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Abstract

This paper investigates the dynamics specific to various firm cohorts and their implications at an aggregate level. Utilizing a structural model that incorporates entry, exit, and selection of heterogeneous firms, we demonstrate that the dynamics of firms from each generation, and thus the historical economic landscape, can be reconstructed. Moreover, we estimate generation-specific parameters in both demand and supply within our theoretical model, using Japanese data. Our findings reveal that fixed operational costs for firms established immediately after the Second World War are relatively lower compared to subsequent generations of firms, resulting in an increased market congestion for these early-born enterprises.

Keywords: Heterogeneity, fixed cost, business cycles.

JEL: D24, E23, E32, L11, L60.

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1 Introduction

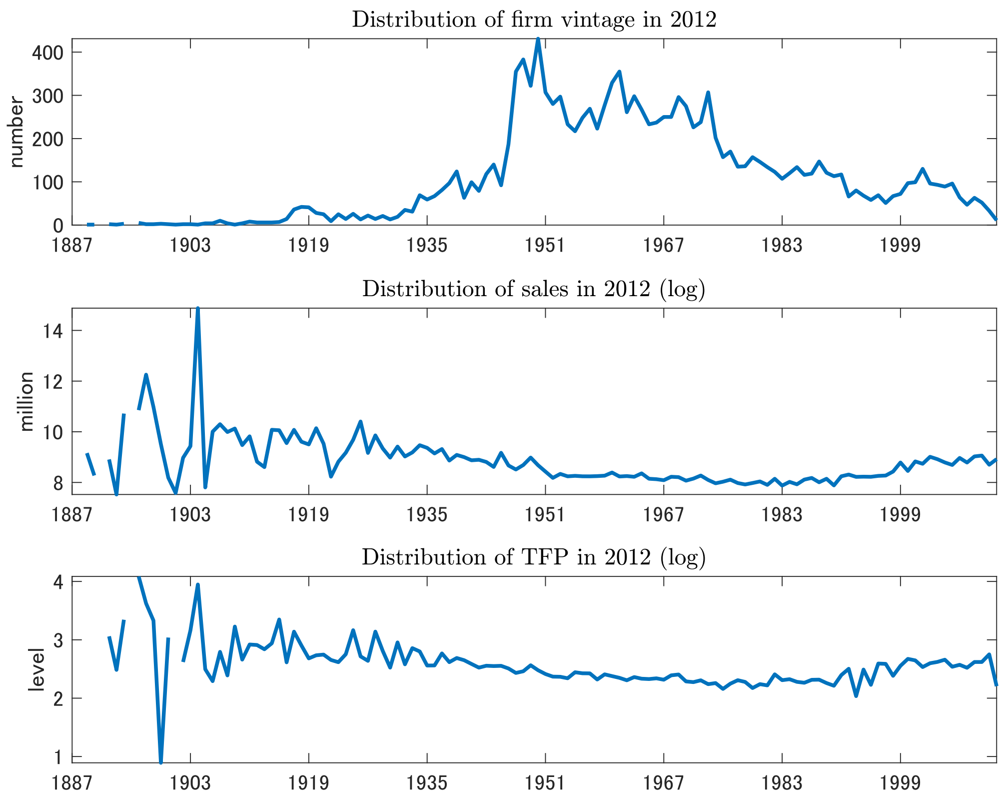
The economic landscape we observe today is a cumulative product of past activities, heavily influenced by firms established during various historical periods. While certain firm cohorts may outperform others in terms of sales and productivity, thus exerting a stronger influence on the economy (Klepper, 1996; Jovanovic and Rousseau, 2005), individual firms' unique traits also play a role, as in the standard model (Hopenhayn, 1992). This leads to the question: How much do the specific attributes of a particular firm cohort matter? In other words, how do generation-specific dynamics in history shape the aggregate macro-level conditions we observe today? To answer these questions, we explicitly model "generation," referring to the year of a firm's foundation, and explore its significance in characterizing the actual data.

To illustrate this, Figure 1 includes three panels depicting the age-specific distribution of Japanese firms established between 1887 and 2012. These panels also display corresponding sales and Total Factor Productivities (TFPs) based on a 2013 survey. We derived our data from the Basic Survey of Japanese Business Structure and Activities (BSJBSA), provided by the Ministry of Economy, Trade, and Industry of Japan (METI).[1] Notably, firms established during the mid-ranges of the study period (after the Second World War until the early 1970s) constitute a significant proportion but display lower sales and TFPs compared to both older and more recent cohorts. Sales and TFPs were notably higher for firms established in the initial periods, lasting until approximately the 1929 Great Depression, which corresponds to the Meiji (1868-1912) and Taisho (1912-1926) eras. We refer to these latter firms as "superstars," accounting for their strong influence on the current macroeconomy (Gabaix, 2011; Crouzet and Mehrotra, 2020; Autor et al., 2020).

Examining the distribution of generations of firms and their sales in the 2013 survey, it becomes challenging to explain the observed convex distribution for the number of firms and concave distribution for sales and TFPs without considering cohort-specific condi-

[1] For a detailed description of the data, please refer to Appendix A. Total Factor Productivities (TFPs) were estimated following the method outlined by Olley and Pakes (1996). In calculating tangible capital, we utilized book-to-market ratio data from Hosono et al. (2017).

Figure 1: Distribution of generation firms and their sales in the 2013 survey



The first panel shows the distribution of generation firms in the 2013 survey. The second panel shows the distribution of the sales of these generation firms in the 2013 survey. The last panel shows the total factor productivity estimated following Olley and Pakes (1996). The horizontal axis present the period of establishement. In the calculation of tangible capital, we use the data of book-to-market value ratio by Hosono et al. (2017)

tions. If we assume constant depreciation and homogeneous selection for all generations of firms, we would expect to see a stationary distribution: For the number of firms, we would anticipate a fat-tailed distribution leaning towards recent times, as recently-established firms tend to survive more while older generations get replaced. For sales and TFPs, we would expect a hump-shaped distribution since older firms have difficulty surviving over time, and younger firms have insufficient time to grow significantly. Solely considering idiosyncratic characteristics of firms falls short in understanding these observed patterns. Therefore, it is necessary to assume that cohort-specific conditions, persisting over time in both supply and demand, could influence firms' subsequent performance and thereby shape the economic landscape in later periods.

More specifically, we propose a DSGE model in which the interaction between generations of firms and macroeconomic aggregates occurs based on the entry and exit of heterogeneous firms with varying idiosyncratic productivities. The macroeconomic dynamics in our model result from the aggregation of all generation firms, both past and present, while the current aggregate outcome influences their sales, production, and employment. This feature allows us to reconstruct the history of each generation as a mirrored operation. We demonstrate that once we have knowledge of generation-specific conditions—such as the generation-specific parameters in demand and supply, the aggregate state of the economy at their birth period, as well as macroeconomic conditions following their birth period—we can fully reconstruct the history of each generation.

We then calibrate our model using Japanese data to illustrate generation-specific dynamics. One notable aspect of the Japanese data is that the current economic landscape, including the distribution of the number of generation firms from 1887 to 2012, their sales, and TFPs, can be obtained from survey data, as shown in Figure 1. To recreate the historical macroeconomic conditions in the Japanese economy, we simulate the theoretical model with structural shocks estimated from the historical GDP data spanning 126 years. This simulation, combined with the simulated aggregate macro dynamics over time and the current economic landscape depicted in Figure 1, enables us to deduce generation-specific structural parameters in both demand and supply. These parameters encompass generation-specific preferences, technologies, and fixed operational costs. Our estimation reveals that the fixed operational costs of firms created right after the Second World War until the early 1970s are significantly low, and the demand conditions for the firms born around the turn of the 20th century are relatively high.

Upon obtaining the conditions specific to each generation and those of the macro economy, we reconstruct the dynamics of each generation, upon which we perform quantitative exercises. Specifically, we provide "snapshots" of the economic landscape at particular moments in history (1923 and 1996). These snapshots result from the simulated generation-specific dynamics given the aforementioned macroeconomic aggregates. In our snapshot of 1923, the share of recent generations of producers from 1914 to 1919 is high, aligning with the historical narrative of the boom period after the First World War. However, their sales, productivity, and employment are not particularly high from the perspective of 1923. Moreover, our simulated snapshots in 1923 and 1996 closely resemble those of the survey data, providing robustness for our estimation strategy.

Furthermore, we conduct a counterfactual analysis concerning the distribution of generation-specific operational fixed costs, which exhibits a pattern opposite to our estimates. By increasing the operational fixed costs for the cohort born after the Second World War, we observe a significant increase in the share of recent generation of firms, while the number of generation firms created during the postwar boom periods decreases. Simultaneously, the sales, productivity, and employment of these postwar generation firms substantially increase. Consequently, our counterfactual fixed costs dramatically alter the economic landscape of generation firms and their characteristics. This also raises suggestions regarding the potential congestion created by these large but inefficient firms.

In the literature, several articles have tabulated the overall age of US companies using *Fortune* data (Corporation, 1996; Stangler, 2009; Luttmer, 2011; Ma et al., 2023) and the US business size distribution data in Kwon et al. (2022). Our paper stands out by providing unique cohort-specific evidence with respect to Japanese firms. While Ma et al. (2023) finds cohort-specific effects only in the industrial sector, it is interesting to note that we discover a similar concave distribution of fixed operational costs for other sectors as well, including light manufacturing, public utilities, telecommunications and transportation, and construction.

Our study builds upon a rich tapestry of research in the realm of firm entry and exits within a real business cycle model, drawing from the foundational works of Ghironi and Melitz (2005), Bilbiie et al. (2012), and Hamano and Zanetti (2017). Specifically, the crucial role of fixed costs in our model aligns with studies such as Cacciatore and Fiori (2016) and Bilbiie et al. (2019).

There is a noteworthy relationship between firm size distribution and firm age established in the literature on firm growth theory (Luttmer, 2007, 2011; Cabral and Mata, 2003). Current research conducted by Pugsley et al. (2018) and Sedlacek and Sterk (2017) underscores the significance of ex-ante firm heterogeneity during the birth period in characterizing the firm's size distribution and age profile, as evident from US firm data. Moreover, our findings resonate with works such as Abbring and Campbell (2005), Foster et al. (2016), and Pugsley et al. (2018), which emphasize the importance of demand accumulation over the firm life cycle. Studies examining cohort effects at the business cycle frequency have discovered that companies established during economic downturns tend to maintain smaller sizes compared to those established during periods of economic prosperity (Moreira 2016; Sedlacek and Sterk 2017). In a recent study, Ma et al. (2023) similarly identifies the presence of generation-specific effects, as we have, in the US industrial sector and provides a theoretical model to characterize the observed age distribution of firms.

Why do we observe generation effects in Japan? Our estimation results potentially encompass many factors from real history. After World War II, the Ministry of International Trade and Industry implemented various policies, including subsidies, tax exemptions, R&D initiatives, and export promotion measures. These policies initially targeted steel and iron industries, later shifting to heavy industries, machinery sectors, and high-tech industries. Government banks also had lending programs focusing on infrastructure, rural development, environmental investments, and energy-saving technology. The notably low fixed operational costs for the generation after World War II until the early 1970s may be linked to these regulatory policies. Additionally, the concentration of "superstars" at the turn of the 20th century may suggest organizational issues (Jovanovic and Rousseau (2005); Loderer et al. (2017); Bowen et al. (2023)).

The rest of the paper is structured as follows: Section 2 unfolds the model. Section 3 delves into our benchmark calibration and estimation. Section 4 marshals the theoretical model with Japanese data, generating the simulated generation-specific technologies and fixed costs. Leveraging these parameters, we go on to reconstruct the macroeconomic dynamics of each generation firm and engage in a counterfactual analysis. Section 5 brings the paper to a close with concluding remarks.

2 The Model

In this section, we present the theoretical model. The model incorporates endogenous firm entry and selection based on heterogeneity among firms. Additionally, we explicitly model generation firms and their products. Consumption at time t is composed of differentiated product varieties produced by generation firms created in previous time periods, $v < t$. Entrants incur a sunk cost to enter the market, and upon entry, they draw generation-specific technology (φ_v) from generation-specific distribution $G_v(\varphi_v)$. Firms are also required to cover generation-specific operational fixed costs (f_v) to engage in production, which may commence from the next time period after entry. The aggregate dynamics in the current time period are reconstructed by aggregating those at each generation level.

2.1 Households

During each time period t, the representative household maximizes the following expected utility:

$$E_t \sum\nolimits_{i=t}^{\infty} \beta^{i-t} \left(\ln C_t - \chi \frac{L_t^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \right), \tag{1}$$

where C_t is consumption, L_t is labor supply, $0 < \beta < 1$ is the discount factor, $\chi > 0$ is the degree of disutility in supplying labor, and ψ is the Frisch elasticity of the labor supply.

Consumption at time t is composed from different "generations" of baskets as follows:

$$C_t = \left(\sum_{v=0}^{t-1} \alpha_v^{\frac{1}{\sigma}} C_{v,t}^{1-\frac{1}{\sigma}} \right)^{\frac{1}{1-\frac{1}{\sigma}}}$$

where $C_{v,t}$ stands for the consumption of basket of generation v, and α_v represents the preference weight for that basket. Furthermore, product varieties within a particular generation v are differentiated as follows:

$$C_{v,t} = \left(\int_{\omega \in \Omega_v} c_{v,t}(\omega)^{1-\frac{1}{\sigma}} d\omega \right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

where $c_{v,t}(\omega)$ represents the demand for each product variety ω of generation v, and $\sigma > 1$ is the elasticity of substitution among product varieties. We maintain simplicity in the model by assuming the same elasticity of substitution across generation firms and within product varieties.

The optimal demand for each product variety ω within a generation v at time t is found to be as follows:

$$c_{v,t}(\omega) = \left(\frac{p_{v,t}(\omega)}{P_{v,t}}\right)^{-\sigma} C_{v,t}, \tag{2}$$

where $p_{v,t}(\omega)$ denotes the price of the product variety ω of a generation v. The price index of a basket of generation v at time t is as follows:

$$P_{v,t} = \left(\int_{\omega\in\Omega_t} (p_{v,t}(\omega))^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}},$$

The optimal demand for a basket of a particuler generation v is found to be as follows:

$$C_{v,t} = \left(\frac{P_{v,t}}{P_t}\right)^{-\sigma} \alpha_v C_t \tag{3}$$

where P_t is the price index of aggregate basket C_t. Finally, the aggregate price index P_t is found to be as follows:

$$P_t = \left(\sum_{v=0}^{t-1} \alpha_v P_{v,t}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

The aggregate price index is thus a weighted average of different generation prices. Throughout the paper, we choose P_t as a numeraire.

