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Why do people who think they have failed want to see the results more? An investigation based on the Ego Utility Model.

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Abstract

This paper examines how ego utility influences decision making and shows that the desire to maintain or enhance one's self-image can lead to the avoidance of useful information if it conflicts with existing beliefs. It challenges the traditional economic view of purely rational decision making focused on economic gain by incorporating ego utility into expected utility theory. The study provides theoretical evidence on how ego utility affects information processing and decision-making, suggesting that self-esteem plays a significant role. This work enriches the field of behavioural economics by shedding light on the reasons behind individuals' reluctance to seek relevant information, highlighting the complex relationship between ego utility and information seeking behaviour.

JEL codes: C11, C65, D91.

Keywords: ego utility, Bayesian updating, beta distribution.

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1 Introduction

Additional information is usually helpful to guide the decision process towards the optimal choice. Evidence for such improvements in decision making can be found in numerous studies (Grieco and Hogarth 2009; Juslin et al. 2000; Ryvkin et al. 2012). However, even when relevant information is widely and freely available, people often fail to seek it out, resulting in suboptimal decisions.

For example, while there is a wealth of accurate information on health and nutrition available on the internet, many people still adhere to superstitions and unfounded diets, ignoring advice based on scientific evidence. Similarly, although there is a wealth of investment education available online, many individual investors do not use this information and instead make decisions based on inadequate research or past performance.

Such phenomena are well known as confirmation bias and cognitive dissonance, but Köszegi (2006) successfully modelled them within the framework of behavioural economics by incorporating ego utility into expected utility theory.Ego utility refers to the satisfaction or pleasure individuals derive from enhancing their own self-esteem or ego. [1]

His utility function consists of two parts: the first part concerns ego utility,that assigns high ego utility to oneself when making risky economic transactions (e.g., investments) and low ego utility when deciding not to do it, and the second part derives from the success or failure of the economic transactions. Köszegi (2006) showed theoretically that the presence of this ego utility could lead people to be overconfident, not to seek information and to make sub-optimal investments.

Möbius et al. (2022) conducted experiments to test the theoretical predictions of Koszegi's model. In stead of the optimal Bayesian reasoning. they assumed a biased Bayesian updating that engages in biased information processing, and showed that

[1] Many traditional economic theories have assumed that individuals' preferences depend solely on the outcomes, such as consumption or profits from work. In contrast, Bénabou and Tirole (2016) argues that beliefs themselves (e.g. beliefs about one's own abilities) are 'assets' with which individuals can consume, invest or produce. The ego utility model contributes to this belief-based theory of utility.

biased information processing can increase belief utility at the cost of increasing the probability of making the wrong decision to invest. Although the functional form of ego utility models has varied since Köszegi (2006), they consistently consist of two parts: the ego utility aspect and the utility derived from the outcomes of economic transactions.

These models suppose situations where collecting information has some instrumental value with regard to economic earnings. However, people often do not seek to obtain accurate information and understand their current situation correctly, when no economic earning is involved. Moreover, their way of processing information is influenced by how well they perform. For example, Amateurs completing a DIY project may eagerly show off their handiwork, hoping for praise and suggestions for improvement. Amateurs completing a DIY project may eagerly show off their handiwork, hoping for praise and suggestions for improvement, while skilled craftspeople may casually display their creations, waiting for others to admire and inquire about their techniques.Similarly, after cooking, people who are not confident in their culinary skills want to hear others' opinions immediately, while those who are confident tend to wait for spontaneous praise.

It is worth pointing out that these phenomena cannot be explained by either confirmation bias or cognitive dissonance. The former suggests that agents underweight disconfirming evidence and overweight confirming evidence. The latter states that that people have a natural drive to maintain consistency in their thoughts, beliefs, and actions.These theories can explain that people prefer information that justifies their actions and beliefs, but they cannot explain the phenomenon described above, where people who believe their performance or abilities are poor are more likely to seek information. Our study shows theoretically that such situations can be explained by a simple ego utility ego model,that supposes beliefs many enter in preferences directly.

Our paper makes two contributions. First, we show that agents who derive utility directly from their self-image will exhibit a range of distinctive and measurable biases in the way they process information. Second, the bias is dependent on the magnitude of the ego utility that people initially possess. The Bayesian framework (combined with the assumption of correct priors) implies that agents are always unbiased in their beliefs about the underlying parameters determining performance. Nevertheless, our research suggests that even a rational agent capable of Bayesian updating may not always pursue the optimal set of information, depending on the magnitude of her ego utility.

