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Abstract

This study examines the two-decade-long low interest rate environment in Japan using the Nelson-Siegel yield curve framework emphasizing the role of decay factor. We find that the decay factor has declined particularly after the global financial crisis, pushing down the entire yield curve as well as the conditional variance of bond yield in Japan. The decay factor was very low when BOJ's yield curve control started in 2016 and remained low with small fluctuations since. Decay factor shocks can be interpreted as long-dated term premium shocks, and these shocks tend to decrease with BOJ's bond purchases, controlling for other possible factors that affect term premia such as business cycles and economic uncertainty. (*JEL*: E58, E52, C32)

KEYWORDS: decay factor, Nelson Siegel, term premium, yield curve control, Japan, nonlinear state space model

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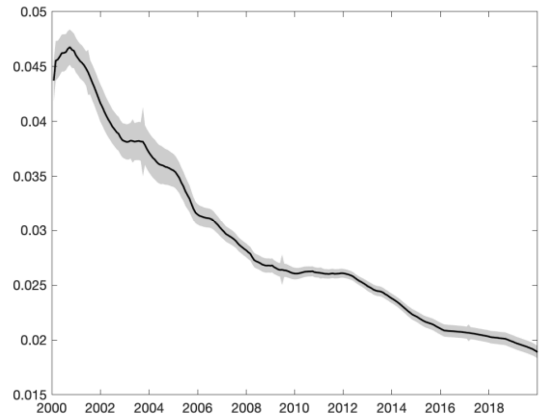
1. Introduction

Low interest rate environments are becoming increasingly common worldwide. In particular, Japan has already been in a zero-rate environment for more than two decades, as depicted in Appendix Figure A1. During these years, the short-term interest rate remained near zero while the ten-year Japanese government bond (JGB) yield fell from about 2 to 0 percent. Further, long-term bond yield remained at very low levels with very small fluctuations since the Bank of Japan (BOJ) has committed to purchase Japanese government bonds to achieve its targets for long- and short-term interest rates under its yield-curve control since September 2016. Is there a novel aspect of yield-curve characteristics that should be examined?

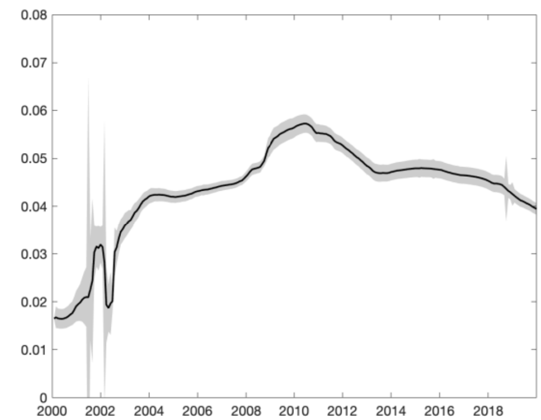
This study analyzes the role of the decay factor in accounting for the Japanese low interest rate environment using a dynamic Nelson-Siegel (NS henceforth, 1987) yield curve model. Figure 1 presents empirical evidence of the decay factor based on 10-year rolling regressions using the dynamic model applied by Diebold and Li (2006). The decay factor has been on a downward trend in Japan for the past two decades (Figure 1a). By comparison, there is minimal evidence of trend decline in the United States (Figure 1b) and only a slight decline has been observed in Germany (which is the country that generally evidences the lowest interest rates in the Euro Area, Figure 1c). Given a clear decline in the decay factor in Japan, this paper treats the decay factor as an additional time-varying yield-curve factor following Koopman, Mallee, and Van der Wel (2010).[1]

[1] Although Diebold and Li (2006) assume a constant decay factor in their estimation, they did not eliminate the possibility that the decay factor varies with time. In fact, these authors present the NS model with the time subscript set to the decay factor.

a) Japan



b) United States



c) Germany

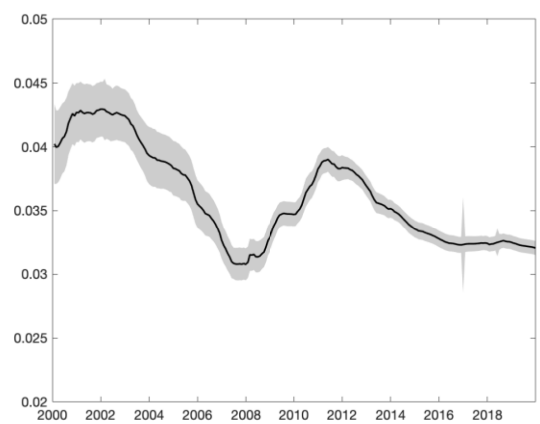


Figure 1 Rolling regression on the decay factor. This figure plots the decay factor estimated by 10-year rolling regressions of the dynamic NS model applied by Diebold and Li (2006). The decay factor is constant within each sample period. The horizontal axis indicates the last month of the data used for each regression. For example, the January 2010 value indicates the estimates obtained using the sample period of February 2000 to January 2010. The gray areas correspond to one-standard error confidence intervals. Figures 1a, 1b, and 1c show the estimated decay factor across time for Japan, the United States, and Germany, respectively.

In the NS model, when the decay factor decreases, the shape of the slope loading (Appendix Figure A2a) flattens and becomes more similar to that of the level loading. Also, the maximum of the curvature loading increases (Appendix Figure A2b). We show that a decay-factor decline flattens Japanese yield curves via the slope and curvature terms in the model because the slope and curvature factors are estimated to be persistently negative in Japan.

This study contributes several new findings. First, the study shows that the declining decay factor has pushed down Japanese long-term interest rates as well as their conditional volatility in Japan, particular after the global financial crisis and the early years of quantitative and qualitative easing policy (QQE). The study finds that the decay factor was particularly low when BOJ's yield curve control started in 2016 and remained low with very small fluctuations since, pushing down the entire yield curve. Second, this study computes the nonlinear impulse responses of model variables to a negative decay-factor shock and finds that the shock reduces long-dated term premiums, rather than the corresponding expected short rate. Thus, a decay-factor shock can be interpreted as a term premium shock. Lastly, this study conducts simple regressions showing that the decay factor shocks are negatively correlated with central bank's bond purchases, more specifically the offered amount of BOJ's JGB purchases. This empirical evidence seems to be consistent with preferred habitat theory (e.g., Vayanos and Vila, 2021) because a negative decay factor shock compresses term premium and flattens yield curve. The regressions also control for other economic variables given that term premia are likely to be related with business cycles (e.g., Cochrane and Piazzesi, 2005, Bauer, Rudebusch, and Wu, 2014, Cochrane 2017), inflation and inflation expectations (e.g. Wright, 2011), and uncertainty (e.g., Hansen, McMahon, and Tong, 2019).

The paper proceeds as follows. Section 2 discusses the role of the decay factor in the NS model. Section 3 explains the data and estimation strategy and presents the estimated results. Section 4 conducts non-linear impulse response analyses. Section 5 discusses why the decay factor has declined in Japan. Section 6 presents the conclusion.

2. The Decay Factor in the NS Model

This section reviews the standard NS model and discusses how changes in the decay factor affect the bond yield. The standard NS model is given by

$$y(\tau) = \beta_0 + \beta_1 g_1(\lambda) + \beta_2 g_2(\lambda), \tag{1}$$

where τ is the maturity date and λ is the decay factor in period *t*. The factor loading of the level factor is one, and the factor loadings of the slope and curvature factors are given by $g_1(\lambda) = \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)$ and $g_2(\lambda) = \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$, respectively. Diebold and Li (2006) interpreted β_0, β_1, and β_2 as the standard yield-curve factors (the level, slope, and curvature factors, respectively). The short rate is the sum of the level and slope factors. Following Koopman et al. (2010), λ is treated as an additional yield curve factor.[2]

The factor loadings of the slope and curvature factors depend on the values of the yield-curve factors. The left panels of Appendix Figures A2 show the slope and curvature loadings with different values of the decay factor. The right panels of Appendix Figures A2 show the slope and curvature factor terms in the yield equation (i.e., the second and third terms in eq. (1)) with

[2] Choi and Kang (2020) propose a generalized NS framework that nest time-varying factor loadings and time-varying variance-covariance of the factor shocks.

different values of the decay factor, setting the values of slope and curvature factors at their averages. A decay-factor decline pushes down the yield curve when these factors take negative values. In the next section, we show that both slope and curvature factors are estimated to be negative throughout the investigated period in Japan.

