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Search frictions in good markets and CPI inflation

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Abstract

We develop a New Keynesian DSGE model to examine how preference shifts between online and brick-and-mortar retail affect pricing dynamics and inflation. Central to our model are goods market search frictions, which govern the interaction between retailers and producers. We introduce distinct search efficiencies for online and brick-and-mortar retailers, capturing the evolving retail landscape. Our analysis reveals two key channels through which these frictions impact inflation: the composition channel, arising from differing search efficiencies, and the arbitrage channel, reflecting changes in market tightness. Both channels operate through the search friction mechanism, altering the wedge between consumer and producer prices. Bayesian estimation identifies that both channels reinforce each other, lowering CPI inflation. This research highlights the critical role of goods market search frictions in understanding modern inflation dynamics.

JEL codes: E31, E52, J64
Keywords: Search and matching friction, CPI inflation, Firm dynamics

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1 Introduction

Retailers play a crucial role in the economy, occupying the final stage in the journey of goods from production to consumption. Their economic value lies in their ability to search for and match household demand with the variety of goods produced in the economy. As intermediaries, retailers charge households a retail margin on top of the wholesale prices paid to producers, compensating for the costs incurred through the search and matching process. This retail margin introduces a wedge between producer and consumer prices, the magnitude of which is determined by the retailers' search efficiency. Higher search efficiency translates into lower search costs and a narrower gap between consumer and producer prices, highlighting the significant impact of retailers' effectiveness in aligning product offerings with consumer preferences on the prices households ultimately pay. Empirical evidence underscores the crucial role retailers play in determining the final prices paid by consumers. Nakamura (2008) conducted an extensive study using detailed price and quantity data from US grocery stores, revealing that the majority of observed price fluctuations originated at the retail level rather than from manufacturers. This study laid the foundation for subsequent research examining retailers' influence on the wedge between consumer and producer prices. Hottman et al. (2016), for example, utilized comprehensive barcode data to measure retail markups across US stores, finding substantial markup dispersions that varied with store characteristics such as size and product variety. These variations in retail markups significantly contribute to aggregate price dispersion and the gap between the prices producers receive and the prices consumers ultimately pay, emphasizing the critical role retailers play in shaping the final costs borne by households.

One of the most significant trends in retail markets over the past 15 years has been the rapid rise of online retail platforms, such as Alibaba, Amazon Marketplace, and eBay. These platforms have become essential to the e-commerce ecosystem, enabling digital search and matching between buyers and sellers through various interfaces. The increasing prominence of online shopping has led to a notable shift away from traditional physical stores, with the share of online retail sales in the UK surging from less than 10% in 2010 to nearly 30% by 2023, highlighting the transformative impact of e-commerce on the retail landscape. The COVID-19 pandemic further accelerated this trend, as online retail became relatively more desirable compared to in-person shopping, initially catalyzing a considerable shift towards online retail sales. However, upon the reopening of brick-and-mortar stores, this shift partially reverted, providing evidence of possible short-run temporary shifts in consumer preferences towards online retail markets (Figure 1).

This observation motivates the development of an economic framework to examine how short-term preference shocks towards online versus brick-and-mortar retail impact pricing dynamics and inflation measures. We construct and estimate a New Keynesian dynamic stochastic general equilibrium (DSGE) model incorporating frictional goods markets. Our model features product market search and matching frictions influencing consumer price inflation. We introduce two distinct retailer types - online and brick-and-mortar - each with different search efficiencies in the product market. This distinction explicitly captures the shift in consumer preferences toward online retailers. The model

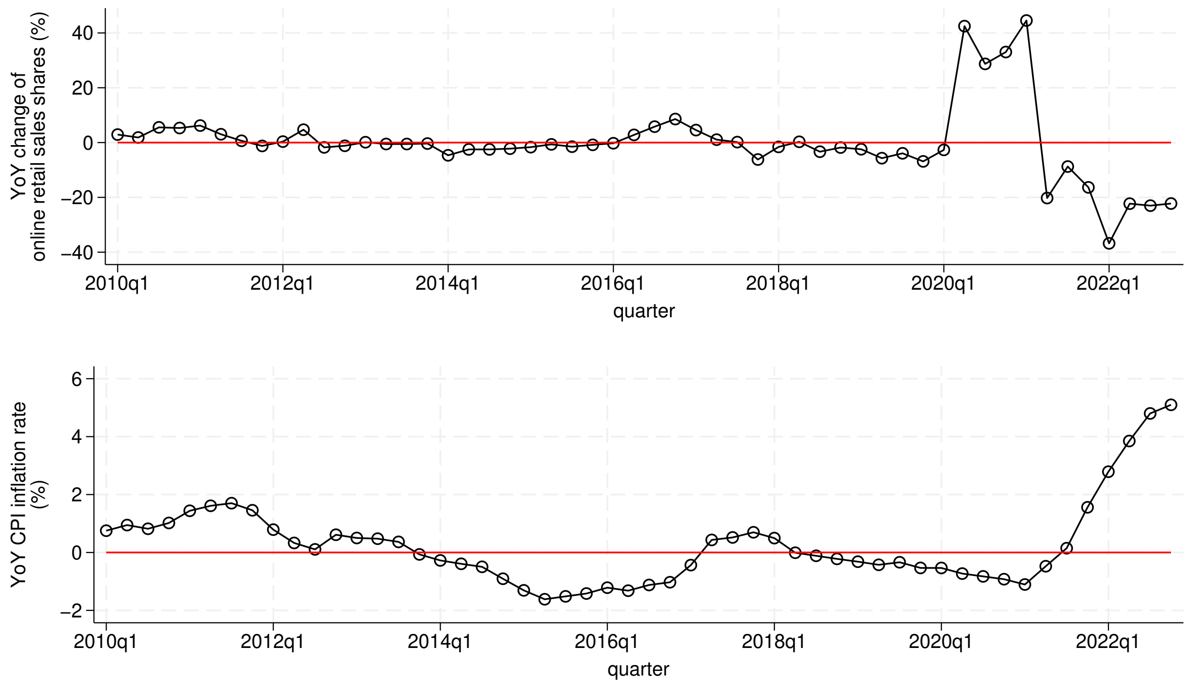

Figure 1: Year-over-year change in the online retail sales share and the year-over-year CPI inflation in the UK

employs a matching function to govern the process between retailers and producers, where retailers exert search efforts to acquire product varieties. Product market tightness, defined as the ratio of efficiency-adjusted matching efforts to the number of producers, emerges as a key variable. Increasing market tightness facilitates producers' ability to sell varieties while forcing retailers to put more effort into searching. This mechanism modulates the wedge between consumer and producer prices, consequently influencing consumer price inflation. We utilise the demand shifts observed during the COVID-19 pandemic to analyze how shocks to online retail sales share affect pricing dynamics and the New Keynesian Phillips Curve relationship between inflation and economic activity.

Our model identifies two primary channels through which shifts in online retail sales share affect inflation dynamics. The composition channel stems from differences in search efficiency between online and brick-and-mortar retailers. Shifts in consumer preference towards online retail sales alter the overall market composition, affecting the weights assigned to each retailer type in calculating the wedge between consumer and producer prices. The arbitrage channel reflects how changes in consumer preferences affect search and matching conditions in both online and brick-and-mortar markets. Specifically, shifts in consumer preferences change product market tightness in each market in opposite ways, leading to adjustments in the wedge between consumer and producer prices. Increased demand for online markets intensifies search efforts from online retailers, tightening this market and prompting higher online search costs. Conversely, reduced demand for brick-and-mortar loosens product market tightness in the brick-and-mortar stores retail sector. These channels operate simultaneously but can have opposing effects on inflation. The net effect on inflation depends on the relative strengths of these channels.

After calibrating the steady-state and conducting Bayesian estimation of our model, our findings indicate that shifts in consumer preferences towards online retailers contribute to reduced CPI inflation. Results suggest that the two channels have reinforcing effects on CPI inflation. The composition channel shows that consumer preference shifting towards online retail sales leads to lower inflation, implying greater search efficiency and a lower wedge between consumer and producer prices in online retail. Meanwhile, the arbitrage channel loosens good market tightness for brick-and-mortar retailers, allowing them to exert lower search efforts and charge a smaller wedge between consumer and producer prices.

Literature Review

Our paper builds upon and extends the growing literature emphasizing the importance of product market frictions and customer relationships in understanding firm dynamics and macroeconomic outcomes. Two key contributions in this area are Gourio and Rudanko (2014) and Petrosky-Nadeau et al. (2016). Gourio and Rudanko (2014) develop a search-theoretic model of firm dynamics with frictional product markets, demonstrating how long-term customer relationships act as a form of intangible capital that affects firm behavior. Their model generates dampened and hump-shaped responses to shocks, and provides a novel explanation for the weak investment-q correlations observed empirically. Using firm-level data from Compustat, they find support for their model's predictions across industries with varying degrees of friction. Petrosky-Nadeau et al. (2016) focus on consumer search behavior in the goods market, using data from the American Time Use Survey to document procyclical patterns in shopping time. They find that aggregate time spent shopping declined during the Great Recession, with the largest drops observed among unemployed individuals. Their analysis reveals a positive correlation between shopping time and state-level GDP changes, as well as a positive association between personal and household income and time spent shopping. Our paper draws rationality from these studies by incorporating their insights on the importance of product market frictions and consumer search behavior.

Second, our work extends the research of Bilbiie et al. (2008, 2014) on firm dynamics in New Keynesian models. Bilbiie et al. (2008) introduced a New Keynesian Phillips curve that incorporates the number of firms as a key variable affecting inflation dynamics and monetary policy transmission. Building on this, Bilbiie et al. (2014) derived optimal monetary policy in this framework. Our research is also closely related to Dong et al. (2021), who developed a New Keynesian DSGE model exploring the relationship between product life cycles and staggered pricing in the retail industry. While we share similar mechanisms for adjusting to demand shocks, our focus is on the shift in consumer preferences towards online retailers and their efficiency in matching supply with demand. Our primary contribution lies in exploring how this shift impacts the pass-through from producer prices to consumer price inflation.

