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Abstract

This paper studies the welfare cost of business cycles under incomplete markets and heterogeneous labour skills for male and female workers. The main goals are to estimate welfare gains and/or losses of economic agents if they could live in an economy without aggregate uncertainty, and to analyse the magnitudes of gains and/or losses among subgroups of agents. These tasks can be realised by calibrating a stochastic general equilibrium model with aggregate productivity shocks, individual skill uncertainty and unemployment risks, and compare the results to a similar model only without aggregate fluctuations. It is found that when business cycles are removed the overall welfare could increase up to almost 6% which is 700 times larger than the famous result in Lucas (1987). However, from a disaggregated perspective, the results are contrary to conventional expectation that subgroups with lower income should gain more benefit from the removal of business cycles due to the more adverse labour market conditions which hinder the ability to smooth consumption particularly under liquidity constraints and aggregate uncertainty. Instead, females gain less benefit than males, low-skilled workers are less better off than high-skilled workers, and unemployed workers gain slightly less than employed workers. Wealth inequality is found to remain fairly unchanged when business cycles are present in the economy although there is a noticeable shift in wealth distribution.

JEL Classification. E24, E32, J24, J64, J65.

Keywords. Business cycles, wealth inequality, worker heterogeneity.

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1 Introduction

It is apparent that a macroeconomy experiences business cycles which hinder the ability of economic agents to plan for their future and maximise their lifetime utilities. It is also evident that key economic policies are aimed to stabilise and protect an economy from these fluctuations. Hence, it is not questionable that one would want to find out how much better off we will be if the aims of these policies are effectively achieved.

Lucas (1987) answered this question basing on a representative agent model with perfect insurance markets where there is no borrowing constraints. He calculated the cost of aggregate instability as a percentage increase in consumption necessary to leave the consumer indifferent between living in an economy with aggregate uncertainty and in an economy without, which he found it to be “extremely low”. It could be as low as 0.008% of average consumption with logarithmic utility function. However, in an actual economy where incomplete markets prevail, this famous result may no longer hold because not only economic agents cannot be fully insured against aggregate risks but heterogeneity in wealth status also implies further idiosyncratic uncertainty.

İmrohoroğlu (1989) explored the possibility that the welfare cost of business cycles could increase significantly if an economic model captures the existence of incomplete markets. Using a general equilibrium model with exogenous liquidity constraints, she compared the results with those from a model without borrowing constraints. She found that the welfare cost of business cycles actually increases significantly with the presence of incomplete markets and it is mainly due to the change in capital accumulation behaviour.[1] Even though the calculated losses in consumption due to aggregate uncertainty are only between 0.3-1.5% of total consumption, this paper sheds light on the importance of imposing incomplete market condition in the analysis of business cycles.

However, İmrohoroğlu (1989)’s model specifies aggregate fluctuations as only in the form of varying employment probabilities which means that the shocks are idiosyncratic. It is assumed that an economy’s production is not directly affected by business cycles and that it varies only with the change in labour supply which is characterised by unemployment rate. Moreover, the price or rate of interest in an economy is exogenously given and constant which potentially causes less variation in macroeconomic aggregates.

To capture more details of a real economy, Krusell & Smith (1998) extended an Aiyagari-Bewley-Huggett-type model by including aggregate productivity shocks which could explain more movement in aggregate income without causing unemployment rates

[1] The cost is around 4-5 times larger than Lucas (1987)’s result, depending on the rate of risk aversion ranging from 1.5 to 6.2.

to fluctuate too unrealistically.[2] Nonetheless, introducing shocks in both aggregate and individual levels makes the analysis much more complicated particularly with respect to the distribution of wealth and other macroeconomic variables. The distribution is no longer stationary when individual decision rule must depend on the state of aggregate shocks in each period. The problem lies in the evolution of the distribution because at a given point in time this distribution is a highly dimensional object and since future prices are functions of it, the agents would need this information to make decisions on consumption and savings. Most importantly, the paper suggests that agents only need to keep track of the first moment of wealth distribution and aggregate uncertainty to analyse the stochastic behaviour of macroeconomic variables. They found that most agents have the same marginal propensities to consume (which is roughly linear) and behave like a permanent-income representative agent without borrowing constraints. As a result, approximate aggregation does hold and makes this class of general equilibrium analysis with aggregate and individual shocks much more computationally manageable.[3]

With this model in Krusell & Smith (1998), Krusell & Smith (1999) studied the welfare cost of business cycles in a more disaggregated fashion. They focused on consumer heterogeneities and a possibility welfare costs of business cycles could be significantly higher for certain subgroups of consumers than others.[4] In their analysis, the heterogeneities are in preference (discount rate), employment status and wealth or captial holdings. They found that the overall welfare costs, though roughly 17 times greater than the result from Lucas (1987), are quite small and they are negative for many subgroups.[5] The very poor, however, could gain substantially up to 2% from having the cycles eliminated but the size of this subgroup is vanishingly small.

Continuing along this disaggregated path, Mukoyama & Şahin (2006) noticed a distinction between labour market conditions of high- and low-skilled workers,[6] and suggested that difference in welfare gains from having business cycles removed for agents with different skills could be of significance the same way the difference in welfare gains of rich and poor agents is regarded as such in Krusell & Smith (1999). In addition to earning

[2] An Aiyagari-Bewley-Huggett-type model is a model of neoclassical production economies with liquidity constraints and individual employment uncertainty where there are heterogeneities in wealth, income and preferences (discount rates) which hereinafter will be referred to as an Aiyagari model.

[3] What's more, they also found that by introducing heterogeneity in preferences the resulted wealth distribution mimics the US data much better than the model without preference heterogeneity but since it is not the main goal of this paper, I exclude this feature from the analysis.

[4] For example, the rich versus the poor, the employed versus the unemployed.

[5] The long-run gain from eliminating business cycles under economies with one state of unemployment and two states of unemployment (short- and long-term) are respectively 0.138% and 0.068%.

[6] High-skilled workers are defined to have some college degree or above and low-skilled workers to have a high-school diploma or below. While in Mukoyama & Şahin (2006) they labelled workers as skilled and unskilled, this paper uses instead high-skilled and low-skilled with the same definitions.

less income, low-skilled workers face a higher level of unemployment and a more volatile unemployment process implied by the average duration of unemployment. It implies that they would suffer more than high-skilled workers with the presence of business cycles and should be considerably better off without. Their analysis was built on Krusell & Smith (1998)'s model with further specifications to accommodate skill heterogeneity. The findings from this paper also support the earlier implication. For example, the welfare gains from removing business cycles of low-skilled unemployed agents (the poorest) are 6-8 times greater than that of an average agent and 6-10 times greater than that of high-skilled employed agents (the richest).[7] Moreover, welfare gains of agents of any type are higher when the business cycles are removed during a bad aggregate state than a good aggregate state.

To this end, I continue to examine further the roles of consumer heterogeneity in determining welfare gains of eliminating business cycles for different subgroups of agents. In particular, I separate workers not only by wealth, employment status and labour skill, but also by gender. Based on the US Current Population Survey and OECD database, male and female labour markets in the US have certain contradicting environments. Females have more desirable labour market conditions for they have shorter average duration of unemployment and their unemployment rates are actually lower than males' for any skill levels, but at the same time it is evident that their earnings are also lower regardless of skill types.[8] This makes it interesting to find out which types of agents would gain more benefit (or suffer a loss) from the elimination of business cycles and how different the magnitude would be. This paper focuses on results from the economy with cycles and the economy without cycles where it has already reached its the steady state, meaning the results from a Krusell & Smith model will be compared directly with those from an Aiyagari model to calculate welfare gains and/or losses.[9]

[7]Welfare costs of business cycles of low-skilled unemployed agents, an average agent and high-skilled employed agents are respectively 0.150%, 0.024% and 0.027% when the cycles are removed during a good aggegate state and they are respectively 0.622%, 0.081% and 0.063% when the removal takes place in a bad aggregate state.

[8]From the US Current Population Survey, the average unemployment rate for female workers during 1992-2010 is 4.55% while it is 4.70% for male workers. The average duration of unemployment, according to the OECD database, for female workers during 1992-2010 is 18.50 weeks while it is 20.19 weeks for male workers.