The remaining sections examine the dynamic versions of NS models where the yield-curve factors depend on their lags. Both cases with a fixed decay factor and with a time varying decay factor are examined. For notational convenience, the NS model with a fixed decay factor will be labeled "NS-LS," and the NS model with a time varying decay factor will be labeled "NS-NL." The vector of the standard yield-curve factors, $X_{\mathrm{t}} = [\beta_{0,t};\ \beta_{1,t};\ \beta_{2,t}]$ for NS-LS and $X_{\mathrm{t}} = [\beta_{0,t};\ \beta_{1,t};\ \beta_{2,t}; \lambda_t]$ for NS-NL follows the VAR(1)

$$X_t = \mu + \rho X_{t-1} + \Sigma v_t, \qquad v_t \sim N(0, I), \tag{2}$$

where the vector of v_t represents the factor shocks and Σ is assumed to be the lower triangular per the normalizing condition and existing studies, such as Joslin, Priebsch, and Singleton (2014) and Diebold, Rudebush, and Aruoba (2006).

The n year term premium TP_t^n is defined as the difference between the n year bond yield and the expected short rate. This rate is calculated as

$$\left(\frac{1}{12n}\right) E_t\left[\sum_{j=0}^{12n} r_{t+j}\right] = \left(\frac{1}{n}\right) E_t\left[\sum_{j=0}^{n} \left(\beta_{0,t+j} + \beta_{1,t+j}\right)\right], \tag{3}$$

where r_t denotes the short rate in month t.

Allowing the decay factor to change over time has important implications for the conditional variance of bond yields. Despite the time invariance of Σ, the conditional variance of the τ-period bond yield (i.e., $V_t(y_{t+1}(\tau))$) becomes time varying. This feature of the model helps explain the decline in the volatility of the long-term interest rates in Japan.

3. Estimation and Estimated Results

3.1. Data

As presented in Appendix Figure A1, the data used are Bloomberg's zero yield-curve data (end-of-period) for Japan from April 1989 to December 2019. The bond yields of 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, and 360 months were used as the maturities for estimation. Appendix Table A1 provides summary statistics on the bond yields and maturities. The data used in the rolling regressions in Figure 1 are the US bond yield data obtained from the Gurkaynak, Sack, and Wright (2006) database, and the German bond yield data obtained from the Bundesbank.[3]

In Section 5, quarterly regressions are conducted to examine the relationship between the identified decay-factor shocks and the following variables: (i) business cycles, (ii) economic uncertainty, (iii) inflation, and (iv) the BOJ policy on JGB purchases. The business-cycle variable is measured by output gaps obtained from the Cabinet Office of Japan (CAO).[4] The uncertainty

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[3] The data and related documents are available at:
https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html.
https://www.bundesbank.de/en/statistics/money-and-capital-markets/interest-rates-and-yields/daily-term-structure-on-listed-federal-securities-651570.

[4] The CAO is the Japanese equivalent of the U.S. Bureau of Economic Analysis. The CAO estimates potential GDP based on the Cobb–Douglas production function with the Hodrick–Prescott filtered Solow residual. The CAO's data are available at: https://www5.cao.go.jp/keizai3/getsurei/getsurei-index.html.

variable is measured by S&P/JPX JGB VIX Index (we denote this index as "VIX" henceforth) obtained from Japan Exchange Group,[5] and is available only from 2008. Inflation is measured by Consumer Price Index (PCI) inflation (the change from the same quarter in the previous year) obtained from Ministry of Internal Affairs and Communications. Two measures are used to measure BOJ's government bond purchases: the offered amount of BOJ purchases on JGB with remaining maturity greater than 1 year and less than equal to 10 years or over 10 years (in trillion yen) and maturity-weighted debt held by BOJ (in the fraction of GDP). The offered amount of BOJ purchases is constructed using publicly available BOJ offer dates and amount of Japanese government bond purchases.[6] The maturity-weighted debt held by BOJ is computed using the JGB maturity structure database of Koeda and Kimura (2021).[7] The maturity-weighted JGB holdings are then divided by the nominal GDP. These series are shown in appendix figures (Appendix Figures A5-A9).

3.2. Estimation Approach

The dynamic NS model can be expressed in a state space representation with the measurement equation given by Eq. (1) and the transition equation given by Eq. (2). The mapping from X_t to a model-implied variable from the term structure model y_t, $y_t = g(X_t;\ \Theta)$ is linear for the NS-LS model and is non-linear for the NS-NL model, and Θ is the model parameter.

[5] S&P/JPX JGB VIX Index measures the implied volatility of Japanese government bonds using options on JGB futures. For details, see methodology document available at https://www.jpx.co.jp/english/markets/derivatives/sp-jpx-jgb-vix.

[6] BOJ operation information is available at: https://www.boj.or.jp/statistics/boj/fm/ope/index.htm/.

[7] From this database, the BOJ's JGB holdings in the face value are computed by aggregating the coupon and principal payments of individual bonds held by the BOJ at each quarter end. Denote s(n) as the BOJ's JGB holdings in the face value with the remaining maturity over n -1 years and less than or equal to n years. The maturity weighted debt held by the BOJ is equal to [s(1), s(2), …, s(40)]*[1 2 …40]'.

To estimate the NS-LS and NS-NL models, a one-step estimation approach is applied. The NS-LS model is estimated via maximum likelihood using Diebold and Li's (2006) state-space representation. The NS-NL model is estimated via maximum likelihood using the state space model with an extended Kalman filter as described in Appendix A.[8] The initial parameter values for the one-step estimation are chosen based on the two step-estimation of Diebold and Li (2006).[9] The sample period for the benchmark estimation starts in November 2000 and ends in December 2019.[10] The standard errors are obtained by numerically computing the Hessian matrices.

3.3. Estimated Results

Table 1 reports the estimated coefficients for the factor dynamics under NS-LS and NS-NL. The eigenvalues of ρ are positive and less than 1 under either NS-LS or NS-NL. NS-NL has a better model fit to the data particularly under BOJ's QQE since 2013 (Appendix Figure A4). The decay factor process is persistent with the coefficient for the lagged decay-factor equals to 0.95 and the coefficients for other factors are statistically insignificant. Figure 2 shows the estimated decay factor dynamics under NS-NL. The difference between the decay factors in Figures 1a and 2 is that the former is estimated via the 10-year rolling regressions of the dynamic NS model applied by Diebold and Li (2006), whereas the latter is estimated via the one-step estimation described in Section 3.2. The decay factor dropped during the peak of the global financial crisis possibly

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[8] To keep the tractability of the state space model, this study applies the NS framework, rather than the Nelson-Siegel-Svensson (NSS) framework, which includes additional curvature and decay factors. The European Central Bank publishes NSS parameter estimates daily and its estimated yield-curve factors are quite volatile. Such volatile yield-curve factors may be better addressed with a term structure model with regime switches and stochastic volatility (e.g., Ang and Bekaert, 2002).

[9] The two-step estimation proceeds as follows. In the first step, Eq. (1) is estimated by OLS or nonlinear OLS for each month. In the second-step estimation, the VAR of X is estimated.

[10] For the VAR estimation in the second step estimation, November 2000 is chosen as the initial month, based on the multiple breakpoint test results of Bai and Perron (1998) for NS-NL, where the most recent break occurs in the slope factor equation in November 2000.

reflecting a fall in term premium associated with flight to safety. It then has been on a clear declining trend from 2010, and after reaching a very low level (less than 0.015), it experienced only small fluctuations in the last several years of the sample period under BOJ's yield curve control.