2 Model

In developing a general model, we will largely use application-neutral language, but a specific setting is helpful to keep in mind for motivation. Let's consider the following situation. People are given the opportunity to answer questions with objectively clear truth values (such as in mathematics or logic questions), and can choose to see the correct answer after answering them. And, it is assumed that there is no cost associated with viewing the correct answer, and that there is no economic advantage or disadvantage associated with whether the answer is correct or incorrect. If standard expected utility theory were applied to such a situation, without considering ego utility, the theoretical prediction would be that people would be indifferent to whether they saw the answer or not. However, if ego utility is taken into account, the theoretical prediction might be different.

After solving the question, but before seeing the correct answer, people have anticipatory ego utility (Köszegi 2006) in the form of an increasing function of their subjective expectation of performance. By seeing the correct answer, they can realize whether their own answers are correct and thus gain more accurate information about their answering ability. However, seeing the correct answer can either damage their self-image (ego) if they find out they were wrong when they thought they were right, or improve their self-image (ego) if they find out they were right when they thought they were wrong. The decision maker will decide whether or not to view the answer, taking into account the impact on her self-image. Their optimal strategy can be solved by backward induction.

Our model takes into account the ego utility of the decision maker and assumes that she can perform Bayesian updating. [2] Decision makers form their beliefs about the probability of answering a question correctly by trying to solve it. Here, we assume that the prior probability density function of the correct answer, which varies with the difficulty of the question, is known to the decision maker and the function follows a unimodal beta distribution. That is, the distribution is defined as follows ($B(a,b)$ is a beta function):

$$\frac{1}{B(a,b)}x^{a-1}(1-x)^{b-1} \qquad a > 1 \text{ and } b > 1. \tag{1}$$

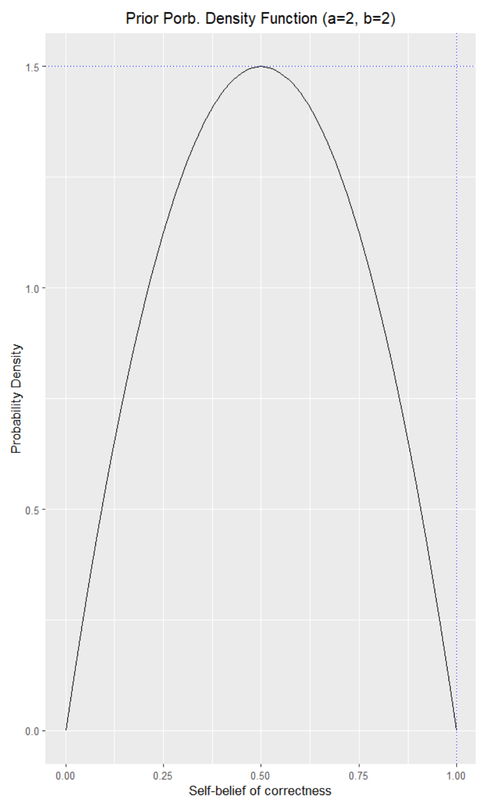

Figure 1: Prior Probability Density Function (a=2, b=2)

[2]A limitation of our setting is that it is difficult to apply to an environment where the decision maker cannot perform Bayesian updating. Some psychologists suggest (Taylor and Brown 1988; Kruger and Dunning 1999) that people often have and want to keep irrationally positive beliefs about themselves hat are far from Bayesian updating. Many psychologists, however, state that the self-image motivation is the main force in decision making process (Kunda 1987; Dweck 2013). Following standard methodology, to highlight the effect of this force, we are introducing no other new element into the model, but the assumption of Bayesian updating.

The assumption of a unimodal distribution is considered natural from the widely accepted phenomenon in education and statistics that performance on problems with objectively determined right or wrong answers, such as in mathematics and physics, tends to follow a unimodal distribution. From another perspective, it could be said that the implications of our model generally hold for problems where the prior probability density function of the correct answer is unimodal.

