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Money and cooperation in small communities

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Abstract

In this note, I investigate the circulation of money in small communities. I build a two-player repeated gift-giving game and then show that players can sustain cooperation by using money. An efficient outcome is obtained when players are able to hold multiple units of currency.

Keywords: primitive money, repeated game.

JEL codes: C73, E42, N10.

1 Introduction

In this note, I analyze the circulation of money in small communities. Since Kiyotaki and Wright (1989), the literature of monetary search theory has emphasized that money is essential under decentralized trade and lack of double coincidence of wants. Kocherlakota (1998) and Araujo (2004) show that the existence of money also needs a lack of public information and contagion process of deviations. These conditions are satisfied when the number of members in a society is large. Hence, theories of money as a medium of exchange usually assume large population economies.

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On the other hand, there are some evidence that money was used in small communities. Some primitive monies were traded in local villages such as stone money, sea shells, crops, etc.[1] Douglas (1958) reports an example from anthropological fieldwork in Lele in Congo. Lele community was small and there was no market; however, there is evidence of the circulation of primitive money. This money is actually a cloth called Raffia. Douglas (1958) describes that this cloth currency was not mainly used as a medium of exchange, but payments for social statuses, e.g., marriage gifts, fines, fees to enter a cult, and so forth. The cloth currency had a role of keeping incentives of service to the community. In this sense, money was used as a medium of cooperation in this small society.

The present paper analyzes this type of cooperative role of money in a two-player repeated gift-giving game. Then I show that money is a simple and efficient device for sustaining cooperation. The game is an example of a small society like Lele. A player randomly receives a chance of gift-giving which is not observed by the other player. Players have an incentive to hide the chance and do not give a gift. I first show that, if there is one unit of fiat money, there exists an equilibrium that a player gives a gift when the opponent pays the money. It is a simple and intuitive strategy for small societies. In this equilibrium, the money is a device of record-keeping as noted in Kocherlakota (1998). Having money means a player cooperated in the past. Next, I consider the case that players are able to hold multiple units of money. Then, the first-best outcome can be obtained when the players' time preference converges to zero. In this sense, this simple strategy has a desired property about efficiency.

The game is classified as a repeated game with two-sided hidden states. This aspect is similar to Athey and Bagwell (2001) in the context of collusion between oligopoly firms. The analysis of holding multiple units of money is related to some search models such as Green and Zhou (1998), Zhou (1999), Camera and Corbae (1999), and Kamiya and Shimizu (2007).

[1] Goldberg (2005) criticizes that economists have considered some primitive currencies as fiat money incorrectly.

2 One unit of money

This section examines a simple case in which there is one unit of money. Suppose a two-player repeated game in continuous time.[2] Each player stochastically gets a chance of giving one unit of gift. The arrival of the chance follows an independent Poisson process with parameter σ. The chance is a private information which is not observed by the other player. If a player gives a gift, it costs c and the opponent acquires a payoff u. Simply assume $u > c$. Each player discounts the payoff with a time preference rate r.

One of the players has a unit of money. Now I consider a strategy that, if a player who does not have the money gives a gift, the owner of the money pays it in exchange. Let V_1 denotes the expected discounted payoff of the money holder and V_0 of a player who does not have. On the equilibrium path, these satisfy

$$rV_0 = \sigma(-c + V_1 - V_0), \tag{1}$$

$$rV_1 = \sigma(u + V_0 - V_1). \tag{2}$$

The incentive constraint for gift-giving is

$$V_1 - V_0 - c \geq 0. \tag{3}$$

Using this strategy, the economy is classified as a Markov game; that is, all history of the game is summarized only as money holding. Hence, the incentive constraint avoids the one-shot deviation condition, which is sufficient to hold an equilibrium. By (1) and (2), Inequality (3) is rewritten as

$$u \geq \left(1 + \frac{r}{\sigma}\right) c. \tag{4}$$

The condition is satisfied when the players are sufficiently patient. In this equilibrium, the

[2] I do not use a standard discrete time repeated game just for simplification.

ex-ante payoff is

$$\frac{r(V_0 + V_1)}{2} = \frac{\sigma(u - c)}{2}, \tag{5}$$

which is less than the efficient outcome $\sigma(u - c)$. This is because the money holder does not give the gift when this player gets a chance. To avoid this inefficiency, I allow players to hold multiple units of money in the next section. It decreases the probability that one of the two players holds no money.

