productivity). *Under the assumption $\theta_2 > 1$, there exists a cutoff productivity level $z_{c,t}$ such that firms are indifferent between adopting green or brown technology:*

$$d_{g,t}\left(z_{c,t}\right) = d_{b,t}\left(z_{c,t}\right). \tag{3}$$

Proof. See Appendix A.1.

When $\theta_2 \leq 1$, no such cutoff exists, and all firms operate as brown. Figure 2 illustrates the endogenous determination of the cutoff productivity.

It is also straightforward to derive the following result regarding the effects of τ_t and $f_{a,t}$ from Proposition 1.

Proposition 2 (Partial-equilibrium effects of $f_{a,t}$ and τ_t). *Under the assumption $\theta_2 > 1$, other things equal, a decrease in $f_{a,t}$ or an increase in τ_t raises the share of green firms in the economy.*

Proof. See Appendix A.2.

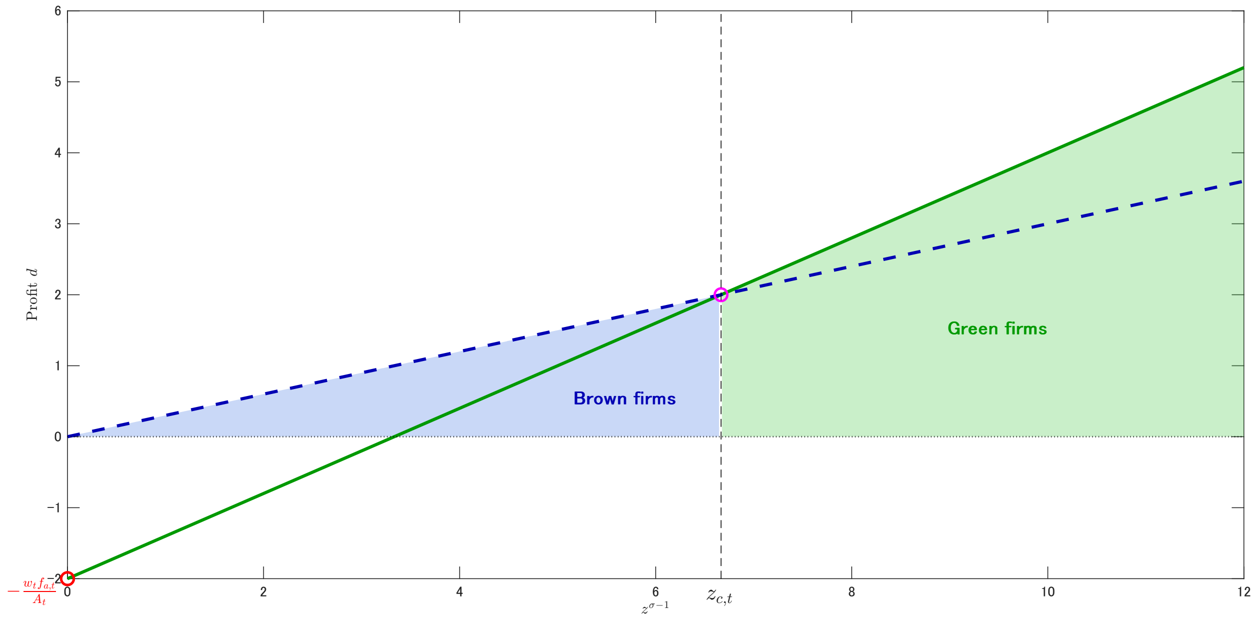
We should be careful in interpreting Proposition 2, since it represents only a partial-equilibrium perspective. In general, changes in $f_{a,t}$ or τ_t also affect equilibrium wages. A full assessment therefore requires a complete general-equilibrium analysis with simulations.

2.2.4 Firm Averages

Given the productivity distribution $G(z)$, the shares of brown and green firms are

$$N_{b,t} = G(z_{c,t})N_t, \qquad N_{g,t} = \left[1 - G(z_{c,t})\right]N_t.$$

Figure 2: Determination of green and brown firms



We also define average productivity levels for each group following Melitz (2003):

$$\tilde{z}_{b,t} \equiv \left[\frac{1}{G(z_{c,t})}\int_{z_{min}}^{z_{c,t}} z^{\sigma-1}\, dG(z)\right]^{\frac{1}{\sigma-1}}, \qquad \tilde{z}_{g,t} \equiv \left[\frac{1}{1-G(z_{c,t})}\int_{z_{c,t}}^{\infty} z^{\sigma-1}\, dG(z)\right]^{\frac{1}{\sigma-1}}.$$

Here, $\tilde{z}_{b,t}$ and $\tilde{z}_{g,t}$ denote the average productivities of brown and green firms, respectively. Using these, we can define the corresponding average real prices $(\tilde{\rho}_{b,t}, \tilde{\rho}_{g,t})$, average real profits $(\tilde{d}_{b,t}, \tilde{d}_{g,t})$, average production $(\tilde{y}_{b,t}, \tilde{y}_{g,t})$, and average emissions $(\tilde{e}_{b,t}, \tilde{e}_{g,t})$.

2.2.5 Firm Entry and Exit

New firms enter at time t, with the number of entrants denoted by H_t. Entry occurs when the expected value of a new firm equals the sunk entry cost. The free-entry condition is therefore

$$v_t = \frac{w_t f_{E,t}}{A_t},$$

where v_t is the value of a firm. v_t turns out to be the expected discounted stream of future dividends:

$$v_t = E_t \sum_{i=t+1}^{\infty} [\beta(1-\delta)]^{i-t} \left(\frac{C_i}{C_t}\right)^{-1} \tilde{d}_i, \tag{4}$$

where $\tilde{d}_t$ denotes the average profits of all firms at time t.

The law of motion for the total number of firms N_t is

$$N_t = (1-\delta)\left(N_{t-1} + H_{t-1}\right),$$

where δ is the firm exit (depreciation) rate.

2.2.6 Parameterization and Productivity Draw

Following Melitz (2003) and Ghironi and Melitz (2005), we assume that firm-specific productivity z is drawn from a Pareto distribution:

$$G(z) = 1 - \left(\frac{z_{\min}}{z}\right)^{\kappa},$$

where $z_{\min}$ is the minimum productivity level and $\kappa > \sigma - 1$ determines the shape of the distribution. We set $z_{\min} = 1$, consistent with the structure of our economy in which all firms produce either as brown or green.