"Ego utility" is the utility derived directly from the self-image, and in the situation we are considering it is necessary to satisfy the following two conditions:

1. It is a monotonically increasing function with respect to the probability x, that she believes her answer is correct.

2. For the same value of x, the utility is higher when the question is difficult than when it is easy.

The second condition means, for example, that the ego utility of believing there is a fifty-fifty chance of having answered a difficult question correctly is greater than the ego utility of believing the same for an easy question. This would be a consistent relationship in terms of the ego utility provided by the self-image.

In our model, we take the cumulative distribution function of the probability x, $F(x)$, as ego utility function. $F(x)$ is a monotonically and continuously increasing function that takes values from 0 to 1 with respect to $x \in [0, 1]$. If the belief is that the answer is completely wrong, $x = 0$, the utility is $0 = F(0)$, and if the belief is that the answer is completely correct, $x = 1$, the utility is $1 = F(1)$. This property satisfies the first condition above. Additionally, assuming that the probability density function of the correct answer is unimodal, for the same value of x, F(x) will be greater when the probability density function of the correct answer is skewed to the left (i.e., when the problem is difficult) than when it is skewed to the right (i.e., when the problem is easy).This property satisfies the second condition above.

It is worth noting that the consideration of the cumulative distribution function as a utility function follows from the empirical and theoretical research below. Van Praag and Kapteyn (1973) pointed out that individuals tend to psychologically compare the income with an imaged worst position and a opposite satiation. Thus, to estimate the individual welfare function of income, which referred to the relative welfare perceived, they selected the cumulative distribution of approximately log-normal distribution for accommodating the characteristic of boundary. Gregory (1980) also applied the idea of relative wealth to support the equivalence between the cumulative distribution of income and the Friedman–Savage utility function. In discussing the decision in educational or employment selection, Chen and Novick (1982) considered a truncated normal and an extended beta cumulative distribution function as a utility function, taking the advantage of flexibility, asymmetry, and bounded range.

Even though we and Köszegi (2006) both chose cumulative distribution function to represent ego utility, there still a point of difference. We employed a continuous cumulative distribution function instead of a binary function. Köszegi (2006) defined the ego utility in association with the cumulative distribution function of the probability of success of investments, which is held as a belief by the decision maker. It is determined by a binary function that is 1 if its value is greater than 1/2 and 0 otherwise. The reason why Koszegi (2006) did not make ego utility a continuous function is presumably because it was necessary to keep the ego utility part as simple as possible in order to include the utility of economic outcomes in the expected utility function. However, since the economic outcomes don't matter in our situation, we decided to consider a continuous cumulative distribution function as the utility function, which allows for easier comparison between different distributions. In other words, it allows us to analyse more precisely the relationship between the values of the parameters a and b, which determine the shape of the beta distribution, and the choices made by the decision-makers. Moreover, as we shall see, even if the ego utility is defined as 1 when the value of the cumulative distribution function is greater than 1/2 and 0 otherwise in line with previous research, the implications of the model do not change.

3 Analysis and results

In this section we explain mathematically the implications of the model in more detail. A decision maker answers a mathematics or logic question and estimates her probability of being correct as x. Suppose the probability distribution of the correct answer rate for this question follows a beta distribution as in (1). The prior cumulative distribution function (here after, $PriorCDF$)derived from (1) can be written as in (2). $B_e(x; a, b)$ is an incomplete beta function.

$$\frac{1}{B(a,b)} \int_0^x t^{a-1}(1-t)^{b-1} dt = \frac{1}{B(a,b)} B_e(x; a, b) \tag{2}$$

When she checks her answer and finds it correct, she updates the probability of being correct based on Bayes' theorem.

$$P(x|True) = \frac{P(x) \wedge P(True)}{P(True)}$$

$$P(x) \wedge P(True) = P(x)\ P(True|x) = (1) * x$$

$P(True)$ represents the expected value of the event that one's answer is correct.

$$\begin{aligned}
\int_0^1 (1) * x\ dx &= \frac{1}{B(a,b)} \int_0^1 x^{(a-1)+1}(1-x)^{b-1} dx \\
&= \frac{1}{B(a,b)} B(a+1, b) \\
&= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \times \frac{\Gamma(a+1)\Gamma(b)}{\Gamma(a+b+1)} \\
&= \frac{a}{a+b}
\end{aligned}$$

Therefore, the posterior probability density function when one's answer is correct can be obtained by normalising $(1) * x$ as (3).