Table 1 Estimated factor dynamics coefficients

	NS-LS			NS-NL			
μ	0.317	-0.306	-0.835	0.133	-0.123	-0.102	0.0002
	(0.099)	(0.115)	(0.184)	(0.123)	(0.116)	(0.168)	(0.0007)
ρ	1.902	0.973	0.086	1.095	0.164	-0.004	0.000
	(0.051)	(0.051)	(0.047)	(0.208)	(0.194)	(0.01)	(0.078)
	-1.026	-0.115	-0.070	-0.129	0.794	0.016	-0.001
	(0.098)	(0.104)	(0.051)	(0.221)	(0.209)	(0.011)	(0.037)
	-0.218	-0.122	0.578	-0.107	-0.146	0.982	-0.001
	(0.075)	(0.062)	(0.086)	(0.307)	(0.383)	(0.091)	(0.026)
				-0.001	-0.002	0.000	0.954
				(0.003)	(0.005)	(0.001)	(0.062)
Σ	0.245	0	0	0.214	0	0	0
	(0.014)			(0.026)			
	-0.255	0.040	0	-0.213	0.034	0	0
	(0.018)	(0.004)		(0.030)	(0.003)		
	-0.312	-0.162	0.455	-0.138	-0.057	0.231	0
	(0.071)	(0.045)	(0.065)	(0.043)	(0.041)	(0.049)	0
				-0.001	0.000	0.000	0.001
				(0.001)	(0.000)	(0.000)	(0.001)
σ_η	0.065			0.054			
	(0.001)			(0.001)			
logL	3703.875			4412.481			

Note: This table presents the estimated coefficients of yield-factor dynamics for the NS-LS and NS-NL. Standard errors are in parentheses.

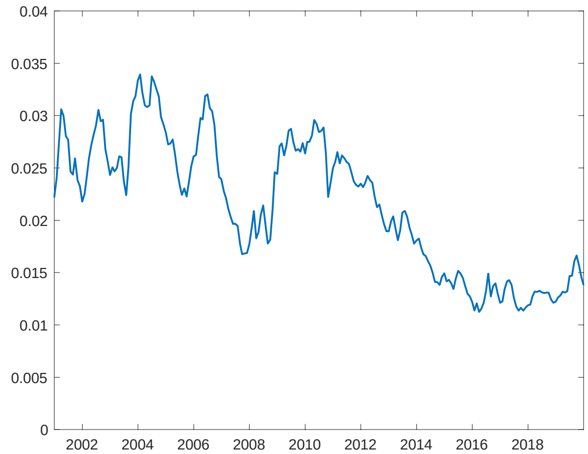


Figure 2 The estimated decay factor at a monthly frequency. This figure plots the decay factor at a monthly frequency. It is estimated using NS-NL.

Figure 3 shows the simulated conditional standard deviation (the square root of conditional variance) of bond yields with different maturities. Specifically, it simulates *t+n* period ahead bond yield for 1000 time given the values of yield curve factors in period t and computes the average of simulated standard deviations. Decay factor declines account for the decline in the long-dated conditional volatility, while the short-dated conditional volatility are close to zero with little movements.

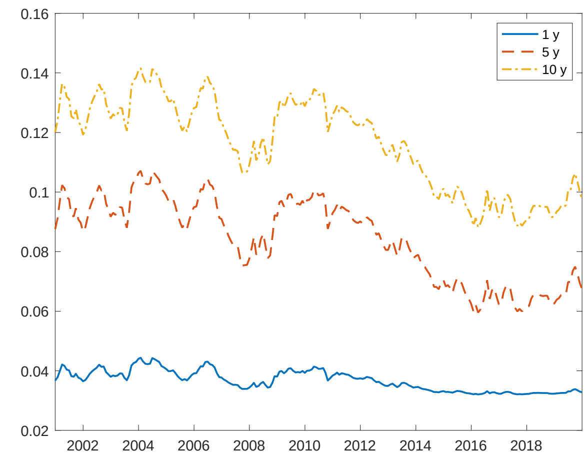


Figure 3 Conditional standard deviation of bond yields. This figure shows the estimated conditional standard deviation of bond yields under NS-NL. The solid, dashed, and dash-dot lines indicate that of 1-year, 5-year, and 10-year bond yields, respectively.

Figure 4 shows the estimated factors under NS-NL.[11] The slope and curvature factors are estimated to be negative throughout the sample period, thus these factors have consistently had a negative effect on bond yields. The level factor was positive but dropped when the negative interest rate policy was introduced in early 2016. The estimated NS-NL level and slope factors mirror each other with the correlation of -0.96. This is consistent with the zero rate on the short end of the yield curve because the prefixed NS factor loadings imply that the sum of the level and slope factors equals the short rate in the NS model.

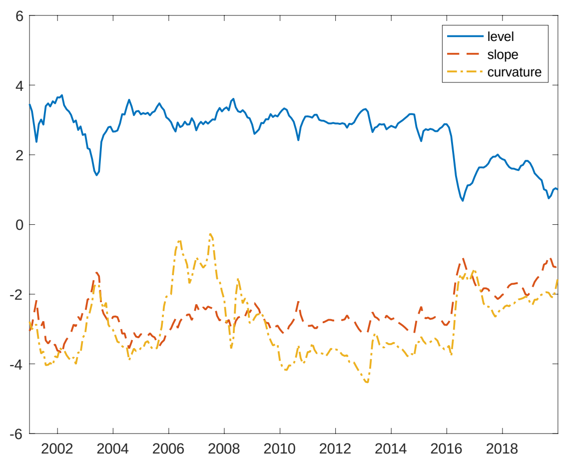


Figure 4 Estimated factors. This figure plots the estimated level-, slope-, and curvature-factors at a monthly frequency based on the NS-NL model.

Figure 5 shows the implied 10-year bond yield shutting down the decay factor shocks. The decay factor shocks are identified with the VAR residuals and the recursive structure of Σ. The counterfactual shows that the difference between the fitted and counterfactual yields. The difference widened under BOJ's quantitative and qualitative easing policy which initiated in April 2013. This implied yield (the counterfactual) would have been consistently higher than the estimated yield (about 60 basis points on average since 2016).

[11] Appendix Figure A3 shows the estimated factors under NS-LS.

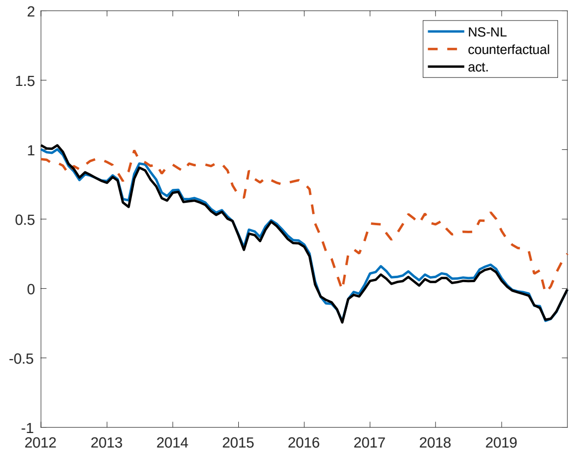


Fig. 5 Model-implied 10-year bond yield shutting down the decay factor shocks. This figure compares the fitted 10-year bond yield under the NS-NL with a counterfactual. The blue line indicates the fitted values. The red line (counterfactual) indicates the implied bond yield shutting down the decay factor shocks. The black line indicates the actual 10-year Japanese government bond yield. In annualized rate in percent.

