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Heterogeneity in Tastes, Productivities, and Macroeconomic Volatility

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Abstract

This paper examines how heterogeneity in product-level tastes and firm-level technologies shapes macroeconomic fluctuations. We develop a general equilibrium model with multi-product firms and endogenous entry, where firms adjust their product mix in response to aggregate shocks. Calibrated to U.S. data, the model replicates key business cycle moments and shows that low taste dispersion amplifies aggregate volatility by limiting per-product profit adjustments, whereas high dispersion dampens fluctuations. While firm-level productivity granularity also affects volatility, its impact is comparatively minor. A simplified analytical model reinforces these findings, highlighting the critical role of aggregate shock propagation to firm- and product-level fixed costs, as well as heterogeneity in tastes and technologies, in determining macroeconomic volatility.

Keywords: Firm Heterogeneity, Multi-Product Firms, Business Cycles, Product Quality
JEL Class.: D24, E23, E32, L11, L60.

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1 Introduction

Firms differ not only in their specific technologies but also in the tastes or qualities associated with their products. Consequently, some products exhibit nearly identical quality and are produced using similar technologies across firms, while others display a wide spectrum of qualities and are produced by firms with significantly different production technologies. The degree of heterogeneity in quality and productivity embedded in products varies across economies and industries (Feenstra and Romalis, 2014; Hottman, Redding, and Weinstein, 2016). Recent studies have examined the reallocation of resources across and within heterogeneous firms and products in the context of economic growth (Lentz and Mortensen, 2008; Acemoglu et al., 2018; Argente, Lee, and Moreira, 2024). However, little is known about the relationship between this firm-product-specific heterogeneity and aggregate fluctuations.

What are the business cycle characteristics of products produced by firms with different technologies? How does heterogeneity in product-level tastes and firm-level technologies shape macroeconomic dynamics? This paper seeks to answer these questions and fill this research gap by developing a stylized general equilibrium model.

Our model incorporates multi-product firms as in Bernard, Redding, and Schott (2010), along with endogenous firm entry and exogenous exit à la Bilbiie, Ghironi, and Melitz (2012). Upon entry, firms draw their specific productivities from a fat-tailed distribution and are subject to fixed operational headquarters costs, which determine the selection of producers, as in Hamano and Zanetti (2017). Similarly, firms in our model draw a quality (taste) level for each product from a fat-tailed distribution. For the same product price, a higher-quality product attracts greater demand than a lower-quality product, as quality or taste acts as a demand shifter. Firms decide whether to produce each product while also incurring firm-specific operational fixed costs and product-specific fixed operational costs, depending on the product's profitability.

In the model, firms adjust their product mix endogenously by adding and removing products in response to aggregate shocks, generating fluctuations in both production and profits at the firm and product levels. The extent of aggregate fluctuations in the economy depends crucially on the dispersion of both tastes and technologies, as well as the propagation of aggregate shocks at the firm and product levels. During economic booms triggered by a positive aggregate technology shock, firms expand their product lines, increasing total profits despite reduced per-product profitability due to intensified competition across different brands. Conversely, in downturns following a negative aggregate technology shock, firms reduce their product lines, focusing on a more profitable set of products to mitigate overall losses.

We calibrate the model using U.S. data to match key U.S. business cycle moments. The parameter values used in the calibration replicate U.S. business cycle moments quite well. Specifically, we find a relatively strong propagation of aggregate technology shocks into product-specific fixed operational costs compared to firm-specific fixed operational costs.

Using impulse response functions, we demonstrate that low quality or taste dispersion amplifies aggregate volatility. A significant increase in the number of products produced by different firms following a positive aggregate technology shock reduces per-product profits due to heightened competition across product varieties, while total firm profits rise as firms expand their product lines. When taste dispersion is low (i.e., products are more homogeneous and concentrated at the lower end of the distribution), smaller downward adjustments in per-product production and profits are required due to the fat-tailed nature of the distribution. As a result, total firm profits expand more, leading to greater aggregate volatility in GDP and consumption. Conversely, high taste dispersion dampens aggregate fluctuations. Additionally, we show that while greater granularity in firm-specific productivities amplifies aggregate volatility, its quantitative impact is relatively minor.

To further clarify the mechanism, we develop a simplified model and analytically demonstrate the intuition behind our numerical results. Specifically, we show that when aggregate technology strongly propagates into product-specific fixed operational costs relative to firm-specific fixed operational costs, lower taste dispersion indeed amplifies aggregate volatility. We also demonstrate that higher granularity in firm-level productivity amplifies aggregate fluctuations, though its impact can be very limited depending on parameter values.

The granular origins of aggregate fluctuations have been extensively studied (Gabaix, 2011; di Giovanni and Levchenko, 2012). This literature argues that with a fat-tailed firm size distribution, firm-specific shocks can account for a significant fraction of aggregate fluctuations. Although our paper does not assume firm-specific shocks, we identify an amplification mechanism for aggregate fluctuations driven by firm heterogeneity.

Minniti and Turino (2013) presents a DSGE model that examines the dynamics of multi-product firms under oligopolistic competition, demonstrating that multi-product firms can amplify aggregate dynamics compared to models with single-product producers. Similarly, Pavlov and Weder (2017, 2022) analyze the implications of multi-product firms for macroeconomic dynamics and indeterminacy within an oligopolistic framework. While these papers analyze business cycles based on multi-product firms, they abstract from both productivity and taste heterogeneity.

Our findings on product dynamics align with those of Broda and Weinstein (2010) and Hottman, Redding, and Weinstein (2016), who analyze product turnover using household-level data for the U.S. Specifically, Hottman, Redding, and Weinstein (2016) finds that differences in firm "appeal" (taste or quality) explain 50–70 percent of the variance in firm size, while the role played by differences in firm marginal costs is limited.

Using Japanese data, Bernard and Okubo (2016) and Dekle et al. (2015) document the business cycle behavior of establishments and their product dynamics. Hamano and Okubo (2021) performs structural estimations of product-specific dynamics and explores the implications of product-specific shocks using Japanese data. Hamano and Oikawa (2021) analyzes general equilibrium interactions across products with non-homothetic preferences and estimates their theoretical model with Japanese data using Bayesian methods. While the latter two papers share a similar modeling framework with ours, our focus is on taste and productivity heterogeneity and its relationship with aggregate volatility, relying on both numerical and analytical methods.

The remainder of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 details the model calibration. Section 4 analyzes the impulse response functions of the theoretical model and shows how heterogeneities in tastes and productivities relate to aggregate volatility. Section 5 presents a simpler model to explore the numerical results analytically. Finally, Section 6 concludes.

2 The Model

The model incorporates heterogeneity in firm-specific productivity and product-specific tastes. Depending on their technology and taste draws, firms decide whether to produce a specific product, thereby capturing the multi-product nature of firms à la Bernard, Redding, and Schott (2010). Furthermore, the model incorporates endogenous firm entry over the business cycle, as in Bilbiie, Ghironi, and Melitz (2012) and Hamano and Zanetti (2017).

2.1 Households

Household j maximizes its expected lifetime utility:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t(j), \tag{1}$$

where $0 < \beta < 1$ is the exogenous discount factor. The utility of household j at time t depends on consumption $C_t(j)$ and labor supply $L_t(j)$, as follows:

$$U_t(j) = \ln C_t(j) - \chi \frac{L_t(j)^{1+\varsigma}}{1+\varsigma},$$

where $\gamma > 1$ denotes the relative risk aversion, $\chi > 0$ represents the disutility of labor supply, and $\varsigma > 0$ is the inverse of the Frisch elasticity of labor supply.

There exists a continuum of products with a total mass of unity. Each product is indexed by i. The consumption basket is defined by the following CES function:

$$C_t(j) = \left(\int_0^1 C_{i,t}(j)^{1-\frac{1}{\theta}} di \right)^{\frac{1}{1-\frac{1}{\theta}}},$$

where $\theta > 0$ represents the elasticity of substitution across products.[1]

[1] We assume that all products have the same elasticity of substitution and do not differentiate between the elasticities of products produced by the same firm. In other words, we abstract from within-firm substitution elasticities. See also Hottman, Redding, and Weinstein (2016).

A *product variety* is defined as a combination of a product and the firm that produces it. Product varieties exist over a continuum Ω, but in each period t, only a subset $\Omega_t \subset \Omega$ is available. Firms are indexed by ω. The consumption basket of a particular product i by household j is given by:

$$C_{i,t}(j) = \left(\int_{\omega \in \Omega_t} \left(\lambda_i(\omega) c_{i,t}(j,\omega) \right)^{1-\frac{1}{\sigma}} d\omega \right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

where $c_{i,t}(j,\omega)$ represents the demand for product i produced by firm ω by household j, and $\lambda_i(\omega)$ represents the taste or quality of the product variety. In particular, $\sigma > 1$ is the elasticity of substitution among product varieties. We assume that $\sigma > \theta > 0$.

The demand for a product variety, $c_{i,t}(j,\omega)$, is given by:

$$\lambda_i(\omega) c_{i,t}(j,\omega) = \left(\frac{p_{i,t}(\omega)/\lambda_i(\omega)}{P_{i,t}} \right)^{-\sigma} C_{i,t}(j), \tag{2}$$

where $p_{i,t}(\omega)$ denotes the price of the product variety. In the above expression, the price index of product i is:

$$P_{i,t} = \left(\int_{\omega \in \Omega_t} \left(\frac{p_{i,t}(\omega)}{\lambda_i(\omega)} \right)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}. \tag{3}$$

Furthermore, the demand for product i by household j is given by:

$$C_{i,t}(j) = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} C_t(j), \tag{4}$$

where the price index of the aggregate consumption basket, P_t, is implicitly defined as:

$$P_t = \left(\int_0^1 P_{i,t}^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \tag{5}$$

We choose P_t as the numeraire.

2.2 Production, Pricing, and Production Decision

In period t, there is a mass N_t of firms. A firm can produce more than one product or remain non-operational. Upon entry, each firm draws a productivity level, φ, from a cumulative distribution $G(\varphi)$, with support on $[\varphi_{\min}, \infty)$, and a consumer taste level for each product, λ_i, from a cumulative distribution $Z_i(\lambda_i)$, with support on $[\lambda_{i\,\min}, \infty)$.

The level of aggregate labor productivity, common to all firms, is denoted by A_t. To produce an amount $y_{i,t}(\varphi, \lambda_i)$ of product i, a fixed operational cost of $\frac{f_i}{A_t^{\theta_i}}$ in effective labor units is required for each product in every period. Additionally, firms must pay a headquarters fixed operational cost, also defined in terms of effective labor units, given by $\frac{f_h}{A_t^{\theta_h}}$. Both f_i and f_h are time-invariant parameters, while θ_i and θ_h determine the propagation of the aggregate technology level on each fixed operational cost.

