WINPEC Working Paper Series No. E2515
June 2025

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March 16, 2025

Abstract

Cooperative game theory addresses two problems: coalition formation and payoff distribution. We hypothesize that the existence of the core, which is a fundamental concept in cooperative game theory, affects coalition formation, and we examine this hypothesis through a laboratory experiment. In the experiment, three subjects in a group bargain with each other on both the coalition formation and the payoff distribution simultaneously. The bargaining protocol is unstructured, i.e., similar to a real bargaining situation. As a result, we obtain the following findings. First, the existence of a core strongly induces the formation of the grand coalition. Second, resulting allocations are frequently in the core when it exists and are at least in the equal division core, which is an extension of the core. Finally, resulting allocations that are outside of the equal division core mostly arise due to ignorance of domination via coalition BC, which is the lowest-value two-person coalition.
Keywords: laboratory experiment, unstructured bargaining, cooperative games, the core, communication
JEL Classification: C71, C91, C92

Declarations

Funding

This work was supported by the JSPS Core-to-Core Program (grant number JPJSCCA20200001); the JSPS KAKENHI (grant numbers JP22H00829,JP18KK0046, JP17H02503).

Declarations of interest

None.

Availability of data and material

See http://expwaseda.sakura.ne.jp/FUNAKI1/data/Data-and-Material_3pUBE1.zip for the raw data and experimental instructions.

Code availability

See http://expwaseda.sakura.ne.jp/FUNAKI1/code/Code_3pUBE1.zip for the zTree programs.

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1 Introduction

In the real world, completing a task by cooperating with others is usually more efficient than completing it alone, but cooperation requires agreement. For example, a group of firms reaches an agreement regarding a business partnership only when each firm believes that the agreement will generate higher profits. This kind of issue is analyzed by cooperative game theory. The simplest cooperative game is Nash's original bargaining problem, in which there are only two individuals. In this case, the options for cooperation are simple: cooperate or do not cooperate. However, as the number of individuals increases, the number of options for cooperation increases exponentially, and the problem becomes more complicated. In cooperative games with three or more players, the most important issues are whom to cooperate with and how to distribute the profits earned through cooperation. These are called the *coalition formation* problem and the *payoff distribution* problem. However, in a textbook on cooperative game theory such as Peters (2008), it is often assumed that individuals will form the grand coalition, i.e., all individuals will cooperate with each other, when the condition of *superadditivity* holds. Then, the main issue becomes the payoff distribution problem.

Cooperative game theory is formalized by the set of all coalitions and the worth of each, which describes how much the coalition members obtain from the coalition. In the case of firms, worth represents the surplus from cooperation among several firms. Superadditivity means that the worth of the union of any two disjoint coalitions is not less than the sum of the worth of the two coalitions. In other words, merging two groups is always better than or at least equal to leaving them separate in terms of social surplus. Under this condition, the grand coalition is formed by iterating merges, and it provides the greatest social surplus. This is why it is usually assumed that the grand coalition is formed when superadditivity holds.[1] However, we question this assumption.

Consider a simple example with three firms: A, B, and C. The worth of the grand coalition is 120, and that of coalitions A and B, A and C, and B and C is 100, 80, and 70, respectively.[2] Singletons, i.e., coalition A, coalition B and coalition C, obtain 0. This situation satisfies the superadditivity condition. Suppose that C suggests forming the grand coalition and sharing the payoff from the grand coalition equally, namely, $(40, 40, 40)$. Will A and B agree to this suggestion? No, because they would be better off by forming coalition AB and implementing another allocation, $(50, 50, 0)$. Note that if coalition AB is formed, C remains alone and receives 0. Is this allocation the final outcome? No, because A and C might refuse it by forming coalition AC with another allocation, $(60, 0, 20)$. In this example, there is no end to such bargaining.

This idea of bargaining is utilized in the concept of the *core*. The core is defined as the set of allocations against which no subcoalition can be made better off by deviating from the grand coalition with a feasible allocation. The core is empty in the above example, so we would expect that the grand coalition will not form even though the example features superadditivity. The best alternative to the grand coalition is coalition AB because it gives the second largest social surplus. However, it is not certain that coalition AB will be formed, as the above bargaining example illustrates. On the other hand, we expect the grand coalition to be formed when the core is nonempty. If some of the firms find and suggest a core allocation, they cannot be better off by forming subcoalitions. Hence, we predict that in such bargaining situations, the likelihood that the grand coalition is formed is higher when the core is nonempty. In other words, it is more likely all individuals will agree to cooperate with each other. This is what we would like to examine through a laboratory experiment.

Although noncore allocations can be dominated by other allocations, subjects might reach an agreement on a noncore allocation in a laboratory setting. There are a few reasons for this: Some groups of subjects may not be able to find a core allocation, and others may reach a compromise even when the core is nonempty. However, such agreements might be explained by the equal division core, which was introduced by Selten (1972). The equal division core is the set of allocations that are not dominated by equal divisions of worth among the coalition members. For instance, in the above example, there are four equal divisions that we consider to be candidate dominating allocations: $(40, 40, 40)$, $(50, 50, 0)$,

[1] See Peters(2008), for example.

[2] Hereafter, coalition $\{A, B\}$, coalition $\{A, C\}$ and coalition $\{B, C\}$ are denoted by coalition AB, coalition AC and coalition BC, respectively.

$(40, 0, 40)$, and $(0, 35, 35)$. In the laboratory experiment, equal divisions are an important focal point, as Selten (1987) emphasized, so we expect that the subjects might be satisfied with an allocation in the equal division core. Then, this group of subjects will form the grand coalition with a noncore allocation. Since the equal division core is a superset of the core, the equal division core exists even when the core is empty. Hence, if the equal division core exists, it can also explain agreements within those groups that form the grand coalition in games with an empty core.

Communication among individuals plays a key role in inducing cooperation. Previous studies have revealed that in laboratory experiments, communication among subjects encourages them to cooperate with each other. For example, Bochet et al. (2006) focus on communication in an experiment based on a public goods game. They find that communication among the subjects encourages them to cooperate with each other even though they neither know whom they are talking to nor are able to see each others' faces. We would also like to examine this phenomenon by running two different experimental treatments. In one treatment, subjects in the same group can communicate with each other through a chat window on the computer screen. In the other treatment, the subjects cannot communicate with each other and can only make, accept and reject offers. In the treatment with communication, the grand coalition may be more likely to be formed even if a subcoalition can be made better off by deviating. This is also what we would like to examine through the two treatments.

There are some previous studies on three-person bargaining experiments, but most such studies employ structured bargaining, which can be modeled as a noncooperative game or an extensive form game. For example, in Okada and Riedl (2005), one of the subjects is chosen to be the proposer, and after his or her proposal is made, the others decide in turn to accept it or reject it. This form of bargaining can be described by an extensive form game. Nash (2008) introduced an original bargaining protocol named the "agencies method" and simulated it with robot players. He found that the protocol was successful in achieving the most efficient outcome, i.e. the formation of the grand coalition. The bargaining protocol is divided into two stages. The players elect the "final agent" by voting for each other in the first stage, and the final agent distributes the payoffs in the second stage. The other players cannot refuse the distribution chosen by the final agent. Then, Nash et al. (2012) implemented the agencies method with human players to find that the bargaining protocol successfully induces the formation of the grand coalition in a laboratory experiment. However, natural and normal bargaining is not like that in the above-mentioned experiments. There is no exogenously defined order of proposals, and anyone can make a proposal at any time in real bargaining situations. In addition, a player who receives a proposal does not have to respond immediately: he or she has time to compare the proposal with other proposals and may strategically postpone his or her reaction. Moreover, if we employ structured bargaining as in these previous studies, the results of the bargaining will depend not only on the worth of the coalitions but also on the structure of the bargaining protocol. For example, if we change the first proposer from player A to player B in a structured bargaining scenario, that change may affect the theoretically predicted outcome, such as the subgame perfect Nash equilibrium. Then, the resulting coalition and allocation would also be affected. On the other hand, if we employ an unstructured bargaining protocol, i.e., if we do not provide the subjects with an exogenously defined order for making proposals, acceptances, and rejections, we can suppose that the resulting coalitions and allocations are unaffected by the order of actions. For these reasons, we employ unstructured bargaining, and we argue that our model can be modeled as a cooperative game. In particular, the core defined by domination is expected to predict the resulting coalitions and allocations in unstructured bargaining.[3] Then, we presume that if the core exists, which is a stronger condition than superadditivity, it can affect the resulting coalition and allocation.

There have been some previous studies on unstructured bargaining experiments. Anbarci and Feltovich (2018) and Navarro and Veszteg (2020) conducted two-person unstructured bargaining experiments. Additionally, Karagözoğlu (2019) presented a survey of the recent trend of shifting from structured bargaining experiments to unstructured experiments and the advantages of utilizing unstructured experiments. While there are simply two options for cooperation in two-person bargaining—cooperate or do not cooperate—there are four options for each subject in a three-person bargaining scenario. From A's point of view, he or she can cooperate with (i) both B and C, (ii) only B, (iii) only C or (iv) neither of

[3]See Berl et al. (1976) for an experimental test of validity of the core in a game without sidepayment.

them. Bolton et al. (2003) focus on the effect of "communication links" in a three-person unstructured bargaining experiment. They examine coalition formation and payoff distribution in a game with several different communication alternatives. For example, in one treatment, each subject can send a message or offer to one or both of the other two subjects. In another treatment, one subject can send a message or offer to the other subjects, but the other two subjects cannot directly send a message or offer to each other. Then, they compare the effect of the communication structure on coalition formation and the payoff distribution. In our experiment, described in detail in Section 3, communication among the subjects is public: all messages and offers are sent to both of the other subjects. Tremewan and Vanberg (2016) also employ three-person unstructured bargaining. One difference between our bargaining protocol and theirs is the criterion for having reached an agreement. In their experiment, acceptance of an offer by two subjects is enough to consider an agreement as being reached even though the subjects may bargain about the allocation for the grand coalition. We find this criterion to be inconsistent with an agreement for the grand coalition because we believe that such an agreement should be formed unanimously. Otherwise, the person who did not agree with the resulting allocation might complain, and if so, the grand coalition would no longer be stable. In contrast, in our experiment, an allocation for a coalition must be accepted by all subjects who are involved in the coalition for us to consider an agreement as having been reached.