2.2 Production, Pricing and Selection

We assume that firms' technologies are conditoned at their birth. Specifically, each entrant draws a productivity level φ_v, from a cumulative distribution, $G_v(\varphi_v)$, with support on $[\varphi_{\min}, \infty)$. Different from Melitz (2003) who assumes a cumulative common distribution which is independent of cohorts, the draw of productivity is conditional on the generation-specific distribution $G_v(\varphi_v)$. Production requires labor and capital as input. It also requires fixed operational costs f_v which is also specific at generation level.[2]

[2] We don't model any transitory ex-post shock at firm level productivity as Hopenhayn and Rogerson (1993), neither the convergence process of the idiosyncratic producivity to its long-run level over time.

Goods produced are demanded for fixed operating costs, consumption, and investment. Denting $l_{v,t}(\varphi_v)$, and $k_{v,t}(\varphi_v)$ as the amount of labor and captial required for the production by a generation firm with productivity level φ_v is thus given by the following:

$$\left(\frac{l_{v,t}(\varphi_v)}{\gamma}\right)\left(\frac{k_{v,t}(\varphi_v)}{1-\gamma}\right)^{1-\gamma}=\frac{y_{v,t}(\varphi_v)}{Z_t\varphi_v}+\frac{f_v}{Z_t}, \tag{4}$$

Here γ represents the weight of labor input in production, and $y_{v,t}(\varphi_v)$ stands for the production of goods demanded for consumption and investment. Z_t is the commen TFP level for all firms. Cost minimization yields the followng demand function for each factor of production:

$$l_{v,t}(\varphi_v)=\left(\frac{w_t}{\lambda_t}\right)^{-1}\gamma\left(\frac{y_{v,t}(\varphi_v)}{Z_t\varphi_v}+\frac{f_v}{Z_t}\right),\ k_{v,t}(\varphi_v)=\left(\frac{r_t^K}{\lambda_t}\right)^{-1}(1-\gamma)\left(\frac{y_{v,t}(\varphi_v)}{Z_t\varphi_v}+\frac{f_v}{Z_t}\right),$$

where w_t and r_t^K represent the real wage and the rental rate of capital, respectively. Also, plugging these demands into the original production function, we obtain the expression for the cost index λ_t as

$$\lambda_t=w_t^{\gamma}r_t^{K1-\gamma},$$

The demand addressed to a firm with productivity level φ_v is characterized by equation (2). Profit maximization yields the following optimal price of the firm:

$$\rho_{v,t}(\varphi_v)=\frac{\sigma}{\sigma-1}\frac{\lambda_t}{Z_t\varphi_v}, \tag{5}$$

In the above expression, $\rho_{v,t}(\varphi_v)$ stands for the real price of firm with specific productivity φ_v.

Because of the fixed operational costs, depending on the level of firm-generation-specific productivity φ_v, firms may or may not produce. Thus, using equation (4), (5),

In contrast, we assume that the long-run steady-state level of firm productivity is achieved immediately upon entry, assuming that φ_v is invariant over time, as in Melitz (2003). Our assumption is supported by the recent empirical evience by Pugsley et al. (2018) who emphasize that ex-ante difference among firms is crucial for thier later performance in the US.

and (3), if production materializes, the following real operational profits are generated:[3]

$$d_{v,t}(\varphi_v) = \frac{1}{\sigma}\rho_{v,t}(\varphi_v)^{1-\sigma}\alpha_v Y_t^C - \frac{\lambda_t}{Z_t}f_v. \tag{6}$$

where $Y_t^C \equiv C_t + I_t$ with the aggregate investment I_t. that results from the goods market clearing such that $y_{v,t}(\varphi_v) = c_{v,t}(\varphi_v) + i_{v,t}(\varphi_v)$. Only generation firms that generate positive dividends $d_{v,t}(\varphi_v) > 0$ produce at time t by covering the operational fixed costs. We thus determine the cutoff productivity level for a particular generation v at time t, $\varphi_{v,t}$, with the following:

$$d_{v,t}(\varphi_{v,t}) = 0.$$

2.3 Average within generation

Given the distribution of the productivity level, $G_v(\varphi_v)$, the mass of firms, $N_{v,t}$, is defined over the productivity levels $[z_{\min}, \infty)$. Among these firms, a subset of firms engage in production. The number of producers of generaiton v is determined by $S_{v,t} = [1 - G_v(\varphi_{v,t})] N_{v,t}$. Following Melitz (2003) and Ghironi and Melitz (2005), we refer to the average with $\sim$ and define the average productivity of producers of the generation $\tilde{\varphi}_{v,t}$ as follows:

$$\tilde{\varphi}_{v,t} \equiv \left[\frac{1}{1 - G_v(\varphi_{v,t})}\int_{\varphi_{v,t}}^{\infty} \varphi_v^{\sigma-1} dG_v(\varphi_v)\right]^{\frac{1}{\sigma-1}}. \tag{7}$$

The term $\tilde{\varphi}_{v,t}$ thus contains all the information about the distribution of productivity. By aggregating across productivity levels and substituting equation (7) into equation (5),

[3] The dividends of firms are expressed as

$$\begin{aligned} d_{v,t}(\varphi_v) = \rho_{v,t}(\varphi_v) y_{v,t}(\varphi_v) - w_t l_{v,t}(\varphi_v) - r_t^K k_{v,t}(\varphi_v) \\ = (\rho_{v,t}(\varphi_v) - \lambda_t(\varphi_v)) y_{v,t}(\varphi_v) - \lambda_t(\varphi_v) f_v \varphi_v \\ = (\rho_{v,t}(\varphi_v) - \lambda_t(\varphi_v)) \left(\frac{p_{v,t}(\omega)}{P_{v,t}}\right)^{-\sigma}\left(\frac{P_{v,t}}{P_t}\right)^{-\sigma}\alpha_v Y_t^C - \frac{\lambda_t}{Z_t}f_v \end{aligned}$$

By plugging equation (5) and the definition of the real price, we get (6).

the average real price of generation producers is found to be as follows:

$$\rho_{v,t}(\tilde{\varphi}_{v,t}) = \frac{\sigma}{\sigma-1}\frac{\lambda_t}{Z_t\tilde{\varphi}_{v,t}}.$$

Similarly, by plugging in the optimal demands, the average real profits of producers of generation v can be expressed as follows:

$$d_{v,t}(\tilde{\varphi}_{v,t}) = \frac{1}{\sigma}\rho_{v,t}(\tilde{\varphi}_{v,t})^{1-\sigma}\alpha_v Y_t^C - \frac{\lambda_t}{Z_t}f_v.$$

2.4 Average across generations

Furthermore, we define the average across firms in different generations at time t. We represent "the average of averages across different generations" of a variable X at time t with $X_{\tilde{v},t}$. There are N_t firms at time t that consist of all generation products, while only a subset of S_t firms produce and operate. Note that, by construction, the number of firms N_t at time t consisting of all generation products is defined as $N_t = \sum_{v=0}^{t-1} N_{v,t} = tN_{\tilde{v},t}$ and a subset of S_t producers is aggregated from $S_t = \sum_{v=0}^{t-1} S_{v,t} = tS_{\tilde{v},t}$. The average number of producers across different generation firms is given by $S_{\tilde{v},t} = [1 - G(\varphi_{\tilde{v},t})] N_{\tilde{v},t}$ or equivalently by $S_t = [1 - G(\varphi_{\tilde{v},t})] N_t$, where $\varphi_{\tilde{v},t}$ stands for the average cutoff level of productivity across different generation firms. This cutoff level $\varphi_{\tilde{v},t}$ is characterized with a similar zero cutoff profit condition with those within generation sucht that $d_{\tilde{v},t}(\varphi_{\tilde{v},t}) = 0$. Given this cutoff level, we define the average productivity of producers across different generations as follows:

$$\tilde{\varphi}_{\tilde{v},t} \equiv \left[\frac{1}{1-G(\varphi_{\tilde{v},t})}\int_{\varphi_{\tilde{v},t}}^{\infty} \varphi_v^{\sigma-1} dG(\varphi_v)\right]^{\frac{1}{\sigma-1}}.$$

Based on the above cutoff level, the average real price of producers at time t is expressed as follows:

$$\rho_{\tilde{v},t}(\tilde{\varphi}_{\tilde{v},t}) = \frac{\sigma}{\sigma-1}\frac{\lambda_t}{Z_t\tilde{\varphi}_{\tilde{v},t}}, \tag{8}$$

Note that using the definition of the price indices, the average real price is also expressed as[4]

$$\rho_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right) = S_t^{\frac{1}{\sigma-1}}.$$

Given the above average real price, the average dividends of producing firms at time t are expressed as follows:[5]

$$d_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right) = \frac{1}{\sigma}\frac{Y_t^C}{S_t} - \frac{\lambda_t}{Z_t}f.$$

where $f \equiv \sum_{v=0}^{t-1} \frac{S_{v,t}}{S_t} f_v$ represents the amount of effective operational costs that hold on average. As expected, the average dividends of all producing firms are expressed as they are independent of generation-specific characteristics.

[4]Note that the price index is rewrittens as

$$P_t^{1-\sigma} = \sum_{v=0}^{t-1} \alpha_v P_{v,t}^{1-\sigma} = \sum_{v=0}^{t-1} S_{v,t}\alpha_v p_{v,t}^{1-\sigma}\left(\tilde{\varphi}_{v,t}\right),$$

where$p_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ stands for the average price of product varieties within a particuler generation v. By defining the weighted average price across all generations such that

$$p_{\tilde{v},t}^{1-\sigma}\left(\tilde{\varphi}_{\tilde{v},t}\right) \equiv \sum_{v=0}^{t-1} \frac{S_{v,t}}{S_t}\alpha_v p_{v,t}^{1-\sigma}\left(\tilde{\varphi}_{v,t}\right).$$

Using the above definition in the expression of the price index, we get

$$P_t^{1-\sigma} = S_t p_{\tilde{v},t}^{1-\sigma}\left(\tilde{\varphi}_{\tilde{v},t}\right).$$

So the variety effect is given by with equation (8).

[5]Note that

$$\begin{aligned} d_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right) &\equiv \sum_{v=0}^{t-1} \frac{S_{v,t}}{S_t} d_{v,t}\left(\tilde{\varphi}_{v,t}\right) \\ &= \frac{1}{\sigma}\sum_{v=0}^{t-1} \frac{S_{v,t}}{S_t}\rho_{v,t}\left(\tilde{\varphi}_{v,t}\right)^{1-\sigma}\alpha_v Y_t^C - \sum_{v=0}^{t-1} \frac{S_{v,t}}{S_t} f_v \frac{\lambda_t}{Z_t} \\ &= \frac{1}{\sigma} Y_t^C p_{\tilde{v},t}^{1-\sigma}\left(\tilde{\varphi}_{\tilde{v},t}\right) - \frac{\lambda_t}{Z_t}\sum_{v=0}^{t-1} \frac{S_{v,t}}{S_t} f_v \\ &= \frac{1}{\sigma}\frac{Y_t^C}{S_t} - \frac{\lambda_t}{Z_t} f \end{aligned}$$

where we have used the fact that $\rho_{\tilde{v},t}^{1-\sigma}\left(\tilde{\varphi}_{\tilde{v},t}\right) \equiv \sum_{v=0}^{t-1} \frac{S_{v,t}}{S_t}\alpha_v \rho_{v,t}^{1-\sigma}\left(\tilde{\varphi}_{v,t}\right)$ and $f \equiv \sum_{v=0}^{t-1} \frac{S_{v,t}}{S_t} f_v$.

Finally, we define the average dividends of all firms at time t as follows:

$$d_t \equiv \frac{S_t}{N_t} d_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right).$$

Noting that $d_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right) \equiv \sum_{v=0}^{t-1} \frac{S_{v,t}}{S_t} d_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, this can be also expressed as a weighted average profit of all generations such that $d_t \equiv \frac{S_t}{N_t} \sum_{v=0}^{t-1} \frac{S_{v,t}}{S_t} d_{v,t}\left(\tilde{\varphi}_{v,t}\right) = \sum_{v=0}^{t-1} \frac{S_{v,t}}{N_t} d_{v,t}\left(\tilde{\varphi}_{v,t}\right)$. The weight is the surviving rate at time t with respect to all potential producer firms.

2.5 Capital Accumulation and Firm Entry and Exit

In each period, $N_{E,t}$ number of entrants enters the market. Prior to entry, these new firms are identical and face a sunk entry cost of f_E in effective labor units. The value of a firm is expressed as the sum of their expected discounted profits. Using the stochastic discount factor of households adjusted by exogenous exit-inducing shock δ, we obtain the following:

$$v_t = E_t \sum_{i=t+1}^{\infty} \left[\beta\left(1-\delta\right)\right]^{i-t} \left(\frac{C_i}{C_t}\right)^{-1} d_i. \tag{9}$$

Firm entry occurs until the expected value of entry (9) is equal to the entry cost, which leads to the following free entry condition:

$$v_t = w_t \frac{f_E}{Z_{E,t}}.$$

where $Z_{E,t}$ stands for labor efficiency level specific to entry.

We assume that firms that enter at time t only start producing at time $t+1$. The timing of entry and production implies that the number of products evolves according to the law of motion:

$$N_t = \left(1-\delta\right)\left(N_{t-1} + N_{E,t-1}\right).$$

Given the above motion of firms, the number of firms of a particular generation is as follows:

$$N_{v,t} = \left(1-\delta\right)^{t-v} N_{E,v}. \tag{10}$$

Furthermore, the aggregate stock of capital in the economy is assumed to evolve as:

$$K_{t+1} = \left(1 - \delta^K\right) K_t + I_t,$$

where δ^K stands for the depreciation rate of capital.

2.6 Parametrization of Productivity Draw

To solve the model, we assume a distribution of productivity levels, φ_v with the following generation-specific Pareto distribution:

$$G_v\left(\varphi_{v,t}\right) = 1 - \left(\frac{\varphi_{\min}}{\varphi_v}\right)^{\kappa_v},$$

where $\varphi_{\min}$ is the minimum productivity level and κ_v determines the shape of distribution of a particular generation v. The dispersion decreases as κ_v increases and is skewed towards the lower bound $\varphi_{\min}$, while it increases as κ_v decreases. When $\kappa_v = \infty$, all products are located at $\varphi_{\min}$, and the products become homogeneous within that generation. To ensure the variance of the productivity distribution is finite, we must assume that $\kappa_v > \sigma - 1$.

