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Masashige Hamano, Francesco Pappadà

Waseda INstitute of Political EConomy
Waseda University
Tokyo, Japan

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Masashige Hamano　　Francesco Pappadà

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Abstract

This paper examines the exchange rate policy in a two-country model with nominal wage rigidities and firm dynamics. We show that a flexible exchange rate is unable to replicate the flexible price allocation under incomplete financial markets. In our setting with heterogeneous firms, a monetary intervention dampens nominal exchange rate fluctuations and stabilizes the firm selection in the export market. The reduction in wage setting uncertainty ensured by a fixed exchange rate is particularly relevant when firms are small and homogeneous, thus providing a rationale for *currency manipulation* in exchange rate policies.

JEL Classification Codes: F32, F41, E40.

Keywords: exchange rate policy, firm heterogeneity, nominal rigidities.

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1 Introduction

Over the past few years, policymakers have adopted measures aimed at protecting industries exposed to trade, as witnessed by the recent trade tensions between US and China. The main objective of these policies is to stabilize exports which are vulnerable to economic shocks. These policy decisions primarily take two forms: i) trade policies, including changes in trade tariffs, or ii) exchange rate policies, as policymakers may opt for a managed floating rather than a fully floating exchange rate. This practice, sometimes dubbed "currency manipulation" in the public debate, has the objective of reducing the volatility of exchange rate thus the volatility of exports. Indeed, a partial management of the exchange rate has the advantage of limiting the fluctuations of profits in the export market, hence stabilizing the selection of exporter firms.

In this paper, we show how the selection of firms in the export market may provide a strong motivation for limiting the fluctuations in the nominal exchange rate. We build a two-country dynamic stochastic general equilibrium model with nominal wage rigidities, endogenous entry and heterogeneous firms. We analyze the exchange rate policy trade-offs in a tractable framework where firm dynamics respond to demand shocks. In our model, a flexible exchange rate absorbs external demand shocks only partially. In fact, fluctuations in the nominal exchange rate modify the selection of exporter firms. In presence of wage rigidities, the uncertainty about future labor demand arising from the selection in the export market translates in higher wage markups. In contrast, a fixed exchange rate dampens the uncertainty about future labor demand arising from the selection of firms in the export market, shifting the burden of adjustment from the export market to the domestic market. A fixed exchange rate achieves the desired composition of consumption coherently with the preference shift but it increases the domestic price level due to the uncertainty faced by incumbents and the endogenous entry of firms in the domestic market.

The main contribution of this paper is to explore the role of firm heterogeneity and nominal rigidities on the exchange rate policy trade-offs, and provide an analytical solution of the general equilibrium model including the optimal exchange rate policy. In our economy, when firm productivity is less dispersed, the fluctuations on external demand may induce a larger fraction of firms to enter or exit the export market. As households expect the labor demand to move substantially in response to demand shocks, they set higher wages. In this context, a fixed exchange rate sterilizes the fluctuations on labor demand due to selection of firms in the export market, and therefore relatively dampens the increase in wage markups. Our results therefore suggest that a fixed exchange rate deals better with wage rigidities when the dispersion of firm productivity is low, that is when many firms are subject to fluctuations in external demand. The reverse is true for a granular economy where firm productivity is highly dispersed.

The exchange rate policy trade-offs arise from the presence of incomplete financial markets. We show that the allocation under flexible prices is distorted in our model because of incomplete financial markets. For this reason, a flexible exchange rate may not be efficient despite its ability to replicate the allocation of flexible prices. On the other hand, a fixed exchange rate may improve welfare by increasing the comovement between the demand shock and the production of preferred goods. We describe the exchange rate policy trade-offs and derive the optimal monetary policy in a Nash equilibrium maximizing the welfare of domestic households. We then compare the welfare under the two polar cases of fixed vs. flexible exchange rate.

This paper belongs to the literature on the "shock absorber" role played by a flexible exchange rate established in the original contributions by Friedman (1953) and Mundell (1961). We show that a flexible exchange rate partially absorbs the shock for the domestic economy, while inducing a substantial volatility in the export market. A monetary policy intervention aimed at dampening the fluctuations in the nominal exchange rate therefore acts as a powerful macroeconomic stabilization tool for the export market. In response to external shocks, our framework captures both the selection of heterogeneous firms into the export market and the endogenous firm entry under alternative exchange rate policies. In line with our setting, several papers emphasize the adjustments occurring at both the intensive and extensive margins of trade with or without firm heterogeneity (see among others Ghironi and Melitz (2005), Alessandria and Choi (2007), Corsetti et al.