Our research is closely related to Dong et al. (2021), who developed a New Keynesian DSGE model to explore the relationship between product life cycles and staggered pricing in the retail industry. The paper incorporates endogenous product entry by integrating the entry new retailers into the frictional goods markets. It demonstrates how demand shocks affect the entry new retailers, the total number of retailers, and the tightness of the frictional goods markets. The level of market tightness then influences the proportion of products undergoing price adjustments. In essence, the model endogenises Calvo's parameter, directly linking product dynamics to price inflation. While our paper shares the same mechanism for adjusting to demand shocks, it diverges by focusing on the shift in consumer preferences towards online retailers, who are more efficient in matching supply with demand than traditional brick-and-mortar retailers. Our primary contribution is to explore how this shift impacts the pass-through from producer prices to consumer price inflation and the monetary transmission.

The paper is structured as follows: Section 2 properly examine the effects of an increase in the share of online retail sales. Section 3 introduces our NK-DSGE model with endogenous product entry and frictional goods markets. Section 4 examines modifications to the New Keynesian Phillips Curve due to these market frictions. Section 5 details our calibration and estimation strategies, respectively, alongside simulation results. The paper concludes in Section 6, summarising our key findings and implications.

2 Empirical Evidence

Before we dive into the structural model, we investigate the relationship between the share of online retail sales and consumer price inflation by examining the impulse response of consumer price inflation to an increase in the share of online retail sales. There are two primary considerations. First, the empirical results are relevant only if consumer price measurement incorporates data from brick-and-mortar and online retailers. According to the *Consumer Prices Indices Technical Manual* (2019), the Office for National Statistics (ONS) collects online prices through mail-order catalogues from retailers with national pricing policies, including online retailers. The second key consideration is that the Consumer Price Index (CPI) should reflect shifts in consumer preferences toward online retailers. As noted by the ONS, outsourced price collectors could not visit physical stores during the pandemic, prompting the office to place greater emphasis on online price collection. Consequently, price collection during COVID-19 has partially accounted for these preference shifts. Since this adjustment is unlikely to fully capture the extent of consumers' shift toward online retailers, our estimation can be considered conservative.

To conduct this analysis, we employ the local projection method (LP) introduced by (Jordà, 2005)[1].

[1]The LP method directly estimates how a targeted variable responds to shocks over time. This approach uses simple regressions for each time period, making it easier to estimate and understand than VAR models, which require

LP allows us to characterize the path of the cumulative response of CPI inflation (CPI) to a percentage change in the share of online retail sales to total retail sales (ONLINE). The cumulative response function can be expressed as:

$$
\begin{aligned}
CR(\Delta_h CPI_{t+h}, \delta) =& E_t \left(\Delta_h CPI_{t+h} | ONLINE_t = \overline{ONLINE} + \delta; X_t, X_{t-1}, ... \right) \\
& - E_t \left(\Delta_h CPI_{t+h} | ONLINE_t = \overline{ONLINE}; X_t, X_{t-1}, ... \right) \qquad (1)
\end{aligned}
$$

Here, $CR(\Delta_h CPI_{t+h}, \delta)$ represents the average cumulative response of CPI inflation given a size δ change in the percentage change of the online retail sales share. The path of CPI inflation is conditional on $X_t, X_{t-1}, ...$, which encompasses the history of CPI inflation, PPI inflation, and the percent change in the share of online retail sales, accounting for the persistent effects of changes in PPI inflation or the share of online retail sales. Our dataset consists of monthly observations spanning from the first quarter of 2013 to the fourth quarter of 2022, comprising 144 data points[2].

The top panel of Figure 2 reports results from OLS estimation, controlling for CPI inflation, PPI inflation, and the share of online retail sales in the previous lags (1, 3, 6). The results suggest that the cumulative response of the CPI inflation rate to a one percent increase in the share of online retail sales is consistently negative for 12 consecutive months, attaining statistical significance within the 90% confidence interval. Throughout a 12-month period, the cumulative response in the UK peaks after 9 months at around -0.15%. The response is robust to the number of lags that capture the history of CPI inflation, PPI inflation, and the share of online retail sales, as our results remain significant at the 10% level in the first 12 months for all lag specifications.

It is crucial to note that share of online retail sales is influenced by a myriad of factors that could simultaneously affect CPI inflation, presenting identification problems. To identify the shock to consumer preferences towards online retail sales, we conduct an instrumental variable local projection (LP-IV) estimation, using the number of deaths due to the COVID-19 pandemic as an instrument for the percentage change in the share of online retail sales. The number of COVID-19 deaths is considered exogenous, as it influences preferences towards online shopping while not being directly subject to CPI inflation. The LP-IV results, reported in the bottom panel of Figure 2, confirm the causal response of CPI inflation to an increase in the share of online retail sales. We find similar point estimates of the impulse responses for every specification, and the results are more significant, with the response being statistically significant at the 90% level for up to 18 months. These findings confirm that shocks to consumer preferences for online retail markets have negative and significant effects on CPI inflation, supporting the relevance of search frictions in retail markets in determining price dynamics.

a lot of parameter estimation and matrix manipulation. Also, the LP method does not need a fully specified model of the whole dynamic system, so it's less likely to have errors from incorrect model specifications. This makes the LP method's estimates of impulse responses more reliable when studying complex relationships between variables.

[2]Consumer price index is Please refer to Appendix A for the description of the dataset.

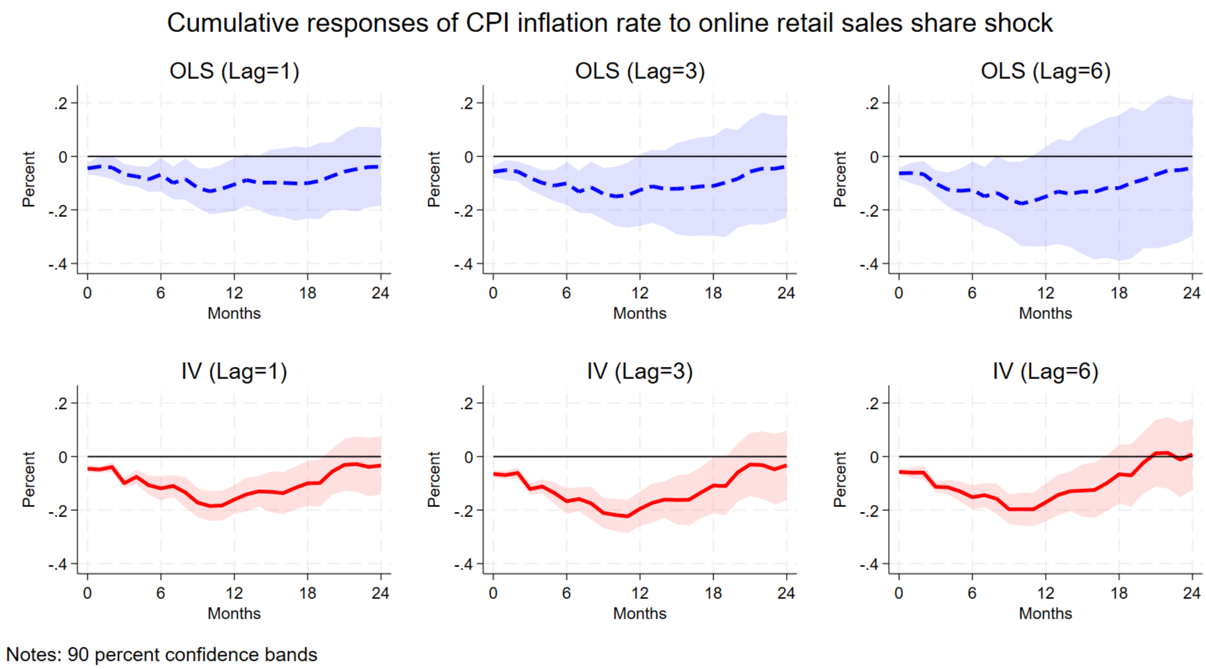

Figure 2: Response of CPI inflation to an increase in the share of online retail sales

3 Model

We move on to the main investigation where we introduce dynamic stochastic general equilibrium model (DSGE) that features search and matching functions in good markets. We build our model upon Bilbiie, Ghironi, and Melitz (2008), the pioneer work that introduces the endogenous linkages between the entry heterogeneous firms and monopolistic competition in a DSGE model with sticky prices. We develop good market search and matching friction from Michaillat and Saez (2015).

3.1 Demand for retail goods

The novel feature of our model relative to Bilbiie, Ghironi, and Melitz (2008) is the introduction of retailing and good market search and matching frictions. The model economy consists of a continuum of atomistic households, each identical. We denominate all contracts and prices in nominal terms. We construct a similar consumer problem to a simplified model. At time t, household consumes final goods offered by two types of retailers: online retailers (O) and brick-and-mortar (indexed by B). The basket of goods is thus defined as

$$C_t = \left(\frac{C_{O,t}}{\alpha_t}\right)^{\alpha_t} \left(\frac{C_{B,t}}{1-\alpha_t}\right)^{1-\alpha_t} \quad ,$$

where α_t is the expenditure share of retail goods from brick-and-mortar retailers, which we assume to be exogenous and follow an AR(1) process in percent deviation from its steady-state level with an $i.i.d.$ normal error term. Let $P_{j,t}$ denotes the price of the retail goods offered by a retailer of type $j \in \{O, B\}$ at time t. The consumption-based price index of the final goods is then

$$P_t = P_{O,t}^{\alpha_t} P_{B,t}^{1-\alpha_t}, \tag{2}$$

and the household's demand for retail goods from each retailer is

$$C_{O,t} = \alpha_t \frac{P_t C_t}{P_{O,t}} \text{ and } C_{B,t} = (1 - \alpha_t) \frac{P_t C_t}{P_{B,t}}.$$