4. Nonlinear Impulse Response Analyses

The impulse response of y_t to a yield-factor shock can be defined by the difference between the following conditional expectations:

$$E_t[y_{t+k}|X_t + \nu_t; \Theta] - E_t[y_{t+k}|X_t; \Theta]. \tag{4}$$

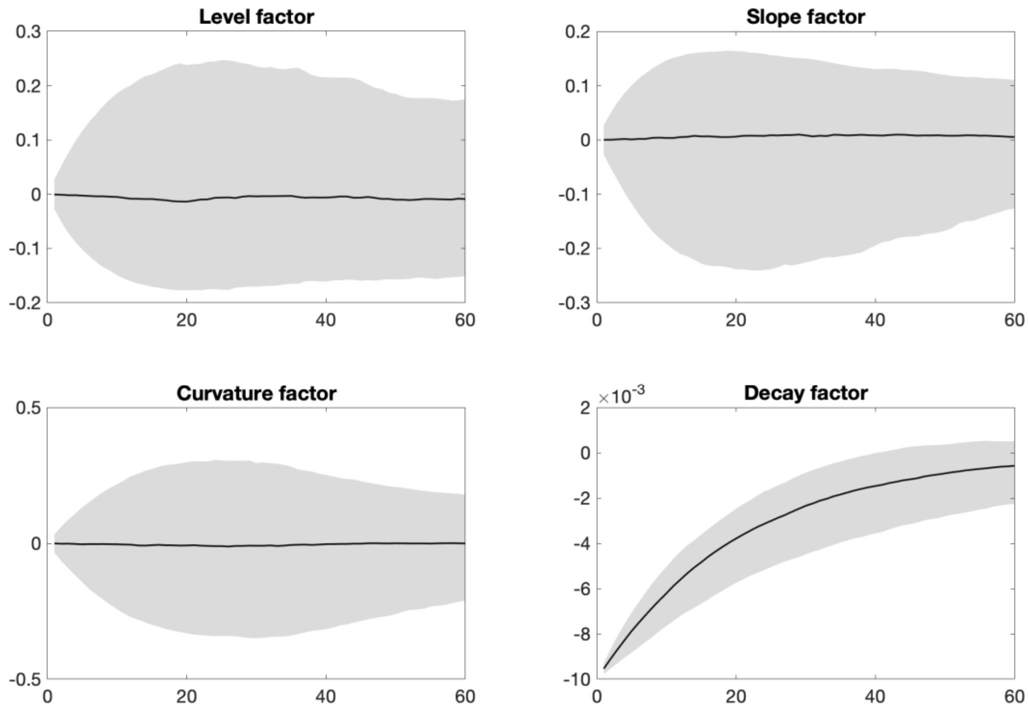
The vector of ν_t represents the factor shocks. The month in which a shock occurs is called the "impacted period." We numerically compute Eq. (4) given model parameter estimates. As in Diebold, Rudebush, and Aruoba (2006), the yield-curve factors were treated as the endogenous variables in the VAR estimation. However, because the decay factor dynamics in the NS framework are considered, nonlinear responses of bond yields to a factor shock, which can depend on the size and sign of the shock as well as initial conditions, were computed. The mapping of X_t to a model-implied variable from the term structure model y_t, $y_t = g(X_t;\ \Theta)$ is nonlinear. The error bands are obtained by drawing parameter vectors from the asymptotic distribution and picking the 84th and 16th percentiles.[12]

4.1. Impulse responses to a decay factor shock

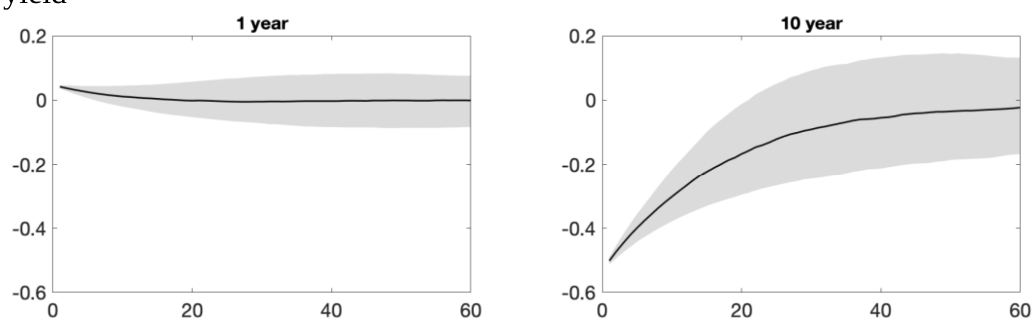
The impulse responses of the decay factor and short- and long-dated yields (1- and 10-years, respectively) to a negative decay factor shock that reduces the decay factor by 0.01 are shown in Figures 6 and 7 with the impacted months set at April 2013 (when the BOJ introduced QQE) and September 2016 (the month that BOJ's yield curve control was introduced by BOJ) respectively. The negative response of decay factor is quite persistent due to the persistence in the decay factor process (Table 1). The magnitude of the drop in the long-dated yields depends on the initial conditions. The 10-year bond yield drops by about 51 and 19 basis points upon impact with the impacted months of April 2013 and September 2016, respectively. Thus, the effect of the decay factor shock on lowering the long-dated bond yield has weakened between the two impacted months as the long-dated yields decreased. The impulse responses of the other (the standard yield curve) factors are irresponsive and insignificant. This implies that the short rate, which is the sum of the level and slope factors, is also irresponsive to the shock. Thus, the changes in bond yield are mostly explained by the corresponding term premium changes. In short, a negative decay factor shock compresses long-term yields via the term premium channel.

[12] See Appendix E in Hayashi and Koeda (2019) for details.

a) Yield curve factors



b) Bond yield



c) Term premium

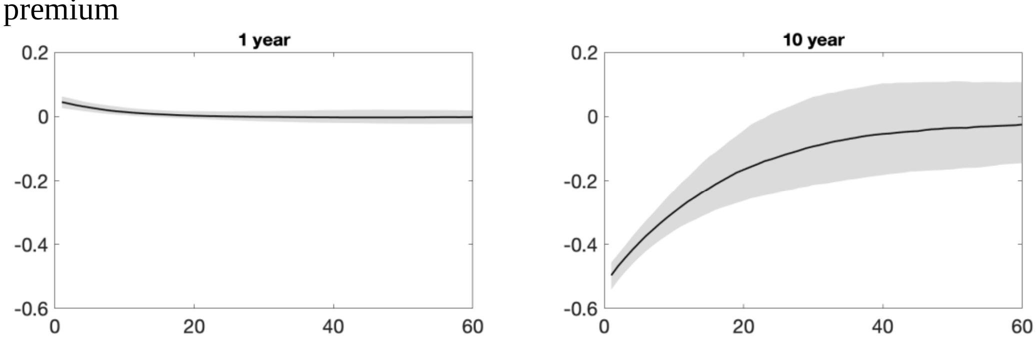
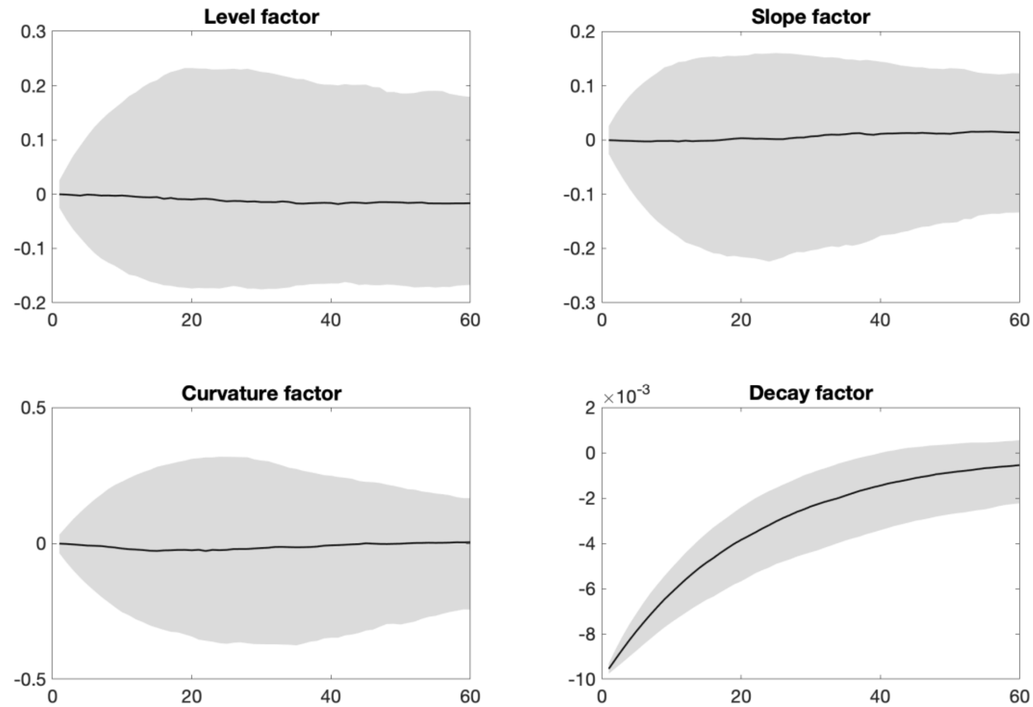
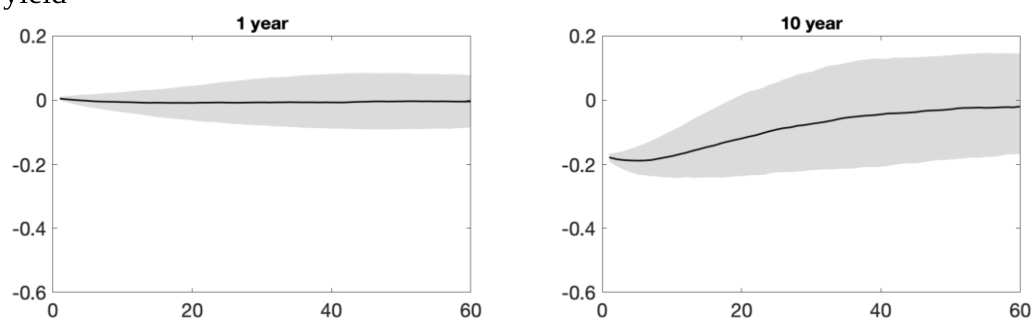