(2007), Pappadà (2011), Corsetti et al. (2013), Cacciatore (2014) and Hamano (2014)). In this paper, we introduce nominal rigidities and discuss the exchange rate policy with endogenous entry of heterogeneous firms.[1] As we assume nominal rigidity in wage setting by households and let firms adjust freely their prices, the producer currency pricing ensures the "expenditure switching effect" at individual firm price level. Our modeling setup is therefore similar to Rodriguez-Lopez (2011) which analyzes the expenditure switching effect with heterogeneous firms and its resulting bias in aggregate price. However, the scope of our paper is different as we focus on the optimal monetary policy and the exchange rate policy under financial market incompleteness. Incomplete financial markets indeed introduce distortions in the flexible price allocation, and raise a chance for the fixed exchange rate to dominate the flexible one. While Devereux (2004) highlights the role of the elasticity of labor supply and Hamano and Picard (2017) the preference for product variety in ranking the exchange rate regime, we study how the heterogeneity in firm productivity shapes the response of the economy to demand shocks.

Our paper is also related to the recent debate on trade integration or protectionism (e.g. Auray et al. (2019); Erceg et al. (2018); Lindé and Pescatori (2019)). Cacciatore and Ghironi (2020) study the Ramsey-optimal cooperative monetary policy in an open economy with firm heterogeneity and search and matching frictions in labor market. They show that the optimal cooperative monetary policy is still an inward-looking one in their open economy setting. While they focus on the consequences of trade linkages on the Ramsey cooperative monetary policy, we show in a closed form solution the distance between the optimal monetary policy and alternative monetary policies that include the adoption of a cooperative currency peg. In a similar setting, Barattieri et al. (2018) study a temporary tariff shock and cast a doubt for its effectiveness as a macroeconomic stabilization tool. While this literature studies the interplay between monetary policy and trade policies, we rather focus on the ability of the monetary policy to act as a powerful macroeconomic stabilization tool depending on the exchange rate policy. In this

[1] Hamano and Zanetti (2020) also explore the link between selection of firms and monetary policy but in a closed economy setting.

respect, our paper is reminiscent of Bergin and Corsetti (2020), which study the impact of monetary policy on the comparative advantage of countries. In Bergin and Corsetti (2020), a perfect consumption risk-sharing is guaranteed and nominal rigidities are the only source of distortion, thus the flexible price allocation is efficient. While Bergin and Corsetti (2020) focus on the stabilization of marginal costs under complete financial markets, we discuss their stabilization under incomplete financial markets. Moreover, in their paper the optimal monetary policy stabilizes marginal costs and allows an expansion of the differentiated goods sector. This effect in turn fosters firm entry in the long run and thus affects the comparative advantage. Instead of the sectoral reallocation, we focus on reallocation within our tradable sector with heterogeneous firms. As in Bergin and Corsetti (2020), monetary policy is therefore crucial in enhancing endogenous firm entry and improving the competitiveness in the long run.

The paper is structured as follows. In the next section, we introduce a two country model with external demand shocks and provide an analytical solution of our model. In section 3, we provide the solution of our model under two polar exchange rate policies. Section 4 reports the welfare analysis and shows i) the optimal monetary policy in a Nash equilibrium as a function of the fundamentals of the economy, and ii) the welfare comparison under the polar cases of flexible and fixed exchange rate. Section 5 concludes.

2 The Model

In this section, we introduce a two country dynamic stochastic general equilibrium model with firm heterogeneity. Both Home and Foreign countries are inhabited by a unit mass of households which provide imperfectly-substituted labor. All goods are tradable but only a fraction of them are exported by firms operating in monopolistic competition, and the number of exporters is determined endogenously. We introduce demand shock to each countries' goods, and study how these shocks interacts with firm dynamics according to the conduct of monetary policy.

There are two important frictions in our model. First, we introduce a nominal rigidity, as households set wages one period in advance based on their expectations of future labor demand. Second, the international asset markets are incomplete, as Home household cannot hold Foreign assets and vice versa.[2] Finally, we show a closed form solution of our dynamic stochastic general equilibrium model without relying on any approximation method.