We can express the consumption-based price index of the final goods as and household's demand for retail goods in real term relative to the consumer price index as

$$1 = \rho_{O,t}^{\alpha_t} \rho_{B,t}^{1-\alpha_t}, \tag{3}$$

where $\rho_{O,t} = P_{O,t}/P_t$ and $\rho_{B,t} = P_{B,t}/P_t$, respectively. Furthermore, we derive the representative household's optimal demand for retail goods as a function of real retail price as

$$C_{O,t} = \frac{\alpha_t C_t}{\rho_{O,t}} \text{ and } C_{B,t} = \frac{(1 - \alpha_t) C_t}{\rho_{B,t}}, \tag{4}$$

respectively

3.2 Retailers and good market search and matching

A retailer of type $j \in \{O, B\}$ purchase varieties indexed ω, $y_t(\omega)$, from a continuum of varieties, Ω, available in each period. They aggregate varieties into retail goods $Y_{j,t}$ using a CES aggregator that takes the form

$$Y_{j,t} = V_{j,t} \left(\int_{\omega_i} y_{j,t} (\omega)^{\frac{\sigma_t - 1}{\sigma_t}} d\omega \right)^{\frac{\sigma_t}{\sigma t - 1}}, \tag{5}$$

where $y_{j,t}$ is the demand of retailer of type j for variety ω and $V_{j,t} \equiv N_{j,t}^{\psi - \frac{1}{\sigma - 1}}$ in which $N_{j,t}$ stands for the number of varieties to which the retailer of type j has access. ψ stands for the marginal utility resulting from a unit increase in the number of varieties as discussed in Benassy (1996).[3] $\sigma_t > 1$ is the stochastic elasticity of substitution between varieties. Importantly, σ_t determines the stochastic markup in the goods market. Following Smets and Wouters (2003), we interpret shock to this parameter as a cost-push shock to the inflation equation. Cost-push shock is exogenous and follows an AR(1) process in percent deviation from its steady-state level with an *i.i.d.* normal error term. We assume that brick-and-mortar and online retailers have access to the same set of varieties and buy all varieties. It implies that $N_{B,t} = N_{O,t} = N_t$ and $V_{B,t} = V_{O,t} = V_t$, respectively. The number of varieties that the retailer purchases is determined by a matching function. We borrow the matching function specification from den Haan et al. (2000) which which takes the form

[3] If we set $\psi = \frac{1}{\sigma - 1}$, the consumption basket dissolved to the one discussed in Dixit and Stiglitz (1977).

$$Y_{j,t}=\left(\left(\zeta_j Y_{j,t}^{Search}\right)^{-\lambda}+N_t^{-\lambda}\right)^{-1/\lambda} \tag{6}$$

where $\zeta_j Y_{j,t}^{Search}$ is defined as efficiency-adjusted search efforts. $Y_{j,t}^{Search}$ is the retail goods that a retailer of type j pays for matching efforts, where

$$Y_{j,t}^{Search}=Y_{j,t}-Y_{j,t}^{Sales}. \tag{7}$$

$Y_{j,t}$ and $Y_{j,t}^{Sales}$ denote the total output purchased from producers and the output sold to consumers and the new entrants. Parameter λ governs the elasticity of substitution between matching efforts and the number of firms. In each period, there are H_t entrant firms who purchase baskets of retail goods to pay for the sunk entry cost f_E. Assume that this basket has the same composition of retail goods as consumption, demand for retail goods O and B from prospective entrants are

$$\frac{\alpha_t H_t f_{E,t}}{\rho_{O,t}} \text{ and } \frac{(1-\alpha_t)\, H_t f_{E,t}}{\rho_{B,t}}$$

respectively, ζ_j is product-market search efficiency. Higher ζ_j implies that a unit increase in $Y_{j,t}^{Search}$ contributes more as an input to the matching process. translating to a higher chance of being matched with producers. The parameter $\lambda > 0$ governs the elasticity of substitution of efficiency-adjusted matching efforts and the number of producers.

We define product market tightness $\mathcal{T}_{j,t}$ as the ratio of the efficiency-adjusted matching efforts to the number of producers, that is, $\mathcal{T}_{j,t}=\zeta_j Y_{j,t}^S/N_t$. Tightness determines the ratio of the total output firms sell to a retailer j over the number of firms, $\mathcal{P}_{j,t}=Y_{j,t}/N_t$, and the ratio of the total output firms firms sell to a retailer j over an efficiency-adjusted matching effort, $\mathcal{Q}_{j,t}=Y_{j,t}/\left(\zeta_j Y_{j,t}^S\right)$. We can show that when the product market tightness increases, producers can sell variety more easily, but it will be harder for retailers to find varieties. Note that we can write a total purchase of final goods,

$$Y_{j,t}=\mathcal{Q}_{j,t}\zeta_j Y_{j,t}^S. \tag{8}$$

We assume that retailers are identical within each type and operate in perfectly competitive markets. The retailer of type j maximises the following profit measured in real terms relative to the consumer price index,

$$d_{j,t}=\rho_{j,t}Y_{j,t}^{Sales}-\int_\omega \rho_t(\omega)\, y_{j,t}(\omega)\, d\omega$$

subject to (5), (7), and (8). The first order condition with respect to total retail goods sold by type-j retailers suggests that real retail prices set by the retailer of type j, are given by

$$\rho_{j,t} = \underbrace{\left(1 - \frac{1}{\mathcal{Q}_{j,t}\zeta_j}\right)^{-1}}_{\equiv \mathcal{M}_{j,t}} \rho_{P,t} \tag{9}$$

where $\rho_{P,t}$ is the real aggregate producer price and $\mathcal{M}_{j,t}$ is interpreted as the markup that retailers j set to cover the cost of search activity, $Y_{j,t}^{Search}$, paid in the unit of retail goods. Hereafter, we will refer to this markup as the search cost. search cost is positively related to the probability of the retailer being able to acquire variety. brick-and-mortar and online retailers are different in the search efficiency parameter. We assume that $\zeta_O > \zeta_B$ online retailers are more efficient in searching for varieties than brick-and-mortar retailers. Lastly, the first order condition states that the retailer j's demand for each variety ω is

$$y_{j,t}(\omega) = V_t^{\sigma-1} \left(\frac{\rho_t(\omega)}{\rho_{P,t}}\right)^{-\sigma} Y_{j,t} \ . \tag{10}$$

3.3 Firm entry and exit

In each period, there exists an unbounded number of potential entrants, who are forward-looking and able to accurately forecast their future expected profits, $d_t(\omega)$, for every period t. We assume a one-period time-to-build lag, that is, the entrants at time B commence production at time $t+1$. Additionally, they are aware of the probability, δ, of encountering a death shock that necessitates firm exit at the end of the period, after production and entrants. B calculate their anticipated post-entry value, represented by the present discounted value of their expected profit stream, $v_t(\omega)$[4].

As mentioned above, prior to entryirms face the sunk entry cost of $f_{E,t}$ units of consumption goods.[5] Entry continues until the anticipated post-entry value matches the cost of entry. It is stated as the free entry condition,

$$v_t(\omega) = f_{E,t}.$$

Measuring sunk entry cost in units of consumption goods instead of units of effective labour implies a positive response of firm entry monetary expansion, assuming that a sunk entry cost is constant.[6] This feature aligns the model with the empirical findings outlined in Bergin and Corsetti (2005) and Lewis (2006). A monetary policy shock that reduces the *ex ante* real interest rate between B and $t+1$ brings about the expansion in consumption demand. Furthermore, since producers do not pay sunk entry cost in units of effective labour, we rule out the sectoral reallocation of labour

[4] $v_t(\omega)$ also represents the average value of incumbent firms subsequent to production, as both new entrants and incumbents face an identical survival probability of $1-\delta$.

[5] The change in $f_{E,t}$ can be interpreted as the changes in product market regulation that facilitate or hinder firm entry.

[6] This assumption does not imply that the post-entry value of the firms in a data-consistent unit or that in nominal term is constant.

between firm entryd the production of existing producers. As a result, the expansionary monetary policy shock induces firm entryootnoteIf we assume that $f_{E,t} = 1$, investment in our model behaves closely to the standard RBC model without capital adjustment cost. To avoid the problem such that the no-arbitrage condition between bonds and shares features only forward variables, exposing the model to indeterminacy, we must restrict our interest rate rules to current inflation rather than expected inflation. Finally, the one-period time-to-build lag dictates that the number of producers during period B follows $N_t = (1-\delta)(N_{t-1} + H_{t-1})$.

3.4 Producers and producing decision

The setup of the producers is similar to that of the firms in Bilbiie, Ghironi, and Melitz (2008). The difference lies in that firms in Bilbiie, Ghironi, and Melitz (2008) sell varieties directly to representative households, whereas our model assumes that producers sell varieties to brick-and-mortar and online retailers . There is a continuum of monopolistically competitive producers. Each producer manufactures a variety, $\omega \in \Omega$. producer ω produces requires labour, $l_t(\omega)$, to produce the output $y_t(\omega) = Z_t l_t(\omega)$ where Z_t denotes the aggregate labour productivity which represents the effectiveness of a unit of labour. Aggregate labour productivity is exogenous and follows an AR(1) process in percent deviation from its steady-state level with an *i.i.d.* normal error term. The unit cost of production is w_t/Z_t, measured in units of consumption goods, where $w_t = W_t/P_t$ is the wage in real terms relative to the consumer price index. We assume no fixed production costs, thus all firms produce.