The total labor demand for a firm with productivity level φ is given by:

$$l_t(\varphi) = \int_0^1 I_i \left[\frac{y_{i,t}(\varphi, \lambda_i)}{Z_t \varphi} + \frac{f_i}{A_t^{\theta_i}} \right] di + \frac{f_h}{A_t^{\theta_h}}, \tag{6}$$

where I_i is an indicator function that takes the value 1 if the firm produces product i, and 0 otherwise.

The demand for each firm-specific product variety is characterized by equation (2). Profit maximization yields the following optimal price:

$$\rho_{i,t}(\varphi, \lambda_i) = \frac{\sigma}{\sigma - 1} \frac{w_t}{A_t \varphi}, \tag{7}$$

where $\rho_{i,t}(\varphi, \lambda_i)$ denotes the real price of product i produced by a firm with productivity φ that has drawn a consumer taste λ_i for this product. w_t represents real wages.

Depending on the level of product-specific productivity φ and consumer taste λ_i, a product may or may not be produced within the firm. Using equations (6), (7), and (4), if production occurs, the following real operational firm-product-specific profits are generated:

$$d_{i,t}(\varphi, \lambda_i) = \frac{1}{\sigma} \left(\frac{\rho_{i,t}(\varphi, \lambda_i)}{\lambda_i} \right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta} \int_0^1 C_t(j) dj - w_t \frac{f_i}{A_t^{\theta_i}},$$

where $\rho_{i,t} \equiv \frac{P_{i,t}}{P_t}$, which represents the real price of the basket of product i. Since the elasticity of substitution among varieties is assumed to be greater than one ($\sigma > 1$), a lower taste-adjusted real price implies higher profits.

The total operational profits of a producing firm with productivity φ are given by:

$$d_{s,t}(\varphi) = \int_0^1 I_i d_{i,t}(\varphi, \lambda_i) di - w_t \frac{f_h}{A_t^{\theta_h}}.$$

where I_i is an indicator function. Each product i may or may not be produced by a firm with productivity φ. Only products satisfying $d_{i,t}(\varphi, \lambda_i) > 0$ are produced. For a firm with productivity φ, there exists a zero-profit consumer taste cutoff $\lambda_{i,t}^*(\varphi)$ for product i, defined as:

$$d_{i,t}\left(\varphi, \lambda_{i,t}^*(\varphi)\right) = \frac{1}{\sigma} \left(\frac{\rho_{i,t}(\varphi, \lambda_i)}{\lambda_{i,t}^*(\varphi)} \right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta} \int_0^1 C_t(j) dj - w_t \frac{f_i}{A_t^{\theta_i}} = 0. \tag{8}$$

Additionally, a firm with productivity φ operates only if $d_{s,t}(\varphi) > 0$. From this condition, we can determine the zero-profit productivity cutoff φ_t^* as:

$$d_{s,t}\left(\varphi_t^*\right) = \int_0^1 \int_{\lambda_{i,t}^*(\varphi_t^*)}^{\infty} d_{i,t}\left(\varphi_t^*, \lambda_i\right) dZ_i(\lambda_i) di - w_t \frac{f_h}{A_t^{\theta_h}} = 0. \tag{9}$$

2.3 Firm Entry and Exit

In each period, a mass H_t of entrants enters the market. Prior to entry, these new firms are identical and face a sunk entry cost of $\frac{f_E}{A_t}\left(\frac{H_t}{H_{t-1}}\right)^{\omega}$ in effective labor units. In the expression for the entry cost, $\omega > 0$ captures the congestion effect due to the increasing number of entrants from the previous period. Entrants are assumed to require one time period before becoming potential producers.

Firm entry occurs until the expected entry value, v_t (defined later, see equation (23)), equals the entry cost, leading to the free entry condition:

$$v_t = w_t \frac{f_E}{A_t}\left(\frac{H_t}{H_{t-1}}\right)^{\omega}. \tag{10}$$

The timing of entry and production implies that the number of firms evolves according to the following law of motion:

$$N_t = (1-\delta)\left(N_{t-1} + H_{t-1}\right), \tag{11}$$

where δ represents the exit rate of firms and entrants from period $t-1$ to t.

2.4 Product Average

A specific average of productivities weighted by consumer tastes for all producers of product i is defined following Bernard, Redding, and Schott (2010),[2]

$$\tilde{\varphi}_{i,t} \equiv \left[\int_{\varphi_t^*}^{\infty} \tilde{\lambda}_{i,t}(\varphi)\frac{dG(\varphi)}{1-G(\varphi_t^*)}\right]^{\frac{1}{\sigma-1}}, \quad \text{where} \quad \tilde{\lambda}_{i,t}(\varphi) \equiv \int_{\lambda_{i,t}^*(\varphi)}^{\infty} (\lambda_i\varphi)^{\sigma-1}\frac{dZ_i(\lambda_i)}{1-Z_i(\lambda_{i,t}^*(\varphi))}.$$

In the above expression, $\tilde{\lambda}_{i,t}(\varphi)$ represents the average productivity-weighted taste of product i for a firm with productivity φ. It summarizes the range of tastes that allow for the production of product i by a firm with productivity φ. The term $\tilde{\varphi}_{i,t}$ thus encapsulates all information about the distribution of productivities and consumer tastes. In short, it can be interpreted as the taste-weighted average productivity of product i.

Using this taste-weighted average productivity, the *taste-adjusted* real price for product i among all firms that produce it is defined as:

$$\tilde{\rho}_{i,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{A_t\tilde{\varphi}_{i,t}}.$$

Based on this real price, we can define the average profits for each product i as:

[2] See their technical appendix for details.

$$\tilde{d}_{i,t} = \frac{1}{\sigma}\frac{\rho_{i,t}\int_0^1 C_{i,t}(j)dj}{M_{i,t}} - w_t \frac{f_i}{A_t^{\theta_i}}, \tag{12}$$

where $M_{i,t}$ denotes the number of product varieties for product i. In deriving the above expression, we use the demand for the basket of product i, $C_{i,t}(j) = \rho_{i,t}^{-\theta} C_t(j)$, and the price index of each product basket i, $\rho_{i,t}^{1-\sigma} = M_{i,t}\tilde{\rho}_{i,t}^{1-\sigma}$.

In the model, only a subset of the N_t firms becomes producers due to fixed operational costs, with the mass of producing firms denoted by S_t. Since $S_t = [1 - G(\varphi_t^*)]\, N_t$, we define the average operational profits among potential producers as:

$$\tilde{d}_t = \left(\frac{S_t}{N_t}\right)\tilde{d}_{s,t}. \tag{13}$$

Finally, the average real profits among surviving producers are given by:[3]

$$\tilde{d}_{s,t} = \int_0^1 \frac{M_{i,t}}{S_t}\tilde{d}_{i,t}di - w_t \frac{f_h}{A_t^{\theta_h}}, \tag{14}$$

where we use the fact that:

$$M_{i,t} = \int_{\varphi_t^*}^{\infty} \left[1 - Z_i(\lambda_{i,t}^*(\varphi))\right] \frac{dG(\varphi)}{1 - G(\varphi_t^*)} S_t.$$

2.5 Parametrization of Productivity and Taste Draw

We assume the following Pareto distributions for $G(\varphi)$ and $Z_i(\lambda_i)$, respectively:

$$G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^{\kappa}, \text{ and } \quad Z_i(\lambda_i) = 1 - \left(\frac{\lambda_{i\min}}{\lambda_i}\right)^{v},$$

where $\varphi_{\min}$ and $\lambda_{i\min}$ are the minimum productivity and taste levels, respectively. The parameters κ and v determine the shape of each distribution. As these parameters increase, dispersion decreases, concentrating productivity and tastes toward the lower bounds $\varphi_{\min}$ and $\lambda_{i\min}$. We set $\varphi_{\min} = \lambda_{i\min} = 1$ without loss of generality. To ensure that the variance of the productivity distribution is finite and that the number of products is positive, we assume $\kappa > v > \sigma - 1$.

With this parametrization, we can express the taste-weighted average productivity, $\tilde{\varphi}_{i,t}$, as[4]

$$\tilde{\varphi}_{i,t} = \left[\frac{v}{v - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}} \varphi_t^* \lambda_{i,t}^*(\varphi_t^*), \tag{15}$$

[3] See Appendix A for a detailed derivation.

[4] Using the zero-profit consumer taste cutoff (8) for a firm with productivity φ_t^*, the consumer taste cutoff of a firm with productivity φ_t, i.e., $\lambda_{i,t}^*(\varphi)$, can be expressed as a function of the cutoff productivity level φ_t^* and the consumer taste cutoff of this cutoff firm, $\lambda_{i,t}^*(\varphi_t^*)$, as $\lambda_{i,t}^*(\varphi) = \frac{\varphi_t^*}{\varphi}\lambda_{i,t}^*(\varphi_t^*)$. This expression implies that the cutoff consumer taste of a firm decreases with respect to its own productivity but increases with respect to φ_t^* and $\lambda_{i,t}^*(\varphi_t^*)$, reflecting intensified competition.

The average number of products per producer and the fraction of producers among total firms are given by

$$\frac{M_{i,t}}{S_t} = \frac{\kappa}{\kappa - v}\lambda^*_{i,t}(\varphi^*_t)^{-v}, \quad \frac{S_t}{N_t} = \varphi_t^{*-\kappa}. \tag{16}$$

By combining (15) and (16), we obtain

$$\tilde{\varphi}_{i,t} = \left[\frac{v}{v-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{S_t}{N_t}\right)^{-\frac{1}{\kappa}} \left(\frac{M_{i,t}}{S_t}\frac{\kappa - v}{\kappa}\right)^{-\frac{1}{v}}. \tag{17}$$

Furthermore, the zero-profit productivity cutoff condition (9), $d_{s,t}\left(\varphi^*_t\right) = 0$, can be rewritten as[5]

$$\tilde{d}_{s,t} = \frac{v}{\kappa - v} w_t \frac{f_h}{A_t^{\theta_h}}. \tag{18}$$

For the firm with cutoff-level productivity φ^*_t, we define the zero-profit consumer taste cutoff condition as $d_{i,t}\left(\varphi^*_t, \lambda^*_{i,t}(\varphi^*_t)\right) = 0$ as in equation (8). As a result, we have the following:

$$\tilde{d}_{i,t} = \frac{\sigma - 1}{v-(\sigma-1)} w_t \frac{f_i}{A_t^{\theta_i}}. \tag{19}$$

2.6 Household Budget Constraints and Intertemporal Problems

Household j receives income by supplying labor $L_t(j)$ at the real wage rate w_t, acquiring average dividend income $\tilde{d}_t$, and selling its initial share position v_t of shareholdings x_t in the firms of the economy. The household spends its income on consumption $C_t(j)$ and on purchasing $x_{t+1}(j)$ shares of the existing firms and entrants at share price v_t. The household budget constraint is thus given by:

$$L_t(j)w_t + x_t(j)N_t\left(v_t + \tilde{d}_t\right) = C_t(j) + x_{t+1}(j)v_t\left(N_t + H_t\right). \tag{20}$$

During each period t, the representative household chooses consumption $C_t(j)$, shareholdings $x_{t+1}(j)$, and labor supply $L_t(j)$ to maximize the expected lifetime utility function (1) subject to the budget constraint (20). The first-order conditions with respect to consumption and labor supply yield the following labor supply equation:

$$\chi L_t(j)^{\varsigma} = w_t \Lambda_t(j), \tag{21}$$

where $\Lambda_t(j) = C_t(j)^{-1}$ represents the marginal utility of consumption.