Figure 6: Impulse responses to a negative decay-factor shock: April 2013. The above panels plot the impulse responses of yield curve factors and model implied variables to a decay-factor shock that reduces the decay factor by 0.01 upon impact. Figure 6a shows the responses of yield-curve factors. Figures 6b and 6c show the responses of bond yields (1- and 10-year), the term premium (1- and 10-year), respectively, in the annualized rate in percent. The grey areas correspond to one-standard error confidence intervals.

a) Yield curve factors



b) Bond yield



c) Term premium

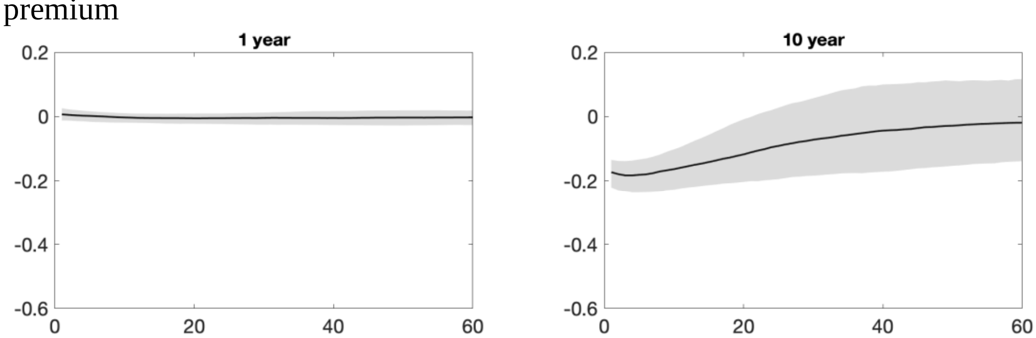


Figure 7: Impulse responses to a negative decay-factor shock: September 2016. The above panels plot the impulse responses of yield curve factors and model implied variables to a decay-factor shock that reduces the decay factor by 0.01 upon impact. Figure 7a shows the responses of yield-curve factors. Figures 7b and 7c show the responses of bond yields (1- and 10-year), the term premium (1- and 10-year), respectively, in the annualized rate in percent. The grey areas correspond to one-standard error confidence intervals.

5. Discussion

5.1. Simple regression results

In order to examine the nature of the decay-factor shock, this subsection conducts simple regressions. Table 2 regresses the identified decay factor shocks (the corresponding VAR residuals imposing the recursive structure of Σ) on its lag and measures for BOJ's bond purchases, business cycles, economic uncertainty, and inflation described in Section 3.1. The table shows that the decay factor shock decreases with the offered amount of BOJ bond purchases. The shock also tends to increase with economic uncertainty measured by the VIX. The coefficient for CPI inflation turns out to be statistically insignificant.

The decay factor shock is not the only shock that are related to BOJ policy. Appendix Table A2 reports the regression results replacing the slope-factor shock as the dependent variable. The slope factor shock also decreases with the offered amount of BOJ bond purchases and increases with output gaps. Contrary to the effects of decay-factor shocks, those of slope shock can notably affect the expected short rate on the shorter end of yield curve (illustrated in the left panels of Appendix Figures A10b and A10c). However, these effects on the long-dated bond yields are short lived. In other words, the decay factor shocks are long-dated term premium shocks that flatten yield curve more persistently than the slope shock. Such shocks tend to decrease with the amount of BOJ bond purchases.

Table 2 Regression of the residuals in the decay factor equation

output gap	-0.1092**	-0.0742	-0.0586	-0.0528	-0.0631
	(0.0534)	(0.0482)	(0.0474)	(0.0465)	(0.0493)
INF	-0.0651	-0.0506	-0.0167	-0.0319	-0.0485
	(0.0820)	(0.0649)	(0.0653)	(0.0619)	(0.0648)
VIX		0.1672***	0.1020	0.1189**	0.1234*
		(0.0549)	(0.0629)	(0.0556)	(0.0686)
BOJoffer (1<, =<10)			-0.0216*		
			(0.0111)		
BOJoffer (>10)				-0.0921**	
				(0.0377)	
MATDEBT					-0.0464
					(0.0437)
decay factor shock (-1)	-0.0235	-0.1698	-0.2669*	-0.2863*	-0.2160
	(0.1182)	(0.1421)	(0.1465)	(0.1427)	(0.1483)
constant	-0.1163	-0.5971***	-0.1643	-0.2427	-0.3374
	(0.0908)	(0.1744)	(0.2796)	(0.2197)	(0.2999)
Adjusted R^2	0.067	0.238	0.284	0.317	0.241
Sample Period	2001Q2–2019Q4	2008Q1–2019Q4	2008Q1–2019Q4	2008Q1–2019Q4	2008Q1–2019Q4

Note: The estimation period is from 2001 Q2 to 2019 Q4, defined based on the breakpoint test results reported in footnote 10. The monthly estimates obtained from the NS-NL model are converted into quarterly estimates based on the average of the end-of-month results. Here, matdebt stands for the maturity-weighted government debt held by the BOJ to nominal GDP. The null of the unit root is rejected at 5% significance for the dependent variable. Standard errors are in parentheses. Here, *, **, and *** respectively indicate the 10%, 5%, and 1% significance levels.

5.2. Limitation of this study

While this study relaxes a common parameter restriction with respect to the decay factor, it still has some limitations. First, it does not apply the NS model in a no-arbitrage framework. Existing no-arbitrage NS models require a constant decay factor when imposing restrictions on factor dynamics under the risk-neutral measure (e.g., Diebold and Rudebush, 2013; Christensen, Diebold, and Rudebusch, 2011). Second, this study does not explicitly introduce a zero lower bound (ZLB). This is because the entire sample period used for estimation falls in a ZLB environment. Third, it examines nominal, rather than real, bond yields, following the approach of Imakubo, Kojima, and Nakajima (2018), who estimate an equilibrium yield curve for Japan using real bond yields and output-gap data in a dynamic NS framework assuming a constant decay factor.

6. Conclusion

This study analyzed the prevailing low interest rate environment in Japanese bond markets and under BOJ's yield curve control using the standard yield curve framework by removing a common parameter restriction. It analyzed how the Nelson-Siegel decay factor affects bond yields and volatility, considering the non-linear effect arising from it. It found that the declined decay factor can account for the decline in the long-term interest rates and their conditional volatilities in the presence of persistently negative slope and curvature factors, particularly from 2012 in Japan. It also found that the negative decay-factor shocks reduce long-dated nominal term premium, and these shocks are negatively correlated with BOJ bond purchases.