With the above Pareto distribution, we can express the average productivity of producers $\tilde{\varphi}_{v,t}$ of generation v in equation (7) as follows:

$$\tilde{\varphi}_{v,t} = \varphi_{v,t}\left[\frac{\kappa_v}{\kappa_v - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}}, \tag{11}$$

and the number of generation producers is given by the following:

$$\frac{S_{v,t}}{N_{v,t}} = \varphi_{\min}^{\kappa_v}\left[\frac{\kappa_v}{\kappa_v - (\sigma - 1)}\right]^{\frac{\kappa_v}{\sigma-1}} \tilde{\varphi}_{v,t}^{-\kappa_v}. \tag{12}$$

In addition, substituting equation (11) into the product's real profits (6), the zero cutoff profit condition $d_{v,t}\left(\varphi_{v,t}\right) = 0$ is expressed as

$$d_{v,t}\left(\tilde{\varphi}_{v,t}\right) = \frac{\sigma - 1}{\kappa_v - (\sigma - 1)} \frac{\lambda_t}{Z_t} f_v.$$

Similarly, using our notion of the average of all generations, we get the following relations:

$$\tilde{\varphi}_{\tilde{v},t} = \varphi_{\tilde{v},t}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}, \quad \frac{S_t}{N_t} = \varphi_{\min}^{\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa_{\tilde{v}}}{\sigma-1}}\tilde{\varphi}_{\tilde{v},t}^{-\kappa},$$

and

$$d_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right) = \frac{\sigma-1}{\kappa-(\sigma-1)}\frac{\lambda_t}{Z_t}f.$$

In the above average expressions, κ stands for the parameter that shapes the Pareto distribution for all the generations of firms.

2.7 Household Budget Constraint and Intertemporal Problems

The household budget constraint is given by the following:

$$B_{t+1} + C_t + x_{t+1}v_t\left(N_t + N_{E,t}\right) + I_t + T_t = \left(1+r_t\right)B_t + L_t w_t + x_t N_t\left(v_t + d_t\right) + r_t^K K_t. \quad (13)$$

where B_{t+1} and x_{t+1} stand for bond holdings and share holdings into $t+1$, respectively. r_t is the real bond retun. T_t is the lump-sum transfer from goverment if any. During each period t, the representative household chooses its consumption C_t, labor supply L_t, bonds B_{t+1}, investments I_t, share-holdings x_{t+1} to maximize the expected utility function (1) subject to the budget constraint (13).

The first-order conditions with respect to consumption and labor supply yield the standard labor supply equation as follows:

$$\chi L_t^{\frac{1}{\psi}} = w_t C_t^{-1}.$$

The first-order condition with respect to bond holdings, investments and share holdings yields the following Euler equations:

$$1 = \beta\left(1+r_t\right)E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1},$$

$$1 = \beta E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(r_{t+1}^K + 1 - \delta^K\right),$$

and

$$v_t = \beta\left(1-\delta\right)E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(v_{t+1} + d_{t+1}\right),$$

Note that once iterated forward, the last equation shows the share price as (9).

2.8 Model Equilibrium and Solution

In equilibrium, the aggregate labor supply, L_t, is employed in either the production of consumption goods (intensive margins, i.e., the production scale) or the creation of new firms (extensive margins):[6]

$$L_t = L_t^C + N_{E,t}\frac{v_t}{w_t},$$

where $L_t^C \equiv S_t l_{\tilde{v},t}(\tilde{\varphi}_{\tilde{v},t})$ stands for the total employment used in production. In the expression, $l_{\tilde{v},t}(\tilde{\varphi}_{\tilde{v},t}) = \left(\frac{w_t}{\lambda_t}\right)^{-1}\gamma\left(\frac{y_{v,t}(\tilde{\varphi}_{\tilde{v},t})}{Z_t\tilde{\varphi}_{\tilde{v},t}} + \frac{f}{Z_t}\right)$ represents the labor demand required for the production on average.

As auxiliary variables, we also define total investment TI_t, real GDP Y_t and real average sales $y_{\tilde{v},t}(\tilde{\varphi}_{\tilde{v},t})$ with $TI_t \equiv v_t N_{E,t} + I_t$, $Y_t \equiv L_t w_t + S_t d_{\tilde{v},t}(\tilde{\varphi}_{\tilde{v},t}) + r_t^K K_t$ and $d_{\tilde{v},t}(\tilde{\varphi}_{\tilde{v},t}) = \frac{\rho_{\tilde{v},t}(\tilde{\varphi}_{\tilde{v},t})}{\sigma} y_{\tilde{v},t}(\tilde{\varphi}_{\tilde{v},t}) - \frac{\lambda_t}{Z_t} f$, respectively. Finally, we assume that the aggregate total factor productivity and entry specific technology follows the same AR(1) process as $\ln(Z_t) = \rho\ln(Z_{t-1}) + \varepsilon_t$, and $\ln(Z_{E,t}) = \rho_E \ln(Z_{E,t-1}) + \varepsilon_{E,t}$, where ε_t and $\varepsilon_{E,t}$ are normally distributed innovation with a zero mean and a variance equal to σ_Z^2 and $\sigma_{Z_E}^2$, respectively. The model at the aggregate average level consists of 24 equations and 24 endogenously determined variables in which N_t and K_t are the state variables. Table 1 summarizes the system of equations (except the productivity processes).

The variables related to a particuler generation v are characterized by the system of equations presented in Table 2. There are eight equations and eight generation specific variables consits of the entire economy. The system of equations presented in Table 1 and Table 2 consisit of the entire economy. We thus have the following proposition.

Proposition 1. *Since $N_t = \sum_{v=0}^{t-1} N_{v,t}$, $S_t = \sum_{v=0}^{t-1} S_{v,t}$ and $K_t = S_t k_{\tilde{v},t}(\tilde{\varphi}_{\tilde{v},t})$, once we know the dynamics of each generation firm in history, we can recover the number of firms N_t and the amount of physical capital K_t which are the state variables in the aggregate dynamics at each point in time. Given the state-space representation of the system, this*

[6] Using the notation of generations, this is equivalent to $L_t = \sum_{v=0}^{t-1} S_{v,t} l_{v,t}(\tilde{\varphi}_{v,t}) + N_{E,t}\frac{v_t}{w_t}$. Also note that the the condition is equivalent to the aggregated budget constraint among households: $C_t + v_t N_{E,t} + I_t = L_t w_t + N_t d_t + r_t^K K_t$.

Table 1: Summary of the benchmark model

Average pricing	$\rho_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)=\frac{\sigma}{\sigma-1}\frac{\lambda_t}{Z_t\tilde{\varphi}_{\tilde{v},t}}$
Real price	$\rho_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)=S_t^{\frac{1}{\sigma-1}}$
Average profits	$d_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)=\frac{1}{\sigma}\frac{Y_t^C}{S_t}-\frac{\lambda_t}{Z_t}f$
Average sales	$y_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)=\frac{\sigma}{\rho_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)}\left(d_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)+\frac{\lambda_t}{Z_t}f\right)$
Average profits	$d_t=\frac{S_t}{N_t}d_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)$
ZCP	$d_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)=\frac{\sigma-1}{\kappa-(\sigma-1)}\frac{\lambda_t}{Z_t}f$
Surviving rate	$\frac{S_t}{N_t}=\varphi_{\min}^{\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}\tilde{\varphi}_{\tilde{v},t}^{-\kappa}$
Free entry condition	$v_t=\frac{wf_{E,t}}{Z_{E,t}}$
Motion of products	$N_{t+1}=(1-\delta)\left(N_t+N_{E,t}\right)$
Euler equity	$v_t=\beta\left(1-\delta\right)E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(v_{t+1}+d_{t+1}\right)$
Euler bond	$1=\beta\left(1+r_t\right)E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1}$
Euler capital	$1=\beta E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(r_{t+1}^K+1-\delta^K\right)$
Capital accumulation	$K_{t+1}=\left(1-\delta^K\right)K_t+I_t$
Aggregate capital	$K_t=S_tk_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)$
Aggregate labor	$L_t^C=S_tl_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)$
Absorption	$Y_t^C=C_t+I_t$
Real wage	$l_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)=\left(\frac{w_t}{\lambda_t}\right)^{-1}\gamma\left(\frac{y_{v,t}\left(\tilde{\varphi}_{\tilde{v},t}\right)}{Z_t\tilde{\varphi}_{\tilde{v},t}}+\frac{f}{Z_t}\right),$
Rental rate	$k_{\tilde{v},t}\left(\tilde{\varphi}_{\tilde{v},t}\right)=\left(\frac{r_t^K}{\lambda_t}\right)^{-1}\left(1-\gamma\right)\left(\frac{y_{v,t}\left(\tilde{\varphi}_{\tilde{v},t}\right)}{Z_t\tilde{\varphi}_{\tilde{v},t}}+\frac{f}{Z_t}\right)$
Defnition of cost index	$\lambda_t=w_t^{\gamma}r_t^{K1-\gamma}$
Optimal labor supply	$\chi L_t^{\frac{1}{\psi}}=w_tC_t^{-1}$
Labor market clearing	$L_t=L_t^C+N_{E,t}\frac{v_t}{w_t}$
Total Investment	$TI_t\equiv v_tN_{E,t}+I_t$
Real GDP	$Y_t\equiv L_tw_t+N_td_t+r_t^KK_t$

Table 2: Summary of the benchmark model (for a particuler generation)

Average pricing	$\rho_{v,t}\left(\tilde{\varphi}_{v,t}\right)=\frac{\sigma}{\sigma-1}\frac{\lambda_t}{Z_t\tilde{\varphi}_{v,t}}$
Average profits	$d_{v,t}\left(\tilde{\varphi}_{v,t}\right)=\frac{1}{\sigma}\rho_{v,t}\left(\tilde{\varphi}_{v,t}\right)^{1-\sigma}\alpha_v Y_t^C-\frac{\lambda_t}{Z_t}f_v$
Average sales	$y_{v,t}\left(\tilde{\varphi}_{v,t}\right)=\frac{\sigma}{\rho_{v,t}\left(\tilde{\varphi}_{v,t}\right)}\left(d_{v,t}\left(\tilde{\varphi}_{v,t}\right)+\frac{\lambda_t}{Z_t}f_v\right)$
Labor demand	$l_{v,t}\left(\tilde{\varphi}_{v,t}\right)=\left(\frac{w_t}{\lambda_t}\right)^{-1}\gamma\left(\frac{y_{v,t}(\tilde{\varphi}_{v,t})}{Z_t\tilde{\varphi}_{v,t}}+\frac{f_v}{Z_t}\right)$
Capital demand	$k_{v,t}\left(\tilde{\varphi}_{v,t}\right)=\left(\frac{r_t^K}{\lambda_t}\right)^{-1}(1-\gamma)\left(\frac{y_{v,t}(\tilde{\varphi}_{v,t})}{Z_t\tilde{\varphi}_{v,t}}+\frac{f_v}{Z_t}\right)$
ZCP	$d_{v,t}\left(\tilde{\varphi}_{v,t}\right)=\frac{\sigma-1}{\kappa_v-(\sigma-1)}\frac{\lambda_t}{Z_t}f_v$
Surviving rate	$\frac{S_{v,t}}{N_{v,t}}=\varphi_{\min}^{\kappa_v}\left[\frac{\kappa_v}{\kappa_v-(\sigma-1)}\right]^{\frac{\kappa_v}{\sigma-1}}\tilde{\varphi}_{v,t}^{-\kappa_v}$
Nb of generation firms	$N_{v,t}=(1-\delta)^{t-v}N_{E,v}$

in turn means that the dynamics at generation levels are built up to establish the entire aggregate dynamics of the economy.

Propsoition 1 confirms our common sense that the entire economic history is dependent of each firm born at different points in time. However, we often don't know the history of each of all generations. Instead we only have information on the aggregate dynamics of the economy. Is there any way to recover the dynamics at generation levels? The answer is yes and it leads us to the following proposition.

Proposition 2. *Given the parameter value of α_v, κ_v, f_v and the number of new entrants in a particuler generation year $N_{E,v}$ as well as the aggregated absorption Y_t^C, wages w_t, the rental rate of captial r_t^K, marginal cost λ_t, the productivity level Z_t and $Z_{E,t}$ in the current time period, the variables that are specific to the generation at the current time period, namely $\rho_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, $d_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, $y_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, $l_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, $k_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, $\tilde{\varphi}_{v,t}$ $S_{v,t}$ and $N_{v,t}$ are recovered from the system of equations.*

Proposition 2 is a mirrored one of Propostion 1. Combined with the system of equations presnted in Table 1, it is possible to solve the system presnted in Table 2. Intuitively, having the information on the aggregate dynamics, we only need to know the number of new entrants at a particuler year as the past information to recover the accutal state of that generation cohort firms. In Section 3, we use the above second proposition to recover the dynamics of generation firms.

Finally, note that Proposition 1 and Proposition 2 hold with or without the long-run growth in productivity level. In the following section, we assume that the process of aggregate productivities is stationary.[7] The above system of equations do not have analytical solutions. Consequently, we approximate the system with the perturbation method around the stationary steady state.

3 Benchmark Calibration and Estimation

We now use the theoretical model developed in the previous section and apply it to Japanese data. and apply it to Japanese data. Our goal here is to reestablish the generation specifc dynamics and performe a counterfactual analysis. The advantage of Japnaese data is to have the distribution of generations with respect to the number of firms, their sales and employment. We use these information to back out the generation specific stuctual parameters, namely κ_v, α_v and f_v.

3.1 Calibration

First, we discuss the calibration of the parameters related to the aggregate economy. The theoretical model is calibrated on an annual basis with benchmark values of the parameters, as shown in Table 3. The annual discount factor is set to 0.96, resulting in a steady-state real interest rate of 4%. The value of the Frish elasticity of labor supply ψ is 2.15, which is taken from Sugo and Ueda (2008), who estimated the elasticity using Japanese data from the postwar period. The elasticity of substitution among varieties and the parameter that shapes the Pareto distribution across generations are set to 3.8 and 3.4, respectively, following Ghironi and Melitz (2005). With $\sigma = 3.8$, this gives a markup of 35 %. Note that these values satisfie the restriction on these parameters such

[7] As we will see later, all variables are stationary including those at generation level except $S_{v,t}$ and $N_{v,t}$. The statinarity is recovered when these varialbes are expressed as a ration such that $S_{v,t}/N_{v,t}$.

that $\kappa > \sigma - 1$. The value of the firm deprecation rate δ and fixed operational cost f are chosen according to Hamano and Oikawa (2021); these values match the average firm creation and operation rates in Japan, as observed in the current production survey. Specifically, we set $\delta = 0.0223$ and $f = 0.04241$, resulting in steady-state firm entry rate N_E/N of 0.00571 and operation rate S/N of 0.987.[8] Further, we set the value of capital depreciation and the labor share in production according to Fujiwara et al. (2005) ,who estimated the values of these parameters for the postwar Japanese economy. The minimum idiosyncratic productivity level $\varphi_{\min}$ and fixed entry cost f_E are set to unity in the steady state. The parameter that determines the disutility of labor supply χ is given so that the value gives the steady-state labor supply of $L = 1$.