Accordingly, the average productivity levels are given by

$$\tilde{z}_{b,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left[\frac{z_{\min}^{\kappa}}{1-\left(\frac{z_{\min}}{z_{c,t}}\right)^{\kappa}} \left(\frac{1}{z_{\min}^{\kappa-(\sigma-1)}} - \frac{1}{z_{c,t}^{\kappa-(\sigma-1)}}\right)\right]^{\frac{1}{\sigma-1}}, \quad \tilde{z}_{g,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} z_{c,t}.$$

Using these definitions, the share of green firms is

$$\frac{N_{g,t}}{N_t} = 1 - G(z_{c,t}) = z_{\min}^{\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}} \tilde{z}_{g,t}^{-\kappa}.$$

Finally, the abatement cutoff profit condition (3) can be rewritten as

$$\tilde{d}_{g,t} = \frac{\kappa}{\kappa-(\sigma-1)}\left(\frac{\tilde{z}_{b,t}}{z_{c,t}}\right)^{1-\sigma} \tilde{d}_{b,t} + \left[\frac{\sigma-1}{\kappa-(\sigma-1)}\right]\frac{w_t f_{a,t}}{A_t}.$$

The average profits of all firms in the economy are then

$$\tilde{d}_t = \frac{N_{b,t}}{N_t}\tilde{d}_{b,t} + \frac{N_{g,t}}{N_t}\tilde{d}_{g,t}.$$

2.3 Intertemporal Optimization

The representative household maximizes expected lifetime utility subject to a budget constraint. The budget constraint is

$$C_t + x_{t+1}v_t(N_t + H_t) = L_t w_t + x_t N_t(v_t + \tilde{d}_t) + T_t, \tag{5}$$

where x_{t+1} denotes share holdings into period $t+1$, and T_t is the lump-sum tax rebate from the government.

In each period t, the representative household chooses consumption C_t, labor supply L_t, and share holdings x_{t+1} to maximize expected utility subject to the budget constraint (5).

The first-order conditions with respect to consumption and labor supply yield the standard labor supply equation:

$$\chi L_t^{\psi} = w_t C_t^{-1}.$$

The first-order condition with respect to equity holdings implies the Euler equation for firm value:

$$v_t = \beta(1-\delta) E_t \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(v_{t+1} + \tilde{d}_{t+1} \right).$$

Once iterated forward, this equation expresses the share price as the discounted stream of future dividends, as defined in (4).

2.4 Pollution and General Equilibrium Conditions

The flow of total emissions at time t, e_t, is given by

$$e_t = N_{b,t}\tilde{e}_{b,t} + N_{g,t}\tilde{e}_{g,t}.$$

Emissions accumulate over time according to

$$s_t = (1-\delta_s)s_{t-1} + e_t,$$

where s_t represents the stock of CO_2 emissions and δ_s is the depreciation rate of the stock between $t-1$ and t.

The stock of emissions is harmful to the economy, as it reduces aggregate labor productivity:

$$A_t = [1 - D(s_t)]\, a_t,$$

where a_t follows an exogenous productivity process:

$$\ln a_t = \rho \ln a_{t-1} + \epsilon_t,$$

with ρ denoting persistence and ϵ_t an i.i.d. technology shock.

The damage function $D(s_t)$ is assumed to take the rational quadratic form

$$D(s_t) = \frac{\gamma_1 s_t^2}{1 + \gamma_2 s_t^2}, \tag{6}$$

which ensures monotonicity and bounded damages ($D(s_t) < 1$) under $\gamma_1 > 0$ and $\gamma_2 > 0$. This formulation is consistent with the DICE-2016R model (Nordhaus, 2017). This specification captures moderate damages at low pollution levels and convex, potentially catastrophic effects at higher stock levels.

The government budget constraint is

$$\tau_t \frac{e_t}{A_t} w_t = T_t.$$

The labor market clearing condition is

$$L_t = N_{b,t}\tilde{l}_{b,t} + N_{g,t}\tilde{l}_{g,t} + H_t \frac{v_t}{w_t},$$

where $\tilde{l}_{b,t}$ and $\tilde{l}_{g,t}$ denote the average labor demand by brown and green firms, respectively. These are given by

$$\tilde{l}_{b,t} = (\sigma - 1)\frac{\tilde{d}_{b,t}}{w_t}, \qquad \tilde{l}_{g,t} = (\sigma - 1)\frac{\tilde{d}_{g,t}}{w_t} + \sigma \frac{f_{a,t}}{A_t}.$$

Finally, real GDP is defined as[1]

$$Y_t \equiv L_t w_t + N_t \tilde{d}_t.$$

Table 1 summarizes the system of equations. There are 27 equations and 27 endogenous variables. The number of product varieties N_t and the stock of CO_2, s_t, serve as the state variables of the economy.

3 Calibration

The calibration is conducted on an annual basis for the world economy over the period 1981–2022, which is characterized by rising GDP and increasing CO_2 emissions.

The subjective discount factor β and the inverse of the Frisch elasticity of labor supply φ are set to 0.96 and 0.5, respectively, in line with standard values in the literature. The elasticity of substitution across varieties σ, the firm exit shock δ, and the Pareto shape parameter κ are set to 3.8, 0.1, and 3.4, respectively, following Ghironi and Melitz (2005) and Hamano and Zanetti (2017). These values are assumed constant over time. The disutility of labor supply is set to $\chi = 0.8509$ so that initial-state labor supply is normalized to unity.

The annual depreciation rate of CO_2, δ_s, and the parameter θ_2 in the abatement cost function are set following Nordhaus (2008) and Heutel (2012). The scaling parameter $\theta_{1,t}$ and its gross growth rate are normalized to one without loss of generality, since its effect is isomorphic to that of τ_t as shown in the optimal abatement condition (2).

Other parameter values are calibrated to match the observed evolution of world GDP and the

[1] It is common in the literature to abstract, to some extent, from fluctuations in the number of product varieties when analyzing real variables. This reflects the fact that empirical price indices are measured imperfectly and often fail to fully capture changes in the number of available varieties (see, for instance, Ghironi and Melitz (2005)). This issue is particularly relevant at short-run frequencies. In our exercise, which focuses on the long run, we assume that the growth in the available set of product varieties is incorporated into real GDP.