A producer ω also faces nominal rigidity in the form of a quadratic price adjustment cost, $pac_t(\omega)$. The price adjustment cost could be interpreted as the quantity of marketing materials a firm must purchase when it changes its prices. Mathematically, we specify the price adjustment cost as the real cost that is incurred when individual price inflation deviates from a steady-state level, which is equal to 0, and assume the cost is proportional to real revenue from production relative to the consumer price index:

$$pac_t(\omega) = \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2 \rho_t(\omega)\, y_t(\omega)\,, \tag{11}$$

where $p_t(\omega)$ is the individual nominal price of the producer ω. For simplicity, we assume that the price adjustment cost is in units of the composite basket that has the same composition as the basket of retail goods. Also, we define the aggregate price adjustment cost, PAC_t, and assume that the price adjustment cost is symmetric across producers. Therefore, we can derive $PAC_t \equiv N_t pac_t$. Total demand for outputs of producer ω thus comes from brick-and-mortar and online retailers , from producers themselves as price adjustment cost, and from the firm entry cost:

$$y_t(\omega) = \left(\frac{\rho_t(\omega)}{\rho_{P,t}}\right)^{-\sigma} Y_t = \left(\frac{\rho_t(\omega)}{\rho_{P,t}}\right)^{-\sigma} (Y_{B,t} + Y_{O,t} + PAC_t)$$

Producer firms choose l_t and p_t to maximise current profit plus their real value at time t. which is the expected present discounted value of the future stream of profits. A one-period time-to-build lag implies that we must start accumulating profits from $t+1$ on. We apply the household's stochastic discount factor on the future profits as the household owns producers:

$$v_t(\omega) = \mathbb{E}_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s(\omega) \tag{12}$$

whereas

$$d_t(\omega) = \rho_t(\omega) y_t(\omega) - w_t l_t(\omega) - \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2 \rho_t(\omega) y_t(\omega).$$

and $\Lambda_{t,s} \equiv [\beta(1-\delta)]^{s-t} U_C(C_s, L_s)/U_C(C_t, L_t)$, subject to total demand. The first-order condition with respect to the individual firm producing price gives the individual producing price equation:

$$\rho_t(\omega) = \mu_t(\omega) \frac{w_t}{Z_t} \tag{13}$$

where

$$\mu_t(\omega) = \frac{\sigma_t y_t(\omega)}{(\sigma_t - 1) y_t(\omega)\left(1 - \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2\right) + \kappa \Upsilon_t} \tag{14}$$

and

$$\Upsilon_t \equiv y_t(\omega) \frac{p_t(\omega)}{p_{t-1}(\omega)}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right) - \mathbb{E}_t\left[\Lambda_{t,t+1} y_{t+1}(\omega) \frac{P_t}{P_{t+1}}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)\left(\frac{p_t(\omega)}{p_{t-1}(\omega)}\right)^2\right] \tag{15}$$

Equation (13) states that individual producer price of production is a markup over marginal costs. In the absence of nominal rigidity, $\kappa = 0$, the markup reduces to $\sigma_t/(\sigma_t - 1)$. producers earn

$$d_t(\omega) = \left(1 - \frac{1}{\mu_t(\omega)} - \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2\right) \frac{\rho_{P,t} Y_t}{N_t}. \tag{16}$$

3.5 Symmetric firm equilibrium

In equilibrium, we assume that all producers make identical decisions. Therefore, $p_t(\omega) = p_t$, $\mu_t(\omega) = \mu_t$, $\rho_t(\omega) = \rho_t$, $y_t(\omega) = y_t$, $pac_t(\omega) = pac_t$, $d_t(\omega) = d_t$, and $v_t(\omega) = v_t$, Under symmetric equilibrium across producers,

$$\rho_t = \mu_t \frac{w_t}{Z_t} \tag{17}$$

where

$$\mu_t = \frac{\sigma_t y_t}{(\sigma_t - 1) y_t \left(1 - \frac{\kappa}{2}\pi_t^2\right) + \kappa \Upsilon_t}$$

and

$$\Upsilon_t = \pi_t (1+\pi_t) - \mathbb{E}_t \Lambda_{t,t+1} \frac{y_{t+1}}{y_t} \frac{1+\pi_{t+1}}{1+\pi^C_{t+1}} \pi_{t+1} (1+\pi_{t+1}).$$

Since in symmetric equilibrium, $\rho_{P,t} Y_t = N_t \rho_t y_t$ where Y_t is the aggregate production, we substitute y_t with $\rho_{P,t} Y_t / (N_t \rho_t)$ such that

$$\Upsilon_t = \pi_t (1+\pi_t) - \mathbb{E}_t \Lambda_{t,t+1} \frac{\rho_{t+1}}{\rho_t} \frac{Y_{t+1}}{Y_t} \frac{N_t}{N_{t+1}} \frac{\rho_t}{\rho_{P,t+1}} \frac{1+\pi_{t+1}}{1+\pi^C_{t+1}} \pi_{t+1} (1+\pi_{t+1}).$$

Put in the definition, $\frac{\rho_t}{\rho_{P,t+1}} \frac{1+\pi_{t+1}}{1+\pi^C_{t+1}} = 1$, and expand the stochastic discount factor $\Lambda_{t,t+1}$ to obtain

$$\Upsilon_t = \pi_t (1+\pi_t) - \beta (1-\delta) \mathbb{E}_t \left(\frac{C_t}{C_{t+1}} \right) \frac{\rho_{P,t+1}}{\rho_{P,t}} \frac{Y_{t+1}}{Y_t} \frac{N_t}{N_{t+1}} \pi_{t+1} (1+\pi_{t+1}).$$

Thus,

$$\rho_t = \frac{\sigma_t}{(\sigma_t - 1)\left(1 - \frac{\kappa}{2}(\pi_t)^2\right) + \kappa \left(\pi_t (1+\pi_t) - \beta (1-\delta) \mathbb{E}_t \frac{C_t}{C_{t+1}} \frac{N_t}{N_{t+1}} \frac{\rho_{P,t+1}}{\rho_{P,t}} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (1+\pi_{t+1}) \right)} \frac{w_t}{Z_t}. \tag{18}$$

The real individual producer price constitutes the aggregate producer price. Following Melitz (2003), in an equilibrium characterised by a mass of firms, N_t, the aggregate producer price in real terms is given by

$$\rho_{P,t} = N_t^{-\psi} \rho_t, \tag{19}$$

which is the variety effect equation. The term $N_t^{-\psi}$ captures variety effects on aggregate producer price.

The setup of our model allows for the decomposition of consumer price into several key elements. We start off by writing Equation (9) in nominal terms and put it in Equation (2):

$$P_t = \mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t} P_{P,t}$$

where $P_{P,t}$ denotes the aggregate producer price in nominal terms. Decomposing $P_{P,t}$ into the individual producer price and variety effects following Equation (19) yields

$$P_t = \mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t} N_t^{-\psi} p_t.$$

The individual producer price p_t can be substituted by the pricing rule from Equation (17), written in nominal terms:

$$P_t = \underbrace{\mathcal{M}_t}_{\text{Aggregate search cost}} \underbrace{N_t^{-\psi}}_{\text{Variety effect}} \underbrace{\mu_t}_{\text{Producer markup}} \underbrace{\frac{W_t}{Z_t}}_{\text{Marginal cost}}, \tag{20}$$

where we define the aggregate search cost, $\mathcal{M}_t = \mathcal{M}_{O,t}^{\alpha_t}\mathcal{M}_{B,t}^{1-\alpha_t}$, and

$$\mathcal{M}_{O,t} = \left(1 - \frac{1}{\mathcal{Q}_{O,t}\zeta_O}\right)^{-1} \text{ and } \mathcal{M}_{B,t} = \left(1 - \frac{1}{\mathcal{Q}_{B,t}\zeta_B}\right)^{-1}. \tag{21}$$

Equation (20) decomposes consumer price into marginal cost, producer's monopolistic markup and variety effects as in Bilbiie, Ghironi, and Melitz (2008). Importantly, we show that search cost, which is the Cobb-Douglas aggregation of brick-and-mortar retailers and online retailers ' search cost, also contributes to consumer price dynamics. The decomposition of consumer price also suggests that the change in consumer price is attributable to the relative expenditure share of the brick-and-mortar retail sales to online retail sales, α_t. The role of good market search friction in determining CPI inflation will be discussed in detail in the following section where we illustrate how good market search friction alters the New Keynesian Philip Curve equation.

3.6 Household

The model economy consists of a continuum of atomistic households, each identical. We denominate all contracts and prices in nominal terms. Each household maximizes an intertemporal utility function given by $E_0 = \sum_{t=0}^{\infty} \beta^t U_t$, where $\beta \in (0,1)$ is the subjective discount factor. The period utility function is separable in consumption, C_t, and labour, L_t, taking the form

$$U_t = \left(\ln C_t - \varepsilon_{L,t}\chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right). \tag{22}$$

In equation (22), χ captures the disutility of labour supply and $\varphi \geq 0$ is the Frisch elasticity of labour supply to wages. The period utility function contains the labour supply shock, $\varepsilon_{L,t}$, which is exogenous and follows an AR(1) process in percent deviation from its steady-state level with an *i.i.d.* normal error term.[7]

Household budget constraint written in real terms follows

$$\frac{\mathcal{B}_{t+1}}{P_t} + C_t + x_{t+1}\left(N_t + H_t\right)v_t = \left(1 + r_t\right)\frac{\mathcal{B}_t}{P_t} + L_t w_t + x_t N_t\left(v_t + d_t\right) + d_{B,t} + d_{O,t}. \tag{23}$$

On the use of budget, the household consumes C_t, buying x_{t+1} shares of the mutual funds constructed from the share of existing firms, N_t, and new firms, H_t, at the share price, v_t, as well as purchasing nominal government bonds $\mathcal{B}_{t+1}$. On the source of budget, the household supplies labour, L_t, to earn income at real wage, w_t. Households hold producers' shares through mutual funds, thus will receive dividend income, d_t, every period they hold the shares and could sell the share at the real value v_t. Households also receive profits from online retailers, $d_{O,t}$, and brick-and-

[7]We choose separable preferences and logarithmic utility from consumption following Bilbiie, Ghironi, and Melitz (2008). Using separable preferences, the logarithmic utility of consumption ensures that the income and substitution effects of the real wage on labour supply neutralise each other when the real wage varies from its steady state value.

mortar retailers, $d_{B,t}$, and on a lump-sum basis. Lastly, the representative household receives the principal and return of bond holdings where $1+r_t \equiv (1+i_{t-1}) / \left(1+\pi_t^C\right)$ designates the gross real interest rate on bond holdings between $t-1$ and t.

In each period t, the representative household chooses consumption, C_t, producer's shareholding for each active retailer, x_{t+1}, and the labour supply, L_t, to maximize the expected utility function subject to the budget constraint. The first-order condition with respect to consumption yields the labour supply equation

$$\varepsilon_{L,t}\chi L_t^{\frac{1}{\varphi}} = \varepsilon_{B,t}\left(1+\tau_L\right)\frac{w_t}{C_t}, \tag{24}$$

which suggests that the representative household will allocate labour efforts until the marginal disutility of labour is equal to the marginal utility from consuming the real wage translated from an additional unit of labour. The Euler equation for bond holding is

$$v_t = \beta\left(1-\delta\right)\mathbb{E}_t\frac{C_t}{C_{t+1}}\left(v_{t+1}+d_{t+1}\right). \tag{25}$$

Lastly, the Euler equation for shareholdings is

$$1 = \beta\mathbb{E}_t\left[\frac{1+i_t}{1+\pi_{t+1}^C}\frac{C_t}{C_{t+1}}\right], \tag{26}$$

where with $1+\pi_t^C \equiv P_t/P_{t-1}$.