The remainder of this paper is organized as follows. In Section 2, we formally introduce cooperative games, superadditivity, the core and the equal division core. After that, we construct four hypotheses regarding the research question. In Section 3, we provide details on the experimental setting, including recruitment, the bargaining procedure, the games and the treatments. In Section 4, we present the results from several points of view. First, we examine the resulting coalitions with graphs and a regression analysis. Next, we move to the resulting payoff distribution not only for the grand coalition but also for smaller coalitions. Finally, we focus on the relationship between the resulting allocation and the core or the equal division core. We reveal why some resulting allocations are not even in the equal division core: domination by the lowest-value coalition is ignored. In Section 5, we summarize and interpret the results and provide some possibilities for future work.

2 Theoretical Model and Hypotheses

We first define a cooperative game with transferable utility. Let a pair (N, v) be a *game*. N is a finite player set and v is a characteristic function. In our experiment, $|N| = 3$. Then, there are three players A, B, and C; hence, $N = \{A, B, C\}$. S, a subset of the player set, is called a *coalition*. In this game, we have several possible coalitions that represent cooperation among the players: $\{A, B, C\}, \{A, B\}, \{A, C\}, \{B, C\}, \{A\}, \{B\}$, and $\{C\}$. The characteristic function v assigns a real number to each coalition and represents the total profit gained by all members of the coalition. We call $v(S)$ the *worth of the coalition* S. We suppose that $v(N) = 120, v(AB) = 90, v(AC) = 70, v(BC) = 50$, and $v(A) = v(B) = v(C) = 0$.[4] This is one of the games implemented in our experiment.

Superadditivity is a fundamental property of cooperative games that provides an advantage for the grand coalition.

Definition 1. *A game (N, v) is superadditive if it satisfies*

$$v(S \cup T) \geq v(S) + v(T) \quad \forall S, T \subseteq N \quad (S \cap T = \emptyset).$$

The game in the above example is superadditive, which can be verified by observing that $v(N) = 120 > v(AB)+v(C) = 90,\ v(N) > v(AC)+v(B) = 70,\ v(N) > v(BC)+v(A) = 50$, and $v(AB), v(AC), v(BC) > 0$. When any two disjoint coalitions merge in a superadditive game, the new coalition generates worth that is greater than or equal to the sum of the worth of the two coalitions. The grand coalition provides the greatest social surplus when the game is strictly superadditive.[5]

[4]We write $v(AB)$ instead of $v(\{A, B\})$ for simplicity.

[5]If the inequality is strict for all nonempty S, T $(S \cap T = \emptyset)$, the game is strictly superadditive. The game in this example is strictly superadditive. Then, the grand coalition is the coalition with the unique greatest surplus and we naturally expect that the grand coalition will hold.

Before defining the core, we first define *domination.* Let a vector $x = (x_A, x_B, x_C) \in \mathbb{R}^3_+$ be an *allocation*, which indicates who gets how much. In our experiment, the set of *feasible allocations* $\mathcal{A}$ is given as follows:[6]

$$\mathcal{A} = \{x \in \mathbb{R}^3_+ | x_A+x_B+x_C=v(N)\} \cup \{x \in \mathbb{R}^3_+ | x_A+x_B=v(AB), x_C=0\} \cup$$
$$\{x\in\mathbb{R}^3_+ | x_A+x_C=v(AC), x_B=0\} \cup \{x\in\mathbb{R}^3_+ | x_B+x_C=v(BC), x_A=0\} \cup \{(0,0,0)\}.$$

Definition 2. *Consider two feasible allocations $x, y \in \mathcal{A}$. Then, x dominates y via coalition S if*

$$\sum_{i\in S} x_i = v(S) \quad and \quad x_i > y_i \quad \forall i \in S.$$

Suppose that C in the example above offers a feasible allocation $\gamma = (40, 40, 40)$, which would result in the formation of the grand coalition. However, A can make a counteroffer that dominates γ via coalition AB: $\alpha = (45, 45, 0)$. In this scenario, A and B will not agree with γ because they can obtain more from α.

Next, suppose that B suggests another feasible allocation $\beta = (65, 30, 25)$ and the grand coalition. No allocation dominates β. The core of a cooperative game is the set of all such feasible allocations.

Definition 3. *The core C(N,v) is defined as*

$$C(N, v) = \{x \in \mathcal{A} \mid \nexists y \in \mathcal{A} \;\; s.t. \;\; y \text{ dominates } x \text{ via some coalition } S \subseteq N\}.$$

We can identify no feasible allocation that dominates the core allocations. In this sense, the core is considered to be stable. However, if the core is empty, for any allocation in the grand coalition, we are guaranteed to find another allocation that dominates the allocation. Then we expect that, in such a scenario, the grand coalition would not be formed. Furthermore, in superadditive games, the core allocation satisfies *efficiency*, that is, $C(N, v) \subseteq \{x \in \mathbb{R}^3_+ | x_A+x_B+x_C=v(N)\}$ in our games. The core is not always nonempty, even in superadditive games, which means that we cannot say that the grand coalition is formed with certainty in such games.

We establish hypotheses for our experiment based on the scenario that we have introduced. First, we predict that when a game is superadditive but the core is empty, the grand coalition will not be formed because, for any allocation in the grand coalition, there always exists another allocation that dominates the allocation. By contrast, when a game has a nonempty core, we predict that the grand coalition will be formed because core allocations are never dominated. Hence, our first experimental hypothesis is as follows:

Hypothesis 1. *The likelihood that the grand coalition is formed is higher when the core is nonempty than when it is empty.*

Because we expect the formation of the grand coalition when the core is nonempty, it is consistent to predict that the resulting allocation is in the core. Hence, the second hypothesis is as follows:

Hypothesis 2. *When the core is nonempty, the resulting allocation to the grand coalition is in the core.*

As we mentioned in the introduction, in a laboratory experiment, the grand coalition could be formed with a noncore allocation in games with a nonempty core, and the grand coalition could also be formed in games with an empty core. Even in these cases, we do not expect that the subjects have ignored the idea of domination. Following Selten (1972), we propose a modified solution called the equal division core, wherein only allocations with an equal division of the worth of the coalitions are possible options for domination. The equal division core is formally defined as follows:

Definition 4. *The equal division core $EDC(N, v)$ is defined as*

$$EDC(N, v) = \{x \in \mathcal{A} \mid \nexists S \subseteq N, \;\; s.t. \;\; e^S \text{ dominates } x \text{ via } S$$

[6] $\mathbb{R}^3_+$ is the nonnegative quadrant of $\mathbb{R}^3$.

where, for $S \subseteq N$, $e^S = ((e_i^S)_{i \in S}, (e_j^S)_{j \in N \setminus S})) =$
$((v(S)/|S|, v(S)/|S|, ..., v(S)/|S|)_{i \in S}, (0, 0, ..., 0)_{j \in N \setminus S}) \in \mathbb{R}^N$. Note that the equal division core exists in every game implemented in our experiment.[7][8] The equal division core always includes the core. In three-person superadditive games wherein $v(A) = v(B) = v(C) = 0$, the equal division core is given as follows:

$$
\begin{aligned}
&EDC(N, v) = \\
&\{x \in \mathcal{A} \mid x_A + x_B + x_C = v(N), \text{ and } [x_A \geq v(AB)/2 \text{ or } x_B \geq v(AB)/2] \text{ and} \\
&\quad [x_A \geq v(AC)/2 \text{ or } x_C \geq v(AC)/2] \text{ and } [x_B \geq v(BC)/2 \text{ or } x_C \geq v(BC)/2]\} \\
&\cup \{(x_A, x_B, 0) \in \mathcal{A} \mid x_A + x_B = v(AB), \text{ and } [x_A \geq v(N)/3 \text{ or } x_B \geq v(N)/3] \text{ and} \\
&\quad [x_A \geq v(AC)/2] \text{ and } [x_B \geq v(BC)/2]\} \\
&\cup \{(x_A, 0, x_C) \in \mathcal{A} \mid x_A + x_C = v(AC), \text{ and } [x_A \geq v(N)/3 \text{ or } x_C \geq v(N)/3] \text{ and} \\
&\quad [x_A \geq v(AB)/2] \text{ and } [x_C \geq v(BC)/2]\} \\
&\cup \{(0, x_B, x_C) \in \mathcal{A} \mid x_B + x_C = v(BC), \text{ and } [x_B \geq v(N)/3 \text{ or } x_C \geq v(N)/3] \text{ and} \\
&\quad [x_B \geq v(AB)/2] \text{ and } [x_C \geq v(AC)/2]\};
\end{aligned}
$$

that is, the equal division core is given by a union of four disjoint sets. The first, second, third, and fourth sets are the equal division cores for the grand coalition, coalition AB, coalition AC, and coalition BC, respectively.