[9]In order to calculate this welfare cost of business cycles, we need to compare results between 2 economies: one with business cycles and one without. There have been many works including Krusell & Smith (1999), Mukoyama & Şahin (2006) and Krusell et al (2009) that study in great detail the transitional periods from the former to the latter. However, integrating out aggregate productivity shocks which are a stochastic process from an economy is not a trivial task especially with a substantial degree of worker heterogeneity as individual employment process depends on it. In order to simulate the true transition path, one must create a new uncorrelated individual stochastic process and, based upon these new realisations, integrate out the aggregate risk from individual unemployment process. For complete details on the integration principle, see Appendix A of Krusell et al (2009).

It is found that an average agent would gain a 5.34% increase in consumption if the business cycles are removed in a good period and a 5.88% increase in consumption in a bad period. When agents are separated by types, it turns out higher income subgeroups gain more than lower income subgroups which is the opposite of what we would expect since agents with lower income should be able to borrow and lend more freely, and hence gain more benefit by being an economy with less uncertainty. One relevant explanation is that the level of aggregate capital is higher when the economy is rid of aggregate fluctuations causing interest rate to fall and wage to rise. Since higher income agents rely on both interest rate and wage unlike agents with lower income who rely more on returns from capital, utilities of agents with higher income are more positively affected from the removal of aggregate risks.[10] It is also found that wealth inequality does not significantly change with the elimination of business cycles, but the distribution of agents over capital holdings does change substantially. The distribution becomes more clustered at lower levels of capital when business cycles are present. Despite adding heterogeneities in labour skills and gender could yield further wealth inequality, the model does not reflect the real wealth distribution in the US, at least without heterogeneity in preference.

The paper is organised as follows. Section 2 presents a model setup. Section 3 describes data and probability structures. Section 4 discusses the calibration. Section 5 analyses the results while Section 6 concludes.

2 Model

2.1 Economy with Business Cycles

The model of an economy with business cycles follows Krusell & Smith (1998) which is a stochastic growth model with fluctuations in employment opportunity as well as in aggregate productivity. In addition, the model includes uncertainty in skill levels following Mukoyama & Şahin (2006) and separates male from female workers.

There is a continuum of infinitely-lived agents of measure one. Each agent has the same logarithmic utility function, $u(c_t) = \ln(c_t)$, and discounts future each period by $\beta \in (0, 1)$. They maximise the discounted lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

[10]Evidently, higher income agents rely on wage even more when labour is specified to have higher income share than capital.

The aggregate production is a Cobb-Douglas function with $\alpha \in [0,1]$. It takes the form

$$\begin{aligned} Y &= zF(K,N) \\ &= zK^{\alpha}N^{1-\alpha} \end{aligned}$$

where $z = \{g, b\}$ is the period's aggregate productivity which takes the value g in a good state and b in a bad state. It follows a two-state Markov chain and $g > 1 > b > 0$.

$$K = \int k_i di$$

is the aggregate capital level where k_i denotes individual i's capital holding. Aggregate labour, N, is given by

$$N = \chi_m N_m + \chi_f N_f$$

such that

$$\begin{aligned} N_m &= \sum_{s=h,l} \theta(s)[(1 - U^z_{m,s}) + \mu U^z_{m,s}] \\ N_f &= \Lambda \sum_{s=h,l} \theta(s)[(1 - U^z_{f,s}) + \mu U^z_{f,s}] \end{aligned}$$

where N_m and N_f are respectively male and female aggregate labour supply. χ_m and χ_f are respectively men's and women's shares of total labour force and treated as constants such that $\chi_m > 0$, $\chi_f > 0$ and $\chi_m + \chi_f = 1$. $s = \{h, l\}$ is the skill of a worker which is either high (h) or low (l). $\theta(s)$ is an exogenous function denoting the amount of labour supply of an s-skilled worker where $\theta(h) > \theta(l) > 0$. $\frac{\theta(h)}{\theta(l)}$ denotes the skill premium or the extra amount of labour supply that high-skilled workers possess on top of that of low-skilled workers. $U^z_{m,s}$ is the unemployment rate of male workers of skill level s when the current aggregate state is z, and $U^b_{m,s} > U^g_{m,s} > 0$. $\mu \in [0, 1)$ denotes the value of home production which is always less than income from employment and could be interpreted as an exogenous level of unemployment benefits.[11] The same notations apply to female workers but there is an additional variable $\Lambda \in (0, 1)$ which denotes the gender pay gap and lowers female labour supply.[12]

[11]This is given so that the unemployed could earn some labour income. It is also used in Mukoyama & Şahin (2006).

[12]This pay gap applies not only to the employed but also to the unemployed female workers since unemployment benefits are generally based on previous earnings according to the US Department of Labour's Unemployment Fact Sheets.

For a production function with constant returns to scale, input prices (interest rate and wage) equal their respective marginal productivities, namely

$$\begin{aligned} r(K,N,z) &= zF_K(K,N) \\ &= \alpha z\Big(\frac{K}{N}\Big)^{\alpha-1} \end{aligned}$$

$$\begin{aligned} w(K,N,z) &= zF_N(K,N) \\ &= (1-\alpha)z\Big(\frac{K}{N}\Big)^{\alpha} \end{aligned}$$

Let $\varepsilon = \{1, 0\}$ denote an employment status where 1 means employment and 0 is for unemployment, and $\delta \in [0,1]$ be the depreciation rate of capital. $\phi(\varepsilon)$ signifies labour supply of an agent where $\phi(1) = 1$ and $\phi(0) = \mu$. Define Γ as the measure or distribution of agents over (k, ε, s). The state variables of a given individual are a set of aggregate variables $\{\Gamma, z\}$ and a set of individual variables $\{k, \varepsilon, s\}$. The law of motion for z is defined by its transition probability matrix which is a two-state, discrete-time Markov chain while the law of motion for Γ is represented by a function $T(\cdot)$, namely

$$\Gamma' = T(\Gamma, z, z')$$

where variables denoted with $'$ are of the subsequent period.

Under this setting, an agent's optimisation problem becomes

$$V(k,\varepsilon,s;\Gamma,z) = \max_{c,k'}\{u(c) + \beta E[V(k',\varepsilon',s';\Gamma',z')|k,\varepsilon,s,\Gamma,z]\}$$

subject to

$$\begin{aligned} c + k' &= \begin{cases} rk + w\phi(\varepsilon)\theta(s) + (1-\delta)k & \text{if male} \\ rk + \Lambda w\phi(\varepsilon)\theta(s) + (1-\delta)k & \text{if female} \end{cases} \\ k' &\geq \underline{k} \\ \Gamma' &= T(\Gamma, z, z') \end{aligned}$$

and the stochastic laws of motion for z, ε and s. $\underline{k}$ denotes the borrowing limit. The decision rule for future capital holdings that solves this problem is $k' = g_m(k,\varepsilon,s;\Gamma,z)$ for male workers and $k' = g_f(k,\varepsilon,s;\Gamma,z)$ for female workers.

2.1.1 Recursive Competitive Equilibrium

Definition:

A recursive competitive equilibrium is a set of prices $\{r(K,N,z), w(K,N,z)\}$, policy functions $g_m(k,\varepsilon,s;\Gamma,z)$ and $g_f(k,\varepsilon,s;\Gamma,z)$, a law of motion for distribution $T(\Gamma,z,z')$, an aggregate capital K, and an aggregate labour N such that

1. $r(K,N,z) = zF_K(K,N)$ and $w(K,N,z) = zF_N(K,N)$
2. Given the aggregate states $\{z,\Gamma\}$, prices $\{r(K,N,z), w(K,N,z)\}$ and the law of motion for distribution $T(\Gamma,z,z')$, $g_m(k,\varepsilon,s;\Gamma,z)$ or $g_f(k,\varepsilon,s;\Gamma,z)$ solves the household's optimisation problem.
3. Γ', which is induced by $g_m(k,\varepsilon,s;\Gamma,z)$, $g_f(k,\varepsilon,s;\Gamma,z)$ and the laws of motion for z, ε and s, is consistent with the law of motion for distribution $T(\Gamma,z,z')$.
4. Capital and labour markets clear:

$$\begin{aligned} K &= \int k_i di \\ N &= \chi_m \sum_{s=h,l} \theta(s)[(1-U^z_{m,s}) + \mu U^z_{m,s}] \\ &\quad + \chi_f \Lambda \sum_{s=h,l} \theta(s)[(1-U^z_{f,s}) + \mu U^z_{f,s}] \end{aligned}$$