3.7 Model equilibrium

We impose good market clearing to derive the aggregate equilibrium. Aggregate accounting identity suggests that

$$C_t + H_t v_t = w_t L_t + N_t d_t + d_{B,t} + d_{O,t}. \tag{27}$$

The model consists of 31 endogenous variables and 31 equilibrium conditions, including the equation that governs the nominal interest rate setting by the monetary authority and the setting of the labour subsidy to eliminate the inefficiency generated by monopolistic competition among producers, in order to achieve the efficient equilibrium. Endogenous variables are ρ_t, μ_t, Υ_t, $\rho_{P,t}$, $\rho_{B,t}$, $\rho_{O,t}$, w_t, π_t, π_t^C, Y_t, $Y_{B,t}$, $Y_{O,t}$, $Y_{B,t}^{Search}$, $Y_{O,t}^{Search}$, $\mathcal{P}_{B,t}$, $\mathcal{P}_{O,t}$, $\mathcal{Q}_{B,t}$, $\mathcal{Q}_{O,t}$, $\mathcal{T}_{B,t}$, $\mathcal{T}_{O,t}$, $\mathcal{M}_t$, $\mathcal{M}_{O,t}$, $\mathcal{M}_{B,t}$, C_t, N_t, H_t, d_t, v_t, L_t, $\tau_{L,t}$, and i_t. N_t constitutes a state variable in the system. Table 1 and 2 summarise model equations.

4 The New Keynesian Phillips Curve

We log-linearise the model around the efficient steady state with zero inflation and derive the New Keynesian Philips Curve. We denote log-linearised variables in San Serif fonts or in Serif fonts capped with Tildes.

Description	Equation
Producer pricing	$\rho_t = \mu_t \frac{w_t}{Z_t}$
Producer markup	$\mu_t = \frac{\sigma_t}{(\sigma_t - 1)\left(1 - \frac{\kappa}{2}(\pi_t)^2\right) + \kappa \Upsilon_t}$
Definition	$\Upsilon_t = \pi_t (1 + \pi_t)$
	$-\beta (1 - \delta) \mathbb{E}_t \frac{C_t}{C_{t+1}} \frac{N_t}{N_{t+1}} \frac{\rho_{P,t+1}}{\rho_{P,t}} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (1 + \pi_{t+1})$.
Variety effects	$\rho_t = N_t^{\psi} \rho_{P,t}$
Search cost (Online)	$\mathcal{M}_{O,t} = \left(\frac{1}{1 - \zeta_{O,t} \mathcal{Q}_{O,t}}\right)^{-1}$
Search cost (Brick-and-mortar)	$\mathcal{M}_{B,t} = \left(\frac{1}{1 - \zeta_{B,t} \mathcal{Q}_{B,t}}\right)^{-1}$
Aggregate search cost	$\mathcal{M}_t = \mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t}$
Retail price (Online)	$\rho_{O,t} = \mathcal{M}_{O,t} \rho_{P,t}$
Retail price (Brick-and-mortar)	$\rho_{B,t} = \mathcal{M}_{B,t} \rho_{P,t}$
Real CPI	$1 = \rho_{O,t}^{\alpha_t} \rho_{B,t}^{1-\alpha_t}$
Producer profits	$d_t = \left(1 - \frac{1}{\mu_t} - \frac{\kappa}{2} (\pi_t)^2\right) \frac{\rho_{P,t} Y_t}{N_t}$
Free entry condition for producers	$v_t = f_{E,t}$
Motion of producers	$N_{t+1} = (1 - \delta)(N_t + H_t)$
Euler equation for producers	$v_t = \beta (1 - \delta) \mathbb{E}_t \frac{C_t}{C_{t+1}} (v_{t+1} + d_{t+1})$
Optimal labour supply	$\varepsilon_{L,t} \chi L_t^{\frac{1}{\varphi}} = (1 + \tau_L) \frac{w_t}{C_t}$
Matching function (Online)	$Y_{O,t} = \left(\left(\zeta_O Y_{O,t}^{Search}\right)^{-\lambda} + N_t^{-\lambda}\right)^{-1/\lambda}$
Matching function (Brick-and-mortar)	$Y_{B,t} = \left(\left(\zeta_B Y_{B,t}^{Search}\right)^{-\lambda} + N_t^{-\lambda}\right)^{-1/\lambda}$
Tightness (Online)	$\mathcal{T}_{O,t} = \frac{\zeta_O Y_{O,t}^{Search}}{N_t}$
Tightness (Brick-and-mortar)	$\mathcal{T}_{B,t} = \frac{\zeta_B Y_{B,t}^{Search}}{N_t}$
Prob. of producer matching (Online)	$\mathcal{P}_{O,t} = \frac{Y_{O,t}}{N_t}$
Prob. of producer matching (Brick-and-mortar)	$\mathcal{P}_{B,t} = \frac{Y_{B,t}}{N_t}$

Table 1: Summary of equations

Description	Equation
Prob. of retailer matching (Online)	$\mathcal{Q}_{O,t} = \frac{Y_{O,t}}{\zeta_O Y_{O,t}^{Search}}$
Prob. of retailer matching (Brick-and-mortar)	$\mathcal{Q}_{B,t} = \frac{Y_{B,t}}{\zeta_B Y_{B,t}^{Search}}$
Use of retail goods (Online)	$Y_{O,t} = C_{O,t} + \alpha_t \frac{H_t f_{E,t}}{\rho_{O,t}} + Y_{O,t}^{Search}$
Use of retail goods (Brick-and-mortar)	$Y_{B,t} = C_{B,t} + (1-\alpha_t) \frac{H_t f_{E,t}}{\rho_{B,t}} + Y_{B,t}^{Search}$
Total production	$\rho_{P,t} Y_t = \rho_{P,t} (Y_{O,t} + Y_{B,t}) + \frac{\kappa}{2} (\pi_t)^2 \rho_{P,t} Y_t$
Good market clearing	$C_t + H_t v_t = w_t L_t + N_t d_t$
CPI inflation	$\frac{1+\pi_t}{1+\pi_t^C} = \frac{\rho_t}{\rho_{t-1}}$
Euler equation for bonds	$1 = \beta \mathbb{E}_t \frac{C_t}{C_{t+1}} \left(\frac{1+i_t}{1+\pi_{t+1}^C} \right)$

Table 2: Summary of equations (Continued)

We start with log-linearizing Equation 2 around the steady state of μ_t which is equal to $\sigma/(\sigma-1)$.

$$\pi_t = \beta (1-\delta) \mathbb{E}_t \pi_{t+1} - \frac{\sigma-1}{\kappa} \widetilde{\mu}_t \tag{28}$$

where individual producer price inflation rate π_t is also percent deviations of gross inflation from zero steady-state while $\widetilde{\mu}_t$ denotes the percent deviations of marginal cost from the steady state. From real CPI equation, and retail prices,

$$\begin{aligned} 1 &= \rho_{O,t}^{\alpha_t} \rho_{B,t}^{1-\alpha_t} \\ 1 &= \mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t} \rho_{P,t}. \end{aligned}$$

Along with pricing rule and $\rho_t = N_t^{\psi} \rho_{P,t}$ and we get

$$\mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t} \mu_t \frac{w_t}{Z_t} = N_t^{\psi} \tag{29}$$

Log linearise Equation (29)[8], we get

$$\hat{\mu}_t = \psi \mathsf{N}_t - \alpha (\ln \mathcal{M}_O - \ln \mathcal{M}_B) \tilde{\alpha}_t + \alpha \tilde{\mathcal{M}}_{O,t} + (1-\alpha) \tilde{\mathcal{M}}_{B,t} - (\mathsf{w}_t - \mathsf{Z}_t).$$

Put this expression in Equation (28),

$$\begin{aligned} \pi_t =& \beta (1-\delta) \mathbb{E}_t \pi_{t+1} + \frac{\sigma-1}{\kappa} (\mathsf{w}_t - \mathsf{Z}_t) - \frac{\sigma-1}{\kappa} \psi \mathsf{N}_t \\ &+ \frac{\sigma-1}{\kappa} \Big(-\alpha (\ln \mathcal{M}_B - \ln \mathcal{M}_O) \tilde{\alpha}_t + \alpha \tilde{\mathcal{M}}_{O,t} + (1-\alpha) \tilde{\mathcal{M}}_{B.t} \Big). \end{aligned} \tag{30}$$

[8]Please refer to B for detailed calculations

Equation 30 represents the New Keynesian Phillips Curve. Variables written in Serif fonts are measured in terms of the deviation from the steady state. The term $E_t(\pi_{t+1})$ stands for expected inflation in the next period, suggesting that current inflation depends on what households and firms think inflation will be in the future. This highlights the forward-looking nature of economic decisions. The expression $w_t - Z_t$ represents the marginal costs of hiring additional units of labour. When real wages increase, it raises the cost of labour, passing on to higher inflation. Conversely, when productivity rises, companies can produce the same output with fewer workers, reducing labour demand and inflationary pressures. As in Bilbiie, Ghironi, and Melitz (2008), our version of the New Keynesian Phillips Curve engenders the marginal cost that captures the number of producers.