Selten (1987) stressed the importance of an equal division of the worth of not only the grand coalition but also the various two-person coalitions in the context of bargaining experiments. We believe that bargaining should proceed following the idea of domination, as we discussed in the introduction. Hence, we predict that the subjects will offer allocations that cannot be dominated by at least the equal division of the worth of the grand coalition and the two-person coalitions. Then, the equal division core should describe both the resulting allocations in games with an empty core and the noncore allocations in games with a nonempty core. Thus, our third hypothesis is as follows:

Hypothesis 3. *Any noncore allocation for the grand coalition is in the equal division core.*

As Bochet et al. (2006) found, people tend to be more cooperative when they can talk to each other; hence, we predict that it is more likely for the grand coalition to be formed when communication among the experimental subjects is possible. Forming a subcoalition rather than the grand coalition can be regarded as exclusive behavior; hence, we expect that the subjects may experience a sense of guilt from excluding others after communicating with them. We also expect that communication will aid bargaining among the subjects because they can easily share their thoughts. As a result, we predict that the subjects will find a core allocation more easily when they can communicate and that the grand coalition is more likely to form. Note that there are two types of communication: public communication and private communication. In this study, we test public communication; hence, all messages are sent to all members of the group. We do not test the scenario in which there are hidden agreements or private messages that are not known to some member of the group. Hence, our fourth hypothesis is as follows:

Hypothesis 4. *The grand coalition is more likely to form when the experimental subjects can communicate publicly with each other than when they cannot.*

3 Experimental Settings

3.1 General Settings

We conducted eight sessions from November 2016 to June 2017 and four additional sessions in May 2023. We recruited 32 students, mainly undergraduates, from several faculties of Waseda University for

[7] See Table 1 for a description of each game, including the existence of an equal division core.

[8] The von Neumann–Morgenstern stable set, bargaining set, Shapley value, and nucleolus also exist in every game, but we do not find that these solutions explain the results of the experiment.

each session. We limited participation to Japanese students because participants were required to read and understand instructions in Japanese. Although the capacity for each session was 30, we recruited two extra participants in case some recruited students dropped out of the study. We recruited students through the portal website for Waseda University students called MyWaseda. First, students who were interested in the experiment sent us an application. After we received each application, we checked for duplication: no one was allowed to join multiple sessions. Those who passed the check received an email from us and participated in the study.

On the day of each session, the participants came to the laboratory just before the starting time. Each participant randomly selected a plastic number plate when entering the laboratory and took a seat according to the number selected. Each seat was in a cubicle surrounded by high partitions so that the subjects could not see each other. When more than 30 participants arrived, we paid 500 JPY as a participation fee to the extra participants and they returned home. After the subjects entered the laboratory, they first read and signed a consent form that discussed how their private personal information would be protected. Then, computer software read the instructions aloud; see Appendix A for the text of these instructions. Next the subjects participated in a practice round whose result did not affect their rewards. The experiment was performed on a computer with z-Tree (Fischbacher, 2007); we instructed the subjects on how to operate the program in the practice round. The subjects played 10 paying rounds after the practice round and answered the post-experiment questionnaires while we prepared their rewards. The reward paid was the sum of the participation fee of 900 JPY and 3 JPY per experimental currency unit (ECU) the subjects accumulated in all paying rounds[9] We paid the subjects in cash at the end of the session. Each session took approximately 90 minutes and the average reward was approximately 1,900 JPY.[10]

3.2 Bargaining Procedures

In each round, each subject played the role of A, B, or C.[11] The 30 subjects were randomly divided into 10 groups of three and given a role and information on the worth of the various coalitions at the beginning of each round.[12] The subjects could not identify which other participants belonged to their group. After the provision of information, the participants moved onto the bargaining stage and were able to make offers.

Any subject could make an offer at any time during the bargaining stage. Each offer consisted of two items: a coalition and allocation. A proposed coalition had to involve the person who made the offer and at least one of the other two subjects. For example, A could offer one of the following coalitions: the grand coalition, coalition AB, or coalition AC. By contrast, A could not offer coalition BC or any single-person coalition. Additionally, the sum of each payoff within the allocation had to be equal to the worth of the coalition, that is, it had to be a feasible allocation. To make an offer, the subjects input the proposed payoffs with nonnegative integers and hit the "OFFER" button on the computer screen. The offer was then sent to the other two members of the group and displayed on their screens. Even if A offered coalition AB, this offer was shown on C's computer screen; hence, we say that the offers were public. Note that each subject could not propose more than one offer at a time. Therefore, if a subject regretted making an offer, the subject had to cancel that offer by choosing the offer and hitting the "CANCEL" button on the computer screen before making another offer.

After each subject received the other participants' offers, the subject could choose a reaction: accept, reject, or even do nothing. Although each subject could see the offers for coalitions that did not involve him or her, the subject could not accept or reject those offers. When the subjects wanted to accept or reject an offer, they chose the offer and hit the "ACCEPT" or "REJECT" button on the computer screen.

[9]We used this method to prevent the subjects' risk attitudes and unfair payment distributions from affecting the experimental outcomes when a round with a large difference between the maximal and minimal points when we happened to choose as a payment round.

[10]1,900 JPY is approximately equal to 13.20 USD at the exchange rate of September 2022.

[11]Each subject played the role of the manager of a virtual firm and was to conduct a project that could be completed by the firm alone or in cooperation with one or two other firms; see the text of the instructions in Appendix A.

[12]To ensure that there was no extreme imbalance in the roles experienced by the subjects, we prepared a randomized table of roles and applied it to all sessions.

One important point is that the subjects had to wait 15 seconds before they accepted an offer, whereas rejection was possible as soon as the offer was made. The subjects could see whether 15 seconds had passed by looking at the "status" column on the computer screen. The offer status changed from "wait" to "active" after 15 seconds had passed since the offer was made. We imposed this restriction to avoid quick acceptance at a glance and to give the other subjects time to make counteroffers. For example, consider player C in the example given in the previous section. Suppose that B offered coalition AB with $(45, 45, 0)$. After looking at this offer, player C makes a counteroffer of coalition AC with $(50, 0, 20)$, which gives A more than B's offer. However, if immediate acceptance is available, A might accept B's offer before player C's counteroffer can be sent. Additionally, it takes time to input numbers and hit the button on the computer screen. This is why we imposed the 15-second wait. If an offer was rejected, the subject who sent that offer could make another offer without canceling the rejected offer. The acceptance and rejection of each offer were shown in the column "status" as, for example, "Accepted by B" or "Rejected by C." Neither acceptance nor rejection of an offer could be withdrawn. The canceled and rejected offers did not disappear from the screen.

A group reached an agreement when an offer was accepted by all subjects involved in the offered coalition, other than the subject who made the offer, and then the round ended. For example, if A offered the grand coalition, the offer would need to be accepted by both B and C for an agreement to be reached. By contrast, if A offered coalition AB, this offer would need to be accepted only by B for an agreement to be reached. No subject could renege on his or her acceptance after an agreement was reached. When a two-person coalition was formed, the subject who was not involved in the coalition was considered to have formed a single-person coalition. The time limit for each round was five minutes. If a group used up its allotted time without reaching an agreement, all members of the group were considered to have formed a single-person coalition and each subject received zero ECU because the worth of each singleton was zero. After all groups finished each round, we provided feedback on the agreed-upon coalition and payoff distribution and each subject's earnings from the round. Subsequently, we randomly divided the subjects into new groups and gave new roles, and the next round began, except for partner matching sessions, which is explained in the following section.

3.3 Games and Treatments

Game	$v(N)$	$v(AB)$	$v(AC)$	$v(BC)$	$v(\cdot)$	Superadditivity	Core	EDC
1		120	100	90				
2		120	100	70				
3	120	120	100	50	0	Yes	Empty	Nonempty
4		120	100	30				
5		100	90	70				
6		100	90	50				
7		100	90	30				
8	120	90	70	50	0	Yes	Nonempty	Nonempty
9		90	70	30				
10		70	50	30				

Table 1: Game Characteristics: Worth of Coalitions, Superadditivity, Core Status, and Equal Division Core Status

Table 1 shows the games the subjects played and indicates whether the games were superadditive and whether the core and equal division core existed in each game. The worth of the grand coalition and of each single-person coalition were fixed to 120 and 0 in all games, whereas that of the two-person coalitions varied from one game to another. As shown in the "Superadditivity" column, all games were superadditive, which means that the grand coalition generated the greatest social surplus. Additionally, the column "Core" shows that the core was empty in games 1–5, whereas it was nonempty in games

6–10. Therefore, if Hypothesis 1 is correct, it should be more likely that the grand coalition will form in games 6–10 than in games 1–5. These games are the same as those of Nash et al. (2012). Although the purpose of this study is not to compare their results and ours, we used the same games. This is because these 10 games were suitable for checking Hypothesis 1 because half of the games had an empty core and the other half had a nonempty core. Finally, unlike the core, the equal division core was nonempty in all 10 games.