2.2 Economy without Business Cycles

The model of a smoothed economy or an economy without aggregate fluctuations is similar to one in Aiyagari (1994) where only idiosyncratic unemployment risks prevail and there is no exogenous shock to the production function of the economy. It is equivalent to an economy in Subsection 2.1 but z is replaced with its average,[13] and unemployment rates and average unemployment durations do not vary with the aggregate state in each period. The problem now becomes stationary since K and N are time-independent and there exists a stationary distribution of agents over (k,ε,s). Moreover, N is already predetermined by the laws of motion for s and ε. The national product and input prices are now

$$\begin{aligned} Y &= F(K,N) = K^\alpha N^{1-\alpha} \\ r(K,N) &= F_K(K,N) = \alpha\Big(\frac{K}{N}\Big)^{\alpha-1} \\ w(K,N) &= F_N(K,N) = (1-\alpha)\Big(\frac{K}{N}\Big)^{\alpha} \end{aligned}$$

Under this setting, an agent's optimisation problem becomes

$$V(k,\varepsilon,s) = \max_{c,k'}\{u(c) + \beta E[V(k',\varepsilon',s')|k,\varepsilon,s]\}$$

[13]As will be specified in Section 4, the average of z is 1.

subject to

$$\begin{aligned} c + k' &= \begin{cases} rk + w\phi(\varepsilon)\theta(s) + (1-\delta)k & \text{if male} \\ rk + \Lambda w\phi(\varepsilon)\theta(s) + (1-\delta)k & \text{if female} \end{cases} \\ k' &\geq \underline{k} \end{aligned}$$

and the laws of motion for ε and s. The decision rule for future capital holdings that solves this problem is $k' = g_m(k, \varepsilon, s)$ for male workers and $k' = g_f(k, \varepsilon, s)$ for female workers. Let us define $\gamma_m(k, \varepsilon, s)$ and $\gamma_f(k, \varepsilon, s)$ as the stationary distributions of male and female agents respectively over (k,ε,s) which satisfy

$$\begin{aligned} \gamma_m(k', \varepsilon', s') &= \sum_k \sum_\varepsilon \sum_s Q_{ss'} \pi^{s'}_{m\varepsilon\varepsilon'} 1\{k' = g_m(k, \varepsilon, s)\} \gamma_m(k, \varepsilon, s) \\ \gamma_f(k', \varepsilon', s') &= \sum_k \sum_\varepsilon \sum_s Q_{ss'} \pi^{s'}_{f\varepsilon\varepsilon'} 1\{k' = g_f(k, \varepsilon, s)\} \gamma_f(k, \varepsilon, s) \end{aligned}$$

where

$$\begin{aligned} Q_{ss'} &= \Pr(s_{t+1} = s' | s_t = s) \\ \pi^{s'}_{m\varepsilon\varepsilon'} &= \Pr(\varepsilon_{t+1} = \varepsilon' | \varepsilon_t = \varepsilon, s_{t+1} = s', \text{male}) \end{aligned}$$

and $1\{k' = g_m(k, \varepsilon, s)\}$ is an indicator function which takes the value 1 if $k' = g_m(k, \varepsilon, s)$ and 0 otherwise. $\pi^{s'}_{f\varepsilon\varepsilon'}$ and $1\{k' = g_f(k, \varepsilon, s)\}$ are analogously defined for female workers.

2.2.1 Stationary Competitive Equilibrium

Definition:

A stationary competitive equilibrium is a set of prices $\{r(K, N)$,$w(K, N)\}$, policy functions $g_m(k, \varepsilon, s)$ and $g_f(k, \varepsilon, s)$, stationary distributions $\gamma_m(k, \varepsilon, s)$ and $\gamma_f(k, \varepsilon, s)$, an aggregate capital K and an aggregate labour N such that

1. $r(K, N) = F_K(K, N)$ and $w(K, N) = F_N(K, N)$
2. Given prices $\{r(K, N), w(K, N)\}$, $g_m(k, \varepsilon, s)$ or $g_f(k, \varepsilon, s)$ solves the agent's optimisation problem.
3. $\gamma_m(k, \varepsilon, s)$ and $\gamma_f(k, \varepsilon, s)$ are induced by the laws of motion for ε and s and respective policy functions $g_m(k, \varepsilon, s)$ and $g_f(k, \varepsilon, s)$.

4. Capital market clears:

$$\begin{aligned} K &= \chi_m \sum_k \sum_\varepsilon \sum_s g_m(k,\varepsilon,s)\gamma_m(k,\varepsilon,s) \\ &\quad + \chi_f \sum_k \sum_\varepsilon \sum_s g_f(k,\varepsilon,s)\gamma_f(k,\varepsilon,s) \end{aligned}$$

2.3 Calculating Welfare Cost

The definition of welfare cost used in this paper follows Lucas (1987) and it is equal to λ that satisfies

$$E_0[\sum_{t=0}^{\infty} \beta^t u((1+\lambda)c_t^o)] = E_0[\sum_{t=0}^{\infty} \beta^t u(c_t^s)]$$

I borrow the notations from Mukoyama & Şahin (2006) where c_t^o is period t's consumption in the original economy, where business cycles are present, and c_t^s is period t's consumption in the smoothed economy, where business cycles are removed. As a result, λ represents a percentage increase or decrease in consumption necessary in each period to make an agent in an economy with business cycles be as satisfied as s/he would be in an economy without business cycles. In other words, λ measures the cost of business cycles befalling the economic agents.

With the log utility function, let us denote $V^o = E_0[\sum_{t=0}^{\infty} \beta^t \ln(c_t^o)]$ and $V^s = E_0[\sum_{t=0}^{\infty} \beta^t \ln(c_t^s)]$. They represent the expected (average) discounted lifetime utilities in the original and smoothed economies respectively. Under this setting, λ could then be solved as follows

$$\begin{aligned} E_0[\sum_{t=0}^{\infty} \beta^t \ln((1+\lambda)c_t^o)] &= E_0[\sum_{t=0}^{\infty} \beta^t \ln(c_t^s)] \\ E_0[\sum_{t=0}^{\infty} \beta^t (\ln(1+\lambda) + \ln(c_t^o))] &= E_0[\sum_{t=0}^{\infty} \beta^t \ln(c_t^s)] \\ \sum_{t=0}^{\infty} \beta^t \ln(1+\lambda) + V^o &= V^s \\ \left(\frac{1}{1-\beta}\right) \ln(1+\lambda) &= V^s - V^o \\ \ln(1+\lambda) &= (1-\beta)(V^s - V^o) \\ 1+\lambda &= \exp[(1-\beta)(V^s - V^o)] \\ \lambda &= \exp[(1-\beta)(V^s - V^o)] - 1 \end{aligned}$$

To calculate λ, value functions in the household's optimisation problem in each model are used. For V^o or the value function in the Krusell & Smith model, I use those of agents in a period where the aggregate capital level is equal or close to the series average. The distribution of agents over (k, ε, s) to be used is also of that period. For V^s or the value function in the Aiyagari model, since the bisection method is employed to find the equilibrium interest rate, the corresponding level of aggregate capital and the stationary distributions, I obtain the value function from using the steady state parameters in the value function iteration.

3 Probability Structures

This section discusses the relevant probability structures used in both economies described in the previous section. Subsection 3.1 on aggregate states obviously applies only to an economy with business cycles while Subsection 3.2 on skill transitions, whose process is exogenous, apply to both economies. An economy with business cycles uses the stochastic unemployment process in Subsection 3.3 while an economy without business cycles uses the one in Subsection 3.4. Subsections 3.5 and 3.6 discuss the individual state transitions with and without the presence of business cycles respectively.

3.1 Aggregate States

The aggregate stochastic process (z) is independent of other macroeconomic variables and follows a two-state Markov chain. Namely, the probability that in period $t+1$ the aggregate productivity will be z' given that z is the aggregate state in period t is defined as

$$P_{zz'} \quad = \quad \Pr(z_{t+1} = z' | z_t = z)$$

and the aggregate transition probability matrix P is

$$P \quad = \quad \begin{bmatrix} P_{gg} & P_{gb} \\ P_{bg} & P_{bb} \end{bmatrix}$$

Following Krusell & Smith (1999), the average business cycle duration is 2 years. As one period in this model is one quarter, the average duration is 8 periods. This means an economy experiences a good state and a bad state on average for 8 periods each which implies

$$\begin{aligned}
\frac{1}{1-P_{gg}} &= 8 \\
\frac{1}{1-P_{bb}} &= 8
\end{aligned}$$

and since

$$\begin{aligned}
P_{gb} &= 1-P_{gg} \\
P_{bg} &= 1-P_{bb}
\end{aligned}$$

this gives

$$\begin{bmatrix} P_{gg} & P_{gb} \\ P_{bg} & P_{bb} \end{bmatrix} = \begin{bmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{bmatrix}$$

while its invariant distribution can be found from the diagonal elements of P^{τ}, $[\chi_g \ \chi_b]$, where τ is a large number, and is equal to [0.5 0.5].