We propose novel implications of goods market search frictions on inflation, specifically the effects of temporary changes in the share of online retail sales to total expenditure on CPI inflation. These effects are captured in the following terms:

$$\frac{\sigma-1}{\kappa}\left(\alpha\left(\ln\mathcal{M}_{O,t}-\ln\mathcal{M}_{B,t}\right)\tilde{\alpha}_t+\alpha\tilde{\mathcal{M}}_{O,t}+\left(1-\alpha\right)\tilde{\mathcal{M}}_{B,t}\right) \tag{31}$$

The effects of temporary changes in the share of online retail sales to total expenditure on CPI inflation can be decomposed into two channels. The first channel, which we call the composition channel, arises from the term

$$\frac{\sigma-1}{\kappa}\alpha\left(\ln\mathcal{M}_O-\ln\mathcal{M}_B\right). \tag{32}$$

This term suggests that an increase in the share of online retail sales (a positive deviation of α_t from the steady state) alters inflation. As online retailers are more efficient at matching than brick-and-mortar retailers, $\ln\mathcal{M}_O - \ln\mathcal{M}_B$ is negative. Given the restrictions on $\alpha - 1$ and κ, this term implies that price inflation decreases in response to a rise in the share of online retail sales. Intuitively, as consumers shift their preferences towards online retail, they purchase from online retailers who charge a lower wedge on top of producer prices, resulting in lower consumer prices and a decline in aggregate consumer prices (or a negative inflation rate). The second channel, which we refer to as the arbitrage channel, is captured by

$$\frac{\sigma-1}{\kappa}\left(\alpha\tilde{\mathcal{M}}_{O,t}+\left(1-\alpha\right)\tilde{\mathcal{M}}_{B,t}\right). \tag{33}$$

As customers shift their preferences towards online retail markets, the marginal benefits of search for online retailers increase, leading them to exert more effort in searching. This increased search effort tightens the online frictional market, decreasing the probability of retailers finding firms and raising search costs. Consequently, online retailers must increase the wedge between consumer and producer prices to cover these costs. Conversely, as customer demand shifts away from brick-and-mortar stores, the marginal benefits of search for these retailers decrease, causing them to exert less effort in searching. This reduced search effort loosens the traditional frictional market, increasing the probability of retailers finding firms and lowering search costs. As a result, brick-and-mortar stores will charge a lower wedge.

Lastly, the New Keynesian Phillips curve for CPI inflation can be expressed as:

$$
\begin{aligned}
\pi_t^C =& \beta(1-\delta) E_t \pi_{t+1}^C + \frac{\sigma-1}{\kappa}(\mathsf{w}_t - \mathsf{Z}_t) - \frac{\sigma-1}{\kappa}\psi \mathsf{N}_t + \frac{\sigma-1}{\kappa}\tilde{\mathcal{M}}_t \\
& - \psi(\mathsf{N}_t - \mathsf{N}_{t-1} - \beta(1-\delta)(\mathsf{N}_{t+1} - \mathsf{N}_t)) \\
& + \left(\tilde{\mathcal{M}}_t - \tilde{\mathcal{M}}_{t-1} - \beta(1-\delta)\left(\tilde{\mathcal{M}}_{t+1} - \tilde{\mathcal{M}}_t\right)\right).
\end{aligned} \tag{34}
$$

Here, π_t^C represents the percentage deviation of gross CPI inflation from steady state. $\tilde{\mathcal{M}}_t$ stands for the percentage deviation of aggregate search wedge from steady state. CPI inflation exhibits greater endogenous persistence than firm-level inflation, as it's directly influenced by the number of firms producing in $t-1$, which was set in $t-2$.

5 Estimation and Interpretation of the results

5.1 Calibration

Table 3 summarises the calibration of the benchmark model, which we calibrate using quarterly data. The discount factor, β, is set to the standard value of 0.99, commonly used in the literature. The Frisch elasticity of labour supply, φ, is set to 2, a value that falls within the range typically used in the literature. The elasticity of substitution among varieties, σ, is calibrated to 11.5 based on estimates of within-brand elasticity from Broda and Weinstein (2010). The exogenous product destruction rate, δ, is set to 0.0588 to match the mean annual product creation rate of 0.25, also from Broda and Weinstein (2010). We assign a value of 0.0952 to the parameter ψ, which governs the marginal utility derived from an increase in product variety. This value aligns with the Dixit-Stiglitz preference specification. Lastly, the disutility of labour supply, χ, is a scaling parameter on the disutility from labour. The choice of this parameter is a normalization that does not affect the model's dynamics. It is calibrated to 1.0802 to deliver a steady-state labour supply equal to 1.

To determine the search efficiency of online and brick-and-mortar stores, we use retail margins as a proxy for the search wedge value in both channels. We measure these margins using industry-wide data and specific company metrics to overcome data availability problems. Firstly, we analyze the retail sector's total gross value added and revenue for the overall retail margin, focusing on firms under ISIC 47[9]. Between 2010 and 2021, the ratio of gross value added to total turnover averaged 0.21, translating to a steady-state industry search wedge of 1.27. Secondly, to estimate

[9] We refer to ISIC, Rev. 4 - Code 47 Retail trade, except motor vehicles and motorcycles

Table 3: Calibrated Parameters

Parameter	**Description**	**Value**
β	Discount factor	0.99
φ	Frisch elasticity of labour supply	2
σ	Elasticity of substitution among varieties	11.5
ψ	Marginal utility from increased product variety	0.0952
δ	Exogenous product destruction rate	0.0588
χ	Disutility of labour	1.0802
ζ_O	Search efficiency of online stores	8.7
ζ_B	Search efficiency of brick-and-mortar stores	4.3

the search wedge for online retailers, we use Amazon's balance sheet as a proxy for UK online retailers[10]. Amazon's gross profit margin averaged 0.11, translating to a search wedge of 1.13. We assign this number to the steady-state search wedge of the online retailers, $\mathcal{M}_O$. Using the average share of online retail sales during the designated period, we differentiate between online and brick-and-mortar retail. Our calculations show search efficiency rates of 8.7 and 4.3 times the amount spent on search activity for online and brick-and-mortar stores, respectively (ζ_O and ζ_B). This data highlights that online retailers are more search-efficient than brick-and-mortar stores.

5.2 Estimation

We estimate the model using quarterly UK data. Our baseline spans the period between 2013:I and 2022:IV. We are restricted to this period because we lack data. The data set consists of five variables: CPI nominal inflation, PPI nominal inflation, the share of online retail sales, real GDP growth, and nominal monetary policy interest rate. All the variables are measured in terms of year-over-year growth rates, except for policy rate which is measured in terms of year-over-year level change.

5.2.1 Priors

This study adopts prior distributions from two seminal works: Smets and Wouters (2007) and Harrison and Oomen (2010). The former provides a widely recognized benchmark in Dynamic Stochastic General Equilibrium (DSGE) modelling, offering a solid foundation for our analysis. However, given its focus on the US economy, modifications are required for UK-specific applications. Harrison and Oomen (2010) address this limitation by estimating an adapted version of the model

[10] Amazon.com, Inc. was selected as the representative model for online retail platforms due to its significant market presence and comprehensive data availability. Amazon is a primarily online retailer, capturing a considerable share of the online retail markets. Its sales constitute 21% of the UK's total online retail sales in 2022. We pick Amazon as a result. Including smaller pure online retailers in our calculations does not materially alter the results

using UK data. By synthesizing priors from these two sources, we maintain comparability with extant literature while ensuring relevance to the UK economic context.

Firstly, we set the prior mean of the producer price adjustment cost, denoted by κ, at 300. This parameter significantly influences the response of real profits and the subsequent adjustment in the number of producers to a monetary policy shock. To attenuate the positive reaction of consumer prices to an expansionary monetary policy shock, allowing for an increase in real profits and encouraging firm entry κ must be sufficiently large. Our choice of prior closely follows the estimates for producer price adjustment costs found in the study by Harrison and Oomen (2010) using UK data. For the monetary policy rule parameters, we use a normal distribution to describe the long-term responses to inflation, ϕ_π, and the output gap, ϕ_c, with means of 1.870 and 0.110, and standard deviations of 0.281 and 0.027, respectively. The rule's persistence is captured by the coefficient on the lagged interest rate, assumed to be normally distributed with a mean of 0.870 and a standard deviation of 0.05. Consequently, an expansionary monetary policy shock results in an increase in the number of firms. For the matching function parameter λ, we lack prior distributional information beyond its non-negativity constraint. So, we experiment on prior specifications. Lastly, we assume the persistence of stochastic processes to be a beta distribution, having an average of 0.5 and a standard deviation of 0.2. We assume the standard deviation of the processes to follow an inverse-gamma distribution, with an average value of 0.10.

Parameter	**Prior Distr.**	**Prior Mean**	**Prior St.Dev.**	**Post. Mode**	**Post. Mean**	**Post. St.Dev.**	**5% HPD**	**95% HPD**
λ	Invgamma	30	3	27.098	28.484	2.728	23.062	33.637
ϕ_π	Normal	1.87	0.281	1.956	1.839	0.275	1.285	2.385
ϕ_c	Normal	0.11	0.027	0.187	0.202	0.025	0.152	0.253
κ	Normal	300	30	299.862	302.333	32.536	243.671	363.220
ρ_r	Beta	0.87	0.05	0.579	0.565	0.055	0.469	0.657
ρ_α	Beta	0.5	0.2	0.721	0.720	0.101	0.560	0.893
ρ_{MONET}	Beta	0.5	0.2	0.687	0.665	0.062	0.556	0.776
ρ_Z	Beta	0.5	0.2	0.183	0.240	0.102	0.055	0.416
ρ_{elastic}	Beta	0.5	0.2	0.954	0.850	0.046	0.726	0.983
$\rho_{L\text{shock}}$	Beta	0.5	0.2	0.500	0.503	0.277	0.181	0.836
$\rho_{B\text{shock}}$	Beta	0.5	0.2	0.339	0.327	0.113	0.146	0.500
σ_α	Invgamma	0.1	2	0.084	0.088	0.010	0.071	0.105
σ_{MONET}	Invgamma	0.1	2	0.023	0.023	0.003	0.018	0.029
σ_Z	Invgamma	0.1	2	0.090	0.081	0.020	0.037	0.120
σ_{elastic}	Invgamma	0.1	2	0.673	1.140	0.186	0.507	1.790
$\sigma_{L\text{shock}}$	Invgamma	0.1	2	0.046	0.075	0.019	0.026	0.131
$\sigma_{B\text{shock}}$	Invgamma	0.1	2	0.179	0.188	0.022	0.148	0.226

Table 4: Parameter estimates with updated prior and posterior distributions

5.2.2 Posterior estimation

Posterior estimates provide insights into key economic parameters and shocks. The producer price adjustment cost parameter is estimated to be 302.333 (SD 32.536), which is close to Harrison and Oomen (2010)'s estimates. The λ parameter (30.5407, SD 2.6960) indicates that retailers' search efforts respond strongly to changes in market conditions. Monetary policy parameters ϕ_π (1.839, SD 0.275) and ϕ_c (0.202, SD 0.025) show that the central bank adheres to the Taylor principle, reacting more than proportionally to inflation while also considering economic growth.