Table 1: Summary of the benchmark model

Average pricing	$\tilde{\rho}_{b,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{A_t\tilde{z}_{b,t}}(1+\tau_t), \quad \tilde{\rho}_{g,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{A_t\tilde{z}_{g,t}}\left[1+\tau_t(1-\Omega_t)+\theta_1\Omega_t^{\theta_2}\right]$
Real price index	$1 = N_{b,t}\tilde{\rho}_{b,t}^{1-\sigma} + N_{g,t}\tilde{\rho}_{g,t}^{1-\sigma}$
Average profits	$\tilde{d}_{b,t} = \frac{1}{\sigma}\tilde{\rho}_{b,t}^{1-\sigma}C_t, \quad \tilde{d}_{g,t} = \frac{1}{\sigma}\tilde{\rho}_{g,t}^{1-\sigma}C_t - w_t\frac{f_{a,t}}{A_t}$
Average production	$\tilde{d}_{b,t} = \frac{1}{\sigma}\tilde{\rho}_{b,t}\tilde{y}_{b,t}, \quad \tilde{d}_{g,t} = \frac{1}{\sigma}\tilde{\rho}_{g,t}\tilde{y}_{g,t} - w_t\frac{f_{a,t}}{A_t}$
Aggregate profits	$\tilde{d}_t = \frac{N_{b,t}}{N_t}\tilde{d}_{b,t} + \frac{N_{g,t}}{N_t}\tilde{d}_{g,t}$
Abatement cutoff	$\tilde{d}_{g,t} = \frac{\kappa}{\kappa-(\sigma-1)}\left(\frac{\tilde{z}_{b,t}}{z_{c,t}}\right)^{1-\sigma}\tilde{d}_{b,t} + \left[\frac{\sigma-1}{\kappa-(\sigma-1)}\right]w_t\frac{f_{a,t}}{A_t}$
Avg. brown productivity	$\tilde{z}_{b,t} = \left(\frac{\kappa}{\kappa-(\sigma-1)}\right)^{\frac{1}{\sigma-1}} z_{min}\left[\frac{1-\left(\frac{N_{g,t}}{N_t}\right)^{1-\frac{\sigma-1}{\kappa}}}{N_{b,t}/N_t}\right]^{\frac{1}{\sigma-1}}$
Avg. green productivity	$\frac{N_{g,t}}{N_t} = z_{min}^{\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}\tilde{z}_{g,t}^{-\kappa}$
Surviving rate	$\frac{N_{b,t}}{N_t} = 1-\left(\frac{z_{min}}{z_{c,t}}\right)^{\kappa}$
Consistency	$\frac{N_{b,t}}{N_t} = 1-\frac{N_{g,t}}{N_t}$
Free entry condition	$v_t = \frac{w_t f_{E,t}}{A_t}$
Firm dynamics	$N_{t+1} = (1-\delta)(N_t+H_t)$
Euler equation (equity)	$v_t = \beta(1-\delta)E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1}(v_{t+1}+\tilde{d}_{t+1})$
Labor supply	$\chi L_t^{\psi} = w_t C_t^{-1}$
Labor market clearing	$L_t = N_{b,t}(\sigma-1)\frac{\tilde{d}_{b,t}}{w_t} + N_{g,t}\left[(\sigma-1)\frac{\tilde{d}_{g,t}}{w_t}+\sigma\frac{f_{a,t}}{A_t}\right] + H_t\frac{v_t}{w_t}$
Firm emissions	$\tilde{e}_{b,t} = \frac{\tilde{y}_{b,t}}{\tilde{z}_{b,t}}, \quad \tilde{e}_{g,t} = (1-\Omega_t)\frac{\tilde{y}_{g,t}}{\tilde{z}_{g,t}}$
Total emissions	$e_t = N_{b,t}\tilde{e}_{b,t} + N_{g,t}\tilde{e}_{g,t}$
Emission dynamics	$s_t = (1-\delta_s)s_{t-1} + e_t$
Productivity externality	$A_t = [1-D(s_t)]\,a_t$
Damage function	$D(s_t) = \frac{\gamma_1 s_t^2}{1+\gamma_2 s_t^2}, \quad \ln a_t = \rho\ln a_{t-1} + \epsilon_t$
Abatement choice	$\Omega_t = \left(\frac{\tau_t}{\theta_1\theta_2}\right)^{\frac{1}{\theta_2-1}}$
Real GDP	$Y_t = L_t w_t + N_t\tilde{d}_t$

accumulation of the CO_2 stock shown in Figure 1. Specifically, we calibrate the growth rates and the initial state values of exogenous technology a_t, fixed abatement costs $f_{a,t}$, and the emission tax τ_t. The estimated gross growth rates are 1.0226, 1.0354, and 1.0661, respectively. The initial state values of fixed abatement costs and the tax rate are $f_a = 3.2458 \times 10^{-4}$ and $\tau = 0.0127$, while technology is normalized to $a = 1$ without loss of generality.

The parameters of the quadratic damage function are calibrated as $\gamma_1 = 9.9921 \times 10^{-7}$ and $\gamma_2 = 1.0097 \times 10^{-6}$. At the initial state, the implied damage level is 0.0019. Figure 9 in Appendix B illustrates the damage function and steady-state damage.

Table 2 summarizes the calibration.

Table 2: Calibration of the model

Parameter	Description	Steady-state value	Gross growth rate
β	Discount factor	0.96	1
φ	Inverse Frisch elasticity of labor supply	0.5	1
σ	Elasticity of substitution across varieties	3.8	1
δ	Exogenous exit shock	0.1	1
κ	Pareto shape parameter	3.4	1
χ	Disutility of labor	0.8509	1
f_E	Fixed entry cost	1	1
δ_s	Depreciation rate of CO_2	0.0089	1
θ_1	Abatement technology parameter	1	1
θ_2	Abatement technology parameter	2.8	1
γ_1	Damage function parameter	9.9921×10^{-7}	1
γ_2	Damage function parameter	1.0097×10^{-6}	1
a	Exogenous technology	1	1.0226
f_a	Fixed cost of abatement	3.2458×10^{-4}	1.0354
τ	Emission tax	0.0127	1.0661

4 Quantitative Analysis

We first examine how our calibration reproduces the observed trends in the stock of CO_2 and world GDP. The analysis also highlights the green transition implied by the theoretical model.

4.1 Data vs. Model

As the first two panels of Figure 3 show, the theoretical model replicates well the observed trends of world GDP and the accumulation of the CO_2 stock. To generate these dynamics, we require rising exogenous technology a, increasing fixed abatement costs f_a, and a gradually increasing emission tax τ. These paths are displayed in the last row of Figure 3. Specifically, the emission tax τ increases by a factor of 14 relative to its initial level in 1988.

The model also sheds light on other variables of interest. In particular, it highlights the dynamics of the number of green and brown firms and their emissions. According to the model, while both the number of green and brown firms (N_g and N_b) expand with economic growth and rising entry H, the number of green firms increases almost 60-fold between 1980 and 2022, compared to only a 2.3-fold increase in the number of brown firms. Moreover, the average emissions of green firms decline much more significantly than those of brown firms. Although emissions per firm decrease, total emissions e continue to rise due to the growing number of producers.