Treatment	Chat	Game Order	Matching	No. of participants	When
T1 NoChat/Asc/Str	-	1→10	Stranger	60	May-Jun, 2017
T2 Chat/Asc/Str	+	1→10	Stranger	60	Nov, 2016
T3 NoChat/Dsc/Str	-	10→1	Stranger	60	May-Jun, 2017
T4 Chat/Dsc/Str	+	10→1	Stranger	60	Jan-Jun, 2017
T5 NoChat/Rdm/Str	-	random	Stranger	30	May, 2023
T6 Chat/Rdm/Str	+	random	Stranger	30	May, 2023
T7 NoChat/Asc/Prt	-	1→10	Partner	30	May, 2023
T8 Chat/Asc/Prt	+	1→10	Partner	30	May, 2023

Table 2: Treatment Description

We tested the eight treatments shown in Table 2 to determine whether there three different effects were present: an effect from the availability of communication, an effect from the order of the games, and an effect from the matching protocol. In the treatments with 'Chat,' the chat window was available during the bargaining stage, whereas it was not available in the treatments with 'NoChat.' The subjects could freely type and send messages to others in their group through the chat window.[13] There were no private messages: all messages were sent to both of the other subjects. Communication through the chat window did not affect the rewards obtained; hence, it can be regarded as a type of cheap talk. If Hypothesis 4 is correct, the likelihood that the grand coalition is formed should be higher in the 'Chat' treatments than in the 'NoChat' treatments. Additionally, because each subject's experiences regarding the coalition and allocation chosen in one round might affect the results in the following round, we presented the games in three different orders. In the treatments with 'Asc,' the subjects played game 1 in the first round through to game 10 in the tenth round, whereas in the treatments with 'Dsc,' they played game 10 in the first round through to game 1 in the tenth round. Moreover, in the treatments with 'Rdm,' the subjects played the games in the following order: 4, 6, 2, 8, 10, 1, 3, 9, 5, 7. Finally, the members of the groups were shuffled at the beginning of each round in the treatments with 'Str,' whereas they remained the same through the experiment in the treatments with 'Prt.' Note that the roles were shuffled even in the 'Prt' treatments.

In each session, we recruited 30 subjects and divided them into 10 groups of three in each game. Each group independently chose its coalition and allocation. Hence, we regard the results as being independent across both games and groups. We count a result for each group in each game as one observation. Each of the 10 groups played 10 games in each session and we ran 12 sessions, which generated 1,200 observations in total.

4 Results

4.1 Variables for Regression Analyses

In this section, we introduce the independent variables for the regression analyses. We create five treatment variables: *Chat*, *Reverse*, *Random*, *Partner*, and *Core*, all of which are dummy variables. The variable *Chat* equals 1 when the chat window is available and 0 when it is not. *Reverse* equals 1 when the game order is 10→1 and 0 otherwise. *Random* equals 1 when the game order is random and 0 otherwise.

[13] Subjects communicating any identifying information, such as their name or student number, was prohibited.

Partner equals 1 when the matching protocol is "partner" and 0 when it is "stranger." Finally, *Core* equals 1 when the core of the game is nonempty (games 6–10) and 0 when it is empty (games 1–5).

We introduce another independent variable, the quota solution, as defined by Shapley (1953). Solutions in cooperative game theory, such as the Shapley value and the nucleolus, determine how much each player should receive; that is, these solutions represent each player's bargaining power. The quota solution is one such solution. It is derived based on the worth of two-person coalitions, which is parameterized in the experiment. We hypothesize that the players' bargaining power is related not only to the resulting payoff but also to the formation of coalitions. We can obtain the quota for each player (Q_A, Q_B, Q_C) by solving the following system of simultaneous equations; it is important to note that we calculate the quota solution for zero-normalized games, where all singletons receive a payoff of zero:

$$\begin{cases} Q_A + Q_B = v(AB) \\ Q_A + Q_C = v(AC) \\ Q_B + Q_C = v(BC) \end{cases}$$

Game	Q_A	Q_B	Q_C	q_A	q_B	q_C	*CoreScale*
1	65	55	35	0.419	0.355	0.226	-35
2	75	45	25	0.517	0.310	0.173	-25
3	85	35	15	0.630	0.259	0.111	-15
4	95	25	5	0.760	0.200	0.040	-5
5	60	40	30	0.461	0.308	0.231	-10
6	70	30	20	0.583	0.250	0.167	0
7	80	20	10	0.727	0.182	0.091	10
8	55	35	15	0.524	0.333	0.143	15
9	65	25	5	0.684	0.263	0.053	25
10	45	25	5	0.600	0.333	0.067	45

Table 3: Quotas, Relative Bargaining Power, and Core Size for Each Game

As Table 3 shows, player A is the strongest, player B is the second strongest, and player C is the weakest across all 10 games. Although (Q_A, Q_B, Q_C) allows us to compare each player's power within a single game, it does not show how power shifts from one game to the next. For instance, Q_B is 25 in both games 4 and 9, but this does not imply that B's bargaining power is identical in both games. This is because A's bargaining power changes between game 4 and game 9. To observe these changes, we calculate the relative bargaining power of each player (q_A, q_B, q_C) using the formula $q_i = Q_i/(Q_A + Q_B + Q_C)$ for all $i \in N$. These variables allow us to track changes in bargaining power across games. For example, both q_B and q_C are higher in game 9 than in game 4, even though Q_B and Q_C remain the same. This indicates that B and C are relatively stronger in game 9 than in game 4. Additionally, we examine not only whether the core is empty or nonempty but also the size of the core, expressed by the index *CoreScale*. We calculate the *CoreScale* index from the quotas as $v(N) - (Q_A + Q_B + Q_C)$. When *CoreScale* is negative (in games 1-5), the core is empty, and as the index becomes smaller, the core moves further from existence. When *CoreScale* is zero (in game 6), the core is a singleton.[14] When *CoreScale* is positive (in games 7–10), the core is nonempty, and as the index increases, the core becomes larger.

[14]The allocation $(x_A, x_B, x_C) = (70, 30, 20)$ is the only core allocation in game 6.

4.2 Coalition Formation

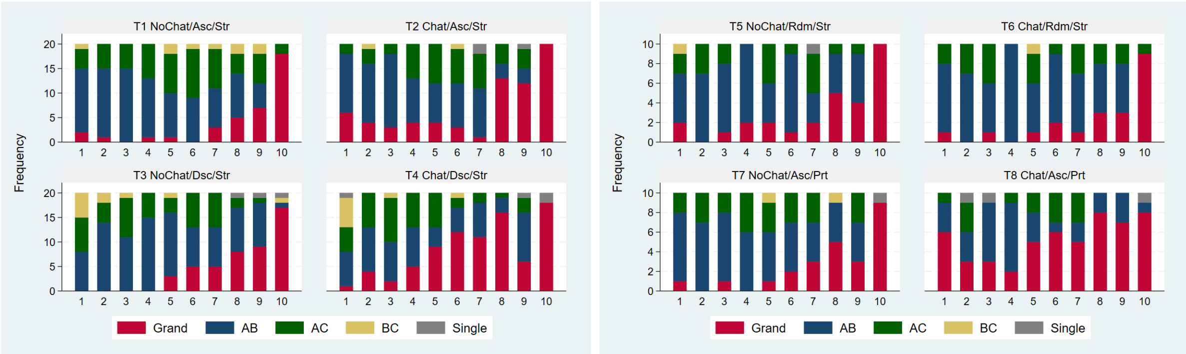

Figure 1: Frequency of Coalitions in Each Game by Treatment

Figure 1 shows the number of groups that form each coalition in each game by treatment. Note that because the number of subjects in T5 to T8 is halved, the total number of observed groups is 10. The grand coalition is more likely to be formed when the core is nonempty (games 6—10) than when it is empty (games 1—5). Moreover, the grand coalition becomes increasingly likely to form as the core expands from game 6 to game 10, with almost all groups forming the grand coalition in game 10, where the core reaches its maximum size. Coalition AB is formed more frequently than coalition AC in most games because $v(AB) > v(AC)$ in every game in the experiment. We note that there are few cases in which coalition BC is formed or where no agreement is reached within five minutes.

We also examine whether there is a treatment effect on coalition formation using Figure 1. When comparing the presence and absence of the chat window, it appears that the grand coalition is more likely to form when the chat window is available than when it is not. Similarly, when comparing across the order of games, reversing or randomizing the game sequence does not seem to have a significant impact on the formation of the grand coalition. Finally, when comparing matching protocols, the use of partner matching does not appear to significantly affect the formation of the grand coalition when the chat window is unavailable. By contrast, when the chat window is available, partner matching seems to increase the likelihood of forming the grand coalition even in games where the core is empty. We verify these effects through the following regression analysis.

	Grand	
Chat	0.425***	0.467***
	(0.147)	(0.155)
Reverse	0.181	0.182
	(0.130)	(0.137)
Random	-0.061	-0.072
	(0.274)	(0.299)
Partner	0.138	0.152
	(0.114)	(0.123)
Coreness	1.088***	-
	(0.100)	
CoreScale	-	0.031***
		(0.003)
Chat×Partner	0.452***	0.467***
	(0.148)	(0.161)
Constant	-1.474***	-0.989***
	(0.132)	(0.122)
Obs.	1,200	

Standard errors (in parentheses) are clustered at the session level.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 4: Probit Regression Analysis of the Formation of the Grand Coalition

Table 4 shows the results of the regression analysis of the formation of the grand coalition using a probit model.[15] *Grand*, the independent variable, is a dummy variable that is equal to 1 if the observed group formed the grand coalition and 0 otherwise. We find that when the core is nonempty and when the core becomes larger, the grand coalition is more likely to be formed at the 1% significance level. This result suggests that Hypothesis 1 is correct: the likelihood that the grand coalition is formed is higher when the core is nonempty than when it is empty. Likewise, when the chat window is available, the grand coalition is more likely to be formed at the 1% significance level. Hence, we conclude that Hypothesis 4 is also correct: the likelihood that the grand coalition is formed is higher when the subjects can publicly communicate with each other than when they cannot. We also note that we do not find any effect of the order in which the subjects play the games on the formation of the grand coalition. Finally, regarding the matching protocol, we cannot say that using partner matching alone has an effect on the formation of the grand coalition. However, we find that when the chat window is available, the likelihood of forming the grand coalition becomes statistically significant at the 1% level.