Table 3: Calibration of the model

β	Discount factor	0.96
ψ	Frisch elasticity of labor supply	2.15
χ	Disutility of supplying labor	0.75508
σ	Elasticity of substitution among varieties	3.8
κ	Distribution parameter	3.4
δ	Depreciation rate of firm	0.0223
δ^K	Depreciation rate of capital	0.2400
γ	Labor share	0.63
f	Fixed operational costs	0.04241
$\varphi_{\min}$	Minimum idiosyncratic productivity level	1
f_E	Fixed entry costs	1
ρ_Z	Persistence of aggregate productivity	0.57749
σ_Z	Standard deviation of productivity shocks	0.0028311

For the productivity process, we assume that $Z_{E,t} = Z_t^{\gamma}$ and we estimate the standard deviation σ_Z and persistence ρ_Z with historical real GDP data for Japan from 1887 to

[8]The data covers from 2001Q to 2017Q. Note at the steady state, $N = (1 - \delta)(N + N_E)$. Then, we have $\delta = (N_E/N) / (1 + N_E/N)$.

2012, the period that corresponds to our sample periods. The data is taken from Maddison Historical Statistics.[9] For the estimation, we consider an empirically consistent measure of the theoretical GDP as $Y_{R,t} \equiv Y_t / S_t^{\frac{1}{\sigma-1}}$ following Ghironi and Melitz (2005).[10] It allows us to capture the fact that fluctuations in the number of product varieties are imperfectly measured in the official statistics. Then we maximize the likelyhood of the theoretical model to estimate σ_Z and ρ_Z. In estimation, we introduce the measurement error.[11]

3.2 Estimation of Generation-Specific Parameters

As stated in Proposition 2, given the aggregate dynamics of the economy and the value of the structual parameters at generation level, it is possible to recover the dynamics of each generation. However, we often don't know the value of these parameters. In this subsection, we propose a GMM approach to estimate them based on Porposition 2. Specifically, to pin down the parameter values of α_v, κ_v and f_v for all generations in the sample period, we use the end of date distiribution of sales, the number of producers and the total factor productivity in the following survery data as observables for $y_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, $S_{v,t}$ and $\tilde{\varphi}_{v,t}$ in the theoretical model.

More precisely, our estimation strategy is as follows. First, we simulate the theoretical model 100,000 times over 126 years with the calibrated parameters and the estimated productivity process for the aggregate economy, as shown in Table 3. Second, using the

[9]The historical Japanese real GDP data is taken from Maddison: https://www.rug.nl/ggdc/historicaldevelopment/maddison/?lang=en

[10]Additionally, we define empirically consistent consumption $C_{R,t} \equiv C_t / S_t^{\frac{1}{\sigma-1}}$ and total investment $TI_{R,t} \equiv TI_t / S_t^{\frac{1}{\sigma-1}}$ accordingly.

[11]To detrend the GDP data, we use an HP filter with a smoothing parameter of $\lambda = 100$. Ravn and Uhlig (2002) suggests $\lambda = 6.25$ for annual data. We find that $\lambda = 100$ is sufficiently good to capture the business cycle over 126 years, while $\lambda = 6.25$ picks up a higher-frequency business cycle. However, the simulation result is qualitatively the same with respect to trend of real GDP. The estimation is also performed with the Bayesian method using established prior information from the literature. The result obtained is isomorphic to the benchmark estimation with uniform prior. The estimation, as well as the approximation of the system of nonlinear equations with the perturbation, are conducted with the RISE toolbox developed by Junior Maih.

current distribution of generation firms, their sales and TFPs in the 2013 survey shown in Figure 1 as observables for $y_{v,t}(\tilde{\varphi}_{v,t})$, $S_{v,t}$ and $\tilde{\varphi}_{v,t}$, together with the number of new entrants in a particuler generation year $N_{E,v}$ as well as the aggregated absorption Y_t^C, wages w_t, the rental rate of captial r_t^K, marginal cost λ_t, the productivity level Z_t and $Z_{E,t}$ obtained at the first stage, we solve the system of equation presnted in Talbe 2 and back-out the generation-specific technology κ_v, the generation-specific preference α_v, and the generation-specific operational fixed cost f_v for each 126 generation. Third, among these 100,000 simulations, we select the one that gives us the closest dynamics of actual real GDP, and choose the corresponding distribution of the generation-specific technologies κ_v, the generation-specific preferences α_v, and the generation-specific operational fixed costs f_v as our best estimate.[12]

Figure 2 shows the results of estimation. While we don't see very much the noticiable difference with respect to the generation-specific technologies κ_v. The generation-specific preferences α_v are relatively high for the firms born around the turn of the 20th century. It is also noticed the distribution of generation-specific fixed costs f_v gives an asymmetric wedge-shaped pattern.[13] From the results of estimation and the current survey data, the

[12]In the simulation, we match the share of a particular generation firm in the total number of generation firms in the 2013 survey with the theoretical counterpart, $S_{v,126}/S_{126}$. The data reports manufacturing firms that have more than 50 employees and those with more than 30 million yen in capital assets. The total number of manufacturing firms as of the 2013 survey was 13,426, according to the BSJBSA data. For the same year, according to the Census of Manufacture by the Ministry of Economy, Trade and Industry ("Kogyo Tokei", in Japanese) and the converter (KogyoTokei Konbata) prepared by RIETI to sum up from plant level to firm level, there were 184,485 firms, based on the census of plants that had more than four employees. Another data source, the Establishment and Enterprise Census of Japan (Ministry of Internal Affairs and Communications), reported the existence of approximately 650,000 published establishments without any restriction regarding the number of employees and any organization. Given that the average reply rate of the BSJBSA survey is approximately 85% and that there are other generation firms that are not captured in the BSJBSA data, the total number of manufacturing firm would be 50 times higher, at most, than the number reported in the 2013 BSJBSA survey. However, our simulation results are robust with respect to scaling in matching the share.

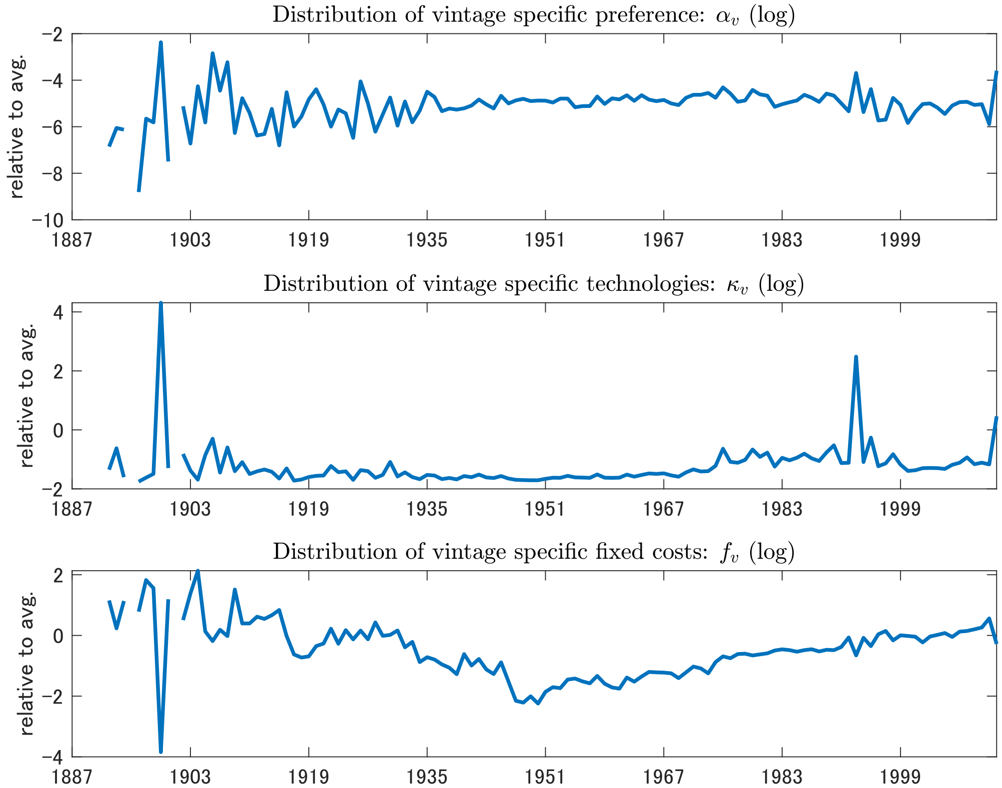
[13]The theoretical model cannot compute the generation-specific preference α_v, technology κ_v and the fixed cost f_v for 1888, 1891 and 1895 because of missing data.

following message would emerge. First, old generation firms (the least popular cohort in the distribution of firm generation), i.e., firms created in the initial time periods up until the approximate time of the 1929 Great Depression, which is roughly the time corresponding to the Meiji (1868-1912) and Taisho (1912-1926) era, are subject to higher fixed costs f_v, yet they are currently more productive and have higher sales compared to other generation firms. Second, generation firms created near the beginning of the Second World War up until the approximate time of the Plaza Accord in 1985 (the most popular cohort in the distribution of generation firms) show low sales and TFPs, although they benefit from low operational fixed costs f_v. Third, generation firms created in the most recent time period after approximately 1985 (the secondly popular cohort in the distribution of generation firms) show lower sales and TFPs and have slightly higher specific fixed costs f_v than the average.[14]

Furthermore, noting that generation-specific fixed costs f_v encompass various types in the real world, our results seem shedding lights on the "industrial policies" in postwar Japan. After the Second World War, the Ministry of International Trade and Industry implemented several industrial policies (Ito et al., 1988; Komiya et al., 1984). The policy scheme consisted of subsidies for specific industries, tax exemptions, R&D policies, and export promotion policies. The targeted sectors were first the steel and iron industries, followed by heavy industries, and then shifted to machinery sectors and high-tech industries. Government banks also had several lending programs that specialized in infrastructure investment, the development of rural areas, investments for the environment, and energy-saving technology (e.g., DBJ, 2002, JASME, 2003). The Japan Development Bank (JDB), which is a government bank housed under the Ministry of Finance, had special lending programs that offered lower interest rates to large companies and major industries. In contrast to the JDB scheme, the Japan Finance Corporation for Small and Medium Enterprise (JASME) (Chusho Kigyo Kinyu Koko) (1953–2008), which was also

[14] Cabral and Mata (2003) argue that time varies the firm size dispersion of firms and found that the distribution becomes skewed to the right for young generation firms. Our results echo their result by simulating generation-specific technologies κ_v that shape the productivity dispersion of firms. Specifically, our paper replicates a similar pattern of the skewness depending on the sample time period.

Figure 2: Estimation of generation-specific parameters



The first, second and third panel show the estimated distribution of generation-specific preferences α_v, technologies κ_v and their fixed costs f_v at the end of periods, respectively.

a government bank, specialized in helping SMEs. The central strategy for several lending programs consisted of lowering interest rates for investments.[15][16]

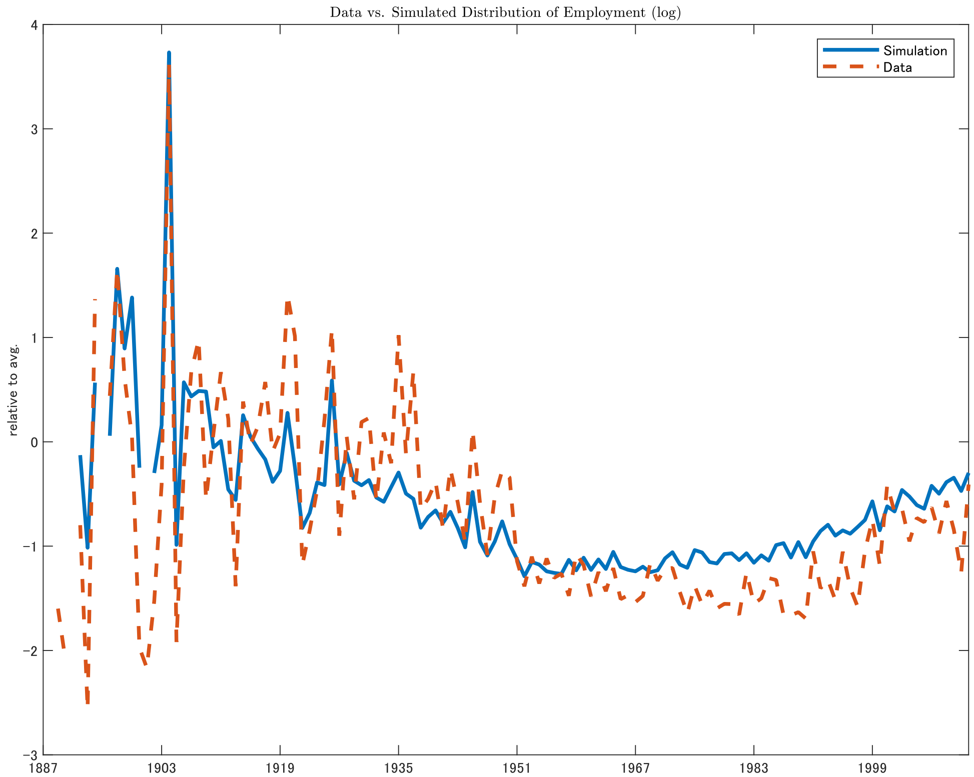
To see how our results are comparable with the data, we use the distribution of the number of regular employees. It is striking to see that our simulated distribution of employment provides a similar pattern as that in data, as shown in Figure 3.

Finally, we provide the simulation for other sectors in Appendix B. The data for the light manufacturing, public utilities, telecommunication and transportation and construc-

[15] Elliott and Okubo (2016) found that these special lending programs offered lower interest rates than the market rates, which promoted abatement investment in Japan.

[16] See for instance Miwa and Ramseyer (2001) for the counterargument on the presence of postwar industrial policies in Japan.

Figure 3: Comparison with the data distribution of employment



The figure shows the simulated distribution of generation employment $l_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ (solid line) and the distribution of generation employment in the data (dashed line) at the end of the periods, respectively.

tion sectors are available from the same source, i.e., BSJBSA. While we see substantial variation across the sectors, it is noticed the share of the firms created in the middle range of the time period spanning 126 years is relatively high, while these firms show lower sales across all sectors. Accordingly, we confirm a similar pattern for the simulated fixed costs, which produces an asymmetric wedge-shaped pattern, as is the case for the manufacturing sector.

4 Macroeconomic Dynamics of Generations

In this section, we demonstrate how the theoretical model whose specificity is summurized in Proposition 1 and Proposition 2 is useful to uncover unobservable past economies.

Specifically, we reestablish the dynamcis of all generations based on the estimation results in the previous section. Further, with the recovered generation dynamcis, we reestablish the past landscape of the economy. Specifically, we provide “snapshots” of it in 1923 and 1996. We choose these these specific years since they allow us to compare our artificially recovered snapshots with the true exisiting snapshots.

4.1 Generation dynamics

Figure 8 in Appendix shows the macroeconomic dynamics for the major aggregate variables that are directly required to compute the pass of each generation firm. The simulated pass of real GDP captures well the business cycles in the data. Overall, the cyclical properties of other aggregate variables, including firm entry and exit, are similar to the ones discussed in Hamano and Zanetti (2017) and are in line with those discussed in the literature of real business cycle models.[17] Specifically, the aggregate macroeconomic codnition that leads to a specific number of entrants for each generation ($N_{E,v}$) is crucial for it persitently influences the number of potential producers of that specific cohort at time t ($N_{v,t}$) through equation (10).