The economic shocks analyzed exhibit distinct characteristics in terms of persistence and volatility. Shocks to the share of online retail sales are moderately persistent in the medium term ($\rho_\alpha = 0.720$, SD 0.101), with moderately volatile innovations ($\sigma_\alpha = 0.088$, SD 0.010) relative to other shocks. Cost-push shocks, in contrast, are both the most persistent ($\rho_{\text{elastic}} = 0.850$, SD 0.046) and exhibit the highest volatility ($\sigma_{\text{elastic}} = 1.140$, SD 0.186), indicating their long-lasting and significant effects on the economy. In comparison, productivity shocks demonstrate lower persistence with an AR(1) coefficient of 0.240 (SD 0.102) and relatively small innovations of 0.081 (SD 0.020). The persistence of online retail sales shocks is similar to that of monetary policy shocks ($\rho_{\text{MONET}} = 0.665$, SD 0.062); however, their innovations are significantly smaller, being approximately one-third the size ($\sigma_{\text{MONET}} = 0.023$, SD 0.003). These observations imply that monetary policy adheres closely to the Taylor rule, with interest rate decisions exhibiting moderate persistence and modest economic impacts.

5.3 Shock decomposition

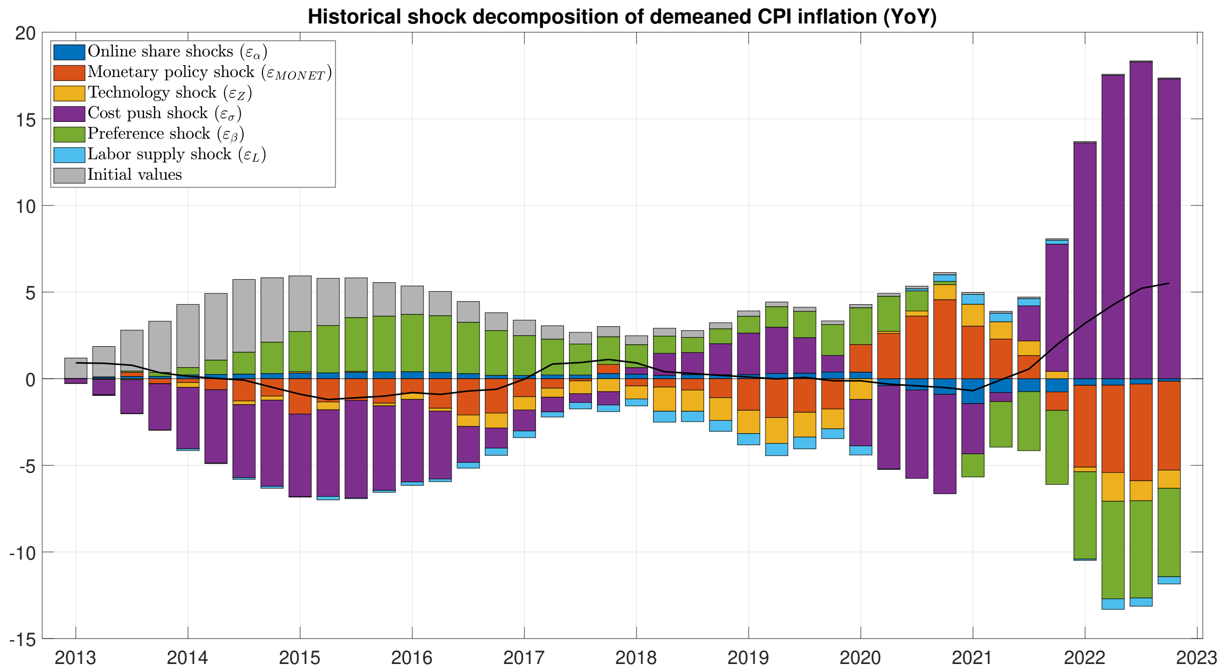

Figure 3: Historical shock decomposition of CPI inflation

Figure 3 illustrates the historical contributions to CPI inflation from various shocks of interest. These include shocks to the share of online retail sales relative to total retail sales (represented by blue bars), monetary policy shocks (orange bars), and cost-push shocks (yellow bars). Contributions from other shocks, including aggregate productivity and labour supply shocks, are aggregated and illustrated by the purple bars. The analysis spans the period of the COVID-19 pandemic in 2020-2021, providing insight into the predominant factors influencing CPI fluctuations during this time frame.

Notably, the shifts towards online retail sales negatively impacted CPI inflation from the second quarter of 2020 (2020:II) to the fourth quarter of 2021 (2021:IV), aligning with the onset of the COVID-19 health and economic crisis. The magnitude of this shock's contribution increased from 2020:II to 2020:IV, subsequently declining and transitioning to an inflationary impact by the first quarter of 2022. Our finding underscores the role of search and matching friction in good markets in determining CPI inflation.

5.4 Impulse responses

Figure 4 shows how key variables respond to a 1 percent increase in online retail sales share. The figure displays changes in CPI inflation (π_t^C), individual price inflation (π_t), number of producers (N_t), consumption (C_t), real wage (w_t), and market tightness and search costs for both online ($\mathcal{T}_{O,t}$, $\mathcal{M}_{O,t}$) and brick-and-mortar retailers ($\mathcal{T}_{B,t}$, $\mathcal{M}_{B,t}$). The increase in online retail sales mainly lowers CPI inflation (π_t^C), matching empirical evidence. It also boosts consumption (C_t). This occurs because online retailers are estimated to have higher search efficiency than brick-and-mortar stores. More online sales shift demand to these more efficient retailers. They charge lower search costs ($\mathcal{M}_{B,t} > \mathcal{M}_{O,t}$) and offer lower retail prices ($\rho_{O,t} < \rho_{B,t}$). Lower prices increase consumer demand, raising consumption (C_t). Higher demand increases producer profits and share prices. This encourages new firms to enter the market, increasing product variety (N_t).

To better understand the mechanisms at work, we examine the channel through which the shocks to online retail sales share affect the CPI inflation rate. For clarity, we focus our analysis on the response of individual price inflation to a 1% increase in the share of online retail sales. We decompose this response into three key elements within our newly derived New Keynesian Phillips Curve (NKPC) model: the search wedge, variety, and real wage channels. We further subdivide the search wedge channel into compositional and arbitrage channels. Our model expands the expected individual price inflation at time t+1 into a function of future NKPC component values, which are added to contemporaneous values in the NKPC. Figure 4 illustrates these results graphically.

The analysis shows that an increase in online retail sales share exerts deflationary pressure through variety channels and search wedge channels. Within the search wedge channels, compositional channels are more potent but shorter-lived, playing a significant deflationary role in the initial periods. This likely reflects the medium persistence of online retail sales shocks. In contrast, arbitrage channels persist longer and become more influential later than compositional channels,

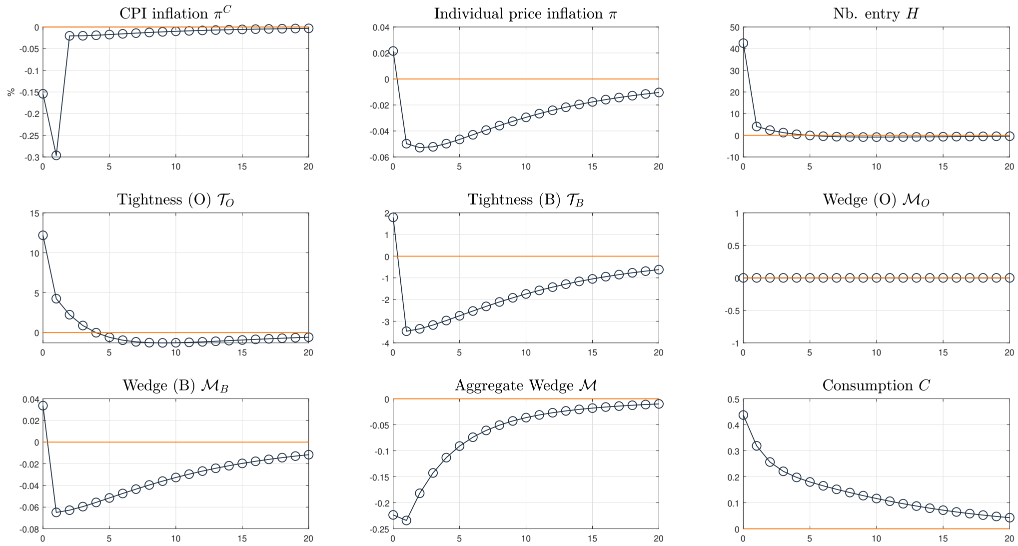

Figure 4: Response to 1% increase in the share of online retail sales

continuing as long as online retail sales shares deviate from their steady state. The variety channel reflects increased product options available online, lowering prices.

6 Conclusion

This study develops an economic framework to examine how short-term preference shocks towards online versus brick-and-mortar retail impact pricing dynamics and inflation measures. We construct and estimate a New Keynesian DSGE model incorporating frictional goods markets, featuring product market search and matching frictions. The model distinguishes between online and brick-and-mortar retailers, each with different search efficiencies, to capture shifts in consumer preferences.

We identify two primary channels affecting inflation dynamics: the composition channel and the arbitrage channel. The composition channel stems from differences in search efficiency between retailer types, while the arbitrage channel reflects how preference changes affect search and matching conditions in both markets. Using Bayesian estimation, our findings indicate that shifts in consumer preferences towards online retailers contribute to reduced CPI inflation. Both channels have reinforcing effects: the composition channel shows that preference shifts towards online retail lead to lower inflation due to greater search efficiency, while the arbitrage channel loosens market tightness for brick-and-mortar retailers, allowing them to charge a smaller wedge between consumer and producer prices.