3.2 Skill Transitions

The transition process for labour skills is independent of aggregate productivity shocks and other variables. Like aggregate shocks, it also follows a two-state Markov chain where the probability that in period $t+1$ an individual labour skill will be s' given that s is the individual labour skill in period t is defined as

$$Q_{ss'} = \Pr(s_{t+1} = s' | s_t = s)$$

and the skill transition probability matrix Q is

$$Q = \begin{bmatrix} Q_{hh} & Q_{hl} \\ Q_{lh} & Q_{ll} \end{bmatrix}$$

The data used in this calculation is from the results of de Broucker & Underwood (1998). They found that in the US the proportion of people who attained post-secondary education given that their parents had also attained post-secondary education is 64.2% while the proportion of people who attained post-secondary education given that their parents had attained only secondary education is 35.7%. In addition, they also found that it did not matter which (gender of the) parent had had post-secondary education. What did affect the younger generation's educational attainment was the highest level of education of both parents, and this was the case regardless of gender (and age) of the younger generation.

As a result, skill transition probability matrices of male and female workers can be represented by the same matrix Q.[14] Define a generation to be 30 years (120 periods) apart, this implies

$$\begin{bmatrix} Q_{hh} & Q_{hl} \\ Q_{lh} & Q_{ll} \end{bmatrix}^{120} = \begin{bmatrix} 0.64 & 0.36 \\ 0.36 & 0.64 \end{bmatrix}$$

or

$$\begin{bmatrix} Q_{hh} & Q_{hl} \\ Q_{lh} & Q_{ll} \end{bmatrix} = \begin{bmatrix} 0.9947 & 0.0053 \\ 0.0053 & 0.9947 \end{bmatrix}$$

and the invariant distribution of high- and low-skilled workers can be found from the diagonal elements of Q^{τ}, $[\chi_h\ \chi_l]$, where τ is a large number, and is equal to [0.5 0.5].

3.3 Unemployment Shocks with Business Cycles

Based upon Mukoyama & Şahin (2006), an idiosyncratic employment process depends on the previous and current aggregate states of the economy as well as the current skill level. But, in addition to that, gender of a worker must also be identified as unemployment rate and average duration of unemployment vary between two genders in this model. Conditioned on all appropriate determinants, a stochastic employment process then follows a two-state Markov chain with the corresponding employment transition probability matrix in case of a male worker, without loss of generality, being

$$\Pi_m^{zz's'} = \begin{bmatrix} \pi_{m11}^{zz's'} & \pi_{m10}^{zz's'} \\ \pi_{m01}^{zz's'} & \pi_{m00}^{zz's'} \end{bmatrix}$$

where

$$\pi_{m\varepsilon\varepsilon'}^{zz's'} = \Pr(\varepsilon_{t+1} = \varepsilon' | \varepsilon_t = \varepsilon, z_t = z, z_{t+1} = z', s_{t+1} = s', \text{male})$$

Unemployment rates are obtained from the US Current Population Survey between 1992-2010 of workers of age 25 and above.[15] To label a given year as good or bad, the

[14] From Appendix B in de Broucker & Underwood (1998), although the correlation between females' educational level and their parents' (0.43) is slightly higher than males' and their parents' (0.36), this information still does not change the distribution of high- and low-skilled agents of either gender under the law of large numbers, i.e., the invariant distribution of male and female workers is not affected.

[15] Ideally we would want 30 years of data on unemployment rates to match the length of a generation but this is the longest possible range of data available on unemployment rates that are categorised by level of educational attainment and gender.

annual total unemployment rates are sorted and the lowest half is considered to be the good years and the rest the bad years.[16] Using this information, we find the unemployment rate for a given type of workers and a given aggregate state by averaging the unemployment rates of those workers for the corresponding years by which the following 8 unemployment rates are obtained: $\{U^g_{mh}, U^g_{fh}, U^g_{ml}, U^g_{fl}, U^b_{mh}, U^b_{fh}, U^b_{ml}, U^b_{fl}\}$. For example, U^b_{fl} is the unemployment rate of low-skilled female workers in a bad aggregate state.

Data on average durations of unemployment for different types of workers are obtained from the OECD database and measured in weeks.[17] According to Mincer (1991), the average unemployment durations for workers of different skill levels are relatively similar. Hence, the duration will be the same across workers of any skill level and only differ by gender. To this end, we obtain 4 average durations of unemployment: $\{D^g_m, D^g_f, D^b_m, D^b_f\}$. For example, D^b_f is the average duration of unemployment for female workers in a bad aggregate state.

3.3.1 Identification & Restriction

In order to obtain a unique set of employment transition probability matrices for all types of workers, we focus first on how workers of a certain type become employed and unemployed in the next period when the aggregate state moves from z in the current period to z' in the next period. After we obtain the expressions on transitions to employment and unemployment from the current period, we equate them with $\chi_{s'}(1-U^{z'}_{ms'})$ and $\chi_{s'}U^{z'}_{ms'}$ respectively in case of male workers because they represent the number of employed and unemployed male workers of s'-skilled in the next period where the aggregate state is z'. Below, we will proceed with the identification in case of male workers without loss of generality.

In the current period, the number of high-skilled employed male workers equals $\chi_h(1-U^z_{mh})$ and these workers will become s'-skilled in the next period with probability $Q_{hs'}$. As a result, the number of males who will become s'-skilled when they are high-skilled and employed in the current period is $\chi_h(1-U^z_{mh})Q_{hs'}$. Similarly, the number of males who will become s'-skilled when they are low-skilled and employed in the current period is $\chi_l(1-U^z_{ml})Q_{ls'}$. This means that the total number of males who are employed in the current period and will be s'-skilled in the next period is $\chi_h(1-U^z_{mh})Q_{hs'}+\chi_l(1-U^z_{ml})Q_{ls'}$. Analogously, the total number of males who are unemployed in the current period and will be s'-skilled in the next period is then $\chi_h U^z_{mh}Q_{hs'} + \chi_l U^z_{ml}Q_{ls'}$. From this, we can represent the employment transitions of male workers who become s'-skilled when the

[16] Since there is an odd number of years, the year of median unemployment rate is labelled as a good year for it is closer to the average of the lower half.

[17] The data used are for workers of age 25-54 to match the subject of data on unemployment rates.

aggregate state moves from z to z' as follows

$$\begin{bmatrix} \chi_h(1-U^z_{mh})Q_{hs'} + \chi_l(1-U^z_{ml})Q_{ls'} \\ \chi_h U^z_{mh} Q_{hs'} + \chi_l U^z_{ml} Q_{ls'} \end{bmatrix}' \begin{bmatrix} \pi^{zz's'}_{m11} & \pi^{zz's'}_{m10} \\ \pi^{zz's'}_{m01} & \pi^{zz's'}_{m00} \end{bmatrix} = \begin{bmatrix} \chi_{s'}(1-U^{z'}_{ms'}) \\ \chi_{s'} U^{z'}_{ms'} \end{bmatrix}'$$

From the above expression, we have 2 unknowns, $\pi^{zz's'}_{m00}$ and $\pi^{zz's'}_{m10}$,[18] and one equation from equalling the second entries of both sides. The equality of first entries is actually automatically satisfied by the equality of the second, given the invariant distribution of high- and low-skilled male workers. Now we need one more restriction for the model to be just-identified. We use information on the average duration of unemployment for male workers in a z year, D^z_m, and the fact that the average duration of unemployment for male workers in a z period is related to the probability that a male worker (of any skill level) remains unemployed in both periods when the aggregate state stays z or simply[19]

$$\frac{1}{1-\pi^{zzs'}_{m00}} = \frac{D^z_m}{13}$$

which always holds for any pair $(\pi^{zzs'}_{m00}, D^z_m)$ where $z = \{g, b\}$ and $s' = \{h, l\}$.