The first-order condition with respect to shareholdings yields:

[5] The detailed derivation is provided in Appendix B.

$$v_t = \beta(1-\delta)E_t\left[\frac{\Lambda_{t+1}(j)}{\Lambda_t(j)}\left(v_{t+1}+\tilde{d}_{t+1}\right)\right]. \tag{22}$$

In deriving the above expression, we have used the law of motion (11).

By iterating equation (22) forward, it follows that share prices are the expected discounted sum of future dividends:

$$v_t = E_t \sum_{k=t+1}^{\infty} [\beta(1-\delta)]^{k-t}\left(\frac{\Lambda_t}{\Lambda_k}\right)\tilde{d}_k. \tag{23}$$

2.7 Model Equilibrium

In equilibrium, the labor market clears: Aggregate labor supply $\int_0^1 L_t(j)dj$ is employed in either the production of consumption goods—including both fixed operational headquarters and product-specific costs—or the creation of new firms:

$$\int_0^1 L_t(j)dj = \int_0^1 M_{i,t}\left(\frac{\tilde{y}_{i,t}}{A_t\tilde{\varphi}_{i,t}}+\frac{f_i}{A_t^{\theta_i}}\right)di + S_t\frac{f_h}{A_t^{\theta_h}} + H_t\frac{f_E}{A_t}. \tag{24}$$

Note that the following equation determines the average scale of production at product level across firms, $\tilde{y}_{i,t}$.

$$\tilde{d}_{i,t} = \frac{\tilde{\rho}_{i,t}}{\sigma}\tilde{y}_{i,t} - w_t\frac{f_i}{A_t^{\theta_i}}.$$

Using the above relation, we can rewrite equation (24) as

$$\int_0^1 L_t(j)dj = \int_0^1 M_{i,t}\left[(\sigma-1)\frac{\tilde{d}_{i,t}}{w_t}+\sigma\frac{f_i}{A_t^{\theta_i}}\right]di + S_t\frac{f_h}{A_t^{\theta_h}} + \frac{H_t v_t}{w_t}. \tag{25}$$

Equation (25) is equivalent to the aggregated accounting identity of GDP obtained by aggregating household budget constraints.

In equilibrium, households are symmetric, implying that $\int_0^1 C_t(j)dj = C_t(j) = C_t$, $\int_0^1 L_t(j)dj = L_t(j) = L_t$, $C_{i,t}(j) = C_{i,t}$, and $\Lambda_t(j) = \Lambda_t$. Furthermore, all products are assumed to be symmetric in equilibrium, leading to

$$\int_0^1 M_{i,t}di = M_{i,t}, \quad \int_0^1 \tilde{d}_{i,t}di = \tilde{d}_{i,t}.$$

Additionally, we define GDP and investment as $Y_t \equiv w_tL_t + N_t\tilde{d}_t$ and $I_t \equiv v_tH_t$. Measurement errors may arise from variations in product varieties. Following Ghironi and Melitz (2005) and Hamano (2022), we assume that statistical agencies fail to capture certain fluctuations in the number of product varieties when computing the price index. The empirical counterpart of the price index is denoted by $\hat{P}_t$. Consequently, any real variable X_t in the model is transformed into its data-consistent counterpart as:

Table 1: Summary of the Benchmark Model

Average pricing of product i	$\tilde{\rho}_{i,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{A_t\tilde{\varphi}_{i,t}}$
Real taste-adjusted price	$\rho_{i,t}^{1-\sigma} = M_{i,t}\tilde{\rho}_{i,t}^{1-\sigma}$
Demand for product i	$C_{i,t} = \rho_{i,t}^{-\theta}C_t$
Aggregate price index	$1 = \rho_i^{1-\theta}$
Average product profits	$\tilde{d}_{i,t} = \frac{1}{\sigma}\frac{\rho_{i,t}C_{i,t}}{M_{i,t}} - w_t\frac{f_i}{A_t^{\theta_i}}$
Average product production	$\tilde{d}_{i,t} = \frac{\tilde{\rho}_{i,t}}{\sigma}\tilde{y}_{i,t} - w_t\frac{f_i}{A_t^{\theta_i}}$
Average producer's profits	$\tilde{d}_{s,t} = \frac{M_{i,t}}{S_t}\tilde{d}_{i,t} - w_t\frac{f_h}{A_t^{\theta_h}}$
Average profits	$\tilde{d}_t = \frac{S_t}{N_t}\tilde{d}_{s,t}$
Consumer taste cutoff	$\tilde{d}_{i,t} = \frac{\sigma-1}{\upsilon-(\sigma-1)}w_t\frac{f_i}{A_t^{\theta_i}}$
Productivity cutoff	$\tilde{d}_{s,t} = \frac{\upsilon}{\kappa-\upsilon}w_t\frac{f_h}{A_t^{\theta_h}}$
Taste-weighted productivity	$\tilde{\varphi}_{i,t} = \left[\frac{\upsilon}{\upsilon-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{S_t}{N_t}\right)^{-\frac{1}{\kappa}}\left(\frac{M_{i,t}}{S_t}\frac{\kappa-\upsilon}{\kappa}\right)^{-\frac{1}{\upsilon}}$
Labor market clearing	$L_t = M_{i,t}\left[(\sigma-1)\frac{\tilde{d}_{i,t}}{w_t} + \sigma\frac{f_i}{A_t^{\theta_i}}\right] + S_t\frac{f_h}{A_t^{\theta_h}} + \frac{H_t v_t}{w_t}$
Free entry condition	$v_t = w_t\frac{f_E}{A_t}\left(\frac{H_t}{H_{t-1}}\right)^{\omega}$
Motion of firms	$N_{t+1} = (1-\delta)(N_t + H_t)$
Euler equation	$v_t = \beta(1-\delta)E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(v_{t+1} + \tilde{d}_{t+1}\right)\right]$
Optimal labor supply	$\chi L_t^{\varsigma} = w_t\Lambda_t$
Marginal utility of consumption	$\Lambda_t = C_t^{-1}$
GDP	$Y_t \equiv w_t L_t + N_t\tilde{d}_t$
Investments	$I_t \equiv v_t H_t$

$$X_{R,t} \equiv \frac{P_t X_t}{\hat{P}_t} = \frac{X_t}{M_{i,t}^{\psi}},$$

where the parameter $0 \leq \psi \leq \frac{1}{\sigma-1}$ determines the extent of measurement errors. When $\psi = \frac{1}{\sigma-1}$, statistical agencies fully capture fluctuations in the number of product varieties, whereas when $\psi = 0$, they fail to capture any of these fluctuations.

Finally, we assume that the log of aggregate labor productivity follows an AR(1) process with persistence ρ_A. The productivity shock $\varepsilon_{A,t}$ is normally distributed with zero mean and variance σ_A^2.

The model consists of 19 equations and 19 endogenous variables, among which the number of firms N_t is a state variable. Table 1 summarizes the benchmark model.

3 Calibration

We calibrate the parameters of the theoretical model to replicate US macroeconomic and product dynamics. We follow Bilbiie, Ghironi, and Melitz (2012) in calibrating the discount factor β, the inverse of the Frisch elasticity of labor supply ς, and the exogenous firm destruction rate δ, using standard values in the literature for the US economy on a quarterly basis. The elasticity of substitution across product varieties σ is set according to Broda and Weinstein (2010), who estimate its value using US micro product data. The parameter κ, which determines the productivity distribution, is set to 11.507 following Hamano and Zanetti (2017), based on the estimates in Broda and Weinstein (2010). The parameter v, which governs the taste distribution, is set to 11 to satisfy the restriction $\kappa > v > \sigma - 1$.

Following Hamano and Zanetti (2017), we set the headquarters fixed costs f_h to replicate the steady-state survival rate $S/N = 0.94$, which corresponds to the mean quarterly destruction rate documented in Broda and Weinstein (2010). Since our calibration is at the product level, we assume that relatively few firms produce any given product. Accordingly, the operational fixed cost for the production of product i, f_i, is set to match $M_i/S = 0.01$, implying that 1% of operating firms produce the product i in the steady state. Finally, the parameter governing the disutility of labor supply, χ, is set to 0.9588 to ensure that the steady-state labor supply is normalized to unity. The entry fixed cost is set to $f_e = 1$ without loss of generality.[6].

The AR(1) process of log A_t follows King and Rebelo (1999), with $\sigma_A = 0.0072$ and $\rho_A = 0.979$, as in Bilbiie, Ghironi, and Melitz (2012) and Hamano and Zanetti (2017). The remaining parameters, which relate to the propagation of the technology shock, are calibrated to match empirical moments. Specifically, to determine the entry adjustment cost ω, technology propagation to operational costs at the firm level θ_S, and at the product level θ_i, we minimize the distance between data moments and those implied by the theoretical model. We target the standard deviation of GDP, consumption, investment, hours worked, and the first-order autocorrelation of hours worked in US data.[7]

As a result, we calibrate the entry adjustment cost parameter as $\omega = 2.0509$, which is consistent with the value used in Bergin, Feng, and Lin (2018) and Lewis and Poilly (2012) in explaining US investment dynamics. We find that the technology propagation to firm-level costs is relatively small, with $\theta_S = 0.1207$, while the product-level propagation θ_i is large, calibrated at 7.2682. Finally, we assume that statistical agencies fail to capture all fluctuations in product varieties by setting $\psi = 0$, following Corsetti, Martin, and Pesenti (2007).

Table 2 summarizes our calibration results.

[6] A detailed derivation of the steady state is provided in Appendix C

[7] We use the `fmin` function in MATLAB to minimize the distance. Equal weights are assigned to each targeted moment in the estimation.