2.1 Households

The representative household maximizes her life time utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t(j)$, where β $(0 < \beta < 1)$ is the exogenous discount factor. Utility of individual household j at time t depends on consumption $C_t(j)$ and labor supply $L_t(j)$ as follows

$$U_t(j) = \ln C_t(j) + \chi \ln \frac{M_t(j)}{P_t} - \eta \frac{[L_t(j)]^{1+\varphi}}{1+\varphi}, \tag{1}$$

where χ and η represent the degree of satisfaction (unsatisfaction) from real money holdings and labor supply respectively, while the parameter φ measures the inverse of the Frisch elasticity of labor supply.

The basket of goods $C_t(j)$ is defined as

$$C_t(j) = \left(\frac{C_{H,t}(j)}{\alpha_t} \right)^{\alpha_t} \left(\frac{C_{F,t}(j)}{\alpha_t^*} \right)^{\alpha_t^*},$$

where α_t and α_t^* are the preference attached to the bundle of goods produced respectively in Home $C_{H,t}(j)$ and imported goods $(C_{F,t}(j))$, as we denote Foreign variables with an asterisk (*). These preferences are assumed to be stochastic. Furthermore, these baskets are defined over a continuum of goods Ω as

$$C_{H,t}(j) = \left(\int_{\varsigma \in \Omega} c_{D,t}(j,\varsigma)^{1-\frac{1}{\sigma}} d\varsigma \right)^{\frac{1}{1-\frac{1}{\sigma}}}, \; C_{F,t}(j) = \left(\int_{\varsigma^* \in \Omega} c_{X,t}(j,\varsigma^*)^{1-\frac{1}{\sigma}} d\varsigma^* \right)^{\frac{1}{1-\frac{1}{\sigma}}}.$$

In each time period, only a subset of variety of goods is available from the total universe of variety of goods Ω. We denote $N_{D,t}$ and $N_{X,t}^*$ as the number of domestic and

[2] In line with related literature, for the sake of tractability we opt for this extreme source of market incompleteness which implies financial autarky.

imported product varieties, respectively. $c_{D,t}(j,\varsigma)$ and $c_{X,t}(j,\varsigma^*)$ represent the demand addressed for individual product variety indexed by ς and ς^*. σ denotes the elasticity of substitution among differentiated goods and is greater than 1.

The optimal consumption for each domestic basket, imported basket and individual product variety are found to be

$$C_{H,t}(j) = \left(\frac{P_{H,t}}{P_t}\right)^{-1} \alpha_t C_t(j), \quad C_{F,t}(j) = \left(\frac{P_{F,t}}{P_t}\right)^{-1} \alpha_t^* C_t(j),$$

$$c_{D,t}(j,\varsigma) = \left(\frac{p_{D,t}(\varsigma)}{P_{H,t}}\right)^{-\sigma} C_{H,t}(j), \quad c_{X,t}(j,\varsigma^*) = \left(\frac{p_{X,t}^*(\varsigma^*)}{P_{F,t}}\right)^{-\sigma} C_{F,t}(j).$$

In the above expressions, $p_{D,t}(\varsigma)$ stands for the price of product variety ς which is domestically produced. In particular, $p_{X,t}^*(\varsigma^*)$ denotes the price of imported product variety ς^*, denominated in currency unit in Home. $P_{H,t}$ and $P_{F,t}$ are the price of basket of goods produced in Home and that of imported, respectively. P_t is the price of aggregated basket. Price indexes that minimize expenditures on each consumption basket are

$$P_t = P_{H,t}^{\alpha_t} P_{F,t}^{\alpha_t^*},$$

$$P_{H,t} = \left(\int_{\varsigma\in\Omega} p_{D,t}(\varsigma)^{1-\sigma} d\varsigma\right)^{\frac{1}{1-\sigma}}, \quad P_{F,t} = \left(\int_{\varsigma^*\in\Omega} p_{X,t}^*(\varsigma^*)^{1-\sigma} d\varsigma^*\right)^{\frac{1}{1-\sigma}}.$$

Similar expressions hold for Foreign. Crucially, the subset of goods available to Foreign during period t, $\Omega_t^* \in \Omega$, can be different from the subset of goods available to Home $\Omega_t \in \Omega$.