In short, the world economy undergoes a partial transformation toward an environmentally friendly structure, captured by the declining cutoff productivity z_c.

How would these outcomes change under different paths of technological progress (through a and f_a) or stronger social pressure for environmental action (through τ)? In the next subsection, we conduct counterfactual analyses to evaluate how alternative exogenous trends affect economic growth and emissions.

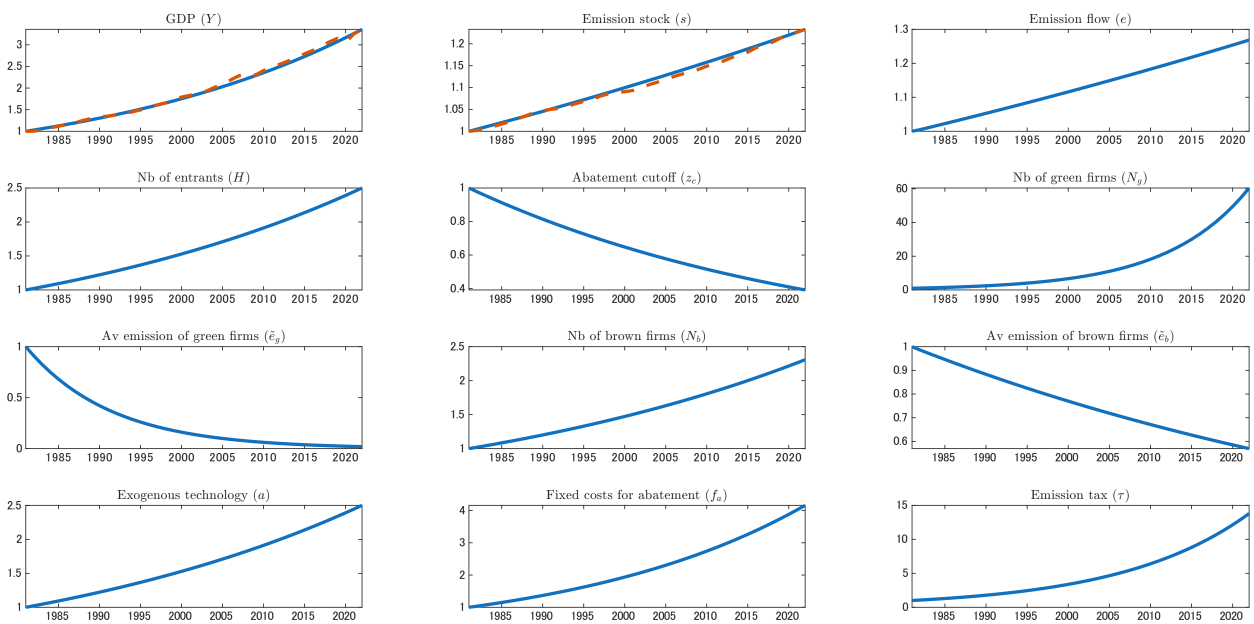


Figure 3: Data vs model

Note: The figure presents the implied trend dynamics of the theoretical model under the benchmark calibration reported in Table 2 (solid lines). It also shows the observed trend dynamics of the stock of CO_2 and world GDP (dashed lines). All variables are normalized to their initial values in 1981.

5 Counterfactual Experiments

In this section, we investigate the impact of alternative growth rates of exogenous technology a, fixed abatement costs f_a, and the emission tax τ. For each case, we compare the benchmark calibration with counterfactual growth paths of each three variables that are one percentage point higher and lower than the baseline trend.

5.1 Counterfactual Technological Development

Figure 4 illustrates the counterfactual paths of exogenous technology. The dashed line corresponds to higher growth and the dotted line to lower growth compared with the benchmark. Faster technological improvement generates stronger economic growth, as reflected in a much larger increase in GDP Y and firm entry H (around 600% and 400% higher, respectively, compared to their initial levels). At the same time, higher productivity also induces a sharper rise in emissions e and the accumulation of CO_2 s (about 1600% higher).

Although the economic–environmental trade-off is evident, an important finding is that the share of green firms is almost identical across the different productivity scenarios. Specifically, the abatement cutoff z_c decreases in all cases, ensuring a comparable extent of green transformation. Emissions per green $\tilde{e}_g$ and brown firm $\tilde{e}_b$ remain very similar across scenarios.

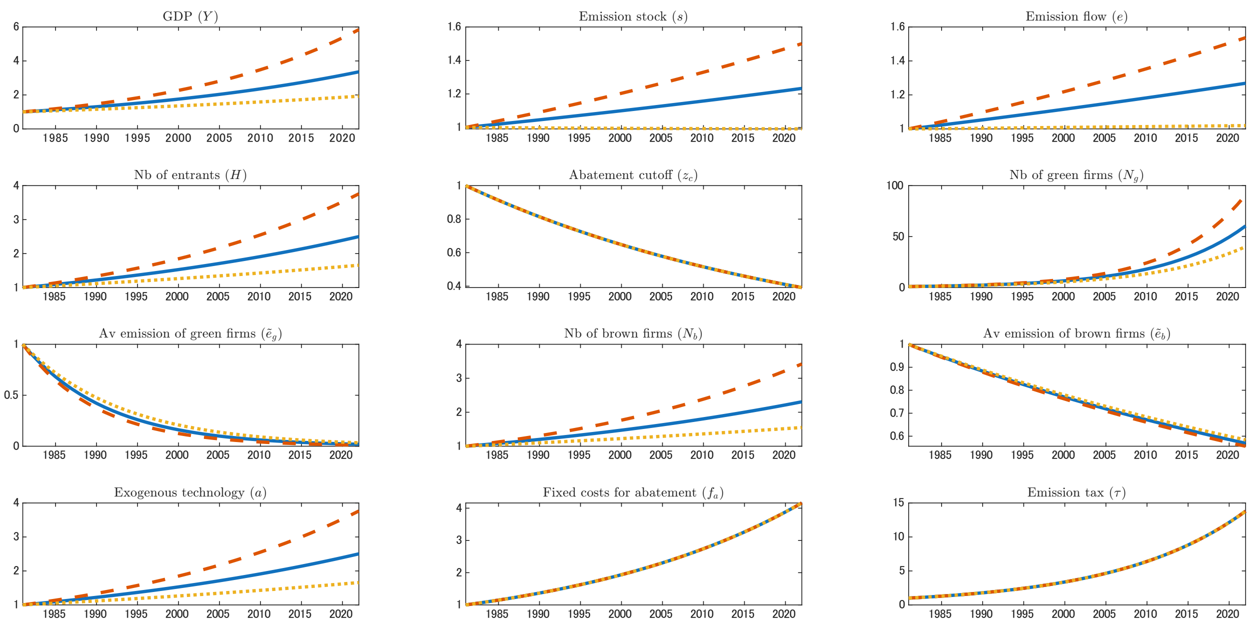


Figure 4: Counterfactual for a

Note: The figure presents the implied trend dynamics of the theoretical model under the benchmark calibration reported in Table 2 (solid lines), together with counterfactual paths of exogenous productivity growth a that are one percentage point higher (dashed lines) or lower (dotted lines) than the benchmark. All variables are normalized to their initial values in 1981.