Table 2: Calibration Results

Parameter	Description	Value
β	Discount factor	0.99
ς	Inverse of Frisch elasticity of labor supply	0.25
δ	Exogenous death shock	0.025
σ	Elasticity of substitution of product varieties	11.5
κ	Productivity dispersion	11.507
υ	Taste dispersion	11
f_e	Fixed cost for firm entry	1
f_h	Operational fixed cost for firm	0.001765
f_i	Operational fixed cost for product i	0.190782
χ	Disutility in supplying labor	0.977166
σ_A	Standard deviation of technology shock	0.0072
ρ_A	Persistence of technology shock	0.979
ω	Entry adjustment cost	2.05091
θ_S	Technology spillover to firm-level costs	0.12072
θ_i	Technology spillover to product-level costs	7.26818
ψ	Measurement error in the number of product varieties	0

4 Quantitative Assessment

In this section, we first document the second moments implied by the theoretical model using the benchmark calibration. We then analyze the propagation mechanism of the model by examining impulse response functions following an aggregate technology shock. In particular, we highlight the role played by heterogeneities in technology and consumer tastes.

4.1 Second Moments

Table 3 reports the second moments from both the data and the theoretical model. The model successfully replicates key features of US business cycle dynamics. Specifically, consumption volatility is lower than output volatility in the theoretical model, while investment exhibits the highest volatility, consistent with empirical data.[8] This alignment holds not only for the targeted standard deviations but also for other moments, such as first-order autocorrelations and correlations with GDP. However, the model underestimates the volatility of labor supply and its first-order autocorrelation compared to the data.

The theoretical model also provides insights into several unobservable moments. In particular, we document the standard deviations, first-order autocorrelations, and correlations with output for the number of firms N_t, the number of entrants H_t, the number of producers S_t, and the number of product varieties $M_{i,t}$. As expected, the number of entrants H_t exhibits high volatility, with a standard deviation of 7.01%, since it is a key component of investment $I_{R,t}$. In

[8] The data moments for the US economy are taken from King and Rebelo (1999).

Table 3: Second Moments

		$Y_{R,t}$	$C_{R,t}$	$I_{R,t}$	L_t	N_t	H_t	S_t	$M_{i,t}$
St. dev. (%)	Data	1.81	1.35	5.30	1.79				
	Model	1.62	1.48	5.38	0.21	4.27	7.01	0.40	25.64
Relative to Y_R	Data	1.00	0.74	2.93	0.99				
	Model	1.00	0.91	3.33	0.13	2.64	4.34	0.25	15.87
First-order auto	Data	0.84	0.80	0.87	0.88				
	Model	0.99	0.99	1.00	1.00	1.00	1.00	0.97	0.98
Corr(Y_R, X_t)	Data	1.00	0.88	0.80	0.88				
	Model	1.00	0.99	0.80	0.63	0.83	0.88	0.98	0.99

contrast, the number of firms N_t, which is a state variable in the theoretical model, displays lower volatility than the number of entrants H_t.

Consistent with the relative magnitudes of the calibrated technology propagation parameters ($\theta_h = 0.12$ and $\theta_i = 7.27$), the number of producing firms S_t exhibits limited volatility at 0.40%, whereas the number of product varieties $M_{i,t}$ is highly volatile at 25.64%. The standard deviation of product varieties is nearly 16 times higher than that of GDP. All these variables (N_t, H_t, S_t, and $M_{i,t}$) exhibit high first-order autocorrelation and strong procyclicality.

4.2 Role of Heterogeneity in Technology and Taste

In this subsection, we explore the role of heterogeneity in firm-specific technologies and product-specific tastes. The theoretical model's intuition is best illustrated by the impulse response functions (IRFs). Figure 1 displays the IRFs of key economic variables in response to a one percent increase in the standard deviation of the technology shock $\varepsilon_{A,t}$. The benchmark calibration is represented by solid lines, while the calibration with lower dispersion in both tastes and technologies is shown with dashed lines.

Specifically, in the alternative calibration, we set $\kappa = 50$ and $v = 45$, ensuring that $\kappa > v$. In this less granular economy, characterized by lower dispersion in both taste and productivity, macroeconomic aggregates exhibit greater variability. For the same magnitude of the technology shock, Y_R, C_R, and I_R fluctuate more both on impact and in transitory dynamics. Notably, Y_R and C_R increase by more than twice as much on impact. Additionally, the variability of labor supply L_t remains muted, amounting to less than 0.1 percent under both parameterizations.

Which parameter, κ or v, or both, drives the amplification in the volatility of macroeconomic aggregates? To investigate this, Figure 2 plots the standard deviation of empirically consistent GDP, σ_{Y_R}, against κ and v. The results indicate that the standard deviation increases with higher values of v, while it remains nearly unchanged with respect to κ.

Why does a higher value of v lead to greater aggregate volatility? The explanation is as follows. Following a positive aggregate technology shock, the presence of strong propagations in the operational fixed costs for product production, as captured by our calibration ($\theta_i = 7.27$), causes the number of product varieties $M_{i,t}$ to rise significantly—that is, the number of firms producing product i increases. However, these newly introduced product varieties have lower taste-adjusted productivities, as they are produced by firms with lower firm-specific technology and weaker consumer tastes (as represented by the decline in $\tilde{\varphi}_{i,t}$ in Figure 1).

On one hand, the sharp increase in product varieties intensifies competition, reducing both the scale of production and the profits associated with each product variety (reflected by the decline in $\tilde{d}_{i,t}$ and $\tilde{y}_{i,t}$). On the other hand, despite the drop in per-product profits, the increase in the number of product varieties raises the total profits earned by firms on average ($\tilde{d}_{s,t}$ increases).

The key observation, as shown in Figure 1, is that when the dispersion in tastes is low (i.e., a larger value of v), the downward adjustments in $\tilde{y}_{i,t}$ and $\tilde{d}_{i,t}$ are smaller, as indicated by the dashed lines. This occurs because when product varieties are less differentiated in terms of taste, given a similar expansion in the number of product varieties $M_{i,t}$ (driven by the large value of θ_i), a more substantial downward adjustment of the taste-weighted cutoff, represented by $\tilde{\varphi}_{i,t}$, is required. Consequently, the contraction in production scale and per-product profits is smaller, leading to larger profit expansions at the firm level. In contrast, when the granularity in taste is higher (i.e., a smaller value of v), variations in $M_{i,t}$ induced by strong propagations necessitate greater adjustments in per-product production scale, thereby absorbing more aggregate volatility.

The analysis of IRFs is instrumental in understanding the mechanisms of the theoretical model. However, it also reveals some limitations: Why is aggregate volatility more sensitive to v than to κ? How do the granularity of taste and productivity depend precisely on other economic conditions, such as θ_h and θ_i? The next section addresses these questions through analytical solutions.

5 Analytical Investigation with a Simpler Model

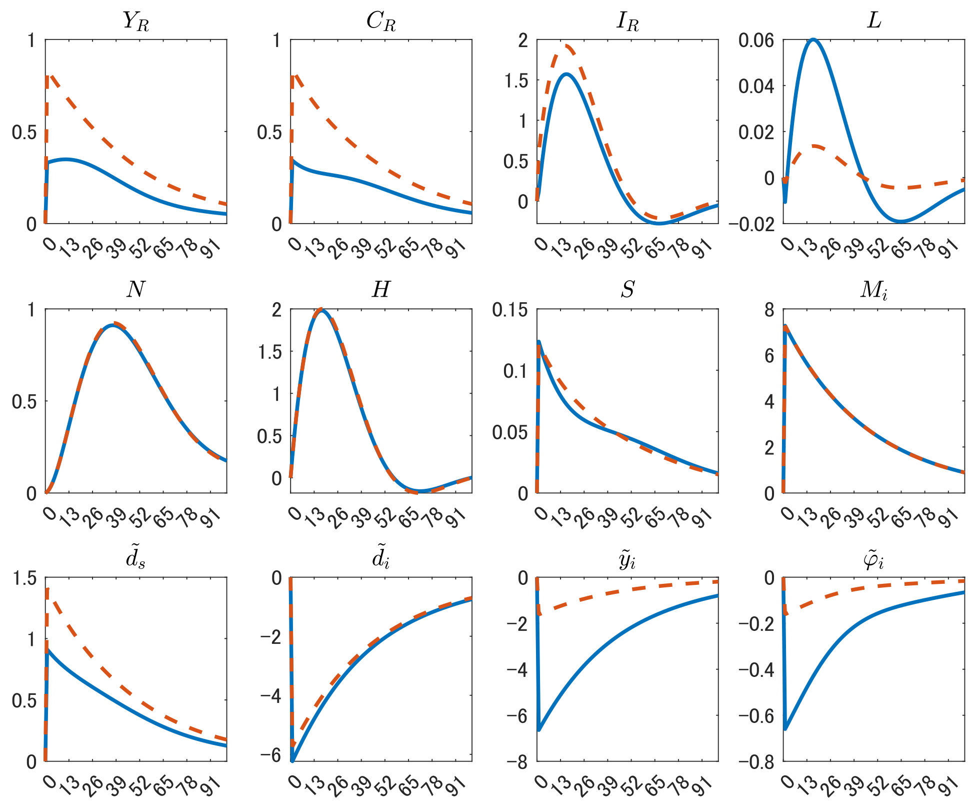
The benchmark model is solved quantitatively due to its non-linearity, which complicates analytical investigation. The intuition developed in the previous section suggests that taste heterogeneity plays a more significant role than productivity heterogeneity in driving higher volatility in macroeconomic aggregates. Is this a general conclusion? How do taste and productivity heterogeneity interact with other parameters? Is there a way to analyze these relationships more precisely? To address these questions, we develop a simpler model.[9]

Specifically, we make the following two modifications to the benchmark model (1):

1) We assume that firms survive for only one period after entry and fully depreciate, so the equation of motion for firms (11) is replaced by $H_t = N_{t+1}$. Consequently, the budget constraint takes a slightly different form:

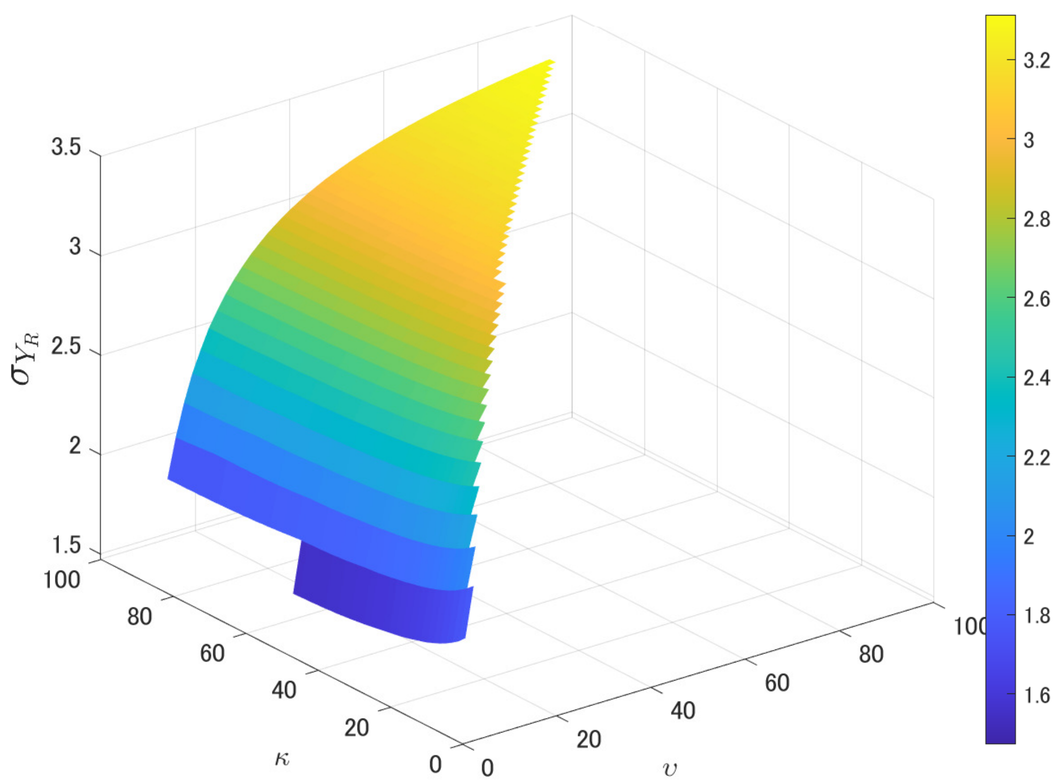
[9]The modeling techniques used to obtain an analytical solution are similar to those presented in Bilbiie (2021), Hamano and Zanetti (2022), and Hamano and Pappadà (2023).

Figure 1: Impulse Response Functions



Note: Figure shows the IRFs of major economic variables of the model following a one percent increase in the standard deviation of the technology shock $\varepsilon_{A,t}$. Solid lines represent the benchmark calibration as in Table 2. Dashed lines represent the alternative calibration with $\kappa = 50$ and $\upsilon = 45$.

Figure 2: Standard Deviation of GDP



Note: Figure shows the standard deviation of empirically consistent GDP Y_R against different values of κ and υ. The model is solved under the restriction $\kappa > \upsilon > \sigma - 1$.

$$L_t w_t + x_t N_t \tilde{d}_t = C_t + x_{t+1} v_t N_{t+1}.$$

As a result, the Euler equation (22) with respect to shareholdings simplifies to:

$$v_t = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \tilde{d}_{t+1} \right]. \tag{26}$$

2) The entry adjustment costs are eliminated by setting $\omega = 0$.

The simplified model developed in this section allows for a closed-form solution while retaining the key features of the benchmark model. The solutions of the simplified model are presented in Table 4.[10]

5.1 Equilibrium in the Simple Model

In the simplified model, the equilibrium labor supply L_t remains constant over time, while the wage w_t increases when the productivity level A_t is high. The primary mechanism of the theoretical model mirrors that of the benchmark model analyzed using IRFs: A positive technology shock raises income, increases consumption, and shifts the labor supply schedule: $L_t = \left(\frac{w_t}{\chi C_t}\right)^{\frac{1}{\varsigma}}$, which reduces labor supply at each wage level.

Simultaneously, the positive technology shock that increases income also raises labor demand for production, as reflected on the right-hand side of equation (25). In equilibrium, wages rise while labor supply remains constant in the simplified model due to a specific wealth effect that offsets the increase in labor supply. These results are consistent with the muted response of labor supply observed in the IRFs of the benchmark model.

Furthermore, similar to the benchmark model, the number of entrants N_{t+1} increases with aggregate labor productivity A_t. The number of producers S_t and product varieties $M_{i,t}$ also rise with higher aggregate productivity, although these variables also depend on the propagation coefficients θ_h and θ_i, respectively.

5.2 Aggregate Volatility and Heterogeneities

As pointed out in the previous section, granularity in tastes and technologies can be an important determinant of aggregate volatility in the economy. To illustrate this point, we focus on the volatility of GDP. Specifically, the log of the empirically consistent measure of GDP can be rewritten as follows,[11]

$$\log Y_{R,t} = \frac{1}{\kappa} \log N_t + \left[1 + \theta_h \left(\frac{1}{v} - \frac{1}{\kappa} \right) + \theta_i \left(\psi - \frac{1}{v} \right) \right] \log A_t + cst,$$

[10] Appendix D provides detailed derivations.

[11] See Appendix E for derivation.

Table 4: Solution of the Simple Model

$$L_t = \left\{ \frac{1}{\sigma} \left[\frac{\upsilon-(\sigma-1)}{\upsilon} \left[\frac{(\sigma-1)^2}{\upsilon-(\sigma-1)} + \sigma \right] + \frac{\sigma-1}{\upsilon} \frac{\kappa-\upsilon}{\kappa} + \beta \frac{\sigma-1}{\kappa} \right] \frac{1}{\chi} \right\}^{\frac{1}{\varsigma+1}}$$

$$\Theta w_t = N_t^{\frac{1}{\kappa}} \left(\frac{1}{L_t^{\varsigma}} \right)^{\frac{1}{\sigma-1} - \frac{1}{\kappa}} f_h^{-\left(\frac{1}{\upsilon} - \frac{1}{\kappa}\right)} f_i^{-\left(\frac{1}{\sigma-1} - \frac{1}{\upsilon}\right)} A_t^{1+\theta_h\left(\frac{1}{\upsilon} - \frac{1}{\kappa}\right)+\theta_i\left(\frac{1}{\sigma-1} - \frac{1}{\upsilon}\right)}$$

$$\Theta \equiv \frac{\sigma}{\sigma-1} \left\{ \left[\frac{\upsilon}{\upsilon-(\sigma-1)} \right]^{\frac{1}{\sigma-1}} \left(\frac{\kappa-\upsilon}{\kappa} \right)^{-\frac{1}{\upsilon}} \left[\frac{1}{\sigma} \frac{\sigma-1}{\upsilon} \frac{\kappa-\upsilon}{\kappa} \right]^{\left(\frac{1}{\upsilon} - \frac{1}{\kappa}\right)} \left[\frac{1}{\sigma} \frac{\upsilon-(\sigma-1)}{\upsilon} \right]^{\frac{1}{\sigma-1} - \frac{1}{\upsilon}} \right\}^{-1} \chi^{\frac{1}{\sigma-1} - \frac{1}{\kappa}}$$

$$C_t = \frac{w_t}{\chi L_t^{\varsigma}}$$

$$N_{t+1} = \frac{\beta}{\sigma} \frac{\sigma-1}{\kappa} \frac{C_t}{w_t} \frac{A_t}{f_E}$$

$$S_t = \frac{1}{\sigma} \frac{\sigma-1}{\upsilon} \frac{\kappa-\upsilon}{\kappa} \frac{C_t}{w_t} \frac{A_t^{\theta_h}}{f_h}$$

$$M_{i,t} = \frac{1}{\sigma} \frac{\upsilon-(\sigma-1)}{\upsilon} \frac{C_t}{w_t} \frac{A_t^{\theta_i}}{f_i}$$

$$\tilde{\varphi}_{i,t} = \left[\frac{\upsilon}{\upsilon-(\sigma-1)} \right]^{\frac{1}{\sigma-1}} \left(\frac{S_t}{N_t} \right)^{-\frac{1}{\kappa}} \left(\frac{M_{i,t}}{S_t} \frac{\kappa-\upsilon}{\kappa} \right)^{-\frac{1}{\upsilon}}$$

$$\tilde{\rho}_{i,t} = \frac{\sigma}{\sigma-1} \frac{w_t}{A_t \tilde{\varphi}_{i,t}}$$

$$\tilde{d}_t = \frac{S_t}{N_t} \tilde{d}_{s,t}$$

$$\tilde{d}_{s,t} = \frac{\upsilon}{\kappa-\upsilon} w_t \frac{f_h}{A_t^{\theta_h}}$$

$$\tilde{d}_{i,t} = \frac{\sigma-1}{\upsilon-(\sigma-1)} w_t \frac{f_i}{A_t^{\theta_i}}$$

$$v_t = w_t \frac{f_E}{A_t}$$

$$Y_t = \left(L_t + \frac{1}{\sigma} \frac{\sigma-1}{\kappa} \frac{1}{\chi L_t^{\varsigma}} \right) w_t$$

$$I_t = v_t N_{t+1}$$

where $N_t = \frac{\beta}{\sigma}\frac{\sigma-1}{\kappa}\frac{C_{t-1}}{w_{t-1}}\frac{A_{t-1}}{f_E}$, and *cst* includes time-invariant constant terms. The variance of the log of empirically consistent GDP can be computed from the above expression, leading to the following proposition.

Proposition 1. *Volatility of GDP and taste heterogeneity. The volatility of empirically consistent GDP, $Y_{R,t}$, increases with respect to v when θ_i is sufficiently larger than θ_h within reasonable bounds.*

Proof. The variance of the empirically consistent measure of GDP, denoted as $\sigma^2_{\log Y_{R,t}}$, is expressed as

$$\sigma^2_{\log Y_{R,t}} = \left\{ \left(\frac{1}{\kappa}\right)^2 + \left[1 + \theta_h\left(\frac{1}{v} - \frac{1}{\kappa}\right) + \theta_i\left(\psi - \frac{1}{v}\right)\right]^2 + \frac{2}{\kappa}\left[1 + \theta_h\left(\frac{1}{v} - \frac{1}{\kappa}\right) + \theta_i\left(\psi - \frac{1}{v}\right)\right]\rho_A \right\}\sigma^2_{\log A_t}, \quad (27)$$

where $\sigma^2_{\log A_t} = \frac{\sigma_A^2}{1-\rho_A^2}$.

Taking the derivative of the above expression with respect to v, we obtain

$$\frac{\partial \sigma^2_{\log Y_{R,t}}}{\partial v} = -\frac{2\left(\theta_h - \theta_i\right)}{v^2}\left[1 + \theta_h\left(\frac{1}{v} - \frac{1}{\kappa}\right) + \theta_i\left(\psi - \frac{1}{v}\right) + \frac{1}{\kappa}\rho_A\right]. \quad (28)$$

The first term before the square brackets is positive when $\theta_h < \theta_i$. For the second term in the square brackets, $1 + \theta_h\left(\frac{1}{v} - \frac{1}{\kappa}\right) + \theta_i\left(\psi - \frac{1}{v}\right)$, let us consider the case with $\psi = 0$, which gives its smallest value for given parameter values. Even in this case, under the parameter restriction $\kappa > v$, the second term remains positive unless θ_i becomes unrealistically large. Thus, we conclude that the derivative is positive when θ_i is sufficiently larger than θ_h within reasonable bounds.