$$\frac{a+b}{a}\left[\frac{1}{B(a,b)}x^{a}(1-x)^{b-1}\right] \tag{3}$$

Similarly, the posterior probability density function when one's answer is wrong can be obtained by normalising (1) $*$ $(1-x)$ as shown in (4).

$$\frac{a+b}{b}\left[\frac{1}{B(a,b)}x^{a-1}(1-x)^{b}\right] \tag{4}$$

From the above, if she checks her answer, she faces the posterior probability density function of (3) with a probability of x and the posterior probability density function of (4) with a probability of $1-x$.

$$\frac{a+b}{a}\left[\frac{1}{B(a,b)}x^{a+1}(1-x)^{b-1}\right]+\frac{a+b}{b}\left[\frac{1}{B(a,b)}x^{a-1}(1-x)^{b+1}\right]$$

By the normalizing the equation above, we can have an expected posterior probability density function.

$$\frac{a+b+1}{a+b+2}\frac{1}{B(a,b)}\left[\frac{a+b}{a}x^{a+1}(1-x)^{b-1}+\frac{a+b}{b}x^{a-1}(1-x)^{b+1}\right]$$

By integrating this expected posterior probability density function indefinitely, the expected posterior cumulative distribution function (here after, $PosteriorCDF$) can be firstly obtained.

$$\frac{a+b+1}{a+b+2}\frac{1}{B(a,b)}\left\{\frac{a+b}{a}B_e(x;a+2,b)+\frac{a+b}{b}B_e(x;a,b+2)\right\}$$

By opening B_e function and rearrangement, we can obtain a simple expression (please see the appendix for proof).

$$\frac{1}{B(a,b)}B_e(x;a,b) + \frac{1}{(a+b+2)B(a,b)}x^a(1-x)^b\left\{\frac{a-b}{ab} + \frac{a+b}{b}(1-x) - \frac{a+b}{a}x\right\} \quad (5)$$

In our model, since $PriorCDF$ (2) represents the prior ego utility and $PosteriorCDF$ (5) represents the expected posterior ego utility, the decision maker decides whether to check the answer by comparing and evaluating these two. Let us focus on the difference between these functions.

$$PosteriorCDF - PriorCDF = f(x) =$$
$$\frac{1}{(a+b+2)B(a,b)}x^a(1-x)^b\left\{\frac{a-b}{ab} + \frac{a+b}{b}(1-x) - \frac{a+b}{a}x\right\}$$

Therefore, the difference between the two becomes zero at three points: x=0, 1 and $\frac{a-b+a^2+ab}{(a+b)^2}$. Let's call this third point C. The fact that the difference becomes zero at x=0 and 1 is obvious because both the posterior CDF and the prior CDF take values of 0 and 1 respectively at each point, but C is actually contained in the open interval (0, 1) under the assumption of unimodal beta distribution (a and b > 1).

$$\text{C} = \frac{a-b+a^2+ab}{(a+b)^2} = \frac{(a-1)b+a+a^2}{(a+b)^2} \quad > 0$$

The denominator is positive. Let us focus on the numerator.

$$a - b + a^2 + ab = (a+b)^2 + a(1-b) - b(b+1)$$

As $a(1-b) - b(b+1)$ is inferior to 0, $a - b + a^2 + ab$ is smaller than $(a+b)^2$. We can, hence, derive $c \in (0,1)$.

By differentiating $PosteriorCDF - PriorCDF$ by x, we can obtain,

$$f'(x) = \frac{1}{(a+b+2)B(a,b)} \frac{(a+b)^2}{ab} x^{a-1}(1-x)^{b-1} \left\{ac - (ac+bc+a+1)x + \Big(a+b+1)x^2\right\}$$ [3].

From $f'(0) = 0$, $f'(1)$ =0, and $f'(C) < 0$, we can draw a graph of the $PosteriorCDF - PosteriorCDF$ as figure 2 for intuitive explanation[4].