[15]The marginal effects are reported in Appendix B.

4.3 Payoff Distribution

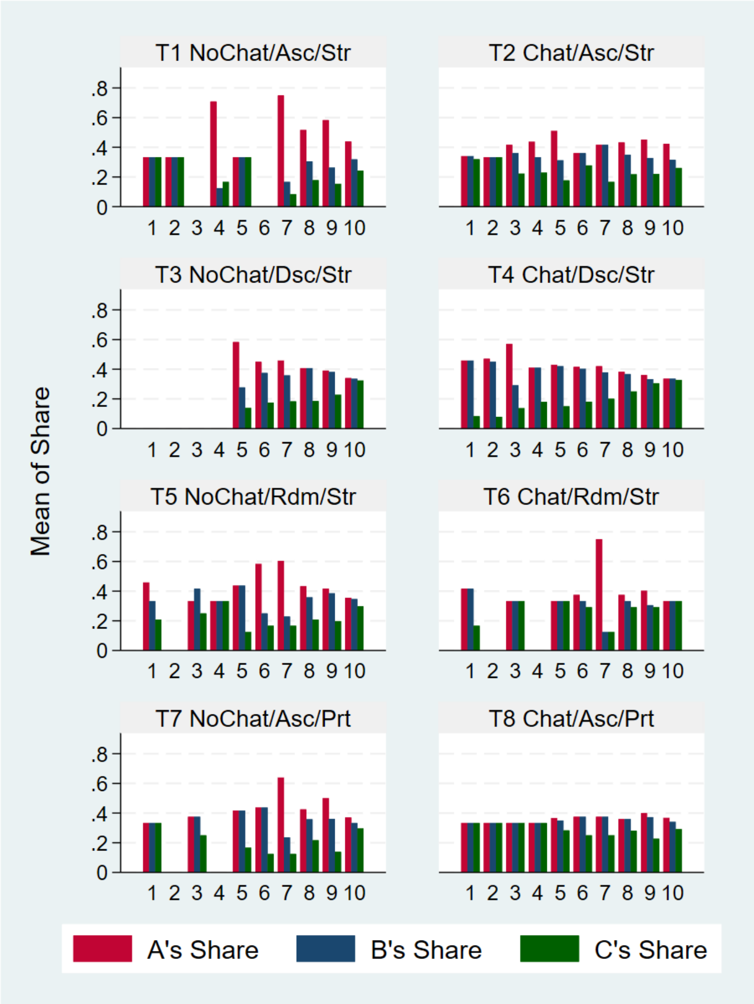

Figure 2: Mean of Each Player's Share in the Grand Coalition in Each Game by Treatment

Next, we examine the resulting allocations from one coalition to another. Let x_A, x_B, and x_C denote the payoffs of players A, B, and C in the resulting allocations, respectively. We calculate player i's share in coalition S as $x_i/v(S)$. Figure 2 shows the mean payoff share of each player in the grand coalition in each game by treatment. Note that the number of observations varies across games. For example, in game 3 of Treatment T1, we have no data because no group formed the grand coalition in that game.

We find that almost all the resulting allocations in the grand coalition can be classified into one of three types: (i) $x_A > x_B > x_C$, (ii) $x_A = x_B > x_C$, or (iii) $x_A = x_B = x_C$. We observe type (i) 73 times, type (ii) 115 times, type (iii) 148 times, and other types 31 times out of the 367 grand coalitions observed. As noted in Table 3, in theory, A is the strongest player, B is the second strongest, and C is the weakest in all games. Therefore, we assumed that type-(i) payoffs, such as $(50, 40, 30)$, would be the most frequently observed in the experiment. However, in reality, the other two types were observed more frequently than this type. For example, regarding the frequent observation of type-(ii) payoffs, such as $(50, 50, 20)$, we interpret this as follows: The subjects perceived A and B as having equal bargaining power, whereas C was in a weaker position in terms of bargaining power. The type most frequently observed was an equal division of type-(iii) payoffs, that is, $(40, 40, 40)$. This type was particularly common in the early stages of the experiment. We infer that the subjects did not initially consider differences in bargaining power when making decisions, but as the experiment progressed, they became aware of these differences and began negotiating accordingly. Furthermore, an equal division was frequently observed in game 10. This can be attributed not only to the fact that game 10 was in the first round in some treatments but also to the fact that, uniquely in game 10, an equal division was included in the core.

(a) A and B's Share in Coalition AB

(b) A and C's Share in Coalition AC

Figure 3: Mean of Each Player's Share in Coalition AB and AC in Each Game by Treatment

Figures 3(a) and (b) show each player's payoff share in coalition AB and coalition AC, respectively. We observe that player A received, on average, between 50% and 70% of the worth of coalition AB and typically received anywhere from 60% to 80% of the worth of coalition AC. This result aligns with the findings for the payoff distribution in the grand coalition: players A and B were generally considered equally strong, whereas player C was perceived as weaker. Why does player A form a coalition with C rather than with B, despite the fact that $v(AB)$ is always greater than $v(AC)$? The answer lies in the fact that, in 242 out of the 265 groups that formed coalition AC, player A received more than half of $v(AB)$. Thus, if player B proposed an equal division of the worth of coalition AB, whereas player C offered more than half of $v(AB)$ to player A, it would be rational for A to accept C's offer. However, this does not necessarily mark the end of the bargaining process, as player B could make a counteroffer that provides a higher payoff than C's offer. This competitive bargaining between players B and C results in significant inequality: the payoff shares deviate further from an even split as the experiment progresses, as shown in Figures 3(a) and 3(b). We observe a similar tendency in the grand coalition, as illustrated in Figure 2.

Selten (1987) primarily aimed to construct a new predictive theory for the results of bargaining experiments, known as the "equal division payoff bounds." Although he did not conduct his own experiment in that article, his theory successfully predicted the results of the experiment run by Murnighan and Roth (1977). A key feature of this theory is its emphasis on the equal division of the worth not only of the grand coalition but also of smaller coalitions. In our experiment, the subjects first equally divided the worth of the grand coalition but quickly realized that the worth of coalition AB was more influential when divided equally.[16] However, although coalition AB was frequently formed with an equal division of worth, coalition AC was also formed when the resulting allocation gave player A more than half of $v(AB)$. In this sense, we argue that the subjects regarded the equal divisions of the worth of the grand coalition and coalition AB as focal points in their bargaining, and it appears that Selten's predictive theory can also explain this outcome.

	x_A	x_B	x_C
Chat	-4.294***	3.210***	3.564***
	(1.088)	(1.021)	(0.874)
Reverse	-2.965**	1.993*	1.473
	(1.216)	(1.015)	(0.941)
Random	-3.338**	5.898***	-0.735
	(1.459)	(1.724)	(1.287)
Partner	-3.280***	4.783***	0.562
	(0.720)	(0.951)	(0.837)
Chat × *Partner*	-1.746	-1.930*	4.996***
	(1.088)	(1.021)	(0.874)
CoreScale	-0.330***	-0.072	0.287***
	(0.042)	(0.043)	(0.016)
q_A, q_B, q_C	54.799***	44.170***	53.042***
	(7.871)	(13.973)	(7.003)
constant	28.389***	18.673***	5.330***
	(4.266)	(4.527)	(1.398)
Obs.		1200	
R^2	0.193	0.034	0.131

Standard errors (in parentheses) are clustered at the session level.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 5: OLS Regression Analysis of the Resulting Payoffs

Table 5 shows the results of the regression analysis of each player's payoff in the resulting allocations, x_A, x_B, and x_C, using an OLS model. The independent variables, *Chat*, *Reverse*, *Random*, *Partner*, *Chat* × *Partner*, and *CoreScale*, are the same as those in the coalition formation regressions. We add each player's relative bargaining power, q_A, q_B, and q_C, to each estimation of x_A, x_B, and x_C, respectively.

The availability of the chat window is negatively correlated with A's payoff and positively correlated with B's and C's payoffs at the 1% significance level. This appears to be because this variable is positively correlated with the formation of the grand coalition. When the grand coalition is formed, A has to share the worth of the grand coalition with B and C. By contrast, when either coalition AB or AC is formed, A only needs to share the worth of the coalition with either B or C. Hence, A's payoff decreases when the grand coalition is formed, and the formation of the grand coalition is correlated with *Chat*, as shown in

[16] In Treatments 3 and 4, the subjects played game 10 in the first round, where the equal division of the worth of the grand coalition, $(40, 40, 40)$, is a core allocation. In this case, almost all groups agreed on this equal division of the worth of the grand coalition.

the previous regression. A similar interpretation can explain B's and C's payoffs. When either coalition AB or AC is formed, the player who is not involved in the coalition receives nothing. By contrast, when the grand coalition is formed, all subjects obtain positive payoffs. This is the reason that the availability of the chat window is positively correlated with x_B and x_C. The same interpretation should explain the negative correlation with A's payoff and the positive correlation with C's payoff of *CoreScale* at the 1% significance level. The insignificant correlation between x_B and *CoreScale* arises because, when *CoreScale* is small, the likelihood that either coalition AB or AC is formed is high, and B can obtain a high payoff from coalition AB, but receives nothing from coalition AC. By contrast, the larger the index, the higher the likelihood that the grand coalition is formed, and then B obtains a middle payoff because B has the second-highest bargaining power. Therefore, we find no significant difference in B's payoff between the games with a small *CoreScale* and those with a large *CoreScale*. Another finding is that A, B, and C's relative bargaining power is positively correlated with the payoffs at the 1% significance level for x_A, x_B, and x_C, respectively. A player should obtain more payoffs in cooperative game theory when the player has stronger bargaining power. We find that the subjects in the experiment also obtain more payoffs accordingly.