For other employment transition probabilities that represent a change in aggregate state ($z \neq z'$), I will follow Krusell & Smith (1998) in imposing further the following restrictions[20]

$$\begin{aligned} \pi^{gbs'}_{m00} &= 1.25 \cdot \pi^{bbs'}_{m00} \\ \pi^{bgs'}_{m00} &= 0.75 \cdot \pi^{ggs'}_{m00} \end{aligned}$$

which now allow us to determine the remaining unknown $\pi^{zz's'}_{m10}$ in each employment transition probability matrix. By equalling the second entries of our earlier expressions for employment transitions of s'-skilled male workers when the aggregate state moves from z to z', $\pi^{zz's'}_{m10}$ is equal to

$$\pi^{zz's'}_{m10} = \frac{\chi_{s'} U^{z'}_{ms'} - \pi^{zz's'}_{m00}(\chi_h U^z_{mh} Q_{hs'} + \chi_l U^z_{ml} Q_{ls'})}{\chi_h(1-U^z_{mh})Q_{hs'} + \chi_l(1-U^z_{ml})Q_{ls'}}$$

As a result, 16 employment transition probability matrices are obtained (8 for each gender) and can be found in Appendix A.1. From there, it can be seen that given the current and next period's aggregate states, high-skilled workers are more likely to remain in employment than low-skilled workers although their chances of escaping unemployment

[18] The other two parameters, $\pi^{zz's'}_{m01}$ and $\pi^{zz's'}_{m11}$, are implied from the fact that each row of matrix $\Pi^{zz's'}_m$ sums to 1.

[19] As there are 13 weeks per one quarter (period), data measured in weeks are divided by 13.

[20] Another approach was done by İmrohoroğlu (1989) where she used $\pi^{gbs'}_{m00} = \pi^{bbs'}_{m00}$ and $\pi^{bgs'}_{m00} = \pi^{ggs'}_{m00}$.

are the same due to Mincer (1991). Female workers have notably higher possibility of escaping unemployment than male workers but their probabilities of maintaining a job are slightly less.

3.4 Unemployment Shocks without Business Cycles

When there is no aggregate fluctuations in the economy, an idiosyncratic employment process is reduced to depend only on the current skill level and gender (which is time-invariant). It follows a two-state Markov chain with the corresponding employment transition probability matrix in case of a male worker without loss of generality being

$$\Pi_m^{s'} = \begin{bmatrix} \pi_{m11}^{s'} & \pi_{m10}^{s'} \\ \pi_{m01}^{s'} & \pi_{m00}^{s'} \end{bmatrix}$$

where

$$\pi_{m\varepsilon\varepsilon'}^{s'} = \Pr(\varepsilon_{t+1} = \varepsilon' | \varepsilon_t = \varepsilon, s_{t+1} = s', \text{male})$$

Data on unemployment rates and average durations of unemployment are the same as in the previous subsection but the years are no longer distinguished as good or bad. As a result, there are 4 unemployment rates: $\{U_{mh}, U_{fh}, U_{ml}, U_{fl}\}$ where U_{fl} denotes the unemployment rate of low-skilled female workers, and 2 average unemployment durations: $\{D_m, D_f\}$ where D_f is the average duration of unemployment for female workers.

Similar to the previous subsection, the elements in employment transition probability matrices can be uniquely identified by the expressions on employment transitions between 2 periods and a restriction on average unemployment duration which are respectively

$$\begin{bmatrix} \chi_h(1-U_{mh})Q_{hs'} + \chi_l(1-U_{ml})Q_{ls'} \\ \chi_h U_{mh} Q_{hs'} + \chi_l U_{ml} Q_{ls'} \end{bmatrix}' \begin{bmatrix} \pi_{m11}^{s'} & \pi_{m10}^{s'} \\ \pi_{m01}^{s'} & \pi_{m00}^{s'} \end{bmatrix} = \begin{bmatrix} \chi_{s'}(1-U_{ms'}) \\ \chi_{s'} U_{ms'} \end{bmatrix}'$$

and

$$\frac{1}{1-\pi_{m00}^{s'}} = \frac{D_m}{13}$$

where the remaining unknown $\pi_{m10}^{s'}$ can be solved for and is equal to

$$\pi_{m10}^{s'} = \frac{\chi_{s'} U_{ms'} - \pi_{m00}^{s'}(\chi_h U_{mh} Q_{hs'} + \chi_l U_{ml} Q_{ls'})}{\chi_h(1-U_{mh})Q_{hs'} + \chi_l(1-U_{ml})Q_{ls'}}$$

As a result, 4 employment transition probability matrices are obtained (2 for each gender) and can be found in Appendix A.2.

3.5 Individual State Transitions with Business Cycles

In an economy with aggregate uncertainty, there are in total 8 individual states in which a male or female agent could be at a given point in time and they are {G1H, B1H, G0H, B0H, G1L, B1L, G0L, B0L}.[21] The transitions between these states are represented by 8×8 individual state transition matrices Π_m^i for male workers and Π_f^i for female workers. For example, the $(1, 8)^{th}$ entry of Π_m^i is the probability that a male individual will be low-skilled and unemployed, and that the aggregate state next period will be bad (B0L) given that he is high-skilled and employed in the current period where the aggregate state is good (G1H), and it is equal to $P_{gb} Q_{hl} \pi_{m10}^{gbl}$. The invariant distribution of male workers in 8 individual states can be found from the diagonal elements of $(\Pi_m^i)^\tau$, $[\chi_{m1}^{gh}\ \chi_{m1}^{bh}\ \chi_{m0}^{gh}\ \chi_{m0}^{bh}\ \chi_{m1}^{gl}\ \chi_{m1}^{bl}\ \chi_{m0}^{gl}\ \chi_{m0}^{bl}]$, where τ is large, and is equal to [0.24 0.24 0.01 0.01 0.24 0.23 0.01 0.02]. When rounded to two digits, the invariant distribution of female workers in 8 individual states, $[\chi_{f1}^{gh}\ \chi_{f1}^{bh}\ \chi_{f0}^{gh}\ \chi_{f0}^{bh}\ \chi_{f1}^{gl}\ \chi_{f1}^{bl}\ \chi_{f0}^{gl}\ \chi_{m0}^{bl}]$, is the same as male workers'.

3.6 Individual State Transitions without Business Cycles

In an economy without aggregate uncertainty, there are in total 4 individual states in which a male or female agent could be at a given point in time and they are {H1, H0, L1, L0}. The transitions between these states are represented by 4×4 individual state transition matrices P_m^i for male workers and P_f^i for female workers. For example, the $(4, 1)^{th}$ entry of P_m^i is the probability that a male individual will be high-skilled and employed (H1) in the next period given that he is low-skilled and unemployed (L0) in the current period, and it is equal to $Q_{lh} \pi_{m01}^h$. The invariant distribution of male workers in the 4 states can be found from the diagonal elements of $(P_m^i)^\tau$, $[\chi_{m1}^h\ \chi_{m0}^h\ \chi_{m1}^l\ \chi_{m0}^l]$, where τ is a large number, and is equal to [0.48 0.02 0.47 0.03]. When rounded to two digits, the invariant distribution of female workers in 4 individual states, $[\chi_{f1}^h\ \chi_{f0}^h\ \chi_{f1}^l\ \chi_{f0}^l]$, is the same as male workers'.

4 Calibration

A summary of parameters is shown in Table 1. Main parameters are standard and follow primarily those in Krusell & Smith (1998) including discount rate β, capital income share α, capital depreciation rate δ, positive productivity shock g, negative productivity shock b, and borrowing limit $\underline{k}$. Home production μ and skill premium $\frac{\theta(h)}{\theta(l)}$ are the same as in Mukoyama & Şahin (2006). In particular, the skill premium is 1.5 and based upon the results from Murphy & Welch (1992). Since in Mukoyama & Şahin (2006) the values for a pair $\{\theta(h), \theta(l)\}$ that satisfies the skill premium are not specified, I normalise $\theta(l)$ to

[21]The notations are as follows: G = a good period, B = a bad period, 1 = being employed, 0 = being unemployed, H = being high-skilled, L = being low-skilled.

1. For gender pay gap Λ, I use data from the US Current Population Survey from 2000 to 2010 on men's and women's median weekly earnings. As the pay gap does not differ more than 2-3% with level of educational attainment, I use the overall gender pay gap which is approximately 0.8. The ratio between male and female labour shares is from ILO's Labour Statistics database (LABORSTA) which shows that the ratio has been fairly stable since the early 1990s and men's share of total labour force is approximately 54%. Relaxation parameter ζ measures how close a new guess for the law of motion for the distribution Γ would be to the implied law of motion after each simulation. N_i and T are respectively the numbers of agents and time periods in the simulated economy.[22]

Parameter	Value	Description
β	0.99	Discount Rate
α	0.36	Capital Share of Total Income
δ	0.025	Depreciation Rate
g	1.01	Positive Productivity Shock
b	0.99	Negative Productivity Shock
$\underline{k}$	0	Borrowing Constraint
μ	0.1	Value of Home Production
$\theta(h)$	1.5	Labour Supply of High-Skilled Individual
$\theta(l)$	1	Labour Supply of Low-Skilled Individual
χ_m	0.54	Male Labour Share
χ_f	0.46	Female Labour Share
Λ	0.8	Gender Pay Gap
ζ	0.2	Relaxation Parameter
N_i	1,000	Number of Simulated Agents
T	7,000	Number of Simulated Periods

Table 1: Parameters for Calibration

4.1 Computational Algorithms

4.1.1 Economy with Business Cycles

1. Create a series of aggregate states for T periods using aggregate transition probability matrix P. I also control for the law of large numbers meaning that the numbers of good and bad periods must be equal or very close to the respective invariant distribution.