With the simulated distribution of generation-specific preferences α_v, technologies κ_v, and fixed costs f_v, together with the above mentioned aggregate macroeconomic dynamics in hand, we can reestablish the history of each generation firm as stated in Proposition 2. Figure 9 in Appendix provides the simulated number of producers $S_{v,t}$, the generation specific average of real prices $\rho_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, sales $y_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, productivites $\tilde{\varphi}_{v,t}$ employment $l_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ and capital $k_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ from 1887 to 2012.[18] We observe considerable heterogeneity across generation firms for the level of these variables that inherit the cohort specific

[17]The standard deviation of GDP in the simulated model is 0.0555, while that in the data is 0.0596. The simulated consumption is less volatile than GDP (its standard deviation with respect to GDP is 0.4185), while the simulated investments are equally as volatile as GDP (the standard deviation with respect to GDP is 0.9014).

[18]With the estimated values of parameters, we find that unstable dynamics occur for some generations, thereby violating the surviving condition such that $S_{v,t}/N_{v,t} < 1$. In generating Figure 9, we exclude the generations that show unstable dynamics.

characteristics. The number of producers $S_{v,t}$ tends to be high for the generation firms that correspond to the boom around 1950's. Note also that by construction, this variable is non-stationary while $S_{v,t}/N_{v,t}$ is stationary. Without generation-specific macroeconomic shocks, all these variables fluctuate in a synchronized manner with each other and with the aggregate dynamics, as shown in Figure 8.

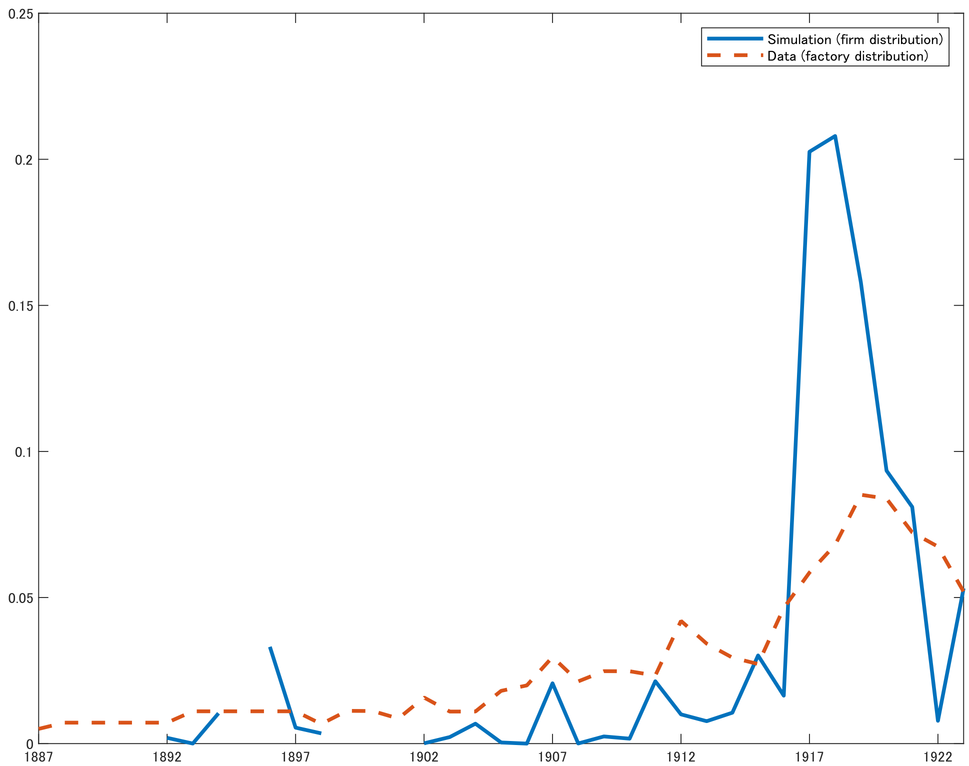
4.2 Snapshots

4.2.1 1923

To see the history of generation firms in a compact way, we provide "snapshots" at particular moments in history. These snapshots are the result of our simulation given the abovementioned aggregates, as well as generation-specific dynamics. Figure 10 provides a snapshot for 1923. Because of the economic boom during the First World War ("Taisen-keiki" in Japanese), the share of generation producers from 1914 and 1919 is high (first panel in the second row in the figure). At the moment of 1923, these firms are relatively new generation firms. The sales, productivity, employment of labor and capital these new-born firms during the war period are not particularly high in the economic landscape of 1923. We observe also a low entry in 1922 and a rebound in 1923, which might reflect the impact of the great Kanto earthquake hit in September 1923.

To check the relevance of our simulation, we compare our snapshot of the distribution of generation firms with the Census of Manufacture by the Ministry of Commerce and Agriculture ("Kojyo Tokei Hyo", in Japanese). The Census of Manufactures in the prewar period records the number of "factorys" in terms of their founding year. The red dotted line in Figure 4 shows the distribution of generation factorys in 1923. While our simulation and snapshot are firm basis, it gives a qualitatively similar pattern as our simulated distribution of generation firms in 1923, which is shown by the solid blue line in the figure.

Figure 4: Distribution of factories in data and simulated snapshot of generation firms in 1923

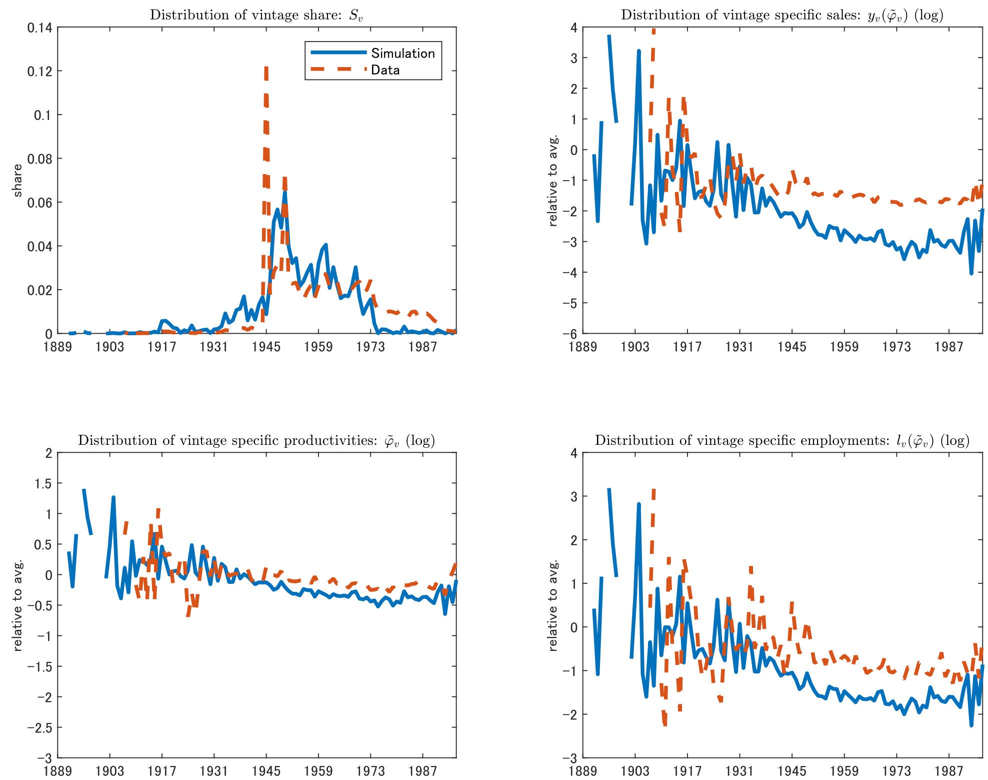


The figure shows the simulated distribution of generation firms $S_{v,t}$ and the distribution of factory generation firms in the data (dashed line) in 1923.

4.2.2 1996

Figure 11 gives a snapshot for 1996 based on our simulated generation-specific technologies and fixed costs. We present the simulated distribution of the number of producers $S_{v,t}$, real prices $\rho_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, sales $y_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, productivites $\tilde{\varphi}_{v,t}$, employments $l_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ and capitals $k_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ based on the estimated distribution of generation-specific technologies κ_v, preferences α_v and their fixed costs f_v in the economic landscape for 1996. It is particularly interesting to have a snapshot for this year since the Census of Manufacture provided by the Ministry of Economy, Trade and Industry also has a snapshot for 1996. Figure 5 provides the comparison. In the figure, the dashed lines show the data and the

Figure 5: Simulated vs. Census Snapshots in 1996



The solid lines in the figure show the simulated distribution of generation firms $S_{v,t}$, the distribution of their sales $y_{v,t}$ $(\tilde{\varphi}_{v,t})$, the distribution of their productivity $\tilde{\varphi}_{v,t}$ and the distribution of their employment $l_{v,t}$ $(\tilde{\varphi}_{v,t})$ in 1996, respectively. The dashed lines in the figure show these distributions in actual data collected in 1996.

solid lines show our simulated snapshot for the number of firms, sales, TFPs and employments. While the census covers medium-sized firms (all plants with more than four employees), our snapshot and the census provide a qulitatively similar landscape. Other simulated snapshots (1914, 1931, 1945, 1960, 1985 and 2009) are given in Appendix C.

4.3 Counterfactual Analysis

As the last exercise, we explore the consequence of a counterfactual distribution of generation-specific fixed costs f_v. We thus consider a counterfactual pass of cohort dynamics. Although it is highly stylized in the theoretical model, the fixed cost for operation ranges from physical costs to legal procedures in the real world. Importantly, we believe that a part of these costs would be a subject of industrial policy instruments, as mentioned before. This is the reason why we consider a counterfactual distribution of fixed costs f_v rather than a counterfactual distribution of technologies κ_v or preferences $\alpha_v$which seems more difficult to be influenced by policies.

Our counterfactual distribution of generation-specific fixed costs f_v is shown by the dotted line in Figure 6. To obtain this, we first approximate the simulated distribution of generation-specific fixed costs f_v with a quadratic function (shown by the smoothed line). The counterfactual is computed as a flipped symmetric distribution against the horizontal axis of the smoothed quadratic distribution. Our counterfactual distribution thus shows the opposite pattern as the simulated distribution of the fixed costs. Specifically, it increases steadily, peaks at approximately 1920, and then decreases.

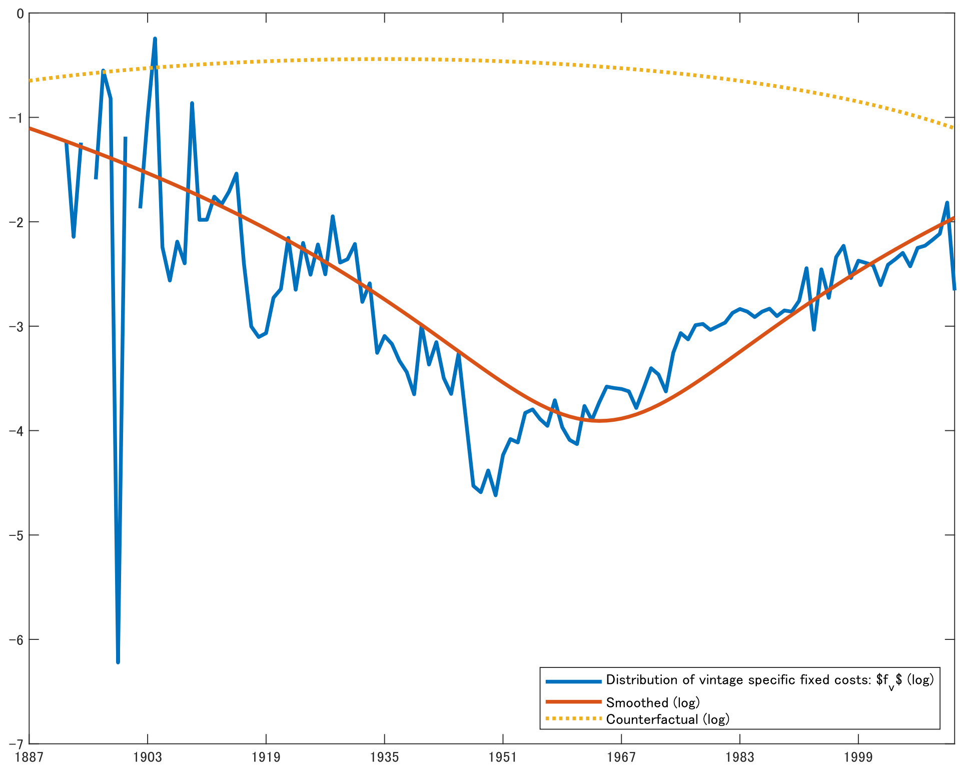
The result of our counterfactual simulation is shown in Figure 7. With the counterfactual distribution, it is striking to see that the share of recent generation firms increases significantly while dramatically reducing the number of generation firms created in the postwar boom (the first panel in the figure). At the same time, the sales $y_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, productivity $\tilde{\varphi}_{v,t}$ and employment $l_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ of these postwar generation firms become substantially high. Our counterfactual fixed costs thus dramatically change the landscape of generation firms, as well as their characteristics.

5 Conclusion

This study revitalizes the past dynamics of firms across various generations by developing a theoretical model centered on the entry, exit, and selection of heterogeneous firms. By examining the current distribution of generation firms and their sales, we estimate generation-specific preferences, technologies, and fixed operational costs. Particularly, the distribution of generation-specific fixed costs, drawn from Japanese data, presents an asymmetric wedge-shaped pattern.