5.2 Fixed Costs for Abatement and Emission Tax

Figure 5 shows the effect of alternative paths for fixed abatement costs, f_a. When f_a is lower (dotted lines), the number and share of green firms rise, as reflected in a declining cutoff z_c. Lower abatement costs allow firms to adopt greener technologies more easily. Average emissions of both green $\tilde{e}_g$ and brown firms $\tilde{e}_b$ fall, contributing to lower total emissions e and a smaller CO_2 stock s. However, the impact on GDP Y and firm entry H is limited, since the reduction in damages only marginally improves aggregate productivity.

A similar pattern emerges for the emission tax τ (Figure 6). Raising the growth of tax rate by one percentage point relative to the benchmark increases the share of green firms z_c and lowers total emissions e, but its impact on GDP Y is again limited.

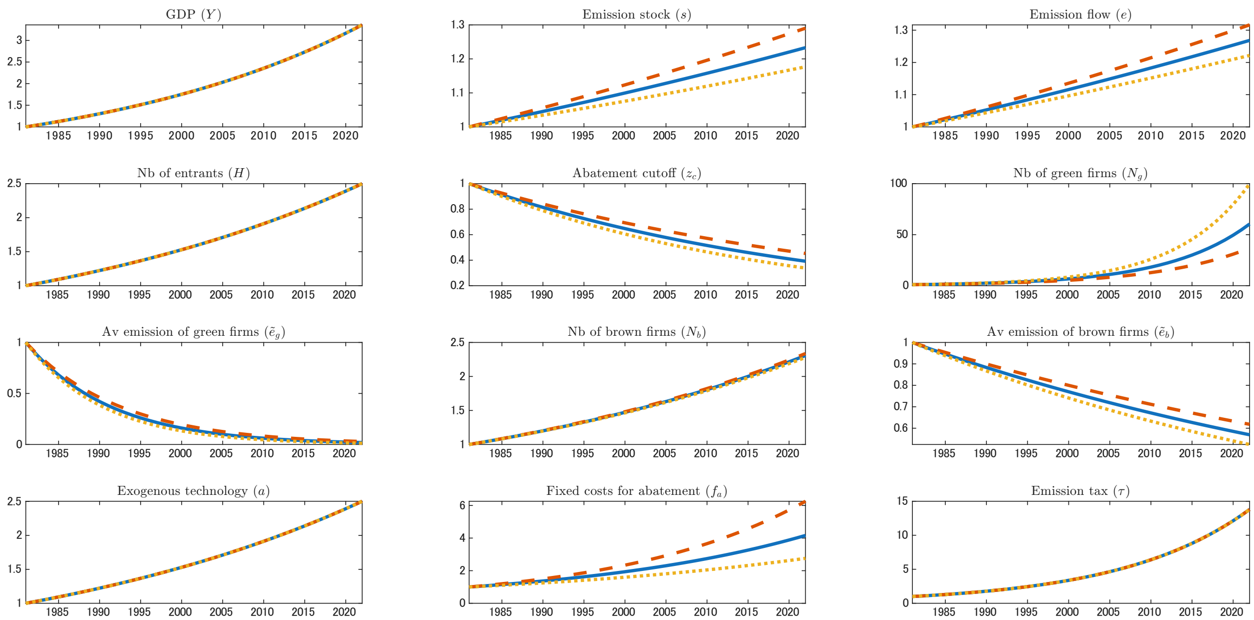


Figure 5: Counterfactual for f_a

Note: The figure presents the implied trend dynamics of the theoretical model under the benchmark calibration reported in Table 2 (solid lines), together with counterfactual paths of exogenous productivity growth f_a that are one percentage point higher (dashed lines) or lower (dotted lines) than the benchmark. All variables are normalized to their initial values in 1981.

These counterfactual analyses show that technological progress has a much stronger effect on both GDP and emissions, while changes in the emission tax and abatement costs primarily affect emissions only. To substantially alter the economic trajectory, more drastic changes in τ or f_a would be required. Importantly, because of negative externalities, emissions harm economic growth itself. This raises the question: is the observed transformation of the world economy optimal, or could better outcomes be achieved through alternative policy interventions? We turn to this issue in the next section.

6 Optimal Policies

What is the optimal tax rate that fully internalizes the negative externalities from emissions? In this section, we analyze three policy scenarios:

First, we determine the emission tax rate and its growth path that maximize expected welfare. Second, we consider the tax rate and its growth path that minimize environmental damages. Third, we examine the case in which expected welfare is maximized jointly with respect to both the emission tax rate and the fixed costs of abatement, along with their growth paths.