Although *Reverse* and *Random*, which are related to the game order, are not correlated with the formation of the grand coalition, they show a negative correlation with A's payoff and a positive correlation with B's payoff. No correlation is observed with C's payoff. This suggests that game order was not important for C. However, it was important for C to make offers that prevent C from being excluded from the resulting coalition. By contrast, for A and B, it was important which game was played early in the experiment. It is understandable that A's payoff was lower in Treatments 3–6, where the subjects had some experience before playing, than in Treatments 1 and 2, where games 1–4, which tend to result in an equal split in coalition AB, could be played early on. It also becomes clear that using partner matching decreases A's payoff and increases B's payoff. This is likely to be because of the fact that fixing group members reduced A's bargaining power and increased B's bargaining power in the experiment. By contrast, C's payoff increased only when the chat window was available and partner matching was used. This is probably because the grand coalition becomes easier to form, which makes it less likely that C will be excluded from the resulting coalition.

4.4 Core, Equal Division Core and Equal Division

Game	Core (%)	*EDC* (%)	Equal Division (%)	Others (%)	Grand Coalition
1		0 (0.0%)	15 (78.9%)	4 (21.1%)	19
2		1 (8.3%)	8 (66.7%)	3 (25.0%)	12
3	empty	3 (27.3%)	5 (45.5%)	3 (27.3%)	11
4		2 (14.3%)	8 (57.1%)	4 (28.6%)	14
5		16 (61.5%)	6 (23.1%)	4 (15.4%)	26
6	1 (3.2%)	20 (64.5%)	8 (25.8%)	3 (9.7%)	31
7	6 (19.4%)	24 (77.4%)	4 (12.9%)	3 (9.7%)	31
8	37 (58.7%)	48 (76.2%)	13 (20.6%)	2 (3.2%)	63
9	32 (62.7%)	39 (76.5%)	12 (23.5%)	0 (0.0%)	51
10	109 (100.0%)	109 (100.0%)	69* (63.3%)	0 (0.0%)	109
Total	185 (64.9%)	262 (71.4%)	148 (40.3%)	26 (7.1%)	367

Table 6: Frequency and Share of Allocations in the Core and in the *EDC* and of Equal Divisions of the Grand Coalition Worth. Shares are in parentheses and * indicates that the equal division is in the core.

Finally, we focus on the relationships between the resulting allocations, the core, and the equal division core, which are related to Hypotheses 2 and 3. Table 6 presents, for each game, the number and share of core allocations, allocations in the equal division core, equal divisions of the worth of the grand coalition, and other allocations, along with the number of times the grand coalition was agreed on. Note that the equal division of the worth of the grand coalition is in the core in game 10, which is marked with an asterisk, whereas it is not even in the equal division core in games 1—9. Additionally, note that the core is included in the equal division core in all games. As shown in the column "Others," an important point is that fewer than 10% of the resulting allocations for the grand coalition were neither in the core nor in the equal division core and were not an equal division of the worth of the grand coalition. To summarize, the resulting allocations for the grand coalition are mostly in the core or the equal division core, but when they are not in either core, the worth is equally divided. When we focus on the core, we find that approximately two-thirds of the resulting allocations for the grand coalition are in the core in games 6-–10. The size of the core increases from game 6 to game 10 as the worth of the two-person coalitions decreases. As a result, we observe only one core allocation in game 6, where the core consists of only one allocation. By contrast, we observe core allocations in all groups that form the grand coalition in game 10, where the core is the largest among the five games. Hence, we conclude that Hypothesis 2 is *conditionally* correct: when the core is nonempty *and somewhat large*, the resulting allocation for the grand coalition is in the core.

Game	Allocations in $EDC \setminus Core$ (%)	Noncore Allocations
1	0 (0.0%)	19
2	1 (8.3%)	12
3	3 (27.3%)	11
4	2 (14.3%)	14
5	16 (61.5%)	26
6	19 (63.3%)	30
7	18 (72.0%)	25
8	11 (42.3%)	26
9	7 (36.8%)	19
10	0 (-)	0
Total	77 (42.3%)	182

Table 7: Frequency of Allocations in the *EDC* out of the Core and Noncore Allocations. Percentages are in parentheses.

Table 7 shows the number of resulting allocations that are in the equal division core but not in the core, in addition to the percentage of such allocations out of all noncore allocations. From game 5 to game 7, where the core is empty or relatively small but the grand coalition is sometimes formed, this percentage is relatively high because finding a core allocation is impossible (game 5) or difficult (games 6 and 7) in those games. However, it appears that the subjects were mindful not to be dominated by equal division in each two-person coalition. As a result, we can interpret that they reached an agreement on allocations that, although not included in the core, were included in the equal division core. In the other games, where the core is either empty or sufficiently large, the resulting noncore allocations tend to be an equal division of the worth of the grand coalition rather than an allocation in the equal division core. In short, while Hypothesis 3 does not hold universally, our findings suggest that when the core is small, the resulting noncore allocations for the grand coalition frequently align with the equal division core, highlighting the subjects' tendency to seek allocations that are at least not dominated by equal division.

Game	EDC/Coalition AB (%)	EDC/Coalition AC (%)
1	57/62 (91.9%)	empty
2	71/73 (97.3%)	5/31 (16.1%)
3	69/74 (93.2%)	15/32 (46.9%)
4	75/75 (100.0%)	22/31 (71.0%)
5	45/51 (88.2%)	9/38 (23.7%)
6	49/52 (94.2%)	20/34 (58.8%)
7	45/48 (93.8%)	33/37 (89.2%)
8	36/39 (92.3%)	1/14 (7.1%)
9	44/44 (100.0%)	11/20 (55.0%)
10	2/2 (100.0%)	empty
Total	493/520 (94.8%)	116 / 237 (48.9%)

Table 8: Frequency of Allocations in the *EDC* for Coalitions AB and AC, and the Number of Times Coalitions AB and AC Formed. Percentages are in parentheses.

The core is defined only for the grand coalition, whereas the equal division core is defined not only for the grand coalition but also for two-person coalitions. Table 8 presents the number of resulting allocations in the equal division cores for coalition AB and coalition AC, along with the corresponding percentage relative to the total number of times coalitions AB and AC were formed.[17] The resulting allocations for coalition AB were mostly in the equal division core. This is because the worth of coalition AB is the highest among the two-person coalitions in all games, which implies that the equal division core is relatively large. For example, an equal division of the worth of coalition AB, $(v(AB)/2, v(AB)/2, 0)$, is never dominated by an equal division of the worth of coalitions AC or BC in any game and is not dominated by the equal division of the worth of the grand coalition in games 1-–9. Hence, as previously mentioned, it is easy for the subjects to find an allocation in the equal division core for coalition AB, particularly the equal division of the worth of coalition AB. By contrast, the equal division of the worth of coalition AC, $(v(AC)/2, 0, v(AC)/2)$, is dominated by the equal division of the worth of coalition AB in all games because $v(AB)$ is larger than $v(AC)$. Thus, it is more difficult for the subjects to find an allocation in the equal division core for coalition AC than to find one for coalition AB. This is one reason that approximately half of the resulting allocations for coalition AC are outside the equal division core for coalition AC.

When we examine the 157 allocations for coalition AC that fall outside the equal division core for coalition AC, we find that 145 are dominated by an equal division of the worth of coalition BC, whereas only 12 are dominated by an equal division of the worth of coalition AB. This suggests that C gave A too much to avoid being excluded from coalition AB. When C proposed coalition AC, C focused on domination via coalition AB and disregarded the possibility of domination via coalition BC. C's primary concern was to avoid exclusion from coalition AB; hence, C gave A more than half the worth of coalition AC, even though C would obtain more from an equal division of the worth of coalition BC. A similar pattern occured in coalition AB: 24 out of 27 resulting allocations that were outside the equal division core for coalition AB were dominated solely by an equal division of the worth of coalition BC. As previously mentioned, we frequently observed an equal division of the worth of coalition AB, but the scenario changed when C made a counteroffer to A that exceeded an equal division of the worth of coalition AB. To prevent exclusion by coalition AC, B had to make a competing offer that gave A more than C's counteroffer. Consequently, although B ultimately secured the agreement, B sometimes gave A significantly more than half the worth of coalition AB, disregarding the possibility of domination via coalition BC. This mechanism explains why some resulting allocations for coalitions AB and AC fell outside the equal division core. The players did not take domination via coalition BC into account.