Employing these simulated parameters along with the cyclical component of GDP

Figure 6: Counterfactual fixed-cost distribution (log)

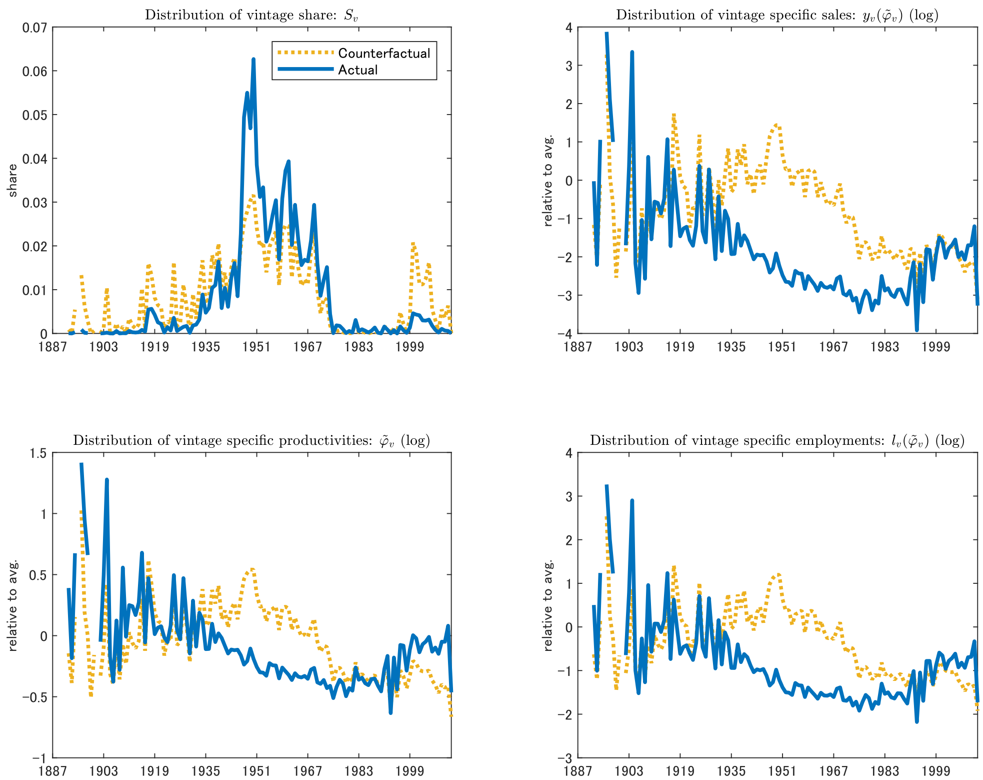


The solid blue line shows the simulated generation-specific fixed costs f_v . The smoothed red line shows the trend of f_v estimated with the quadratic function. The dashed line shows the counterfactual generation-specific fixed costs f_v .

spanning 126 years, we successfully reconstruct the macroeconomic dynamics specific to each generation of firms. This exercise allows us to capture snapshots of the economic environment at specific junctures in history. Furthermore, our study showcases how hypothetical changes in fixed costs can drastically alter the current landscape of generation firms, thereby influencing their characteristics within the Japanese economy.

Despite striving for simplicity in our theoretical model, we acknowledge the potential for future studies to incorporate additional firm characteristics, such as multi-product lines and the effect of economic growth at both the firm and aggregate levels. We are confident that the insights derived from our study could contribute significantly to policy discussions on optimal resource allocation and the dynamism of firms.

Figure 7: Actual vs. counterfactual distribution



The solid lines in the figure show the simulated distribution of generation firms $S_{v,t}$, the distribution of their sales $y_{v,t}\,(\tilde{\varphi}_{v,t})$, the distribution of their productivity $\tilde{\varphi}_{v,t}$ and the distribution of their employment $l_{v,t}\,(\tilde{\varphi}_{v,t})$, respectively. The dashed lines in the figure show these distributions obtained with the counterfactual generation-specific fixed costs f_v presented in Figure 6.

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A Data

This paper uses Japanese firm-level data from the Basic Survey of Japanese Business Structure and Activities (BSJBSA), provided by the Ministry of Economy, Trade and Industry of Japan (METI). The data contain a wide variety of firm-level variables such as founding year, number of employees, sales, profit, and tangible capital. The survey covers all firms with more than 50 employees and with more than 30 million yen of capital asset and has an approximate 85% reply rate. The survey was conducted in 2013.

In addition, the paper uses Japanese plant-level data from the Census of Manufacture ("Kogyo Tokei" in Japanese), provided by the Ministry of Economy, Trade and Industry. The data covers all plants with more than 4 employees.

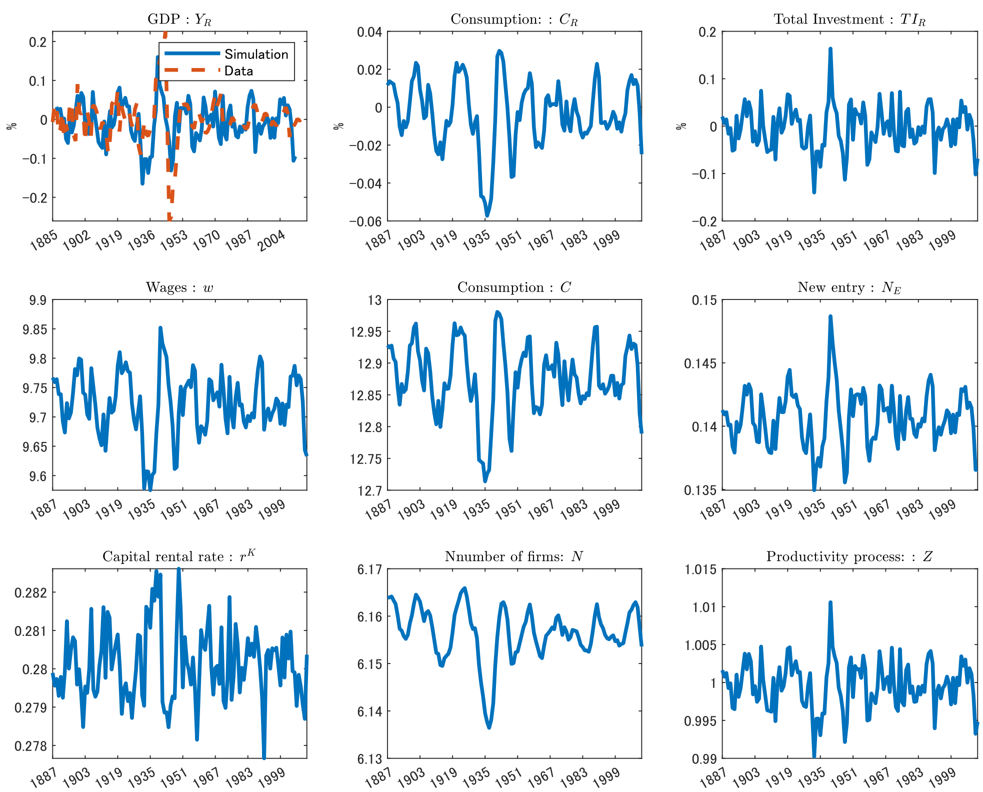
B Generation dynamics

C Simulated Snapshots

D Sectors

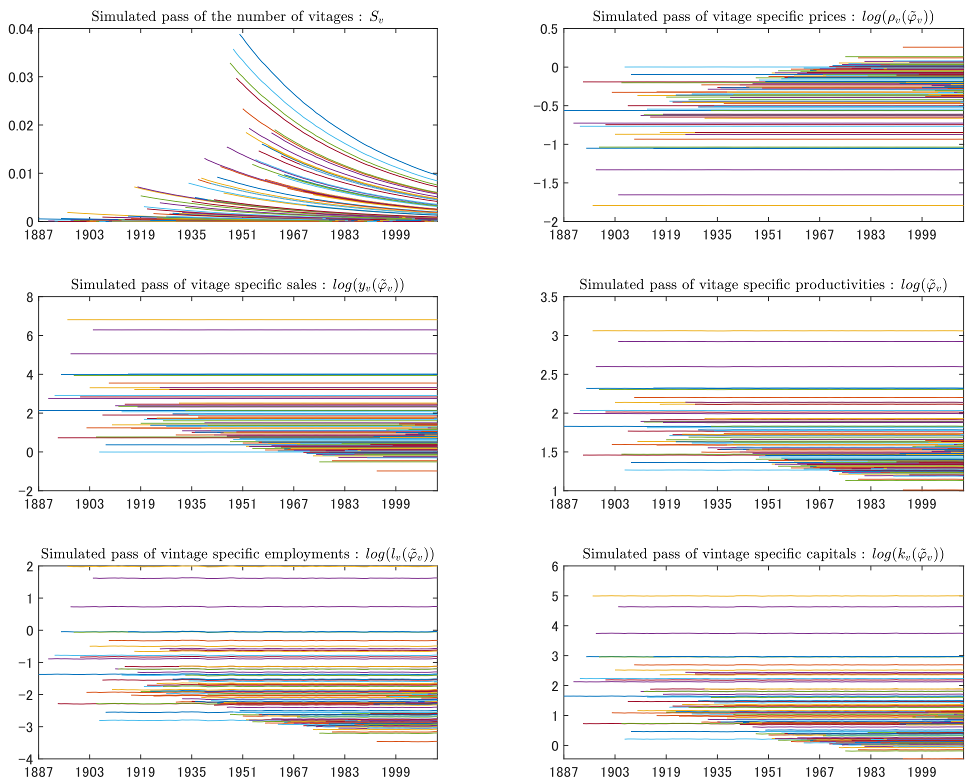
E Snapshots

Figure 8: Simulation for aggregate variables



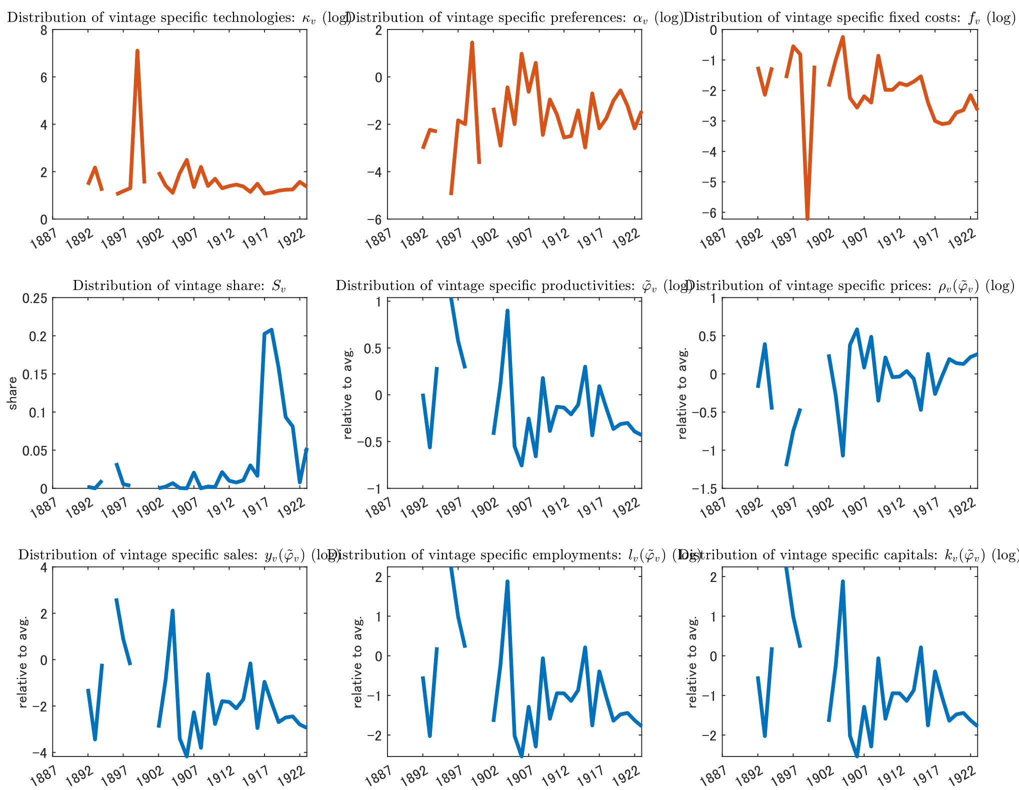
Each entry in the first row shows the percentage-point response in the simulated economy over 126 years. Each entry in the second and third row shows the simulated values expressed in levels over 126 years.

Figure 9: Simulated pass of generations



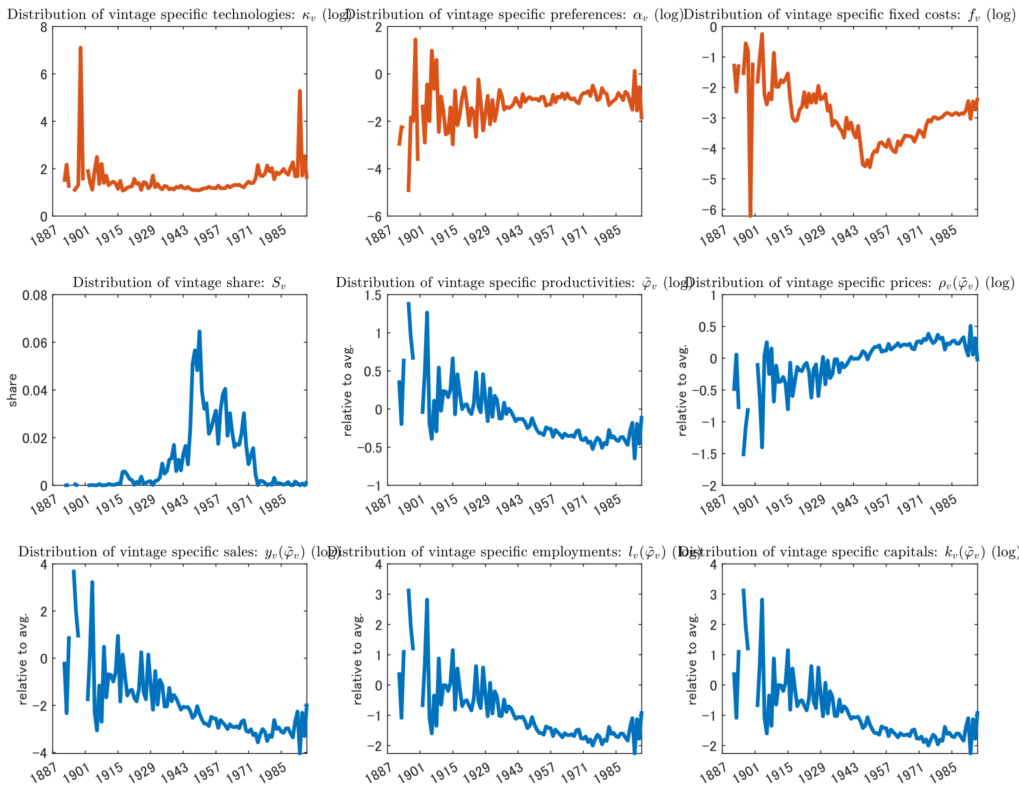
Each entry shows the simulated dynamics of the number of producers $S_{v,t}$, real prices $\rho_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, sales $y_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, productivity $\tilde{\varphi}_{v,t}$, employment $l_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ and capital $k_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ for generation firms over 126 years.

Figure 10: Simulated Snapshot in 1923



The first panel in the first row show the estimated distribution of generation-specific technologies κ_v, preferences α_v and their fixed costs f_v in the economic landscape of 1923. Other panels show the simulated distribution of the simulated dynamics of the number of producers $S_{v,t}$, real prices $\rho_{v,t}(\tilde{\varphi}_{v,t})$, sales $y_{v,t}(\tilde{\varphi}_{v,t})$, productivites $\tilde{\varphi}_{v,t}$, employments $l_{v,t}(\tilde{\varphi}_{v,t})$ and capitals $k_{v,t}(\tilde{\varphi}_{v,t})$ in the economic landscape of 1923.