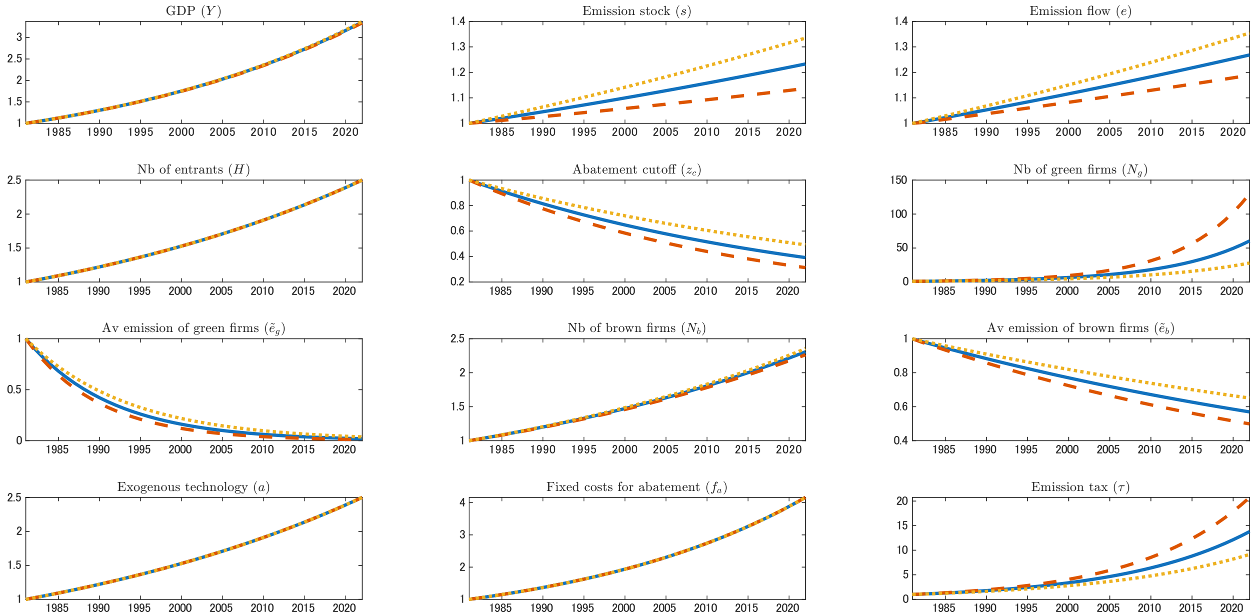


Figure 6: Counterfactual for τ

Note: The figure presents the implied trend dynamics of the theoretical model under the benchmark calibration reported in Table 2 (solid lines), together with counterfactual paths of exogenous productivity growth τ that are one percentage point higher (dashed lines) or lower (dotted lines) than the benchmark. All variables are normalized to their initial values in 1981.

6.1 Optimal Emission Tax

We define the optimal emission tax as the combination of τ and its growth rate g_τ that maximizes welfare, i.e.,

$$\max_{\tau, g_\tau} E_t \sum_{s=t}^{\infty} \beta^{s-t} U(\tau, g_\tau).$$

Figure 7 presents the simulation results. The dashed lines correspond to the optimal policy, while the solid lines show the benchmark calibration. The optimal policy requires a dramatic increase in τ: nearly 500 times higher than its initial level. This sharp rise in the emission tax triggers a major green transformation. The number of green firms N_g engaged in abatement rises by about 50,000 times, whereas the number of brown firms N_b follows a similar trajectory to the benchmark case, increasing by only 2.3 times. Consequently, the abatement cutoff z_c declines substantially, and average emissions from both green $\tilde{e}_g$ and brown firms $\tilde{e}_b$ fall more than in the benchmark.

The enhanced green transformation successfully curbs emissions and reduces the CO_2 stock. By 2022, the stock of CO_2 is 30% lower than in 1980. This decline mitigates the negative externalities, allowing the economy to achieve fourfold GDP growth compared to 3.3-fold growth in the benchmark. Entry of new firms H is also amplified under the optimal tax policy.

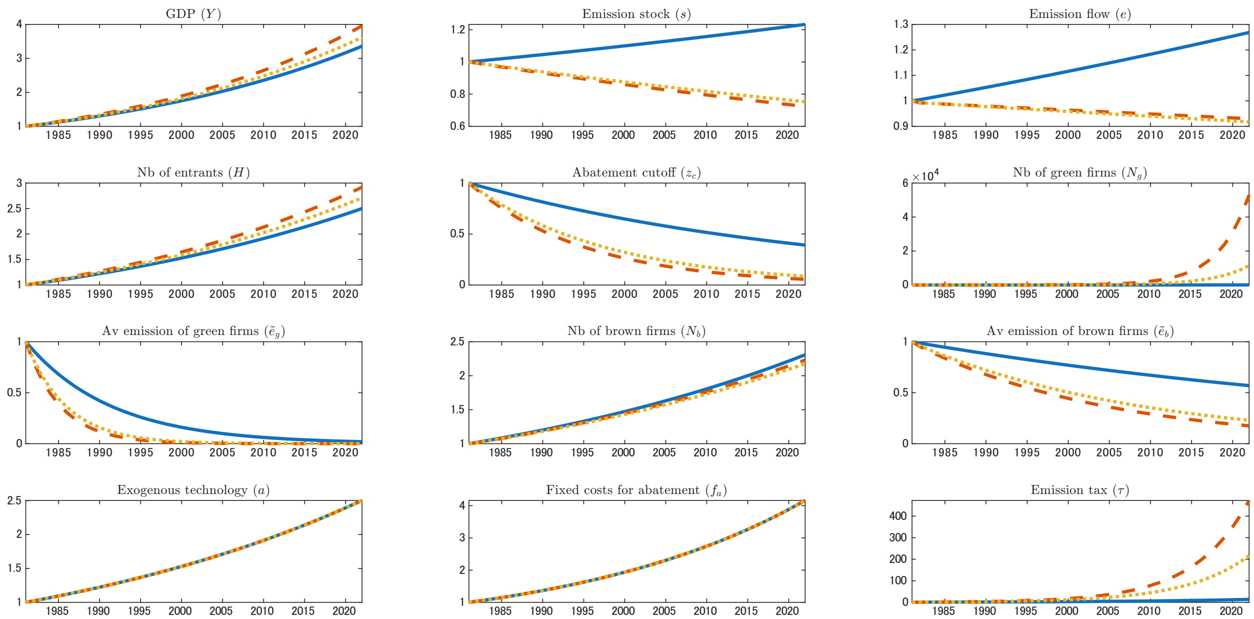


Figure 7: The optimal emission tax

Note: The figure shows the implied trend dynamics of the theoretical model under the benchmark calibration reported in Table 2 (solid lines), along with the paths implied by the optimal emission tax under welfare maximization (dashed lines) and damage minimization (dotted lines). All variables are normalized to their initial values in 1981.

6.2 Damage Minimization

In practice, however, it may be difficult for a benevolent planner to know households' exact preferences. A more feasible approach might be to target emissions. Instead of maximizing expected utility, we consider a minimization policy in which the planner sets τ and g_τ to minimize damages:

$$\min_{\tau, g_\tau} E_t \sum_{s=t}^{\infty} \beta^{s-t} D(\tau, g_\tau).$$

Figure 7 also reports the outcomes of this minimization policy. Here, the green transformation is less dramatic: τ increases by 200% rather than 500%. The number of green firms N_g rises by 10,000 times, while the number of brown N_b firms follows the same trend as in the benchmark. Although the transformation is milder, the minimization policy still delivers higher growth in Y and lower CO_2 emissions s compared to the benchmark—albeit with smaller gains than under the fully optimal tax (shown with dashed lines in the figure).