Again, we examine Hypothesis 3 for coalitions AB and AC. Note that all allocations for coalitions AB

[17] The equal division core for coalition BC is empty in all games; hence, we do not discuss it here.

and AC are noncore allocations, as only grand coalition allocations can be core allocations. For coalition AB, we conclude that Hypothesis 3 is correct: the resulting allocation for coalition AB is almost always in the equal division core. By contrast, for coalition AC, it is difficult to affirm that Hypothesis 3 is correct because only approximately half of the resulting allocations are in the equal division core for coalition AC.

5 Discussion and Concluding Remarks

Cooperative game theory explores the problems of coalition formation and payoff distribution. However, to focus on the payoff distribution problem, cooperative game theorists typically assume that if a game is superadditive, the grand coalition will be formed. Superadditive games do not necessarily have a nonempty core, which implies that when the core is empty, there always exists a subcoalition that can improve its payoff by deviating from the grand coalition. As a result, the grand coalition may not be formed in games with an empty core, even if the game is superadditive. We formulated our hypotheses based on this reasoning and tested them through a laboratory experiment.

We examined two hypotheses regarding coalition formation. Our first experimental result is that the likelihood of the grand coalition forming is higher when the core is nonempty than when it is empty. When the core is empty, any allocation for the grand coalition is dominated by a two-person coalition, which reduces the likelihood that the grand coalition forms. Conversely, when the core is nonempty, core allocations are not dominated by any two-person coalition; therefore, the grand coalition is more likely to form. Hence, we conclude that the existence of a nonempty core strongly facilitates the formation of the grand coalition. Additionally, because all games implemented in the experiment were superadditive, we conclude that superadditivity does not necessarily guarantee the formation of the grand coalition. Our second result is that the likelihood of the grand coalition forming is higher when the subjects can publicly communicate with each other. Communication among the subjects enables them to share their thoughts more easily and discourages the exclusion of a player through the formation of a two-person coalition. As a result, social surplus increases when the grand coalition forms, which highlights the importance of communication. Both the existence of a nonempty core and the availability of a chat window facilitate the formation of the grand coalition, which confirms that Hypotheses 1 and 4 are correct.

We also developed and examined two hypotheses regarding the relationships between the agreed-upon allocations, and the core and equal division core. When the core is relatively large, the chosen allocation for the grand coalition is in the core. We expected this because we hypothesized that the grand coalition forms when the core is nonempty, given that core allocations are never dominated. This occurs when the core is large because the subjects can more easily arrive at a core allocation during bargaining. By contrast, when the core is small, the subjects fail to agree on a core allocation, which results in an allocation outside the core. However, such allocations frequently fall within the equal division core, which means that they are at least not dominated by an equal division of the worth of any possible coalition. Hence, we conclude that the bargaining over allocations proceeds in a manner consistent with the principle of domination. Regarding the equal division core for coalitions AB and AC, the chosen allocation for coalition AB is almost always in the equal division core, whereas the allocation for coalition AC is often in the equal division core. In coalition AB/AC, B/C tends to give A an excessively large share of the coalition's worth to avoid exclusion from coalition AC/AB, disregarding the possibility of domination by an equal division of the worth of coalition BC. As the results for coalition formation show, coalition BC is seldom formed because its worth is the lowest among the two-person coalitions. The subjects, particularly C, tend to assume that coalition BC will not be formed and therefore do not consider it a viable option. We conclude that Hypotheses 2 and 3 are *conditionally* correct: when the core is nonempty, the resulting allocation for the grand coalition is in the core *if the core is large*, and the resulting noncore allocation for the grand coalition is in the equal division core *if the core is small*.

In addition to testing the above hypotheses, we also obtained several results regarding payoff distributions. First, the resulting allocations reflect the subjects' bargaining power, to some extent: the strongest player, A, receives the largest payoff, the second-strongest player, B, receives the second largest, and the weakest player, C, receives the smallest. This finding aligns with cooperative game theory, which suggests that stronger players should receive larger payoffs. Second, we identified two focal points: the equal division of the worth of coalition AB and the equal division of the worth of the grand coalition. In the laboratory experiment, the subjects perceived A and B as equally strong, despite A being theoretically stronger than B. As a result, when either the grand coalition or coalition AB was formed, A and B frequently received equal payoffs. By contrast, when coalition AC was formed, C had to offer A more than half of the worth of coalition AB to avoid exclusion from coalition AB. Consequently, the allocation between A and C was unequal. Unlike the case of A and B, A was regarded as stronger than C not only in theory but also in practice during the experiment.

Although we use the same parameters for the worth of all coalitions as Nash et al. (2012), the primary goal of our research is not to compare our results with theirs. This is because our experimental settings differ from theirs in two key ways. First, our bargaining protocol is unstructured, whereas theirs follows a structured format, as described in Section 1. In their experiment, the subjects first vote to select a splitter, who then distributes the payoffs in the second stage of the bargaining process. Second, we primarily adopt stranger matching, whereas they use partner matching. In their experiment, group members and roles remain fixed throughout all rounds. Under such conditions, if a splitter takes the entire payoff or behaves selfishly in one round, the individual risks punishment from other members in subsequent rounds. Conversely, in most of our treatments, group members and roles were shuffled at the beginning of each round. As a result, if A and B deviated from the grand coalition in one round, they would face no repercussions from C in later rounds. These structural differences explain why our findings diverge from those of Nash et al. (2012) in both coalition formation and payoff distribution. Regarding coalition formation, nearly all groups in their experiment formed the grand coalition, whereas in our experiment, coalition AB was the most frequently formed. In terms of payoff distribution, more than half of the groups that formed the grand coalition in their experiment divided the payoff equally, whereas in our experiment, only about two-fifths of such groups agreed on an equal division. Even in the treatment where we adopted partner matching, the overall trends remained similar. However, in treatments where a chat window was available and we adopted partner matching, the subjects agreed on the grand coalition more frequently and were more likely to reach an allocation close to an equal division. This suggests that the ability to communicate freely made it easier for participants to discuss not only the current round but also their strategies for future rounds. Indeed, a review of the messages exchanged confirms that some groups explicitly engaged in such discussions. Because Nash et al. (2012) did not provide a chat window in their experiment, we conclude that the differences in our results stem primarily from variations in the bargaining protocol.

Although Hypothesis 4 is correct, the subjects sent very few messages through the chat window. As a result, we cannot fully analyze what the subjects discussed or how their bargaining was conducted via the chat window. One possible reason for this is that the subjects were too focused on making offers and reacting to others' offers to send messages. As mentioned in Section 3, messages sent through the chat window are a form of cheap talk. Even if a verbal promise is made in the chat, it is not binding. By contrast, agreements made through the proposal and acceptance of an offer are executed with certainty. Therefore, the subjects prioritized making offers and reacting to others' offers over sending messages through the chat window. To better understand what the subjects discussed and how their bargaining took place via the chat window, another treatment could be designed. For instance, the chat window could be available only in the first half of each round, after which the subjects would be able to make and react to offers. In this setting, the subjects would be more likely to engage in conversation via the chat window, thereby allowing for linguistic analysis. Additionally, the results of this study and those from the new treatment could differ in terms of coalition formation and payoff distributions. This will be explored in future work.

Another possible extension is to have the subjects play other games. Games 1–5 in our study have an empty core, but $v(N) = v(AB)$ holds simultaneously in games 1–4. Although it is true that the formation of the grand coalition was less likely when the core was empty in this study, this equality might also have discouraged the subjects from forming the grand coalition. To ensure the validity of our results regarding the core, more games in which $v(N) > v(AB)$ and in which the core is empty should be played in the experiment. For example, a game in which $v(N) = 120$, $v(AB) = 110$, $v(AC) = 90$, $v(BC) = 70$, and $v(A) = v(B) = v(C) = 0$ would satisfy strong superadditivity and have an empty core. If the subjects frequently form the grand coalition in such games, we might conclude that strong superadditivity is more important than the existence of the core. This extension is also left for future work.[18]

Another possible extension regarding the type of game played would be to introduce symmetry into the subjects' bargaining power. The 10 games in this study all feature asymmetric bargaining power: A is stronger than B, and B is stronger than C. For example, a game in which $v(AB) = v(AC) = v(BC) = 100$ would give A, B, and C equal bargaining power. In such games, each subject should receive 40 from the grand coalition, in theory. However, as the results show, the grand coalition would not necessarily be formed in such an experiment because the game above has an empty core. In that case, we could predict that a two-person coalition would be formed, but we could not predict which coalition would be formed because all two-person coalitions generate the same amount of profit. The subjects might finally agree on an equal division of the worth of the grand coalition after becoming tired of repeating offers to equally divide the worth of the two-person coalitions. Introducing "partial" symmetry to the subjects' bargaining power is also a possible extension. For example, a game in which $v(AB) = v(AC) = 100$, and $v(BC) = 60$ would give B and C equal bargaining power, while giving A more than either of them. In the 10 games in this paper, player C was the most frequently excluded from the resulting coalitions because C had the least bargaining power. However, in the above example, both B and C are equally weak relative to A; hence, we predict that the resulting coalition and allocation should change. Further experimental research is needed to examine fully and partially symmetric games.