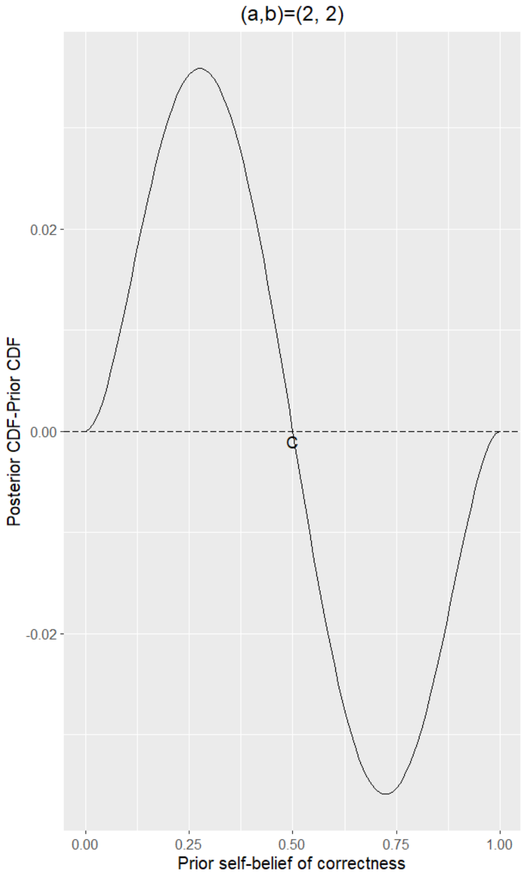

Figure 2: (a,b)=(2, 2)

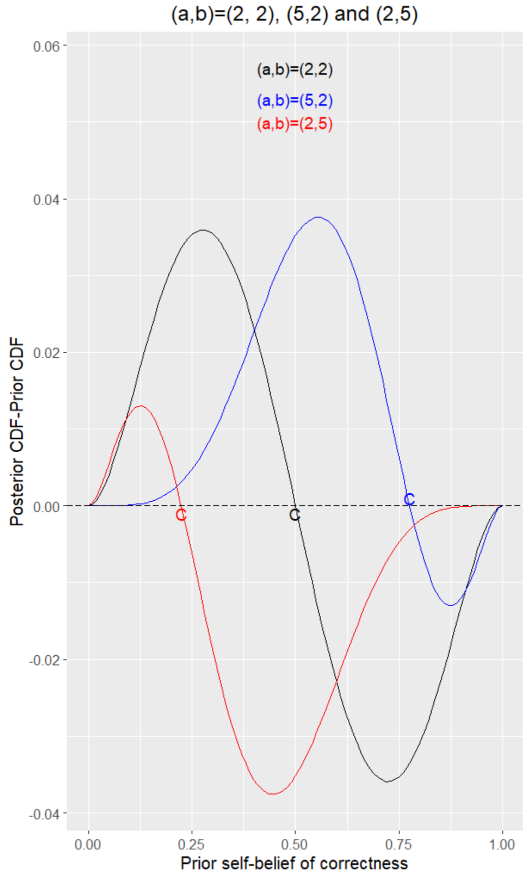

Figure 3: (a,b)=(2, 2), (5,2) and (2,5)

Therefore, for decision makers in the range $(0, C)$ in the prior probability density function, the difference between the posterior CDF and the prior CDF becomes positive, so they want to know the answer. For decision makers in the range $[C, 1)$, as this difference becomes negative, they do not want to know the answer. It means that people who lack confidence in their answers are *more eager to* see the solution than

[3] $PosteriorCDF - PriorCDF$ is continuous and differentiable for $c \in (0, 1)$.

[4] To obtain $f'(c) < 0$, it is sufficient to examine a signal of $ac - (ac + bc + a + 1)x + (a + b + 1)x^2$, because $x^{a-1}(1 - x)^{b-1}$ is larger than 0 under $x \in (0, 1)$. Substituting c for x, the calculation shows $c(c - 1)$, and $c(c - 1) < 0$ for $c \in (0, 1)$. Thus $f'(c)$ is negative under $c \in (0, 1)$

those who are confident.In other words, this model can accommodate the phenomenon mentioned in the introduction that the worse the performance, the more people want to know the result.[5]

Now we show that the result above would not change at all, if we supposed the Köszegi (2006)'s type binary ego utility function taking 1 where $F(x)$ is greater than $\frac{1}{2}$, and 0 otherwise. In this setting, individuals who have a $PriorCDF$ of below $\frac{1}{2}$ and expect the $PosteriorCDF$ to increase above $\frac{1}{2}$ will end up viewing the solution, because this is only the case where the ego utility increases.As their $PriorCDF$ is below $\frac{1}{2}$, it is reasonable to think that such individuals are supposed to be those who lack confidence in their answers.