6.3 Optimal Emission Tax with Innovation for Abatement

The optimal emission tax should be set at a significantly high level when implemented in isolation. However, when combined with improvements in abatement technologies—captured by a reduction in f_a—the role of the tax becomes milder. Specifically, we compute the optimal paths of both τ_t and $f_{a,t}$ that maximize welfare:

$$\max_{\tau, g_\tau, f_a, g_{f_a}} E_t \sum_{s=t}^{\infty} \beta^{s-t} U(\tau, g_\tau, f_a, g_{f_a}).$$

Figure 8 illustrates the benchmark calibration (solid lines) and the jointly optimal paths of τ_t and $f_{a,t}$ (dotted lines). Compared to the case of an optimal emission tax alone (dashed lines), the required increase in the tax is now more moderate. At the same time, the quantitative results remain very similar to the case of the emission tax alone. In the literature, the extent of policy intervention remains controversial (Dietz and Stern, 2015). While we do not explicitly model subsidies for environmentally friendly technologies, our analysis highlights the importance of complementary policy instruments that reduce the costs of adopting such technologies.[2]

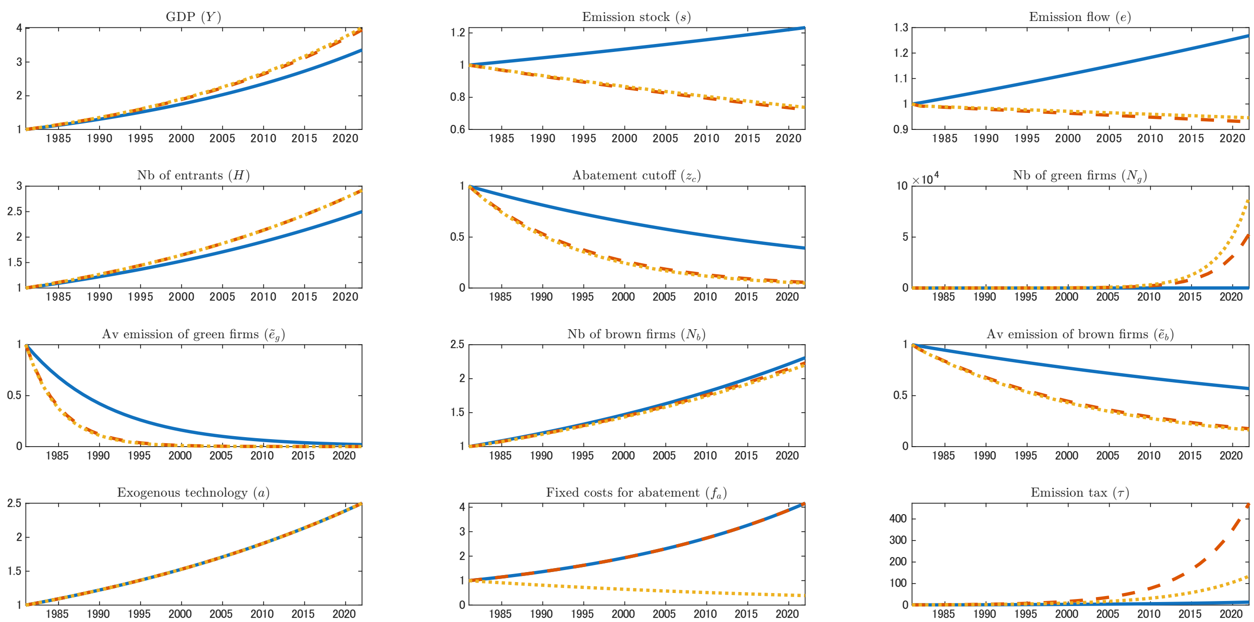


Figure 8: Optimal Emission Tax with Abatement Innovation under Welfare Maximization

Note: The figure shows the implied trend dynamics of the theoretical model under the benchmark calibration reported in Table 2 (solid lines), together with the paths implied by welfare maximization through the optimal emission tax alone (dashed lines) and by the joint policy of the optimal emission tax and abatement cost innovation (dotted lines). All variables are normalized to their initial values in 1981.

7 Conclusion

This paper has developed a dynamic stochastic general equilibrium model with heterogeneous firms to study the interplay between economic growth, CO_2 emissions, and the global green transition. By calibrating the model to world data from 1981 to 2022, we have shown that while emissions continue to rise, the share of green firms in the economy is increasing. Our

[2]Figure 10 in Appendix C shows a similar exercise under damage minimization rather than welfare maximization. As in the welfare maximization case, the role of the emission tax becomes more moderate once abatement technology costs are allowed to change.

counterfactual analysis indicates that faster productivity growth amplifies the trade-off between growth and emissions, whereas slower growth attenuates it, with the composition of green firms remaining relatively stable. Welfare analysis highlights the necessity of a substantially higher emission tax relative to current levels. When combined with abatement innovation, the required increase in the tax is moderated, underscoring the importance of complementary policies. Together, these findings point to the central role of policy in sustaining growth while mitigating environmental externalities.

Several avenues for future research remain open. First, while our model captures long-run dynamics under perfect foresight, extending the framework to allow for uncertainty and short-run fluctuations would improve its ability to analyze cyclical policies, such as countercyclical carbon taxation or subsidies for green innovation. Second, the model abstracts from international heterogeneity, yet climate policies are implemented in a multi-country context with uneven development, diverse institutional capacity, and cross-border spillovers. Embedding the green transition into a multi-country setting with trade, capital flows, and policy coordination would enrich the analysis. Third, while we have emphasized emission taxes, other policy instruments such as subsidies, green bonds, or regulatory standards are increasingly relevant in practice. Exploring the interaction of these tools with taxation would help to clarify the policy mix needed for an effective transition. Finally, linking the model more explicitly to empirical measures of firm-level innovation, sectoral shifts, and technological diffusion would provide additional discipline and external validation.

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A Proofs

A.1 Proof of Proposition 1

Figure 2 illustrates the profit lines of brown and green firms. For an intersection to occur under $f_{a,t} > 0$, it must hold that

$$\rho_{g,t}(z)^{1-\sigma} > \rho_{b,t}(z)^{1-\sigma}.$$

By substituting the equilibrium prices, this condition becomes

$$\frac{\sigma}{\sigma-1}\frac{w_t}{A_t z}\left[1+\tau_t(1-\Omega_t)+\theta_1\Omega_t^{\theta_2}\right] < \frac{\sigma}{\sigma-1}\frac{w_t}{A_t z}\left[1+\tau_t\right].$$

Recall that

$$\Omega_t = \left(\frac{\tau_t}{\theta_1\theta_2}\right)^{\frac{1}{\theta_2-1}}.$$

Substituting this expression, the above condition reduces to

$$\theta_2 > 1.$$

A.2 Proof of Proposition 2

Under $\theta_2 > 1$, other things equal, a decrease (increase) in $f_{a,t}$ raises (lowers) the profits of green firms because the fixed cost of abatement, $w_t\frac{f_{a,t}}{A_t}$, becomes smaller (larger). As a result, the share of green firms increases (decreases).