2. Create N_i skill transition paths for N_i agents and T periods using the skill transition probability matrix Q. Again I control for the law of large number meaning

[22] In Krusell & Smith (1998), they use bigger numbers at 5,000 for N_i and 11,000 for T but I have to tone them down so that my computer could handle the programming within reasonable 12 hours without crashing.

the numbers of high- and low-skilled workers must comply with the corresponding invariant distribution. Since male and female workers have the same skill transition probability matrix, Q, half of male and female workers are high-skilled.

3. Simulate N_i employment paths for N_i agents and T periods using the employment transition matrices as well as the paths of aggregate productivity shocks and skill transitions previously created. I also make sure the law of large numbers hold in each period for each type of workers using the relevant invariant distribution.

4. Discretise capital state space and solve for the agent's maximisation problem by value function iteration under an initial guess on the law of motion for aggregate capital.[23] As in Krusell & Smith (1998), I will approximate the distribution Γ by its first moment. Using a log linear functional form, I have

$$\ln K' = \begin{cases} a_0 + b_0 \ln K & \text{if } z = z_g \\ a_1 + b_1 \ln K & \text{if } z = z_b \end{cases}$$

   For initial values, I use 0 for a_0 and a_1, and 1 for b_0 and b_1.

5. Once policy functions for male and female agents are obtained, the initial distribution used at the start of a simulation is the stationary distribution in the Aiyagari model. To simulate a path of aggregate capital, I firstly sum individual decisions for next period's capital holdings in the first period and use it as the aggregate capital state in the next period. Then I use the policy functions again to find and sum the implied individual capital holdings for the subsequent period to use as the aggregate capital state in the period after. I do this for T periods.

6. Regress an AR(1) model as postulated in step 4 on the resulted series of aggregate capital. The series of aggregate capital is divided into 2 series, one that corresponds to the evolution of aggregate capital in good periods and one for bad periods. The first 1,000 periods are ignored to rule out a possibility of initial condition dependence.

7. Compare the OLS coefficients with the guess for the law of motion of aggregate capital. If they are not close enough, I use a weighted average for a new guess and repeat the procedure from step 4 until convergence is reached.

4.1.2 Economy without Business Cycles

For an economy without business cycles which corresponds to an Aiyagari model, I employ the method of endogenous capital grid points and policy function iterations, and

[23]While Krusell & Smith (1998) allow tomorrow's capital to take any values not necessarily on the grid points, I specify it to be only on the grid points to manage the computational time.

find the equilibrium interest rate with the bisection method. Below is its outline.

1. Discretise tomorrow's capital state space (policy function). Guess an initial interest rate and solve for the household's optimisation problem by policy function iteration from the Euler equation and the budget constraint.

2. Iterate on the distribution of capital induced by the policy function and the endogenous capital state space until it becomes stationary.

3. Find the implied interest rate from the stationary distribution of capital and the invariant distribution for labour supply. Use the bisection method on interest rate to obtain the equilibrium interest rate.[24]

4. Iterate on the value function using the equilibrium prices to get the converged value function.

5 Results

5.1 Model Solution

From the prediction rule using the approximate aggregation method in Krusell & Smith (1998), the resulting OLS coefficients and statistics are

$$\begin{aligned} \ln K' &= 0.34 + 0.91 \ln K, \\ R^2 = 0.9996, &\qquad \sigma^2 = 0.000003 \end{aligned}$$

in good times and

$$\begin{aligned} \ln K' &= 0.04 + 0.99 \ln K, \\ R^2 = 0.9998, &\qquad \sigma^2 = 0.00000007 \end{aligned}$$

in bad times. R^2 figures indicate that the approximate aggregation method works very well given that both current and future state spaces for individual capital holdings are discretised whereas the future state space for capital holdings is set to take any possible values in the original Krusell & Smith (1998) algorithm.

The average interest rate in the original economy is higher than the steady state interest rate in the smoothed economy at 1.89% to 0.92% net of depreciation rate. The average wage in the original economy is lower than that of the smoothed economy at 2.09 to 2.40. This is because the level of aggregate capital in the Aiyagari model is generally higher than in the original economy which will be discussed in the next subsection.

[24] See Aiyagari (1994) for complete details on this procedure.

5.2 Welfare Cost of Business Cycles

Type of Agents	Aggregate State	Welfare Gain in % (λ)
Overall	Good	5.34
Men	Good	7.88
Women	Good	2.43
High-skilled	Good	8.61
Low-skilled	Good	2.16
Employed	Good	5.43
Unemployed	Good	4.84
Overall	Bad	5.88
Men	Bad	8.56
Women	Bad	2.81
High-skilled	Bad	9.37
Low-skilled	Bad	2.49
Employed	Bad	5.92
Unemployed	Bad	4.07

Table 2: Selected Welfare Gains from the Removal of Business Cycles

5.2.1 Overall

Table 2 shows the main welfare gains from the removel of business cycles. It is found that on average agents would need a 5.88% increase in consumption to make up for being in an economy with stochastic aggregates in a bad period and a 5.34% increase in consumption in a good period. The fact that one would require higher compensation in consumption when the aggregate state is bad is consistent with conventional results on business cycle literature even though the gains of having cycles removed are higher than those of existing literature. If we compare the overall welfare gains to that in Lucas (1987), here are very sizeable and up to 600-700 times larger. If comparing to İmrohoroğlu (1989), the results here are 4-20 times larger depending on the risk aversion coefficient. With respect to Krusell & Smith (1999), they are around 40 times larger. Lastly, when comparing with results from Mukoyama & Şahin (2006), welfare gains here are 70 times larger in a bad period and 220 times larger in a good period.

Most definitely, each paper employs different settings which lead to significant difference in results. They could be from different assumptions on utility function, labour supply decision or borrowing constraints as well as different data set employed in the analyses. For example, the average unemployment duration in Krusell & Smith (1999), a little more than 2 quarters, is much longer than the one used in Mukoyama & Şahin (2006) which is less than a quarter. How the model in this paper overlooks a procedure to properly integrate out aggregate shocks could potentially amplify the gains of removing business cycles as well. However, for our analysis, what could be studied from these papers are the relative magnitudes of welfare gains and losses among groups of agents.

5.2.2 Within Subgroups

One potentially crucial result is that the level of aggregate capital in the smoothed economy is actually higher than that in the original economy. This is contrary to results in both Krusell & Smith (1999) and Mukoyama & Şahin (2006). Figure 1 shows a simulated series of aggregate capital over time. The initial value at 41.94 is from the steady state level in the Aiyagari model. We can see stark transitions to a lower average level at 31.69 around which the series fluctuates.[25] Here, precautionary savings is not valid as an explanation for a higher level of aggregate capital in the smoothed economy because less variation in income stream should translate to less incentive to save. It could instead be argued that more uncertainty in the economy hinders the ability of agents to hold a higher amount of capital in each period which, as a result, brings down the average level of capital.

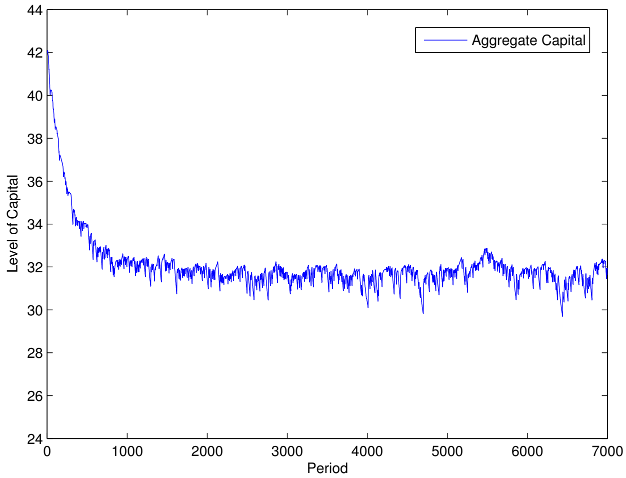

Figure 1: Simulated Path of Aggregate Capital

The above result might explain why, when we look at the magnitudes of welfare gains among different types of agents, the ones with lower income (unemployed, low-skilled and/or female) gain less benefit than what the higher income agents do.[26] Approximately male workers gain 3 times more than female workers, high-skilled workers gain 4 times more than low-skilled workers, and employed workers could gain up to 1.5 times more than unemployed workers. The fact that aggregate capital drops when business cycles

[25] The average value of aggregate capital here is obtained by ignoring the first 1,000 observations.