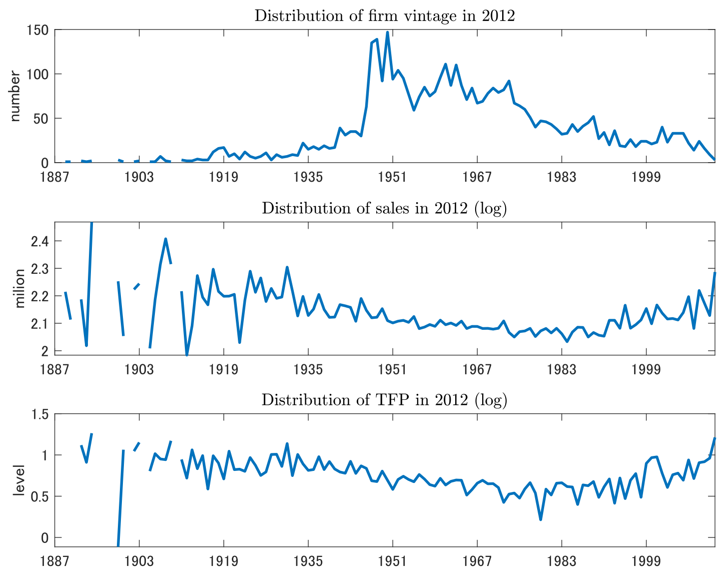
Figure 11: Simulated Snapshot in 1996



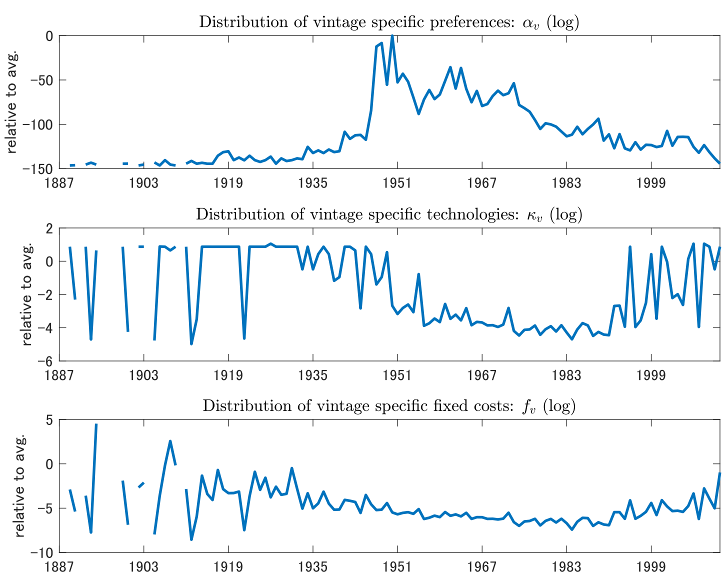
The first panel in the first row show the estimated distribution of generation-specific technologies κ_v, preferences α_v and their fixed costs f_v in the economic landscape of 1996. Other panels show the simulated distribution of the simulated dynamics of the number of producers $S_{v,t}$, real prices $\rho_{v,t}(\tilde{\varphi}_{v,t})$, sales $y_{v,t}(\tilde{\varphi}_{v,t})$, productivites $\tilde{\varphi}_{v,t}$, employments $l_{v,t}(\tilde{\varphi}_{v,t})$ and capitals $k_{v,t}(\tilde{\varphi}_{v,t})$ in the economic landscape of 1996.

Figure 12: Data and Simulation of generation-specific parameters: light manufacturing

(a) Data



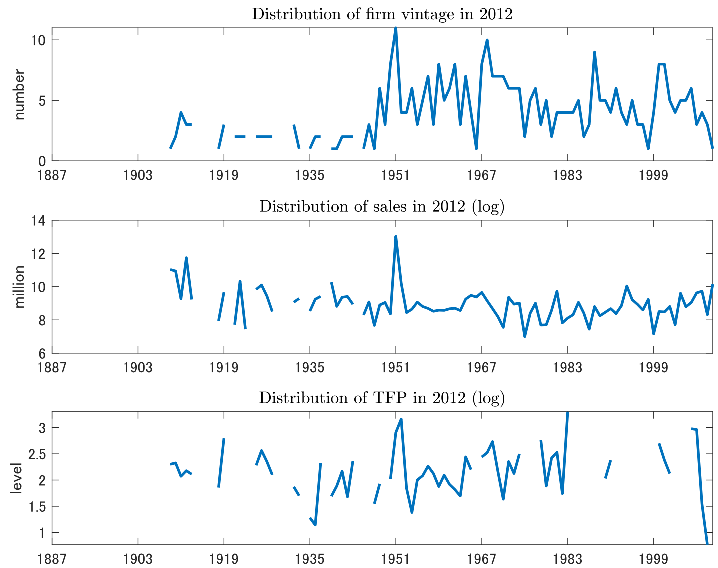
(b) Simulation of generation-specific parameters



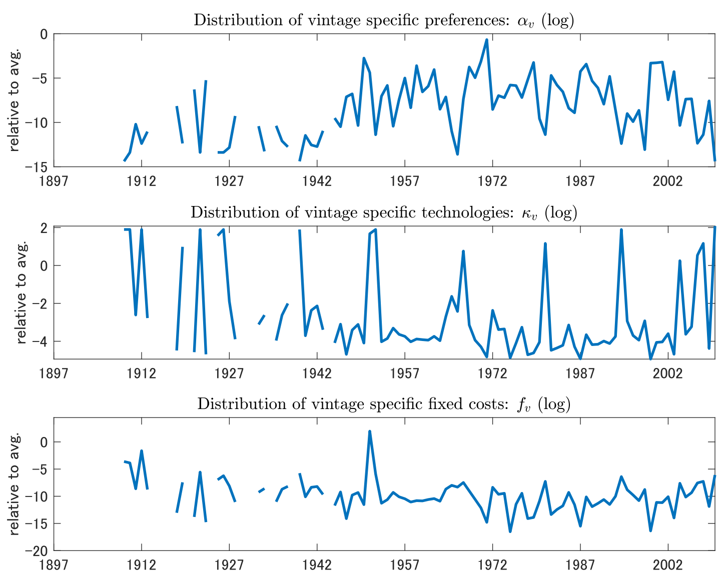
The first and the second panel in the first row show the distribution of generation firms and the distribution of their sales in the 2013 survey, respectively. The first and the second panel in the second row show the simulated distribution of generation-specific technologies κ_v and fixed costs f_v at the end of periods, respectively. The first and the second panel in the third row show the simulated distribution of generation firm productivity $\tilde{\varphi}_{v,t}$ and employment $l_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ at the end of the periods, respectively.

Figure 13: Data and Simulation of generation-specific parameters: public utilities, telecommunication and transportation

(a) Data



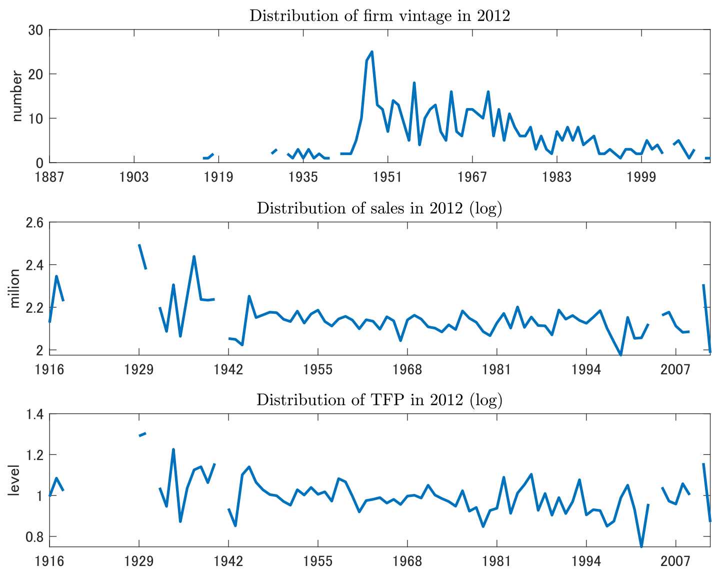
(b) Simulation of generation-specific parameters



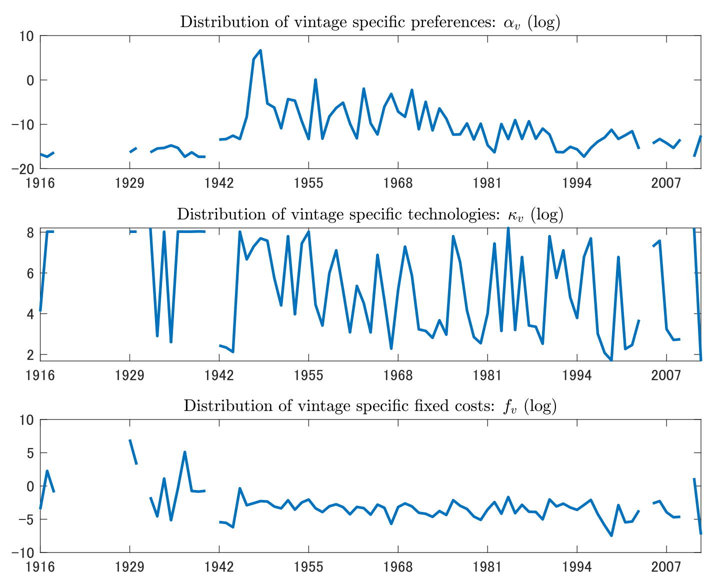
The first and the second panel in the first row show the distribution of generation firms and the distribution of their sales in the 2013 survey, respectively. The first and the second panel in the second row show the simulated distribution of generation-specific technologies κ_v and fixed costs f_v at the end of periods, respectively. The first and the second panel in the third row show the simulated distribution of generation firm productivity $\tilde{\varphi}_{v,t}$ and employment $l_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ at the end of the periods, respectively.

Figure 14: Data and Simulation of generation-specific parameters: construction

(a) Data

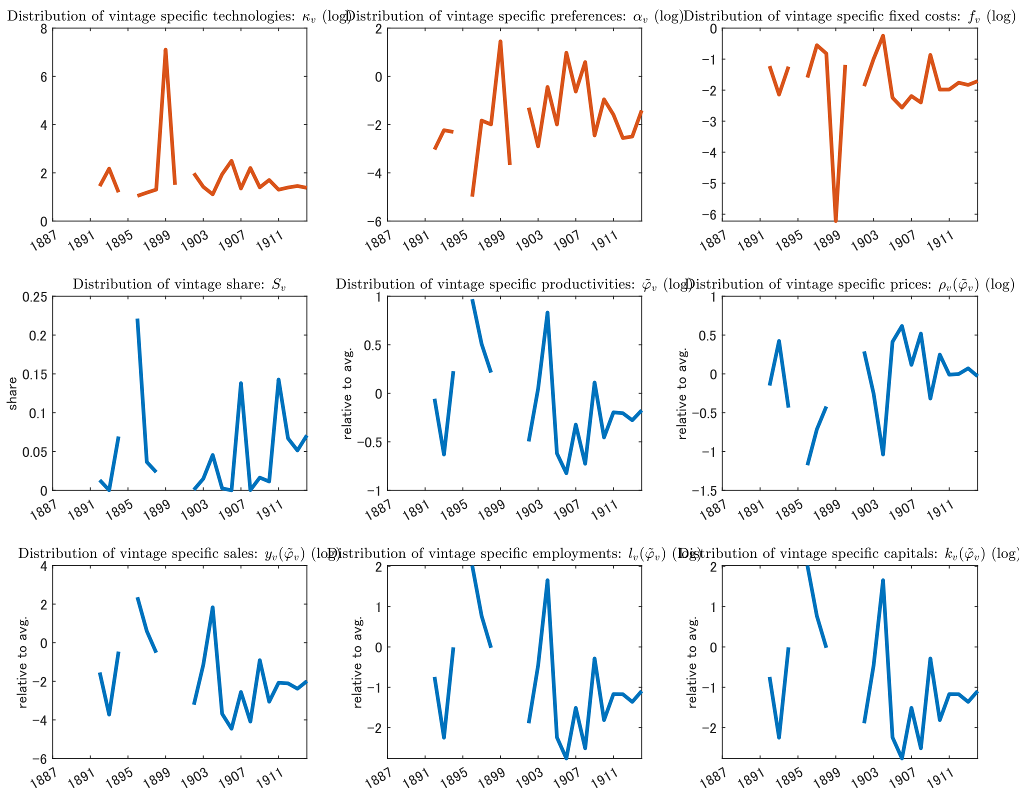


(b) Simulation of generation-specific parameters



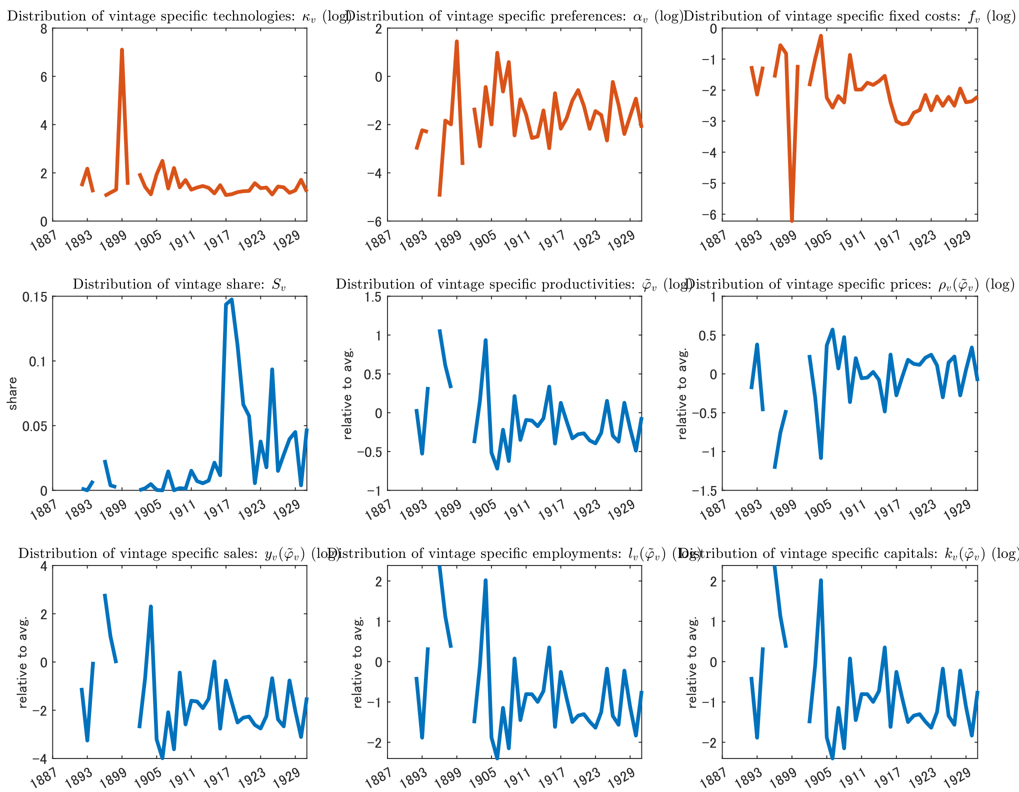
The first and the second panel in the first row show the distribution of generation firms and the distribution of their sales in the 2013 survey, respectively. The first and the second panel in the second row show the simulated distribution of generation-specific technologies κ_v and fixed costs f_v at the end of periods, respectively. The first and the second panel in the third row show the simulated distribution of generation firm productivity $\tilde{\varphi}_{v,t}$ and employment $l_{v,t}(\tilde{\varphi}_{v,t})$ at the end of the periods, respectively.