The proposition highlights that when fluctuations in the number of product varieties $M_{i,t}$ exceed those in the number of producers S_t due to a relatively strong technological spillover ($\theta_h < \theta_i$), GDP volatility tends to be high when the dispersion of tastes is small (i.e., higher v).

The results are consistent with those obtained in the benchmark model in the previous section. Indeed, when we evaluate the derivative using the parameter values in Table 2, we obtain $\frac{\partial \sigma^2_{\log Y_{R,t}}}{\partial v} = 0.0502$.

The above value of the derivative is derived under $\psi = 0$, implying zero capturing of fluctuations in the number of product varieties, which reduces the derivative. In reality, this parameter could be greater than zero. If statistical agencies capture these fluctuations perfectly, i.e., $\psi = \frac{1}{\sigma-1}$, the derivative $\frac{\partial \sigma^2_{\log Y_{R,t}}}{\partial v}$ is larger, leading to the following corollary.

Corollary 1. *The volatility of empirically consistent GDP, $Y_{R,t}$, increases strictly with respect to v when $\theta_h < \theta_i$ and $\psi = \frac{1}{\sigma-1}$.*

Proof. The derivative is strictly positive when $\theta_h < \theta_i$ and $\psi = \frac{1}{\sigma-1}$ since, under the given parameter restrictions, we have $1 + \theta_h \left(\frac{1}{v} - \frac{1}{\kappa}\right) + \theta_i \left(\frac{1}{\sigma-1} - \frac{1}{v}\right) > 0$.

The relationship between aggregate volatility and firm-level productivity heterogeneity has been extensively discussed in the literature as a granular origin of aggregate fluctuations (Gabaix, 2011). According to this strand of the literature, firm-specific shocks cannot fully dissipate in aggregate due to the fat-tailed firm size distribution. Granularity in firm size thus helps explain aggregate fluctuations. In our model, we assume a fat-tailed distribution by applying a Pareto distribution for both firm-specific productivities and tastes. We find that this mechanism is equally relevant in our framework.

Proposition 2. *Volatility of GDP and heterogeneity in productivities. The volatility of empirically consistent GDP, Y_R, decreases with respect to κ when $\theta_h < \theta_i$ and when both θ_h and θ_i are within reasonable bounds.*

Proof. Taking the derivative of equation (27) with respect to κ, we obtain

$$\frac{\partial \sigma^2_{\log Y_{R,t}}}{\partial \kappa} = -\frac{2}{\kappa^2} \left\{ \frac{1}{\kappa} + \left[\rho_A \left(1 + \frac{1}{\kappa}\right) - \theta_h \right] \left[1 + \theta_h \left(\frac{1}{v} - \frac{1}{\kappa}\right) + \theta_i \left(\psi - \frac{1}{v}\right) \right] \right\}. \tag{29}$$

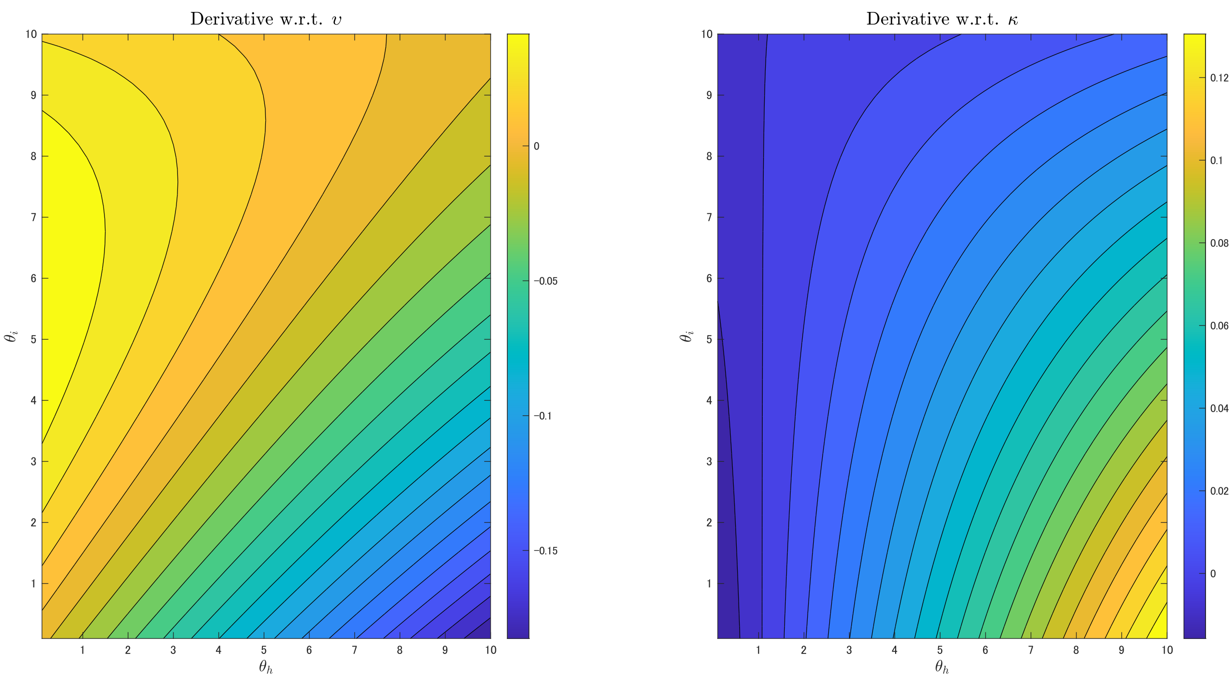
The first term in the braces, $\frac{1}{\kappa}$, is positive. The second term in the square brackets, $\rho_A \left(1 + \frac{1}{\kappa}\right) - \theta_h$, can be positive for realistic parameter values. It becomes negative only when θ_h takes unrealistically large values. For the third term in the square brackets, $1 + \theta_h \left(\frac{1}{v} - \frac{1}{\kappa}\right) + \theta_i \left(\psi - \frac{1}{v}\right)$, let us consider the case with $\psi = 0$, which provides the smallest possible value of the term. Even in this case, under the parameter restriction $\kappa > v$, this term remains positive unless θ_i becomes unrealistically large. Thus, we can safely conclude that the derivative is negative under reasonable values of θ_h and θ_i, assuming $\theta_h < \theta_i$.

The proposition highlights that when fluctuations in the number of product varieties $M_{i,t}$ exceed those in the number of producers S_t due to a relatively strong technological propagation such that $\theta_h < \theta_i$, GDP volatility tends to be lower when the dispersion of firm productivities decreases, corresponding to a higher value of κ.

However, the impact of firm heterogeneity on aggregate volatility is found to be quantitatively small. When we evaluate the derivative using the parameter values in Table 2, we obtain $\frac{\partial \sigma^2_{\log Y_{R,t}}}{\partial \kappa} = -0.00615$. This result is consistent with our findings from the general model, as shown in Figure 2.

Finally, we provide numerical evaluations of both derivatives with respect to different values of θ_h and θ_i in Figure 3. Other parameter values are taken from Table 2. The figure confirms the above two propositions. In summary, both taste dispersion and idiosyncratic product dispersion influence aggregate volatility. Quantitatively, we find that volatility in taste plays a more significant role in determining aggregate volatility to match the US data.

Figure 3: Sensitivity of GDP Volatility w.r.t υ and κ



Note: Figure shows the derivatives of the variance of empirically consistent GDP with respect to κ and υ, i.e., equation (28) and (29) for the parameter space between θ_i and θ_h.

6 Conclusion

This paper examines the role of heterogeneity in product-level tastes and firm-level technologies in shaping macroeconomic fluctuations. By developing a stylized general equilibrium model that incorporates multi-product firms, endogenous firm entry, and exogenous exit, we explore how firms adjust their product mix in response to aggregate shocks and how this adjustment influences business cycle dynamics.

Through calibration with U.S. data, we show that our model successfully replicates key business cycle moments, demonstrating a strong propagation of aggregate technology shocks into product-specific fixed operational costs compared to firm-specific fixed costs.

Our key findings highlight the importance of taste dispersion in amplifying or dampening aggregate fluctuations. When the dispersion of tastes is low, firms experience less downward adjustment in per-product profits, leading to greater expansion in firm-level profits and higher volatility in macroeconomic aggregates such as GDP and consumption. Conversely, high taste dispersion reduces aggregate volatility by distributing higher competitive pressures across firms. While firm-level productivity granularity also contributes to fluctuations, its quantitative impact is found to be relatively minor.

Additionally, we develop a simplified analytical model to reinforce our numerical findings, showing that the extent to which aggregate technology shocks translate into firm- and product-level costs plays a critical role in determining aggregate volatility.

Our results contribute to the growing literature on firm heterogeneity and business cycle dynamics. While previous studies have focused on firm size distributions and firm-specific shocks as sources of aggregate fluctuations, our study identifies a novel amplification mechanism based on taste and productivity heterogeneity. The findings also have implications for policy, suggesting that changes in market structure and product differentiation along taste or quality can influence macroeconomic stability.

Future research could extend this framework by incorporating other frictions such as nominal rigidities, international trade dynamics, or alternative firm competition structures. Empirical validation using micro-level product and firm data across different economies would also provide further insights into the mechanisms driving aggregate volatility.