[26] Nonetheless, Krusell & Smith (1999) found that, apart from very few poorest agents, rich agents often gain the most.

are introduced into an economy means two things. First, interest rate rises as there is less capital. Second, wage becomes lower due to the complimentary nature of inputs in the Cobb-Douglas production function. For higher income workers, their utilities depend on both capital and labour incomes while, in comparison, lower income workers rely more on income from capital holdings. As a result, when we compare two economies with two distinct levels of capital, it is possible that lower income workers would be better off living in a smoothed economy in a smaller magnitude than higher income workers who are impacted more by the lowering of wage in an economy with business cycles.

Looking in a more disaggregated way, it is found that for the most part agents with higher income still gain more benefit than agents with lower income from the removal of business cycles. Tables 3 and 4 in Appendix B show the results on welfare costs of business cycles for all subgroups of agents used in the following analyses. Among the employed, subgroups with higher income gain more from the removal of aggregate fluctuations. Employed males gain 3 times more than employed females in a bad state and 1.5 times more in a good state. High-skilled employed workers gain 4 times more than low-skilled employed workers in both good and bad states. On average, employed workers hold slightly more capital in a good state at 31.90 than in a bad state at 31.78.

Among the unemployed, both male and female workers have positive welfare gains, but males gain 3 times more than females in a good state and 18 times more in a bad state. Unemployed workers with any skill level also receive positive welfare gains. High-skilled unemployed workers gain just as much as low-skilled unemployed workers in a bad state but they gain 14 times more in a good state. On average, unemployed workers hold more capital in a bad state at 29.57 than in a good state where they hold 29.01 of capital.

Among male workers, the unemployed gain almost as much as the employed in both good and bad periods while the high-skilled gain around 1.5-2 times more than the low-skilled. On average male workers hold similar amount of capital in good and bad periods at 36.39 and 36.68 respectively.

Among female workers, the unemployed also gain just as much as the employed in a good period but they gain 7 times less in a bad period. High-skilled female workers also gain more than low-skilled females. In fact, most subgroups of low-skilled females suffer a loss between 1.7-2.6% in consumption from having business cycles removed which could be the result of high reliance on capital income. On average, females hold less capital in a bad period at 25.98 while they hold 26.14 in a good period.

Among high-skilled agents, it turns out that the unemployed gain 1.2 times more than the employed in a good period but the number of these agents is very small (1.2% of the population). In a bad period, the employed gain twice more than the unemployed. High-skilled females gain almost as much as high-skilled males in a bad period and they gain 1.5 times less in a good period. On average, high-skilled workers hold 36.25 of capital in a good period and 37.18 in a bad period.

Among low-skilled agents, the unemployed gain almost twice more than the employed in a bad period, while they gain 3 times less in a good period. Low-skilled males also gain more than low-skilled females and, as indicated earlier, low-skilled females suffer a loss regardless of the state of the economy. On average, low-skilled workers hold 27.10 of capital in a good period and 26.35 in a bad period.

Although it is not clear why differences in welfare gains are more pronounced in a good period for certain subgroups and in a bad period for other subgroups, there is a discernible pattern that among subgroups of higher income agents (employed, male or high-skilled) their welfare gains are relatively closer while in lower income subgroups we can see some large differences regardless of the state of the economy. This suggests that welfare gains of higher income agents are not subject to variance as high as those of lower income agents when aggregate uncertainty is removed.

5.3 Evidence of Precautionary Savings

Despite the fact that precautionary savings may not prevail in the aggregate level, we could see its evidence in the subgroup level when we compare between earnings ratios and future capital holdings ratios. The earnings ratio between the unemployed and the employed is 0.1:1 while the corresponding future capital holding ratio is a lot higher at 0.9:1 suggesting that unemployed workers do not lower significantly the amount of capital they will hold for the next period where their employment status could change for the better which is consistent with the rational behaviour of utility-maximising agents. The same behaviour is also present among the low-skilled workers (as opposed to high-skilled workers) where the earnings ratio is 1:1.5 (or 0.67:1) and the ratio on future capital holding is higher at 0.73:1. Again, since their skill could improve in the next period and they maximise their lifetime utilities, it is not a surprise that low-skilled workers will decide to hold relatively more capital than their earnings. The gap between the two ratios of high- and low-skilled workers is not as large as that of employed and unemployed workers because a probability of escaping unemployment is considerably higher than a probability of escaping low skill level.[27]

[27] This pattern obviously does not extend to the future capital holding ratio between males and females since the model does not allow for the possibility of agents changing their genders.

5.4 Wealth Distribution

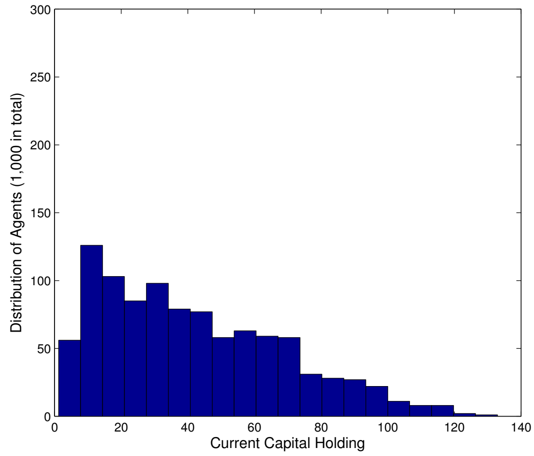

Figure 2: Distribution of Capital Holding in the Smoothed Economy

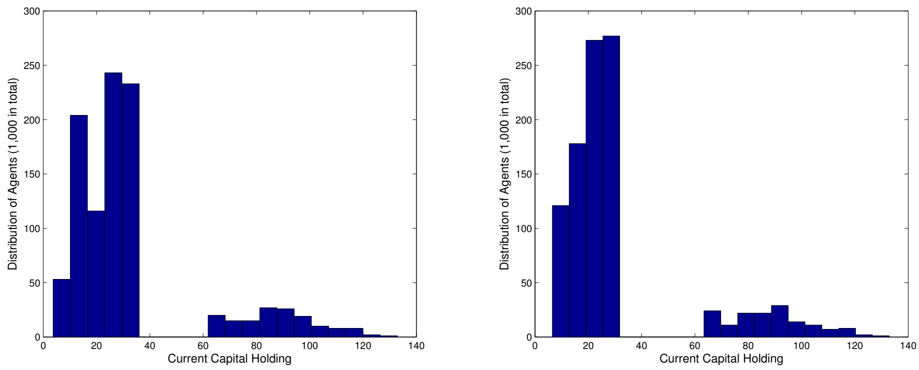

Figure 3: Distributions of Capital Holdings in the Original Economy in Good State and Bad State Respectively

Figure 2 shows the distribution of capital holdings in an economy without aggregate uncertainty and Figure 3 shows the distributions of capital in the original economy with business cycles in a good and a bad period. The distribution in the smoothed economy is more evenly relative to those in the original economy where they are more clustered in the lower levels of capital. This explains why the Lorenz curve of the smoothed economy crosses those of the original economy in Figure 4 which depicts Lorenz curves for different economies. As more than 80% of agents in the original economy hold capital in a very close amount of under 40, it means that the original economy is perceived to have less wealth inequality when agents with lower wealth (whose proportion is high) are in consideration. This is the reason why the Lorenz curves for the original economy at first lie above and later cut the smoothed economy's Lorenz curve when the richer agents are taken into account. The kinks in the original economy's Lorenz curves are the result of discontinuities in distributions as shown in Figure 3.

Introducing aggregate uncertainty to the economy actually does not change the level of inequality significantly. In fact, they have almost the same Gini coefficients of around 0.36.[28] The model may not match the US wealth distribution (with Gini coefficient of 0.8) greatly but this is expected since preference heterogeneity is not present in this model.[29] However, these numbers are still relatively higher than what Krusell & Smith (1998) found in their benchmark model with no preference heterogeneity where Gini coefficient is 0.25 suggesting that adding heterogeneity in skill levels and separating gender could explain further inequality in wealth distribution even though they alone are not enough to capture that of the true economy.