Statement: During the preparation of this paper, the authors used ChatGPT to improve readability and correct grammatical mistakes. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

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[18] In Treatments 5 and 6, after completing the experiments for Games 1–10, we added Games 11 and 12 as additional experiments: Game 11 with $v(N) = 120$, $v(AB) = 100$, $v(AC) = 90$, $v(BC) = 60$, $v(A) = v(B) = v(C) = 0$, and Game 12 with $v(N) = 120$, $v(AB) = 100$, $v(AC) = 80$, $v(BC) = 70$, $v(A) = v(B) = v(C) = 0$. After paying the participants, we observed the following results: out of 40 groups, the grand coalition was formed 5 times, coalition AB 21 times, coalition AC 11 times, and coalition BC 3 times. Hence, even though the games satisfied strong superadditivity, the grand coalition was less likely to form when the core was empty. However, because this was based on a small sample, it is noted here as a footnote.

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Appendix A Instructions

Thank you for your participation. You are about to participate in an experiment related to economic behavior and decision-making. Please put your belongings into your bag.

First, you will read these instructions in order to understand the rules of the experiment. After that, you will make decisions that determine your earnings. Your earnings depend on your decisions and those of the other participants. Your personal data, decisions in the experiment, and earnings will be anonymous and will be shown to no one but the experimenter.

Do not make any noise and do not watch other participants' behavior during the experiment. If you have any questions, please raise your hand. The experimenter will come to you.

This experiment consists of a practice round followed by 10 paying rounds. In each round, you will be randomly assigned to a three-member group by the computer at the beginning of the round.[19] You will not know who is in your group. The next section explains the environment you will face in each round.

<u>General Environment</u>

You and the other two members of your group are running firms A, B, and C and planning to carry out a project. The project can be done by a single firm or through cooperation between two or three firms. Your goal is to complete the project with the other firms and to allocate the profits gained by cooperation to the firms. Your firm will be randomly assigned by the computer at the beginning of each round.

You will be given information on the profits that would be gained by a single firm and by two or three cooperating firms at the beginning of each round. Note that the profit gained by a single firm is

[19]In treatments using partner matching, the text will state: "The group members remain the same throughout the experiment."

0 and the profit gained by cooperation among three firms is 120 points in every round. Hence, only the profits gained by cooperation between two firms will differ in each round. The next section explains the computer screen on which you will bargain with the other participants.

Bargaining Stage

Take a look at the screenshot for the bargaining stage. It will be explained from point (1) to point (6).

(1) Information on the round

You can find your firm's name and the possible profits here. Note that your firm's name and the profits gained by cooperation between two firms will be different in each round.

(2) Input of offers

To carry out the project with other firms and earn profits, make an offer" here. Offers can be of the following two types. Make sure that you input figures into all blanks.

(A) Offer of cooperation among all three firms

Input the profits to be distributed to A, B, and C with integers that are not less than 0 and click the [OFFER A, B, C] button. Make sure that the sum of the distributed profits is 120. If it is not 120, an error message will appear.

(B) Offer of cooperation between two firms

Assume that you are firm C. You want to cooperate with B, which will generate 40 points of profit. Then, input the profits to be distributed to B and to C with integers which are not less than 0 and input -1 into A's column. Then, click the [OFFER B, C] button. Make sure that the sum of the distributed profits is 40. You, as firm C, cannot make an offer of cooperation between only A and B.

(3) Canceling an offer

After you propose an offer, it will be displayed here. You can propose offers as many times as you want to, but you cannot propose more than one offer at a time. If your previous offer has not been rejected and you want to propose a new offer, you mustcancel" the previous offer first. When you cancel your previous offer, click on the offer and the [CANCEL] button. When you have cancelled an offer, its label in the STATUS column becomes canceled".

(4) Accepting/rejecting others' offers

After other members propose an offer, it will be displayed here. Confirm its content and chooseaccept" or reject". The PROPOSER column displays who proposed the offer. If you think that an offer is acceptable, click it and the [ACCEPT] button. For example, if firm B accepts the offer of cooperation among all three firms, the label in the STATUS column for that offer becomesB accepts". If you do not think an offer is acceptable, click it and the [REJECT] button. For example, if firm A rejects an offer, the label in the STATUS column for that offer becomes "rejected by A". Firm C can see offers of cooperation between A and B, but C can neither accept nor reject them.

For 15 seconds after an offer is proposed, its label in the STATUS column is wait". You can reject and cancel such offers, but you cannot accept them during this interval. Wait for the column to showactive" to accept the offer. Therefore, note that even if you propose a new offer with less than 15 seconds of the remaining time, it cannot be accepted.

An offer is approved when it is accepted by all members who are included in it. For example, if you are firm A and you offer cooperation among all three firms, it must be accepted by both B and C to be approved. If you offer cooperation with B only, it has to be accepted only by B to be approved because firm C is not included in that offer.

(5) Time remaining in the round

The time remaining in the round is displayed in seconds here. The time limit for each round is 5 minutes. If no offer is approved in 5 minutes, you will be expected to carry out the project alone and will receive 0 points for that round, so be careful with your remaining time.

(6) Chat window[20]

You can talk with other members through the chat window during the bargaining stage. Type your message and click the [SEND] button. When you use the chat window, never include identifying information (your name, seat number, e-mail address and so on) or write abusive messages.

[20]In the treatments in which the chat window is not available, this part is omitted.

When an offer is approved, the project will be completed and the profits will be distributed according to the offer. That round will then end. If an offer of cooperation between two firms is approved, the firm that is not included in the project will receive profits of 0.

Click the [OK] button after the computer shows how the project is carried out and the profits are distributed. The next round starts when all participants click the [OK] button.

After the experiment, you will be paid 800 Japanese yen as a participation fee plus your earnings, where your total points are exchanged for cash at the following rate: 1 point = 3 Japanese Yen.

This is the end of the instructions. Please raise your hand if you have any questions. You can read the instructions to check the rules during the experiment. You can also use the calculator or the pen set on your table.

The practice round starts now to help you understand how to use the computer during the bargaining stage. The experimenter will instruct you on what to do, so follow those instructions first. The points gained in this practice round will not affect your earnings.

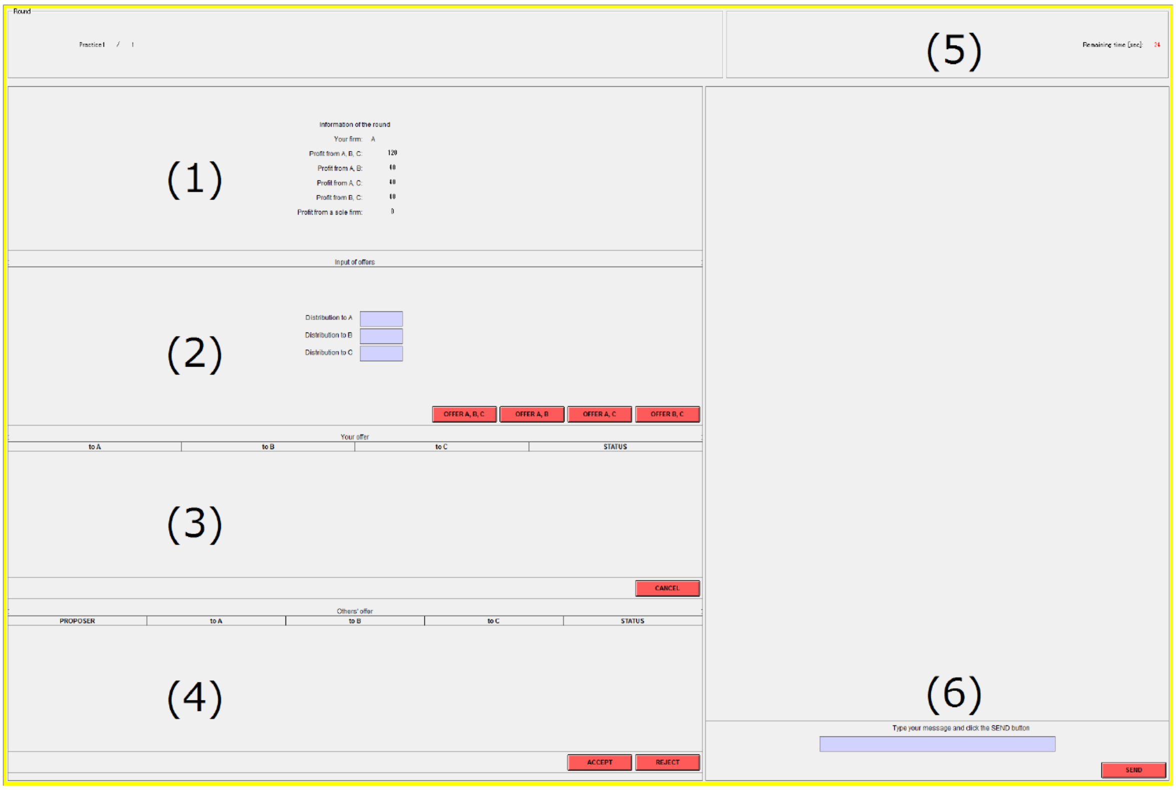

Figure 4: Screenshot of the Bargaining Stage

Appendix B Marginal Effects on Formation of the Grand Coalition

	Grand	
Chat	0.140***	0.152***
	(0.049)	(0.050)
Reverse	0.061	0.061
	(0.045)	(0.047)
Random	-0.020	-0.023
	(0.088)	(0.094)
Partner	0.047	0.051
	(0.040)	(0.042)
Coreness	0.350***	-
	(0.027)	
CoreScale	-	0.010***
		(0.001)
Chat × *Partner*	0.164***	0.169***
	(0.057)	(0.062)
Obs.	1,200	

Standard errors (in parentheses) are clustered at the session level.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 9: Marginal Effects on Formation of the Grand Coalition