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A Average Profits

With $S_t = [1 - G(\varphi_t^*)] N_t$, defining $\tilde{d}_t(\varphi)$ as the expected profits of a firm with productivity φ, the expected profit of potential producers is given by:

$$\tilde{d}_t = [1 - G(\varphi_t^*)] \int_{\varphi_t^*}^{\infty} \tilde{d}_t(\varphi) \frac{dG(\varphi)}{1 - G(\varphi_t^*)} = \frac{S_t}{N_t} \tilde{d}_{s,t}.$$

Similarly, defining $\tilde{d}_{i,t}\left(\varphi, \lambda_{i,t}^*(\varphi)\right)$ as the average realized profits of a firm with productivity φ for product i, the average realized profits of surviving producers are:

$$\begin{aligned}\tilde{d}_{s,t} &= \int_{\varphi_t^*}^{\infty} \tilde{d}_t(\varphi) \frac{dG(\varphi)}{1 - G(\varphi_t^*)} \\ &= \int_{\varphi_t^*}^{\infty} \sum_i^J \left[1 - Z_i(\lambda_{i,t}^*(\varphi))\right] \tilde{d}_{i,t}\left(\varphi, \lambda_{i,t}^*(\varphi)\right) di \frac{dG(\varphi)}{1 - G(\varphi_t^*)} - w_t \frac{f_h}{A_t^{\theta_h}}.\end{aligned}$$

Using the definition of $\tilde{\lambda}_{i,t}(\varphi)$, we express

$$\begin{aligned}\tilde{d}_{i,t}\left(\varphi, \lambda_{i,t}^*(\varphi)\right) &= \int_{\lambda_{i,t}^*(\varphi)}^{\infty} \left[\frac{1}{\sigma}\left(\frac{\rho_{i,t}(\varphi, \lambda_i)}{\lambda_i}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta} \alpha_i C_t^{\epsilon_i} - w_t \frac{f_i}{A_t^{\theta_i}}\right] \frac{dZ_i(\lambda_i)}{1 - Z_i(\lambda_{i,t}^*(\varphi))} \\ &= \frac{1}{\sigma}\left(\frac{\sigma}{\sigma - 1}\frac{w_t}{A_t}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta} \alpha_i C_t^{\epsilon_i} \int_{\lambda_{i,t}^*(\varphi)}^{\infty} (\lambda_i \varphi)^{\theta - 1} \frac{dZ_i(\lambda_i)}{1 - Z_i(\lambda_{i,t}^*(\varphi))} \\ &\quad - \int_{\lambda_{i,t}^*(\varphi)}^{\infty} w_t \frac{f_i}{A_t^{\theta_i}} \frac{dZ_i(\lambda_i)}{1 - Z_{i,t}(\varphi_t^*)}.\end{aligned}$$

Rewriting the above expression,

$$\begin{aligned}\tilde{d}_{s,t} = \int_{\varphi_t^*}^{\infty} \sum_i^J \left[1 - Z_i(\lambda_{i,t}^*(\varphi))\right] \left[\frac{1}{\sigma}\left(\frac{\sigma}{\sigma - 1}\frac{w_t}{A_t}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta} \alpha_i C_t^{\epsilon_i} \tilde{\lambda}_{i,t}(\varphi) - w_t \frac{f_i}{A_t^{\theta_i}}\right] di \frac{dG(\varphi)}{1 - G(\varphi_t^*)} \\ - w_t \frac{f_h}{A_t^{\theta_h}}.\end{aligned}$$

Using the relationship $\tilde{\varphi}_{i,t}^{\sigma-1} = \int_{\varphi_t^*}^{\infty} \tilde{\lambda}_{i,t}(\varphi) \frac{dG(\varphi)}{1 - G(\varphi_t^*)}$, we obtain:

$$\tilde{d}_{i,t} = \frac{1}{\sigma} \tilde{\rho}_{i,t}^{1-\sigma} \rho_{i,t}^{\sigma-\theta} \alpha_i C_t^{\epsilon_i} - w_t \frac{f_i}{A_t^{\theta_i}}.$$

Using equation (4) and the identity $\rho_{i,t}^{1-\sigma} = M_{i,t} \tilde{\rho}_{i,t}^{1-\sigma}$, we further rewrite:

$$\tilde{d}_{i,t} = \frac{1}{\sigma} \frac{\rho_{i,t} C_{i,t}}{M_{i,t}} - w_t \frac{f_i}{A_t^{\theta_i}}.$$

Finally, we express the average realized profits of surviving producers as:

$$\tilde{d}_{s,t} = \sum_{i}^{J} \frac{M_{i,t}}{S_t} \tilde{d}_{i,t} di - w_t \frac{f_h}{A_t^{\theta_h}},$$

where $M_{i,t} = \int_{\varphi_t^*}^{\infty} \left[1 - Z_i(\lambda_{i,t}^*(\varphi))\right] \frac{dG(\varphi)}{1-G(\varphi_t^*)} S_t$.

B Zero Profit Consumer Taste Cutoff and Zero Profit Cutoff

The zero profit consumer taste cutoff (ZPCT) for a firm with cutoff productivity implies:

$$d_{i,t}\left(\varphi^*, \lambda_{i,t}^*\left(\varphi^*\right)\right) = \frac{1}{\sigma}\left(\frac{\rho_{i,t}\left(\varphi^*, \lambda_i^*\right)}{\lambda_{i,t}^*\left(\varphi^*\right)}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta} \alpha_i C_t^{\epsilon_i} - w_t \frac{f_i}{A_t^{\theta_i}} = 0.$$

Substituting the equilibrium price:

$$\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1} \frac{w_t}{Z_t \varphi_t^* \lambda_{i,t}^*(\varphi_t^*)}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta} \alpha_i C_t^{\epsilon_i} = w_t \frac{f_i}{A_t^{\theta_i}}.$$

Using this relation in the average realized product profits, we obtain:

$$\begin{aligned}
\tilde{d}_{i,t} = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1} \frac{w_t}{A_t \tilde{\varphi}_{i,t}}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta} \alpha_i C_t^{\epsilon_i} - w_t \frac{f_i}{A_t^{\theta_i}} \\
= \left[\left(\frac{\varphi_t^* \lambda_{i,t}^*(\varphi_t^*)}{\tilde{\varphi}_{i,t}}\right)^{1-\sigma} - 1\right] w_t \frac{f_i}{A_t^{\theta_i}} \\
= \left[\frac{\sigma-1}{v-(\sigma-1)}\right] w_t \frac{f_i}{A_t^{\theta_i}},
\end{aligned}$$

where we have used the property $\left(\frac{\varphi_t^* \lambda_{i,t}^*(\varphi_t^*)}{\tilde{\varphi}_{i,t}}\right)^{1-\sigma} = \frac{v}{v-(\sigma-1)}$ implied by the Pareto distribution.

The zero profit cutoff condition further implies:

$$
\begin{aligned}
d_{s,t}\left(\varphi_t^*\right) &= \sum_i^J \left[1 - Z_i(\lambda_{i,t}^*(\varphi_t^*))\right] \int_{\lambda_{i,t}^*(\varphi_t^*)}^{\infty} d_{i,t}\left(\varphi_t^*, \lambda_i\right) \frac{dZ_i(\lambda_i)}{1 - Z_i(\lambda_{i,t}^*(\varphi_t^*))} di - w_t \frac{f_h}{A_t^{\theta_h}} \\
&= \sum_i^J \left[1 - Z_i(\lambda_{i,t}^*(\varphi_t^*))\right] \int_{\lambda_{i,t}^*(\varphi_t^*)}^{\infty} \left[\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t\varphi_t^*\lambda_i}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta}\alpha_i C_t^{\epsilon_i} - w_t\frac{f_i}{A_t^{\theta_i}}\right] \\
&\qquad \times \frac{dZ_i(\lambda_i)}{1 - Z_i(\lambda_{i,t}^*(\varphi_t^*))} di - w_t \frac{f_h}{A_t^{\theta_h}} \\
&= \sum_i^J \left[\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t\varphi_t^*\lambda_{i,t}^*(\varphi_t^*)}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta}\alpha_i C_t^{\epsilon_i} \frac{v}{v-(\sigma-1)} - w_t\frac{f_i}{A_t^{\theta_i}}\right] \\
&\qquad \times \lambda_{i,t}^*\left(\varphi_t^*\right)^{-v} di - w_t \frac{f_h}{A_t^{\theta_h}} \\
&= \sum_i^J \left[\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t\tilde{\varphi}_{i,t}}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\theta}\alpha_i C_t^{\epsilon_i} - w_t\frac{f_i}{A_t^{\theta_i}}\right] \frac{\kappa-v}{\kappa}\frac{M_{i,t}}{S_t} di - w_t \frac{f_h}{A_t^{\theta_h}} \\
&= \frac{\kappa-v}{\kappa}\sum_i^J \tilde{d}_{i,t}\frac{M_{i,t}}{S_t} di - w_t \frac{f_h}{A_t^{\theta_h}} = 0.
\end{aligned}
$$

From the first to the second line, we have used the integral property implied by the Pareto distribution, $Z_i(\lambda_i) = 1 - \left(\frac{\lambda_{i\,\min}}{\lambda_i}\right)^v$. From the second to the third line, we used $\frac{M_{i,t}}{S_t} = \frac{\kappa}{\kappa-v}\lambda_{i,t}^*\left(\varphi_t^*\right)^{-v}$ along with $\left(\frac{\varphi_t^*\lambda_{i,t}^*(\varphi_t^*)}{\tilde{\varphi}_{i,t}}\right)^{1-\sigma} = \frac{v}{v-(\sigma-1)}$. Finally, substituting the expression for $\tilde{d}_{s,t}$ derived earlier, we obtain equation (18).

C Steady State

We begin by deriving the steady state of the benchmark model. The Euler equation (22) gives:

$$
\frac{1}{\beta} = (1-\delta)\left(1 + \frac{\tilde{d}}{v}\right). \tag{C.1}
$$

Using the average profit equation (13), the ZCP equation (18), and the free entry condition (10) at the steady state, we can express equation (C.1) as:

$$
\frac{1}{\beta} = (1-\delta)\left(1 + \frac{S}{N}\frac{v}{\kappa-v}\frac{f_h}{f_E}\right),
$$

which determines the steady state endogenous destruction rate, S/N, given operational fixed costs, f_h, with $f_E = 1$.

By substituting (19) and (18) into (14), we obtain:

$$
\frac{\kappa}{\kappa-v} f_h = \frac{\sigma-1}{v-(\sigma-1)}\frac{M_i f_i}{S},
$$

which determines $\frac{M_i f_i}{S}$ given f_h. With a target value of $\frac{M_i}{S}$, this equation also determines f_i.

From the law of motion of products (11), we derive the number of new products:

$$H = \frac{\delta N}{1-\delta}.$$

Using these relationships and substituting (19) into the labor market clearing condition (25), we obtain:

$$\frac{L}{N} = \left[\frac{(\sigma-1)^2}{\upsilon-(\sigma-1)} + \sigma\right] \frac{S}{N}\frac{M_i}{S} f_i + \frac{S}{N} f_h + \frac{\delta}{1-\delta} f_E, \tag{C.2}$$

which, with $L = 1$, determines N.

From (12) and (19), along with $C_i = C$ and $\rho_i = 1$, we get:

$$\left[\frac{\sigma-1}{\upsilon-(\sigma-1)} + 1\right] w f_i = \frac{1}{\sigma}\frac{C}{M_i}.$$

This can be rewritten as:

$$\left[\frac{\sigma-1}{\upsilon-(\sigma-1)} + 1\right] \frac{w}{C} S \frac{M_i f_i}{S} = \frac{1}{\sigma}.$$

Summing over all products and using the definition of the price index (5) along with $\chi L^{\varsigma} = wC^{-1}$, we obtain:

$$\left[\frac{\sigma-1}{\upsilon-(\sigma-1)} + 1\right] \chi L^{\varsigma} \frac{S}{N} N \frac{M_i}{S} f_i = \frac{1}{\sigma}.$$

This equation determines χ.