2.2 Production, Pricing and the Export Decision

There is a mass of $N_{D,t}$ number of firms in Home. Upon entry, firms draw their productivity level z from a distribution $G(z)$ on $[z_{\min}, \infty)$. Since there are no fixed production costs and hence no selection into domestic market, $G(z)$ also represents the productivity distribution of all producing firms. Prior to entry, however, these firms are identical and face a sunk entry cost $f_{E,t} = l_{E,t}$ units of labor.[3] The sunk cost is composed of imperfectly differentiated labor services provided by households (indexed by i) such that

$$l_{E,t} = \left(\int_0^1 l_{E,t} (j)^{1-\frac{1}{\theta}} \, dj \right)^{\frac{1}{1-\frac{1}{\theta}}}, \tag{2}$$

where θ represents the elasticity of substitution among different labor services. We consider $f_{E,t}$ to be exogenous. By defining the nominal wage for type j labor as $W_t (j)$,total cost for a firm to setup is thus $\int_0^1 l_{E,t} (j) \, W_t (j) \, dj$. The cost minimization yields the following labor demand for type j labor service:

$$l_{E,t} (j) = \left(\frac{W_t (j)}{W_t} \right)^{-\theta} l_{E,t}, \tag{3}$$

where W_t denotes the corresponding wage index, which is

$$W_t = \left(\int_0^1 W_t (j)^{1-\theta} \, dj \right)^{\frac{1}{}}.$$

Exporting requires an operational fixed cost of $f_{X,t} = l_{f_X,t}$ amount of labor defined in a similar way as in equation (2). The cost minimization provides a similar demand for each specific labor service as in equation (3).[4]

For the production of each good variety, only composite labor basket is required as input. Thus the production function of firm with productivity z is given by $y_t (z) = z l_t (z)$ where

$$l_t (z) = \left(\int_0^1 l_t (z, j)^{1-\frac{1}{\theta}} \, dj \right)^{\frac{1}{1-\frac{1}{\theta}}}.$$

The cost minimization yields the demand for type j labor for production as

[3] As an alternative, entry cost could be paid in terms of consumption goods as in Corsetti et al. (2010). In that case, monetary policy has an impact on the number of entrants combined with price rigidity. In our setting, we choose to express entry costs in labor units because it is closely related to our source of nominal rigidity which concerns wages. As shown in the model solution in Table 1, with wage rigidity, a positive (negative) monetary shock directly increases (decreases) the entry of firms, in the same fashion as in Corsetti et al. (2010).

[4] The labor demand for exporting are $l_{f_X,t} = \left(\int_0^1 l_{f_X,t} (j)^{1-\frac{1}{\theta}} \, dj \right)^{\frac{1}{1-\frac{1}{\theta}}}$ and $l_{f_X,t} (j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta} l_{f_X,t}$.

$$l_t(z,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} l_t(z).$$

The firm faces a residual demand curve with constant elasticity σ. The production scale is thus determined by the demand addressed to the firm under monopolistic competition. Profit maximization yields the following optimal price $p_{D,t}(z)$ by firm with productivity z:

$$p_{D,t}(z) = \frac{\sigma}{\sigma - 1}\frac{W_t}{z}.$$

If the firm exports, its price of export is $p_{X,t}(z) = \tau p_{D,t}(z)\varepsilon_t^{-1}$ where ε_t is the nominal exchange rate defined as the price of one unit of foreign currency in terms of home currency units. $\tau > 1$ is iceberg trade cost. In our definition, $p_{X,t}(z)$ is thus denominated in terms of foreign currency units.[5]

Total firm profits $D_t(z)$ can be decomposed into those from domestic sales $D_{D,t}(z)$ and those from exporting sales $D_{X,t}(z)$ (if the firm exports) as $D_t(z) = D_{D,t}(z) + D_{X,t}(z)$. Using the demand functions found previously and the aggregate consumption defined as $C_t = \left(\int_0^1 C_t^{1-\frac{1}{\sigma}}(j)\,dj\right)^{\frac{1}{1-\frac{1}{\sigma}}}$, we can write the profits from each market as

$$D_{D,t}(z) = \frac{1}{\sigma}\left(\frac{p_{D,t}(z)}{P_{H,t}}\right)^{1-\sigma}\alpha_t P_t C_t,$$

$$D_{X,t}(z) = \frac{\varepsilon_t}{\sigma}\left(\frac{p_{X,t}(z)}{P^*_{H,t}}\right)^{1-\sigma}\alpha_t P^*_t C^*_t - W_t f_X. \tag{4}$$

Equation (4) implies that a firm exports when z is larger than $z_{X,t}$, the cut-off level of productivity for exporting. Thus, the non-tradedness in the economy arises endogenously with changes in the productivity cutoff $z_{X,t}$.

[5]The practice of pricing to market and dollar pricing has also been emphasized in the literature and become a motivation to limit the fluctuations in the nominal exchange rate (Betts and Devereux (1996), Devereux and Engel (2003), Corsetti et al. (2010), Gopinath et al. (2010) and Gopinath et al. (2019) among others). In the Appendix, we briefly discuss the case for local currency pricing instead of producer currency pricing.