Finally, let's examine the relationship between the parameters a, b, which determine the prior probability density function of correctness (beta distribution), and C. The results of the partial differentiation of C with respect to a and b (a and $b > 1$) are as follows:

$$\frac{\partial C}{\partial a} = (a+b)^{-3}\left[(b-1)a + 3b + b^2\right] \quad > 0,$$
$$\frac{\partial C}{\partial b} = (a+b)^{-3}\left[-a^2 - 3a + (1-a)b\right] \quad < 0.$$

Graphically, as a increases, C approaches 1, and as b increases, it approaches 0 (see figure.3). Other things being equal, an increase in a causes the beta distribution to be skewed to the right, corresponding to a situation where the problems faced by decision makers become easier. Conversely, other things being equal, an increase in b causes the beta distribution to shift to the left, corresponding to a situation where the problems more difficult. The tendency for people who are not confident in their answers to want to see the solution more than those who are confident remains unchanged. However, when comparing the former with the latter, the latter group is more likely to have answered incorrectly, leading to an increasing tendency for people who do not want to

[5]For simplicity, I use the convention that if the decision maker is indifferent, she chooses not to see the answer.

see the solution.

4 Conclusion

In summary, our exploration of the complexities of decision making reveals a nuanced landscape in which the search for information is not only driven by the desire for optimal outcomes, but is also intricately linked to the ego utility derived from one's self-image. This understanding, grounded in behavioural economics and supported by empirical evidence, challenges traditional economic theories that view decision-making as a purely rational process driven by the pursuit of economic gain. Instead, it underscores the profound influence of ego utility on information-seeking behaviour and illustrates how the satisfaction derived from maintaining or enhancing self-esteem can lead individuals to bypass readily available, accurate information in favour of choices that confirm their pre-existing beliefs or self-conceptions.

Finally, we suggest the following directions for development. By pursuing them, we can improve the robustness and applicability of our behavioural economics model and contribute to a deeper understanding of the relationship between ego utility and decision-making behaviour.

- Empirical testing: Conduct experiments to gather empirical data to support the predictions of our model. This could involve designing controlled experiments to observe how individuals with different levels of confidence behave when faced with different decision scenarios. The data collected can then be used to refine and validate the model.

- Incorporate individual differences: Consider how individual differences in personality traits, cognitive abilities and socio-demographic factors may influence the relationship between confidence and solution seeking. Incorporating these factors into the experimental data could provide a more nuanced understanding of the phenomenon.

- Practical applications: Explore potential practical applications of the model in areas such as education, marketing and public policy. For example, our model could inform the design of educational interventions to promote effective learning strategies, or the development of targeted interventions to encourage individuals to seek information when making important decisions.

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A Appendix: Proof of the $PosteriorCDF$ expression

By integrating this expected posterior probability density function indefinitely, the $PosteriorCDF$ can be firstly obtained as

$$\frac{a+b+1}{a+b+2}\frac{1}{B(a,b)}\left\{\frac{a+b}{a}B_e(x;a+2,b)+\frac{a+b}{b}B_e(x;a,b+2)\right\} \tag{A1}$$

From the definition of the regularized incomplete Beta function, we have

$$B_e(x;a,b)=I_x(a,b)B(a,b)$$

Where $I_x(a,b)$ is a regularized incomplete Beta function, and we can derive the relationship between $B_e(x;a+2,b)$ and $B(a,b)$ as follows:

$$\begin{aligned}
B_e(x;a+2,b) &= I_x(a+2,b)\;B(a+2,b)\\
&= \left[I_x(a+1,b)-\frac{x^{a+1}(1-x)^b}{(a+1)B(a+1,b)}\right]B(a+2,b)\\
&= \left[I_x(a,b)-\frac{x^{a}(1-x)^b}{aB(a,b)}-\frac{x^{a+1}(1-x)^b}{(a+1)B(a+1,b)}\right]B(a+2,b)\\
&= \left[\frac{B_e(x;a,b)}{B(a,b)}-\frac{x^{a}(1-x)^b}{aB(a,b)}-\frac{x^{a+1}(1-x)^b}{(a+1)B(a+1,b)}\right]B(a+2,b)
\end{aligned}$$

Since

$$\begin{aligned}
B(a+1,b) &= \frac{\Gamma(a+1)\Gamma(b)}{\Gamma(a+b+1)}\\
&= \frac{a\Gamma(a)\Gamma(b)}{(a+b)\Gamma(a+b}\\
&= \frac{a}{a+b}B(a,b)
\end{aligned}$$