Figure 15: Snapshot: 1914



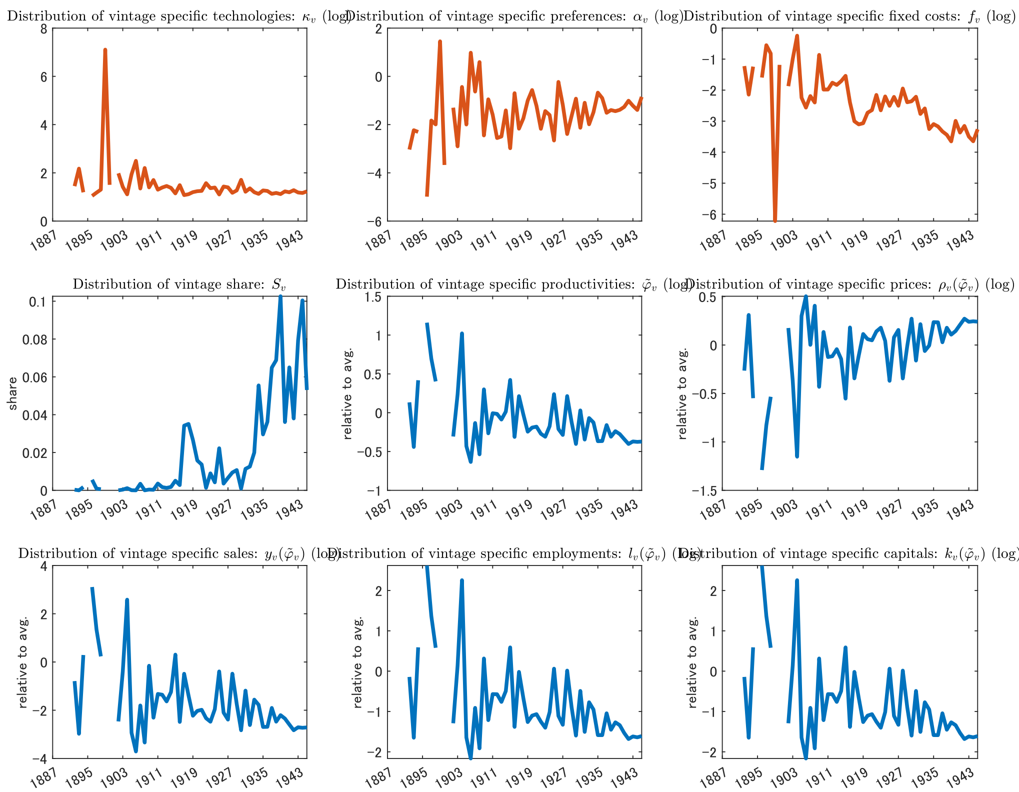
TThe first panel in the first row show the estimated distribution of generation-specific technologies κ_v, preferences α_v and their fixed costs f_v in the economic landscape of 1914. Other panels show the simulated distribution of the simulated dynamics of the number of producers $S_{v,t}$, real prices $\rho_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, sales $y_{v,t}\left(\tilde{\varphi}_{v,t}\right)$, productivites $\tilde{\varphi}_{v,t}$, employments $l_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ and capitals $k_{v,t}\left(\tilde{\varphi}_{v,t}\right)$ in the economic landscape of 1914.

Figure 16: Snapshot: 1931



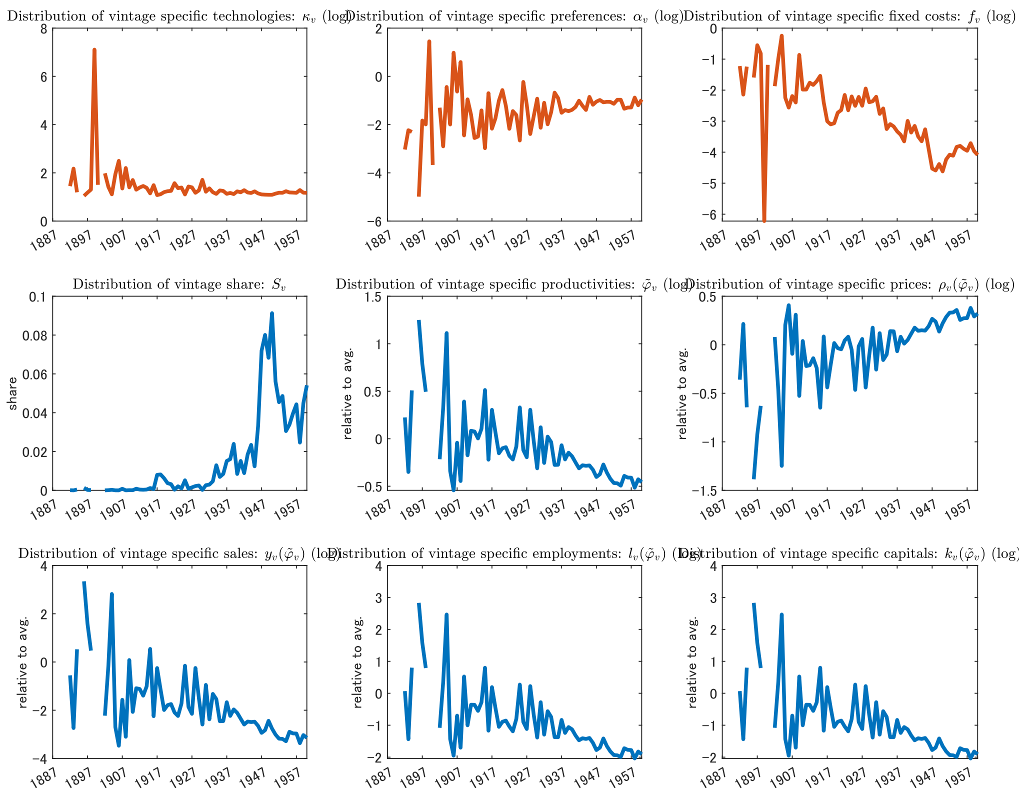
The first panel in the first row show the estimated distribution of generation-specific technologies κ_v, preferences α_v and their fixed costs f_v in the economic landscape of 1931. Other panels show the simulated distribution of the simulated dynamics of the number of producers $S_{v,t}$, real prices $\rho_{v,t}(\tilde{\varphi}_{v,t})$, sales $y_{v,t}(\tilde{\varphi}_{v,t})$, productivites $\tilde{\varphi}_{v,t}$, employments $l_{v,t}(\tilde{\varphi}_{v,t})$ and capitals $k_{v,t}(\tilde{\varphi}_{v,t})$ in the economic landscape of 1931.

Figure 17: Snapshot: 1945



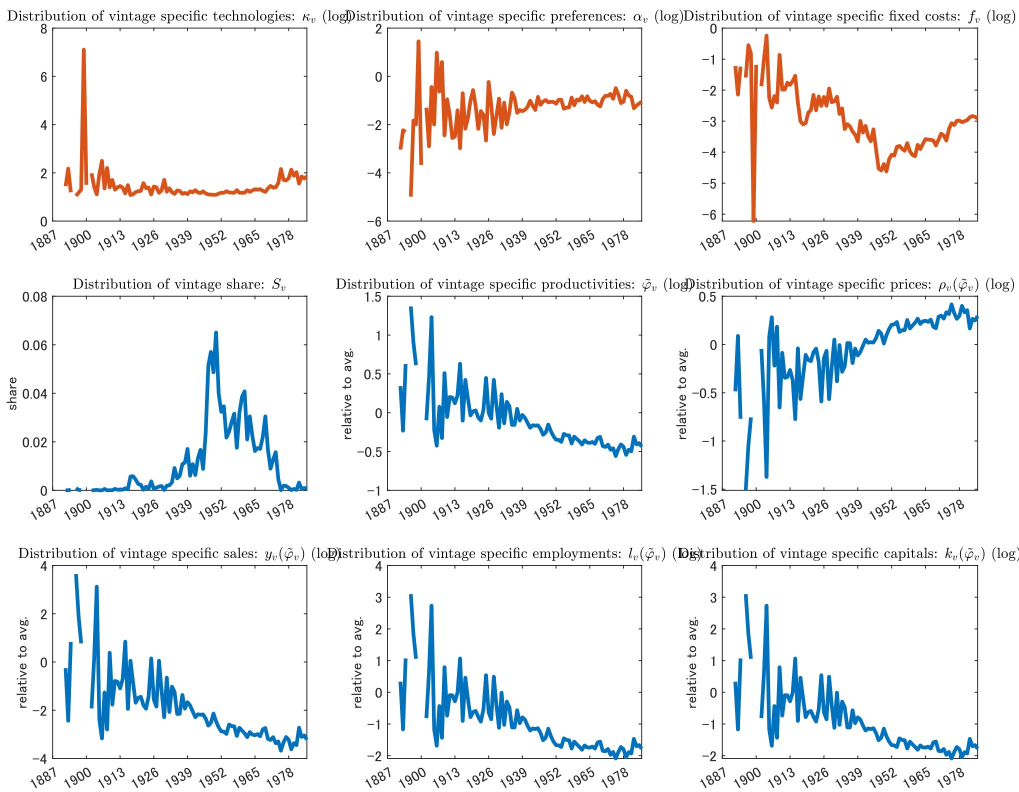
The first panel in the first row show the estimated distribution of generation-specific technologies κ_v, preferences α_v and their fixed costs f_v in the economic landscape of 1945. Other panels show the simulated distribution of the simulated dynamics of the number of producers $S_{v,t}$, real prices $\rho_{v,t}(\tilde{\varphi}_{v,t})$, sales $y_{v,t}(\tilde{\varphi}_{v,t})$, productivites $\tilde{\varphi}_{v,t}$, employments $l_{v,t}(\tilde{\varphi}_{v,t})$ and capitals $k_{v,t}(\tilde{\varphi}_{v,t})$ in the economic landscape of 1945.

Figure 18: Snapshot: 1960



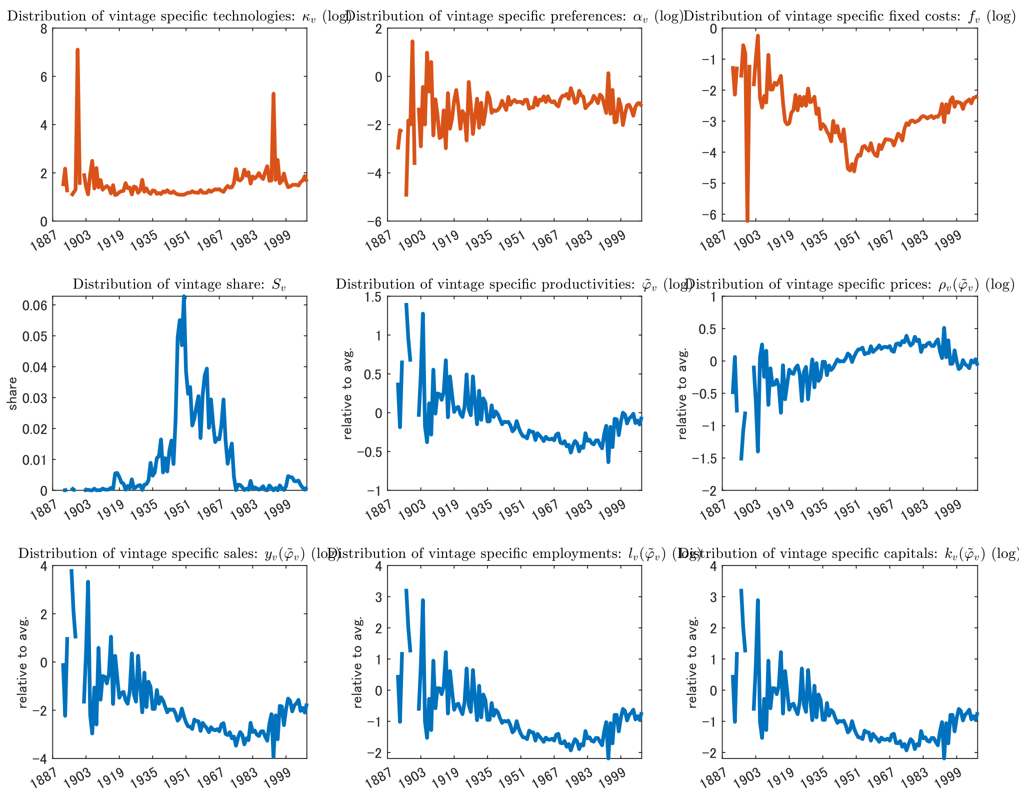
The first panel in the first row show the estimated distribution of generation-specific technologies κ_v, preferences α_v and their fixed costs f_v in the economic landscape of 1960. Other panels show the simulated distribution of the simulated dynamics of the number of producers $S_{v,t}$, real prices $\rho_{v,t}(\tilde{\varphi}_{v,t})$, sales $y_{v,t}(\tilde{\varphi}_{v,t})$, productivites $\tilde{\varphi}_{v,t}$, employments $l_{v,t}(\tilde{\varphi}_{v,t})$ and capitals $k_{v,t}(\tilde{\varphi}_{v,t})$ in the economic landscape of 1960.

Figure 19: Snapshot: 1985



The first panel in the first row show the estimated distribution of generation-specific technologies κ_v, preferences α_v and their fixed costs f_v in the economic landscape of 1985. Other panels show the simulated distribution of the simulated dynamics of the number of producers $S_{v,t}$, real prices $\rho_{v,t}(\tilde{\varphi}_{v,t})$, sales $y_{v,t}(\tilde{\varphi}_{v,t})$, productivites $\tilde{\varphi}_{v,t}$, employments $l_{v,t}(\tilde{\varphi}_{v,t})$ and capitals $k_{v,t}(\tilde{\varphi}_{v,t})$ in the economic landscape of 1985.

Figure 20: Snapshot: 2009



The first panel in the first row show the estimated distribution of generation-specific technologies κ_v, preferences α_v and their fixed costs f_v in the economic landscape of 2009. Other panels show the simulated distribution of the simulated dynamics of the number of producers $S_{v,t}$, real prices $\rho_{v,t}(\tilde{\varphi}_{v,t})$, sales $y_{v,t}(\tilde{\varphi}_{v,t})$, productivites $\tilde{\varphi}_{v,t}$, employments $l_{v,t}(\tilde{\varphi}_{v,t})$ and capitals $k_{v,t}(\tilde{\varphi}_{v,t})$ in the economic landscape of 2009.