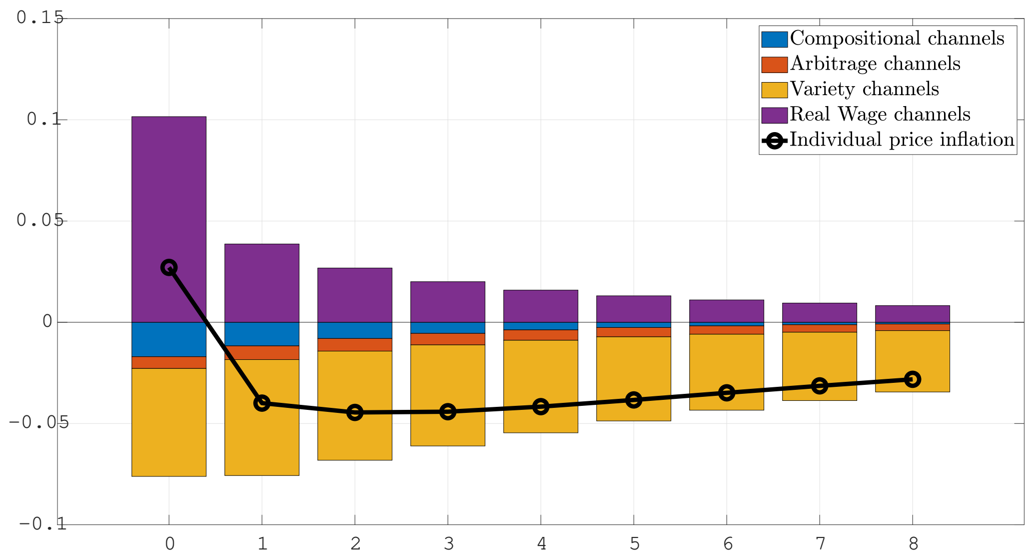

Figure 5: Response to 1% increase in the share of online retail sales

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Appendix for "Search frictions in good markets and CPI inflation

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Contents

A Data

Table 5 and 6 list the data series for local projection estimation and Bayesian estimation, respectively.

B Model

B.1 A first-order Taylor approximation of $\mathcal{M}_t$

To perform a first-order Taylor approximation of the function $\mathcal{M}_t = \mathcal{M}_{O,t}^{\alpha_t}\mathcal{M}_{B,t}^{1-\alpha_t}$ around the steady state $\mathcal{M}_O$, $\mathcal{M}_B$ and α_t, let $f\left(\mathcal{M}_{O,t}, \mathcal{M}_{B,t}, \alpha_t\right) = \alpha_t \log M_{O,t} + (1-\alpha_t)\log M_{B,t}$. Then we calculate

$$\begin{aligned}
f\left(\mathcal{M}_{O,t}, \mathcal{M}_{B,t}, \alpha_t\right) \approx & f\left(\mathcal{M}_O, \mathcal{M}_{B,}, \alpha\right) \\
& + \left.\frac{\partial f}{\partial \mathcal{M}_O}\right|_{\left(\mathcal{M}_O, \mathcal{M}_{B,}, \alpha\right)} \left(\mathcal{M}_{O,t} - \mathcal{M}_O\right) \\
& + \left.\frac{\partial f}{\partial \mathcal{M}_B}\right|_{\left(\mathcal{M}_O, \mathcal{M}_{B,}, \alpha\right)} \left(\mathcal{M}_{B,t} - \mathcal{M}_B\right) \\
& + \left.\frac{\partial f}{\partial \alpha}\right|_{\left(\mathcal{M}_O, \mathcal{M}_{B,}, \alpha\right)} \left(\alpha_t - \alpha\right).
\end{aligned}$$

We get

Variable	Description	Source
Consumer Price	Consumer Price Index excluding energy, food, alcoholic beverages & tobacco	Office for National Statistics
Producer Price	Producer output prices - Domestic manufactured products excluding Duty	Office for National Statistics
Share of Online Retail Sales	Internet sales as a percentage of total retail sales (ratio)	Office for National Statistics
Number of COVID-19 death	Deaths with COVID-19 on the death certificate in the UK	GOV.UK UKHSA data dashboard

Table 5: Data series for local projections

Variable	Description	Source
Consumer Price	Consumer Price Index excluding energy, food, alcoholic beverages & tobacco	Office for National Statistics
Producer Price	Producer output prices - Domestic manufactured products excluding Duty	Office for National Statistics
Share of Online Retail Sales	Internet sales as a percentage of total retail sales (ratio)	Office for National Statistics
Monetary policy rate	Official Bank Rate	Bank of England
Real GDP growth	Gross Domestic Product: chained volume measures	Office for National Statistics

Table 6: Data series for Bayesian estimation

$$\tilde{\mathcal{M}}_t \approx \ln \mathcal{M}_O (\alpha_t - \alpha) + \ln \mathcal{M}_B (\alpha - \alpha_t) + \alpha \tilde{\mathcal{M}}_{O,t} + (1-\alpha) \tilde{\mathcal{M}}_{B,t}$$
$$= -\alpha (\ln \mathcal{M}_B - \ln \mathcal{M}_O) \tilde{\alpha}_t + \alpha \tilde{\mathcal{M}}_{O,t} + (1-\alpha) \tilde{\mathcal{M}}_{B,t}.$$

B.2 Derive Philip's curve for CPI inflation

From the definition of CPI inflation,

$$\frac{1+\pi_t}{1+\pi_t^C} = \frac{\rho_t}{\rho_{t-1}}$$
$$= \frac{N_t^\psi}{N_{t-1}^\psi} \frac{\rho_{P,t}}{\rho_{P,t-1}}$$
$$= \frac{N_t^\psi}{N_{t-1}^\psi} \frac{\mathcal{M}_{t-1}}{\mathcal{M}_t}$$

First-order approximation yields

$$\pi_t = \pi_t^C + \psi (\mathsf{N}_t - \mathsf{N}_{t-1}) - \left(\tilde{\mathcal{M}}_t - \tilde{\mathcal{M}}_{t-1}\right)$$
$$\pi_t = \pi_t^C + \psi (\mathsf{N}_t - \mathsf{N}_{t-1})$$
$$- \left(-\alpha (\ln \mathcal{M}_B - \ln \mathcal{M}_O) \tilde{\alpha}_t + \alpha \tilde{\mathcal{M}}_{O,t} + (1-\alpha) \tilde{\mathcal{M}}_{B,t}\right)$$
$$+ \left(-\alpha (\ln \mathcal{M}_B - \ln \mathcal{M}_O) \tilde{\alpha}_{t-1} + \alpha \tilde{\mathcal{M}}_{O,t-1} + (1-\alpha) \tilde{\mathcal{M}}_{B,t-1}\right)$$

From the expression

$$\pi_t = \beta (1-\delta) E_t \pi_{t+1} + \frac{\sigma-1}{\kappa} (\mathsf{w}_t - \mathsf{Z}_t) - \frac{\sigma-1}{\kappa} \psi \mathsf{N}_t + \frac{\sigma-1}{\kappa} \tilde{\mathcal{M}}_t$$
$$\pi_t^C + \psi (\mathsf{N}_t - \mathsf{N}_{t-1}) - \left(\tilde{\mathcal{M}}_t - \tilde{\mathcal{M}}_{t-1}\right)$$
$$= \beta (1-\delta) E_t \left(\pi_{t+1}^C + \psi (\mathsf{N}_{t+1} - \mathsf{N}_t) - \left(\tilde{\mathcal{M}}_{t+1} - \tilde{\mathcal{M}}_t\right)\right)$$
$$+ \frac{\sigma-1}{\kappa} (\mathsf{w}_t - \mathsf{Z}_t) - \frac{\sigma-1}{\kappa} \psi \mathsf{N}_t + \frac{\sigma-1}{\kappa} \tilde{\mathcal{M}}_t,$$

which can be expressed as

$$\begin{aligned}\pi_t^C =& \beta(1-\delta) E_t \pi_{t+1}^C + \frac{\sigma-1}{\kappa}(\mathsf{w}_t - \mathsf{Z}_t) - \frac{\sigma-1}{\kappa}\psi \mathsf{N}_t + \frac{\sigma-1}{\kappa}\tilde{\mathcal{M}}_t \\ & - \psi(\mathsf{N}_t - \mathsf{N}_{t-1} - \beta(1-\delta)(\mathsf{N}_{t+1} - \mathsf{N}_t)) \\ & + \left(\tilde{\mathcal{M}}_t - \tilde{\mathcal{M}}_{t-1} - \beta(1-\delta)\left(\tilde{\mathcal{M}}_{t+1} - \tilde{\mathcal{M}}_t\right)\right),\end{aligned}$$

where

$$\begin{aligned}\tilde{\mathcal{M}}_t \approx & \ln \mathcal{M}_O(\alpha_t - \alpha) + \ln \mathcal{M}_B(\alpha - \alpha_t) + \alpha \tilde{\mathcal{M}}_{O,t} + (1-\alpha)\tilde{\mathcal{M}}_{B,t} \\ = & -\alpha(\ln \mathcal{M}_B - \ln \mathcal{M}_O)\tilde{\alpha}_t + \alpha \tilde{\mathcal{M}}_{O,t} + (1-\alpha)\tilde{\mathcal{M}}_{B,t}.\end{aligned}$$

B.3 Steady state calculations

In this section, we provide the detailed calculations of the steady state. Firstly, the free entry condition provides the steady state of the post-entry firm value,

$$v = f_E.$$

while the Euler equation for shares provides the steady-state value of the real profits of the producers,

$$d = \frac{1-\beta(1-\delta)}{\beta(1-\delta)} v.$$

From the motion of firms, we can write the number of entrants at the steady state

$$H = \frac{\delta}{1-\delta} N$$

To simplify the steady-state calculation, χ is set to the value such that the steady-state labour supply, L, is equal to one.[1] It simplifies intratemporal optimality,

$$\chi = (1+\tau_L)\frac{w}{C}.$$

Then, from labour market clearing condition,

$$1 = N\left[(\sigma-1)\frac{d}{w}\right],$$

[1] *chi* that satisfies such condition is

$$\chi = (1+\tau_L)\frac{(\sigma-1)d}{\sigma d - \frac{\delta}{1-\delta}v}$$

and the aggregate accounting,

$$C + Hv = w + Nd + d_O + d_B,$$

putting these three steady-state equations together yields

$$N = \left(\frac{Z}{\sigma \mathcal{M} d} \right)^{\frac{1}{1-\psi}}.$$

The steady-state value of other variables can be computed straightforwardly.