The Japanese decay factor was the lowest ever in the last several years of the investigated period under the BOJ's yield curve control. Recently some policy modifications have been made under the yield curve control. In March 2021, the BOJ announced that it would continue to allow the 10-year bond yield to fluctuate around 0%, while it strengthened its commitment on the cap on the 10-year bond yield to be 25 basis points. In addition to these policy modifications, some also see scope for shortening the maturity target under yield-curve control. Besides these policy modifications, the decay factor can increase under monetary policy normalization in the future. In the future research, it may be useful to further examine the effect of monetary policy changes on bond yield and volatility in relation to decay factor changes.

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Appendix Figures and Tables

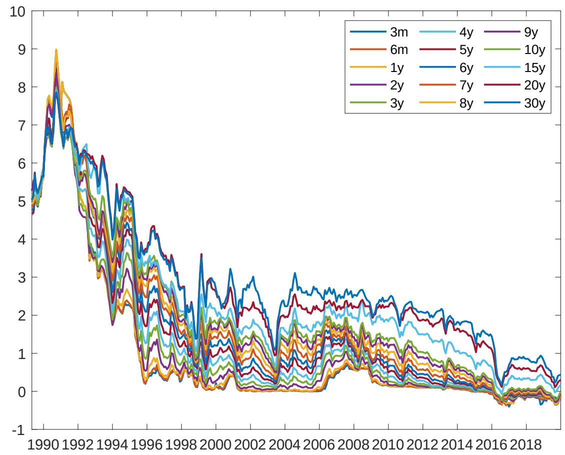
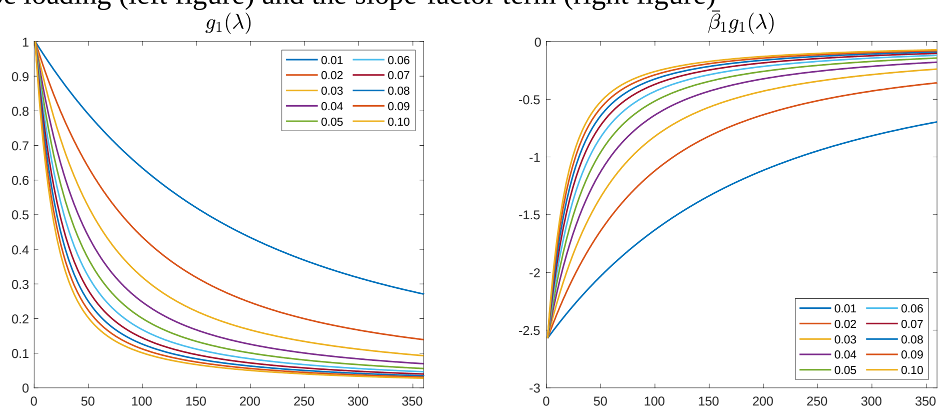


Fig. A1 Japanese government bond (JGB) yields. This figure plots JGB yields from April 1989 to December 2019 in terms of the annualized rate (%). "m" and "y" in the legend stand for "month" and "year".

a) The slope loading (left figure) and the slope-factor term (right figure)



b) The curvature loading (left figure) and the curvature-factor term (right figure)

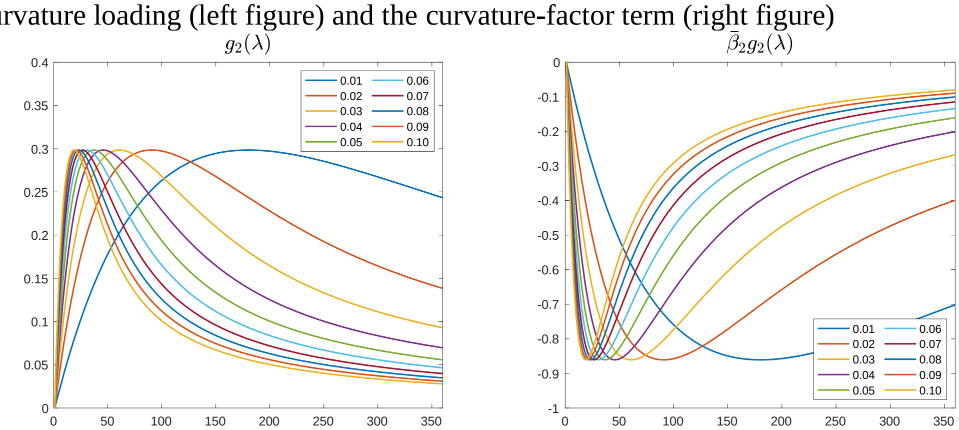


Figure A2 Factor loadings with different values for the decay factor. $\bar{\beta}_1$ and $\bar{\beta}_2$ are the averages of the estimated slope and curvature factors respectively.

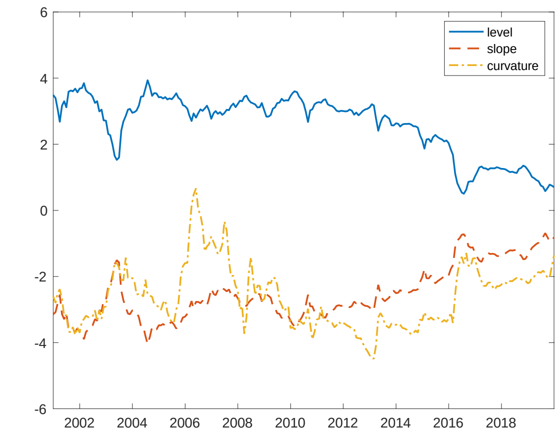


Fig. A3 Estimated factors. This figure plots the estimated level-, slope-, and curvature-factors at a monthly frequency based on the NS-LS model. The constant decay factor is 0.021.

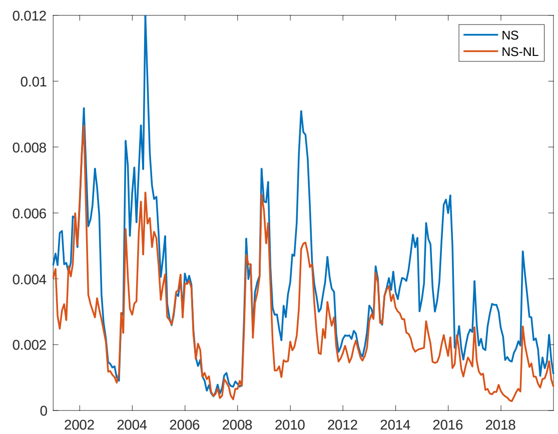


Figure A4 Mean squared error. This figure plots mean squared error computed from the NS-LS and NS-NL models.

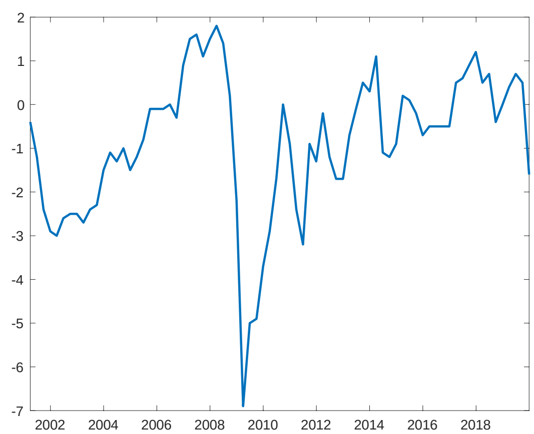


Fig. A5 Output gaps. This figure plots output gaps provided by the Cabinet Office of Japan. In annualized rate in percent.