Furthermore, under $\theta_2 > 1$, other things equal, an increase (decrease) in τ_t reduces (raises) the profits of brown firms more than that of green firms, thereby increasing (decreasing) the share of green firms. Indeed, we have

$$\frac{\partial d_{b,t}(z)}{\partial\tau} < \frac{\partial d_{g,t}(z)}{\partial\tau} < 0,$$

since

$$0 < \frac{\partial \rho_{g,t}(z)}{\partial \tau} < \frac{\partial \rho_{b,t}(z)}{\partial \tau},$$

because

$$0 < \Omega_t.$$

These comparative statics can be also illustrated in Figure 2.

B Damage Function

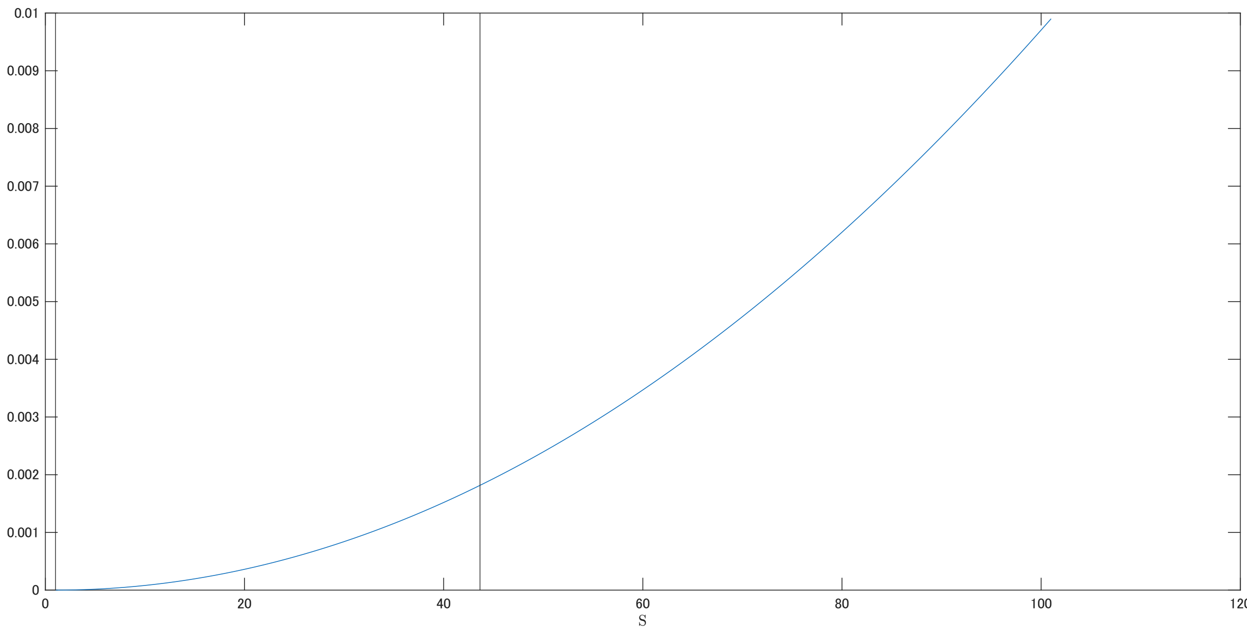

Figure 9: Damage Function

Note: The figure presents the damage function (6) under our calibrated values in Table 2. The horizontal axis measures the stock of emissions at the initial state.

C Minimizing damage with τ and f_a

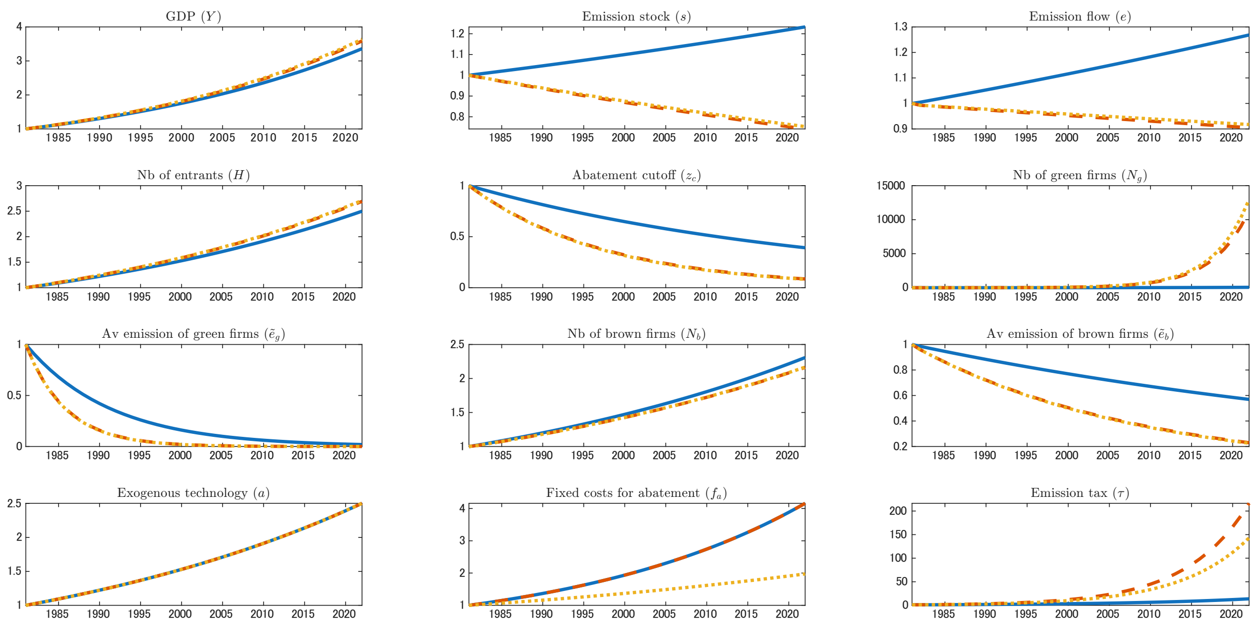

Figure 10: Optimal Emission Tax with Abatement Innovation under Damage Minimization

Note: The figure shows the implied trend dynamics of the theoretical model under the benchmark calibration reported in Table 2 (solid lines), together with the paths implied by damage minimization through the optimal emission tax alone (dashed lines) and by the joint policy of the optimal emission tax and abatement cost innovation (dotted lines). All variables are normalized to their initial values in 1981.