We can derive

$$\begin{aligned} B(a+2,b) &= \frac{a+1}{a+b+1} B(a+1,b) \\ &= \frac{a(a+1)}{(a+b)(a+b+1)} B(a,b) \end{aligned}$$

Thus, $B_e(x; a+2, b)$ can be rearranged as

$$\left[\frac{B_e(x;a,b)}{B(a,b)} - \frac{x^a(1-x)^b}{aB(a,b)} - \frac{(a+b)x^{a+1}(1-x)^b}{a(a+1)B(a,b)} \right] \frac{a(a+1)}{(a+b)(a+b+1)} B(a,b) \tag{A2}$$

Similarly, we can rearrange $B_e(x; a, b+2)$ as follows:

$$\begin{aligned} B_e(x;a,b+2) &= I_x(a,b+2)\ B(a,b+2) \\ &= \left[I_x(a,b+1) + \frac{x^a(1-x)^{b+1}}{(b+1)B(a,b+1)} \right] B(a,b+2) \\ &= \left[I_x(a,b) + \frac{x^a(1-x)^b}{bB(a,b)} + \frac{x^a(1-x)^{b+1}}{(b+1)B(a,b+1)} \right] B(a,b+2) \\ &= \left[\frac{B_e(x;a,b)}{B(a,b)} + \frac{x^a(1-x)^b}{bB(a,b)} + \frac{x^a(1-x)^{b+1}}{(b+1)B(a,b+1)} \right] B(a,b+2) \end{aligned}$$

with

$$\begin{aligned} B(a,b+1) &= \frac{\Gamma(a)\Gamma(b+1)}{\Gamma(a+b+1)} \\ &= \frac{b\Gamma(a)\Gamma(b)}{(a+b)\Gamma(a+b)} \\ &= \frac{b}{a+b} B(a,b) \end{aligned}$$

and

$$\begin{aligned} B(a,b+2) &= \frac{b+1}{a+b+1} B(a,b+1) \\ &= \frac{b(b+1)}{(a+b)(a+b+1)} B(a,b) \end{aligned}$$

Thus, $B_e(x;a,b+2)$ equals to

$$\left[\frac{B_e(x;a,b)}{B(a,b)}+\frac{x^a(1-x)^b}{bB(a,b)}+\frac{(a+b)x^a(1-x)^{b+1}}{b(b+1)B(a,b)}\right]\frac{b(b+1)}{(a+b)(a+b+1)}B(a,b) \tag{A3}$$

According to equations (A2) and (A3), the $PosteriorCDF$ expression (A1) can be rearranged as:

$$\begin{aligned}
(A1) &= \frac{a+b+1}{a+b+2}\,\frac{1}{B(a,b)}\left[\frac{a+b}{a}(A2)+\frac{a+b}{b}(A3)\right] \\
&= \frac{1}{a+b+2}\left[\frac{(a+1)B_e(x;a,b)}{B(a,b)}-\frac{(a+1)x^a(1-x)^b}{aB(a,b)}-\frac{(a+b)x^{a+1}(1-x)^b}{aB(a,b)}\right. \\
&\left.+\frac{(b+1)B_e(x;a,b)}{B(a,b)}+\frac{(b+1)x^a(1-x)^b}{bB(a,b)}+\frac{(a+b)x^a(1-x)^{b+1}}{bB(a,b)}\right] \\
&= \frac{B_e(x;a,b)}{B(a,b)}+\frac{x^a(1-x)^b}{(a+b+2)B(a,b)}\left[\frac{b+1}{b}+\frac{a+b}{b}(1-x)-\frac{a+1}{1}-\frac{a+b}{a}x\right] \\
&= \frac{B_e(x;a,b)}{B(a,b)}+\frac{x^a(1-x)^b}{(a+b+2)B(a,b)}\left[\frac{a-b}{ab}+\frac{a+b}{b}(1-x)-\frac{a+b}{a}x\right]
\end{aligned}$$

Therefore, we derive the simple expression of the $PosteriorCDF$:

$$\frac{1}{B(a,b)}B_e(x;a,b)+\frac{1}{(a+b+2)B(a,b)}x^a(1-x)^b\left\{\frac{a-b}{ab}+\frac{a+b}{b}(1-x)-\frac{a+b}{a}x\right\}$$