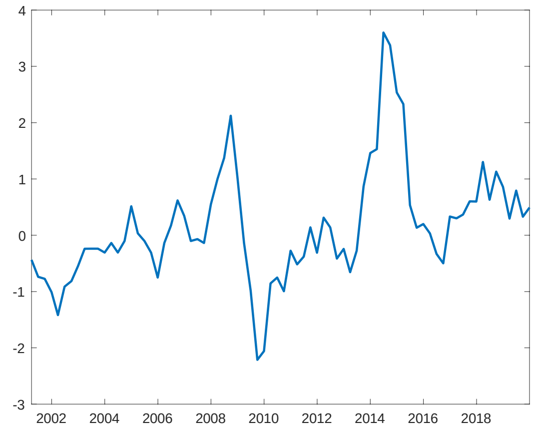


Fig. A6 CPI inflation. In annualized rate in percent.

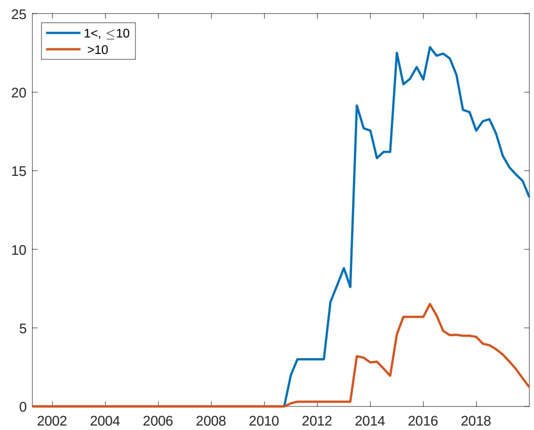


Fig. A7 BOJ offer. This figure plots the offered amount of BOJ purchases on JGB with remaining maturity greater than 10 years (in trillion yen)..

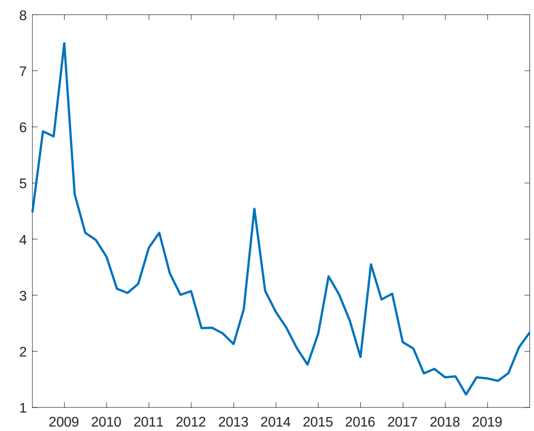


Fig. A8 Economic uncertainty. This figure plots S&P/JPX JGB VIX provided by the Japan Exchange Group.

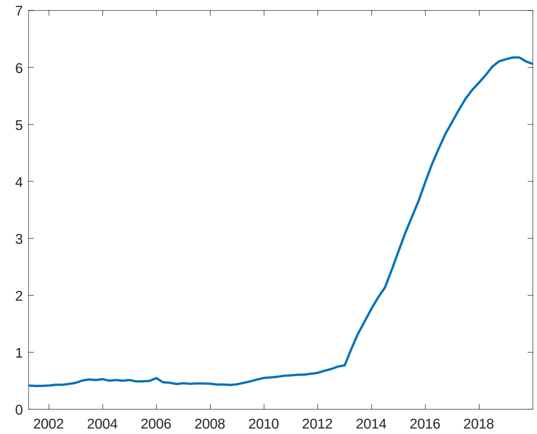
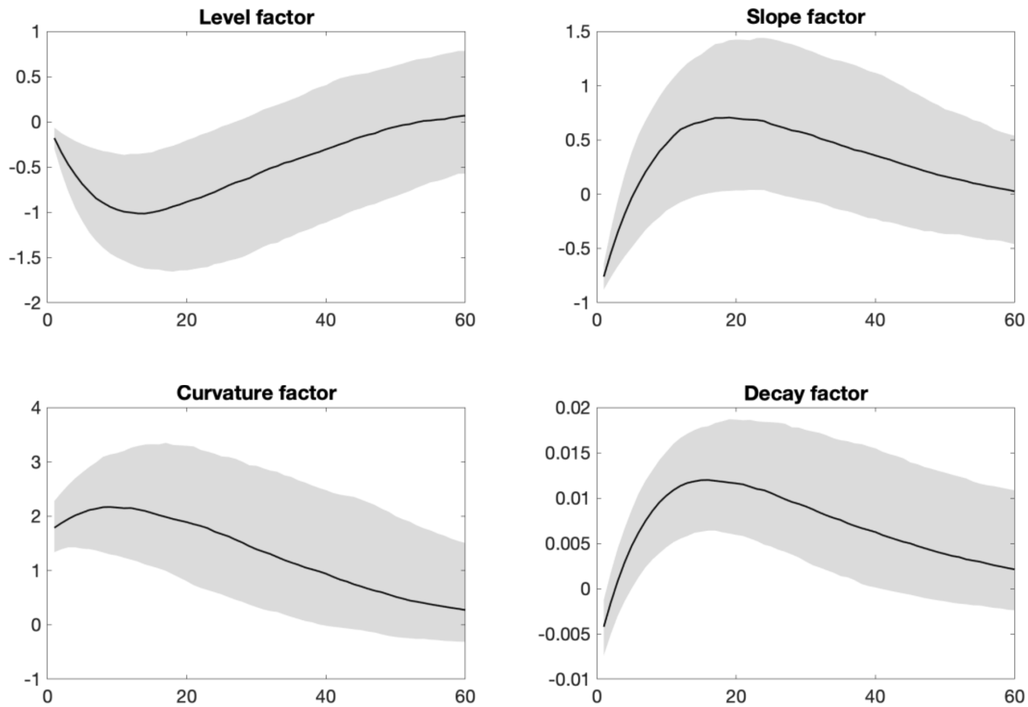
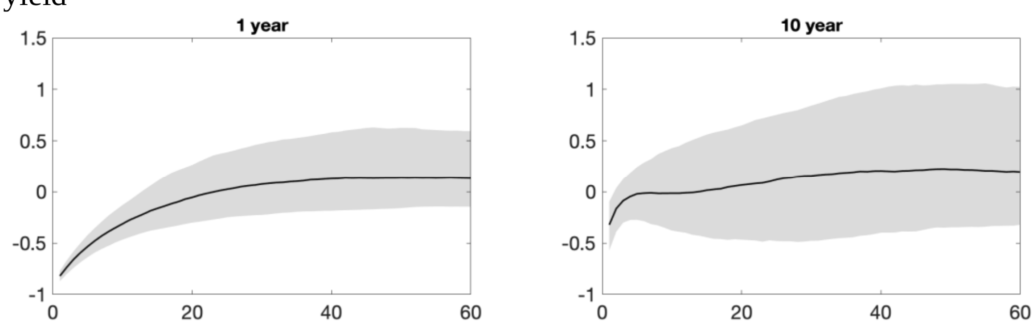


Fig. A9 Maturity-weighted government debt held by the BOJ. In the fraction of nominal GDP.

a) Yield curve factors



b) Bond yield



c) Term premium

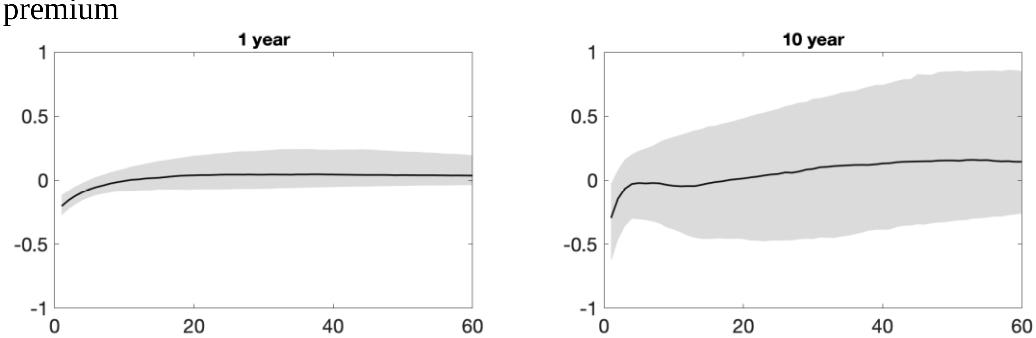


Figure A10 Impulse responses to a negative supply-factor shock: September 2016. The above panels plot the impulse responses of yield curve factors and model implied variables to a slope-factor shock that reduces the slope factor by 0.01 upon impact. Figure A10a shows the responses of yield-curve factors. Figure A10b and A10c show the responses of bond yields (1- and 10-year), the term premium (1- and 10-year), respectively, in the annualized rate in percent. The grey areas correspond to one-standard error confidence intervals.