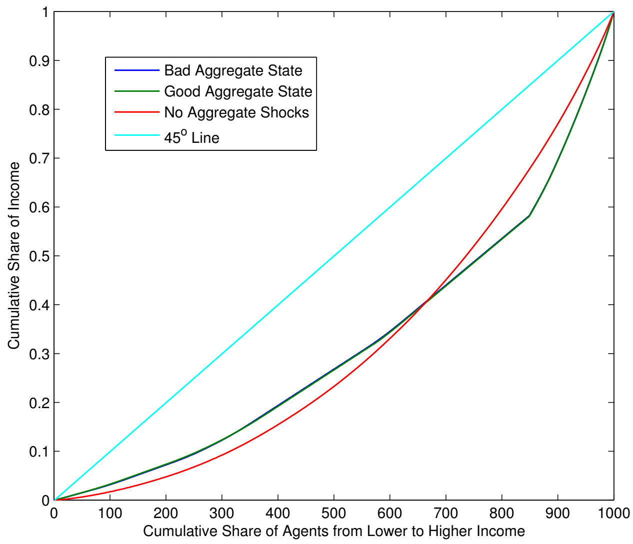

Figure 4: Lorenz Curves

6 Conclusion

This paper is set to find out the welfare cost of business cycles in an economy with liquidity constraints and some heterogeneities among economic agents. These agents are grouped by employment status, skill level and gender. There had been no work on the effect of separating between male and female workers in the calculation of welfare cost of business cycles before and the results show that there is significant difference in welfare

[28]The Gini coefficients for the original economy in a good and a bad period and for the smoothed economy differ only from the third demical place, and are 0.3665, 0.3654 and 0.3662 respectively.

[29]Gini coefficient could vary due to different definitions of wealth but they are quite high around 0.8. For example, Wolff (1994) reported it be 0.84 whereas Budría Rodríguez et al (2002) found it to be 0.80.

gains between two genders, apart from other heterogeneities, where male workers gain between 7.88-8.56% in consumption term from the removal of business cycles while female workers gain only 2.43-2.81%. Overall, welfare costs of business cycles are measured to be around 5.34-5.88% of total consumption depending on the aggregate state being compared to. These numbers are substantially high when compared to other existing literature and are 600-700 times larger than the famous result in Lucas (1987).

Contrary to a priori expectation, main subgroups that benefit the most from the removal of business cycles are high-skilled and male workers whose gains are around 10% of their total consumption while low-skilled female workers gain the least and, most of the time, even suffer from the absence of business cycles equivalent to 1.65-2.60% decrease in consumption. The rather contradicting results are mainly due to the average level of aggregate capital in the economy with cycles being noticeably lower than that of the economy without cycles as well as how agents rely on their two sources of income (interest and wage). It is also found that differences in welfare gains are more pronounced among subgroups of unemployed, low-skilled or female workers who are deemed to have lower income. Wealth inequality barely changes with the introduction of aggregate fluctuations although the distribution of capital becomes more clustered around lower levels of capital.

As labour heterogeneity can help policy makers decide which tools to exercise and goals to focus on, it is worthwhile to incorporate into a model more realistic features in the labour market such as more finely skill levels, endogenous labour supply decision or search and matching theory. For completeness, we may include a government in a model to endogenously determine the optimal taxation and unemployment insurance. We can also study how an income process depends on the aggregate and idiosyncratic uncertainty as well as how (consumption of) agents depend on the permanent and transitory income components as an alternative way to determine the effects of business cycles. These elements could be useful for future research.

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A Employment Transition Probabilities

A.1 Economy with Business Cycles

$$\Pi_m^{ggh} = \begin{bmatrix} 0.9809 & 0.0191 \\ 0.7382 & 0.2618 \end{bmatrix}$$

$$\Pi_m^{ggl} = \begin{bmatrix} 0.9621 & 0.0379 \\ 0.7382 & 0.2618 \end{bmatrix}$$

$$\Pi_m^{gbh} = \begin{bmatrix} 0.9691 & 0.0309 \\ 0.4547 & 0.5453 \end{bmatrix}$$

$$\Pi_m^{gbl} = \begin{bmatrix} 0.9436 & 0.0564 \\ 0.4547 & 0.5453 \end{bmatrix}$$

$$\Pi_m^{bgh} = \begin{bmatrix} 0.9826 & 0.0174 \\ 0.8037 & 0.1963 \end{bmatrix}$$

$$\Pi_m^{bgl} = \begin{bmatrix} 0.9640 & 0.0360 \\ 0.8037 & 0.1963 \end{bmatrix}$$

$$\Pi_m^{bbh} = \begin{bmatrix} 0.9741 & 0.0259 \\ 0.5637 & 0.4363 \end{bmatrix}$$

$$\Pi_m^{bbl} = \begin{bmatrix} 0.9508 & 0.0492 \\ 0.5637 & 0.4363 \end{bmatrix}$$

$$\Pi_f^{ggh} = \begin{bmatrix} 0.9763 & 0.0237 \\ 0.8100 & 0.1900 \end{bmatrix}$$

$$\Pi_f^{ggl} = \begin{bmatrix} 0.9555 & 0.0445 \\ 0.8100 & 0.1900 \end{bmatrix}$$

$$\Pi_f^{gbh} = \begin{bmatrix} 0.9707 & 0.0293 \\ 0.5158 & 0.4842 \end{bmatrix}$$

$$\Pi_f^{gbl} = \begin{bmatrix} 0.9511 & 0.0489 \\ 0.5158 & 0.4842 \end{bmatrix}$$

$$\Pi_f^{bgh} = \begin{bmatrix} 0.9766 & 0.0234 \\ 0.8575 & 0.1425 \end{bmatrix}$$

$$\Pi_f^{bgl} = \begin{bmatrix} 0.9548 & 0.0452 \\ 0.8575 & 0.1425 \end{bmatrix}$$

$$\Pi_f^{bbh} = \begin{bmatrix} 0.9730 & 0.0270 \\ 0.6126 & 0.3874 \end{bmatrix}$$

$$\Pi_f^{bbl} = \begin{bmatrix} 0.9528 & 0.472 \\ 0.6126 & 0.3874 \end{bmatrix}$$

A.2 Economy without Business Cycles

$$\Pi_m^{h} = \begin{bmatrix} 0.9771 & 0.0229 \\ 0.6393 & 0.3607 \end{bmatrix}$$

$$\Pi_m^{l} = \begin{bmatrix} 0.9559 & 0.0441 \\ 0.6393 & 0.3607 \end{bmatrix}$$

$$\Pi_f^{h} = \begin{bmatrix} 0.9744 & 0.0256 \\ 0.6976 & 0.3024 \end{bmatrix}$$

$$\Pi_f^{l} = \begin{bmatrix} 0.9540 & 0.460 \\ 0.6976 & 0.3024 \end{bmatrix}$$

B Tables of Results

The following tables contain welfare gains and losses for different groups of agents when the removal of business cycles takes place in a good and bad period. Types of agents are abbreviated as follows: M = Male, F = Female, 1 = Employed, 0 = Unemployed, H = High-skilled and L = Low-skilled.

Type of Agents	Welfare Gain/Loss in % (λ)
Overall	5.34
M	7.88
F	2.43
H	8.61
L	2.16
1	5.43
0	4.84
HM	10.24
LM	5.56
HF	6.73
LF	-1.70
1M	8.01
0M	6.77
1F	2.50
0F	2.08
H1	8.59
L1	2.26
H0	10.42
L0	0.72
H1M	10.27
H1F	6.70
H0M	10.32
H0F	8.53
L1M	5.71
L1F	-1.65
L0M	3.26
L0F	-2.24

Table 3: Welfare Gains/Losses from the Removal of Business Cycles in a Good Period

Type of Agents	Welfare Gain/Loss in % (λ)
Overall	5.88
M	8.56
F	2.81
H	9.37
L	2.49
1	5.92
0	4.07
HM	10.37
LM	6.78
HF	8.21
LF	-2.32
1M	8.51
0M	7.79
1F	2.90
0F	0.43
H1	9.58
L1	2.34
H0	4.56
L0	3.99
H1M	10.42
H1F	8.58
H0M	8.85
H0F	-0.10
L1M	6.69
L1F	-2.60
L0M	7.38
L0F	1.02

Table 4: Welfare Gains/Losses from the Removal of Business Cycles in a Bad Period