3 Multiple units of money

In the same environment, now I suppose that the sum of money holdings is N units. Let V_i denotes the expected discounted payoff when a player have i units of money. I consider the same strategy that a player pays one unit of money if this player receives a gift. The equilibrium values satisfy

$$rV_0 = \sigma(-c + V_1 - V_0) \tag{6}$$

$$rV_n = \sigma(-c + V_{n+1} - V_n) + \sigma(u + V_{n-1} - V_n) \quad \text{for } n = 1, \ldots, N - 1 \tag{7}$$

$$rV_N = \sigma(u + V_{N-1} - V_N) \tag{8}$$

and the conditions for the equilibrium become

$$\forall i \in \{0, 1, \ldots, N - 1\}, \quad V_{i+1} - V_i - c \geq 0. \tag{9}$$

Proposition 1. *For arbitrary $N \in \mathbb{N}$, there exists $r_N > 0$ such that the equilibrium exists for all r where $0 < r \leq r_N$.*

Given this strategy, a player gives a gift and receives a unit of money to avoid a future possibility that the money holding runs out. This is a risk that the player cannot receive a gift even if the other gets a chance. When N is large, the probability of such cases becomes low, and the welfare is high. On the other hand, since the risk is low, the incentive constraints may be violated. This is because money will be exhausted only in the far future. To sustain the incentive of gift-giving, the time preference r needs to be small enough. This proposition states that in the limit case, $r \to 0$, the incentive constraints hold for all N. In a case $N \to \infty$, the inefficiency of holding no money is eliminated; hence the first best outcome is obtained.

Proof. First, I solve the difference equation of V_n. By (7),

$$\sigma V_{n+2} - (r + 3\sigma)V_{n+1} + (r + 3\sigma)V_n - \sigma V_{n-1} = 0,$$

which is a third-order difference equation. The characteristic equation is

$$\sigma\alpha^3 - (r + 3\sigma)\alpha^2 + (r + 3\sigma)\alpha - \sigma = (\alpha - 1)(\sigma\alpha^2 - (r + 2\sigma)\alpha + \sigma) = 0. \tag{10}$$

I define $k \equiv r/\sigma$. The solutions of (10) are

$$1, \quad 1 + \frac{k}{2} + \sqrt{\frac{k^2}{4} + k} \text{ and } 1 + \frac{k}{2} - \sqrt{\frac{k^2}{4} + k}.$$

Let A and B denote the second and third solutions. Then $A > 1$, $0 < B < 1$, $A + B > 1$. Let C_1, C_2, C_3 be constant coefficients, then the general solution of (10) can be written as

$$V_n = C_1 + C_2 A^n + C_3 B^n. \tag{11}$$

By (7),

$$\sigma(C_2A^{n+2}+C_3B^{n+2})-(r+2\sigma)(C_2A^{n+1}+C_3B^{n+1})+\sigma(C_2A^n+C_3B^n) = rC_1-\sigma(u-c). \tag{12}$$

Since A and B are solution of (10) except 1, the left hand side of (12) is 0. Then

$$C_1 = \frac{\sigma}{r}(u - c).$$

By substituting (11) into (6) and (8),

$$C_2[(1+k)A^N - A^{N-1}] - C_3[B^{N-1} - (1+k)B^N] = c \tag{13}$$

$$C_2[A - (1+k)] - C_3[(1+k) - B] = u \tag{14}$$

I define $D \equiv (1+k)A^N - A^{N-1}$, $E \equiv B^{N-1} - (1+k)B^N$, $F \equiv A - (1+k)$ and $G \equiv (1+k) - B$. It can be easily shown that $D > 0$, $E > 0$, $F > 0$, $G > 0$. By (13) and (14),

$$C_2 = \frac{Eu - Gc}{EF - DG}, \quad C_3 = \frac{Du - Fc}{EF - DG}. \tag{15}$$

It is always holds $EF - DG < 0$ and $D > F$. Assume that $G/E \to 1$ as $r \to 0$, which will be proven later. If r is sufficiently close to 0, $C_2 < 0$ and $C_3 < 0$. By (11),

$$\begin{aligned} c &= C_2[(1+k)A^N - A^{N-1}] - C_3[B^{N-1} - (1+k)B^N] \\ &< C_2[(1+k)A^n - A^{n-1}] - C_3[B^{n-1} - (1+k)B^n] \\ &< C_2[A^n - A^{n-1}] - C_3[B^{n-1} - B^n] = V_n - V_{n-1} \end{aligned}$$

satisfy for all $n \leq N - 1$. The incentive constraint (9) is satisfied.

Finally, I show $G/E \to 1$ as $r \to 0$. Clearly $k \to 0$ as $r \to 0$.

$$\frac{G}{E} = \left[\frac{1}{\left(1 + k/2 - \sqrt{(k^2/4) + k}\right)^{N-1}}\right] \cdot \left[\frac{(1+k) - \left(1 + k/2 - \sqrt{(k^2/4) + k}\right)}{1 - (1+k)\left(1 + k/2 - \sqrt{(k^2/4) + k}\right)}\right]$$

The first bracket converges to 1 and, by applying L'Hopital's rule, it can be shown that the second bracket also converges to 1. By $1+k/2-\sqrt{(k^2/4)+k} = (1+k/2)-\sqrt{(1+k/2)^2-1} < 1$, in order to put G/E close to 1, r must be nearer to 0 as N becomes larger. $\square$

4 Conclusion

This note shows that a simple strategy using money sustains cooperation in a two-player repeated gift-giving game. I also show that, when the time preference converges to 0, the efficient equilibrium can be obtained. This result justifies the circulation of primitive money in small communities.

One possible extension is more than three player cases. It may be technically challenging because the state variable will be changed with the distribution of money holdings. However, theoretical results about the efficiency of monetary trade in this general case are necessary to investigate the role of money in communities.

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