Table A1 Summary statistics

Maturity	Mean	Std. dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	1.029	2.047	-0.402	8.407	0.993	0.833	0.433
6	1.032	2.041	-0.372	8.407	0.993	0.830	0.429
12	1.068	2.037	-0.341	8.407	0.993	0.829	0.434
24	1.129	1.947	-0.330	8.407	0.992	0.826	0.461
36	1.244	1.946	-0.329	8.483	0.992	0.830	0.486
48	1.386	1.962	-0.334	8.531	0.992	0.840	0.524
60	1.538	2.006	-0.338	8.479	0.992	0.850	0.543
72	1.686	2.077	-0.350	8.882	0.993	0.853	0.555
84	1.809	2.076	-0.350	8.959	0.992	0.853	0.561
96	1.920	2.029	-0.334	8.980	0.992	0.848	0.564
108	2.014	1.961	-0.290	8.369	0.991	0.852	0.595
120	2.095	1.904	-0.245	7.983	0.991	0.864	0.620
180	2.398	1.888	-0.068	7.911	0.991	0.873	0.634
240	2.742	1.816	0.110	7.784	0.989	0.863	0.637
360(Level)	2.892	1.719	0.179	7.858	0.988	0.843	0.607
Slope	1.863	0.944	-1.467	3.747	0.973	0.635	0.182
Curvature	-1.663	0.690	-2.830	0.549	0.956	0.480	0.075

This table reports the summary statistics of JGB yields ranging from April 1989 to December 2019. It shows the mean, standard deviation, minimum, maximum, and three autocorrelation coefficients for 1-month, 12-month, and 30-month. The yield curve level is defined as the 360-month bond yield, the slope as the 360-month minus 3-month bond yield, and the curvature as twice the 24-month bond yield minus the sum of the 3-month and 360-month bond yields.

Table A2 Regression of the residuals in the slope factor equation

output gap	0.0567	0.0587	0.1224**	0.1239**	0.1461***
	(0.0382)	(0.0509)	(0.0502)	(0.0505)	(0.0500)
INF	-0.0611	-0.0705	-0.0056	-0.0388	-0.0555
	(0.0597)	(0.0706)	(0.0671)	(0.0648)	(0.0617)
VIX		-0.0183	-0.1276*	-0.0817	-0.1750**
		(0.0600)	(0.0641)	(0.0579)	(0.0665)
BOJoffer (1<, =<10)			-0.0379***		
			(0.0118)		
BOJoffer (>10)				-0.1301***	
				(0.0407)	
MATDEBT					-0.1775***
					(0.0465)
slope shock (-1)	0.4227***	0.4152***	0.2218	0.2299	0.1306
	(0.0964)	(0.1394)	(0.1399)	(0.1392)	(0.1426)
constant	0.0758	0.1227	0.8972***	0.6390***	1.1185***
	(0.0654)	(0.1862)	(0.2941)	(0.2338)	(0.3075)
Adjusted R^2	0.236	0.157	0.308	0.306	0.359
Sample Period	2001Q2–2019Q4	2008Q1–2019Q4	2008Q1–2019Q4	2008Q1–2019Q4	2008Q1–2019Q4

The estimation period is from 2001 Q2 to 2019 Q4, defined based on the breakpoint test results reported in footnote 11. The monthly estimates obtained from the NS-NL model are converted into quarterly estimates based on the average of the end-of-month results. Here, matdebt stands for the maturity-weighted government debt held by the BOJ to nominal GDP. The null of the unit root is rejected at 5% significance for the dependent variable. Standard errors are in parentheses. Here, *, **, and *** respectively indicate the 1%, 5%, and 10% significance levels.

Appendix A: State Space Representation and the Extended Kalman Filter

Define the vector of observations as the vector of observed bond yields with J different maturities: $(n_1, \cdots, n_J)'$

$$Y_t^o = \begin{bmatrix} y_{t,n_1}^o \\ y_{t,n_2}^o \\ \vdots \\ y_{t,n_J}^o \end{bmatrix}.$$

The measurement equations consist of the yield equations, $Y_t^o = G(X_{t+1}) + \eta_{t+1}$, where

$$G(X_{t+1}) = \begin{bmatrix} \beta_{0,t} + \beta_{1,t}\left(\frac{1-e^{-\lambda_t n_1}}{\lambda_t n_1}\right) + \beta_{2,t}\left(\frac{1-e^{-\lambda_t n_1}}{\lambda_t n_1} - e^{-\lambda_t n_1}\right) \\ \vdots \\ \beta_{0,t} + \beta_{1,t}\left(\frac{1-e^{-\lambda_t n_J}}{\lambda_t n_J}\right) + \beta_{2,t}\left(\frac{1-e^{-\lambda_t n_J}}{\lambda_t n_J} - e^{-\lambda_t n_J}\right) \end{bmatrix}, X_t = \begin{bmatrix} \beta_{0,t} \\ \beta_{1,t} \\ \beta_{2,t} \\ \lambda_t \end{bmatrix}, \eta_t \sim N\left(0, \sigma_\eta I\right).$$

The transition equation is

$$X_{t+1} = \Phi_0 + \Phi_1 X_t + \nu_t,$$

where $\Phi_0 = \mu$, $\Phi_1 = \rho$, $\nu_t \sim N(0, \Sigma_\nu)$, $\Sigma_\nu = \Sigma\Sigma'$, $X_0 \sim N(X_0, P_0)$, X_0 and P_0 are known. Predict $X_{t+1|t}$ and $P_{t+1|t}$ as follows:

$$X_{t+1|t} = \Phi_0 + \Phi_1 X_{t|t}$$
$$P_{t+1|t} = \Phi_1 P_{t|t} \Phi_1' + \Sigma_\nu \, .$$

Update $X_{t+1|t+1}$ and $P_{t+1|t+1}$ as follows:

$$X_{t+1|t+1} = X_{t+1|t} + K_{t+1}\left(Y_{t+1}^o - G(X_{t+1|t})\right),$$
$$P_{t+1|t+1} = (I - K_{t+1} H_{t+1}) P_{t+1|t} \, ,$$

where

$$K_{t+1} = P_{t+1|t} H_{t+1}' (H_{t+1} P_{t+1|t} H_{t+1}' + \omega I)^{-1} \, , H_{t+1} = \left(\left.\frac{\partial G(x)}{\partial x}\right|_{X_{t+1|t}}\right)', \text{ and}$$

$$\frac{\partial G(X)}{\partial X} = \begin{bmatrix} \left[1, \frac{1-e^{-\lambda n_1}}{\lambda n_1}, \frac{1-e^{-\lambda n_1}}{\lambda n_1} - e^{-\lambda n_1}, \beta_1\left(\frac{e^{-\lambda n_1}(\lambda n_1 + 1) - 1}{\lambda^2 n_1}\right) + \beta_2\left(\frac{e^{-\lambda n_1}(\lambda n_1 + 1) - 1}{\lambda^2 n_1} + \tau e^{-\lambda n_1}\right)\right] & 0_{1\times n_X} \\ \vdots & \vdots \\ \left[1, \frac{1-e^{-\lambda n_J}}{\lambda n_J}, \frac{1-e^{-\lambda n_J}}{\lambda n_J} - e^{-\lambda n_J}, \beta_1\left(\frac{e^{-\lambda n_J}(\lambda n_J + 1) - 1}{\lambda^2 n_J}\right) + \beta_2\left(\frac{e^{-\lambda n_J}(\lambda n_J + 1) - 1}{\lambda^2 n_J} + \tau e^{-\lambda n_J}\right)\right] & 0_{1\times n_X} \end{bmatrix}.$$