Substituting L from (C.2), we finally obtain:

$$\left[\frac{\sigma-1}{\upsilon-(\sigma-1)} + 1\right] \chi \left\{ \left[\frac{(\sigma-1)^2}{\upsilon-(\sigma-1)} + \sigma\right] \frac{S}{N}\frac{M_i f_i}{S} + \frac{S}{N} f_h + \frac{\delta}{1-\delta} f_E \right\}^{\varsigma} N^{\varsigma+1} \frac{S}{N}\frac{M_i f_i}{S} = \frac{1}{\sigma}.$$

This determines N given f_h, f_E, and χ, since S/N and $\frac{M_i f_i}{S}$ are functions of f_h. Once we solve for N, we can easily determine L and S.

Given $\frac{M_i}{S}$, we can pin down M_i. In calibration, we first set $\frac{M_i}{S}$ based on data. From:

$$\tilde{\varphi}_i = \left[\frac{\upsilon}{\upsilon-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{S}{N}\right)^{-\frac{1}{\kappa}} \left(\frac{M_i}{S}\frac{\kappa-\upsilon}{\kappa}\right)^{-\frac{1}{\upsilon}},$$

we compute $\tilde{\varphi}_i$ and M_i.

Next, we solve for the steady state value of w. Using the price index equation (5), the price index of each product basket i satisfies $1 = M_{i,t}\tilde{\rho}_{i,t}^{1-\sigma}$, which determines $\tilde{\rho}_i$. Since the average price of product i satisfies:

$$\tilde{p}_i = \frac{\sigma}{\sigma - 1}\frac{w}{\tilde{\varphi}_i},$$

With the above equation, we can solve for w. Finally, using the labor supply condition $\chi L^{\varsigma} = wC^{-1}$, we determine consumption.

D Solutions of the Simple Model

In this appendix, we demonstrate the closed-form solution of the model and show how the variability of aggregate variables can be expressed.

D.1 Expressions for S_t, $M_{i,t}$, and N_{t+1}

We first express S_t, $M_{i,t}$, and N_{t+1} as functions of aggregate technology, consumption, wages, and other parameters in the model.

By combining (12) and (19), we obtain:

$$M_{i,t} = \frac{1}{\sigma}\frac{\upsilon - (\sigma - 1)}{\upsilon}\frac{C_t}{w_t}\frac{A_t^{\theta_i}}{f_i}. \tag{D.1}$$

Similarly, by combining (14) and (18), we obtain:

$$\left(\frac{\upsilon}{\kappa - \upsilon} + 1\right)\frac{f_h}{A_t^{\theta_h}} = \frac{M_{i,t}}{S_t}\frac{\sigma - 1}{\upsilon - (\sigma - 1)}\frac{f_i}{A_t^{\theta_i}}.$$

Substituting (D.1) into the above equation, S_t is given by:

$$S_t = \frac{1}{\sigma}\frac{\sigma - 1}{\upsilon}\frac{\kappa - \upsilon}{\kappa}\frac{C_t}{w_t}\frac{A_t^{\theta_h}}{f_h}. \tag{D.2}$$

By combining the free entry condition (10) with $\omega = 0$, the definition of the share price (26), and the average profit equation (13), we get:

$$\beta E_t\left[\frac{C_t}{C_{t+1}}\frac{S_{t+1}}{N_{t+1}}\tilde{d}_{s,t+1}\right] = w_t\frac{f_E}{A_t}.$$

With the zero-profit cutoff (18), the above equation simplifies to:

$$\beta E_t\left[\frac{C_t}{C_{t+1}}\frac{S_{t+1}}{N_{t+1}}\frac{\upsilon}{\kappa - \upsilon}w_{t+1}\frac{f_h}{A_{t+1}^{\theta_h}}\right] = w_t\frac{f_E}{A_t}.$$

Finally, substituting (D.2) into the above expression and transforming, we obtain:

$$N_{t+1} = \frac{\beta}{\sigma}\frac{\sigma - 1}{\kappa}\frac{C_t}{w_t}\frac{A_t}{f_E}. \tag{D.3}$$

D.2 Solution for L_t and w_t

In this subsection, we derive the solutions for L_t, w_t, and other variables in the simple model.

Substituting (19) and the free entry condition with $\omega = 0$ into the labor market clearing condition (25), we obtain:

$$L_t = M_{i,t}\left[\frac{(\sigma-1)^2}{\upsilon-(\sigma-1)}+\sigma\right]\frac{f_i}{A_t^{\theta_i}} + S_t\frac{f_h}{A_t^{\theta_h}} + N_{t+1}\frac{f_E}{A_t}.$$

Substituting (D.3), (D.2), and (D.1) into the above expression, we get:

$$L_t = \frac{1}{\sigma}\left[\frac{\upsilon-(\sigma-1)}{\upsilon}\left[\frac{(\sigma-1)^2}{\upsilon-(\sigma-1)}+\sigma\right]+\frac{\sigma-1}{\upsilon}\frac{\kappa-\upsilon}{\kappa}+\beta\frac{\sigma-1}{\kappa}\right]\frac{C_t}{w_t}.$$

Combining this with the labor supply equation $\chi L_t^{\varsigma} C_t = w_t$, we obtain the solution for L_t:

$$L_t = \left\{\frac{1}{\sigma}\left[\frac{\upsilon-(\sigma-1)}{\upsilon}\left[\frac{(\sigma-1)^2}{\upsilon-(\sigma-1)}+\sigma\right]+\frac{\sigma-1}{\upsilon}\frac{\kappa-\upsilon}{\kappa}+\beta\frac{\sigma-1}{\kappa}\right]\frac{1}{\chi}\right\}^{\frac{1}{\varsigma+1}}.$$

Furthermore, using the price index definition $1 = M_{i,t}\tilde{\rho}_{i,t}^{1-\sigma}$ and the optimal pricing equation $\tilde{\rho}_{i,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{A_t\tilde{\varphi}_{i,t}}$, we obtain:

$$\left[\frac{\sigma}{\sigma-1}\frac{w_t}{A_t\tilde{\varphi}_{i,t}}\right]^{\sigma-1} = M_{i,t}.$$

Substituting (17) into the above equation, we get:

$$\left[\frac{\sigma}{\sigma-1}\frac{w_t}{A_t\left[\frac{\upsilon}{\upsilon-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{S_t}{N_t}\right)^{-\frac{1}{\kappa}}\left(\frac{M_{i,t}}{S_t}\frac{\kappa-\upsilon}{\kappa}\right)^{-\frac{1}{\upsilon}}}\right]^{\sigma-1} = M_{i,t}.$$

Substituting (D.2), (D.1), and the labor supply equation (21) into the above expression and rearranging, we obtain:

$$\Theta w_t = N_t^{\frac{1}{\kappa}}\left(\frac{1}{L_t^{\varsigma}}\right)^{\frac{1}{\sigma-1}-\frac{1}{\kappa}} f_h^{-\left(\frac{1}{\upsilon}-\frac{1}{\kappa}\right)} f_i^{-\left(\frac{1}{\sigma-1}-\frac{1}{\upsilon}\right)} A_t^{1+\theta_h\left(\frac{1}{\upsilon}-\frac{1}{\kappa}\right)+\theta_i\left(\frac{1}{\sigma-1}-\frac{1}{\upsilon}\right)}, \tag{D.4}$$

where:

$$\Theta \equiv \frac{\frac{\sigma}{\sigma-1}\frac{1}{\left[\frac{\upsilon}{\upsilon-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{\kappa-\upsilon}{\kappa}\right)^{-\frac{1}{\upsilon}}}\chi^{\frac{1}{\sigma-1}-\frac{1}{\kappa}}}{\left[\frac{1}{\sigma}\frac{\sigma-1}{\upsilon}\frac{\kappa-\upsilon}{\kappa}\right]^{\left(\frac{1}{\upsilon}-\frac{1}{\kappa}\right)}\left[\frac{1}{\sigma}\frac{\upsilon-(\sigma-1)}{\upsilon}\right]^{\frac{1}{\sigma-1}-\frac{1}{\upsilon}}}.$$

This gives the solution for w_t, as we have solved for L_t and N_t as state variables of the economy. Other variables can be easily determined.

E Volatility of GDP

By substituting the solutions from Table 4, GDP is expressed as:

$$Y_t = \left[L_t + \frac{1}{\sigma} \frac{\sigma - 1}{\kappa} \frac{1}{\chi L_t^{\varsigma}} \right] w_t.$$

The empirically consistent GDP is then given by:

$$Y_{R,t} = \frac{\left[L_t + \frac{1}{\sigma} \frac{\sigma-1}{\kappa} \frac{1}{\chi L_t^{\varsigma}} \right] N_t^{\frac{1}{\kappa}} \left(\frac{1}{L_t^{\varsigma}} \right)^{\frac{1}{\sigma-1} - \frac{1}{\kappa}} f_h^{-\left(\frac{1}{v} - \frac{1}{\kappa}\right)} f_i^{-\left(\frac{1}{\sigma-1} - \frac{1}{v}\right)} A_t^{1+\theta_h\left(\frac{1}{v}-\frac{1}{\kappa}\right)+\theta_i\left(\psi - \frac{1}{v}\right)}}{\Theta \left[\frac{1}{\sigma} \frac{v-(\sigma-1)}{v} \frac{1}{\chi L_t^{\varsigma}} \frac{A_t^{\theta_i}}{f_i} \right]^{\frac{1}{\sigma-1} - \psi}}.$$

where we have used the solutions from (D.1) and (D.4). Taking the logarithm and noting that L_t is constant in equilibrium, we obtain:

$$\log Y_{R,t} = \frac{1}{\kappa} \log N_t + \left[1 + \theta_h \left(\frac{1}{v} - \frac{1}{\kappa} \right) + \theta_i \left(\psi - \frac{1}{v} \right) \right] \log A_t + cst.$$

As a result, the variance of the empirically consistent GDP is given by:

$$\begin{aligned} \sigma^2_{\log Y_{R,t}} = \left(\frac{1}{\kappa} \right)^2 Var\left(\log N_t\right) + \left[1 + \theta_h \left(\frac{1}{v} - \frac{1}{\kappa} \right) + \theta_i \left(\psi - \frac{1}{v} \right) \right]^2 Var\left(\log A_t\right) \\ + 2\frac{1}{\kappa} \left[1 + \theta_h \left(\frac{1}{v} - \frac{1}{\kappa} \right) + \theta_i \left(\psi - \frac{1}{v} \right) \right] Cov(\log N_t, \log A_t). \end{aligned}$$

This can be further rewritten as equation (27).