2.3 Firm Averages

Given a distribution $G(z)$, the productivity level of a mass of $N_{D,t}$ domestically producing firms is distributed over $[z_{\min}, \infty)$. Among these firms, there are $N_{X,t} = [1 - G(z_{X,t})] N_{D,t}$ exporters in Home. Following Melitz (2003) and Ghironi and Melitz (2005), we define two average productivity levels, $\widetilde{z}_D$ for domestically producing firms and $\widetilde{z}_{X,t}$ for exporters as follows

$$\widetilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}, \quad \widetilde{z}_{X,t} \equiv \left[\frac{1}{1 - G(z_{X,t})} \int_{z_{X,t}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}.$$

These average productivity levels summarize all the information about the distribution of firm productivity. Given these averages, we define the average real domestic and export price as $\widetilde{p}_{D,t} \equiv p_{D,t}(\widetilde{z}_D)$ and $\widetilde{p}_{X,t} \equiv p_{X,t}(\widetilde{z}_{X,t})$, respectively. We also define average profits from domestic sales and export sales as $\widetilde{D}_{D,t} \equiv D_{D,t}(\widetilde{z}_D)$ and $\widetilde{D}_{X,t} \equiv D_{X,t}(\widetilde{z}_{X,t})$. Finally, average profits among all firms is given by $\widetilde{D}_t = \widetilde{D}_{D,t} + (N_{X,t}/N_{D,t}) \widetilde{D}_{X,t}$.

2.4 Firm Entry and Exit

Firm entry takes place until the expected value of entry is equalized with entry cost, leading to the following free entry condition:

$$\widetilde{V}_t = f_{E,t} W_t,$$

where $\widetilde{V}_t$ is the expected value of entry which is discussed below. In what follows, we assume i) that entrants at time t only start producing at time $t+1$ (*one-period to build*), and ii) that firms' production plants are assumed to fully depreciate after one period.

2.5 Parametrization of Productivity Draws

We assume the following Pareto distribution for $G(z)$:

$$G(z) = 1 - \left(\frac{z_{\min}}{z} \right)^{\kappa},$$

where $z_{\min}$ is the minimum productivity level and $\kappa > \sigma - 1$ is the shape parameter. With this parametrization, we have

$$\widetilde{z}_D = z_{\min}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}}, \qquad \widetilde{z}_{X,t} = z_{X,t}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}}.$$

The share of exporters in the total number of domestic firms is then given by

$$\frac{N_{X,t}}{N_{D,t}} = z_{\min}^{\kappa}\left(\widetilde{z}_{X,t}\right)^{-\kappa}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{\frac{\kappa}{\sigma-1}}. \tag{5}$$

Finally, there exists a firm with a specific productivity cutoff $z_{X,t}$ that earns zero profits from exporting, as $D_{X,t}\left(z_{X,t}\right) = 0$. With the above Pareto distribution, this implies that the average profits of exporter firms are

$$\widetilde{D}_{X,t} = W_t f_{X,t}\frac{\sigma - 1}{\kappa - (\sigma - 1)}.$$

Note that there is no feedback of the cutoff level productivity to the initial distribution $G(z)$, which is fixed and time invariant. However, the equilibrium cutoff level $z_{X,t}$ and hence the average productivity of exporters $\widetilde{z}_{X,t}$ change over time.

2.6 Household Budget Constraints and Intertemporal Choices

A household j in Home faces the following budget constraint at time t:

$$\begin{aligned} &P_t C_t\left(j\right) + B_t\left(j\right) + M_t(j) + x_t(j) N_{D,t+1}\widetilde{V}_t \\ &\qquad = \left(1+\xi\right) W_t\left(j\right) L_t\left(j\right) + (1 + i_{t-1}) B_{t-1}\left(j\right) + M_{t-1}(j) + x_{t-1}(j) N_{D,t}\widetilde{D}_t + T_t^f, \end{aligned}$$

where $B_t\left(j\right)$ and $x_t(j)$ denote bond holdings and share holdings of mutual funds, respectively. $1+\xi$ is the appropriately designed labor subsidy which aims to eliminate distortions due to monopolistic power in labor markets. i_t represents nominal interest rate between t and $t+1$ and T_t^f represents a transfer from domestic government, which can be positive or negative.

We assume that wages are sticky for one time period.[6] Specifically, the household j sets wages at $t-1$ by maximizing her expected utility at t knowing the following demand for her labor:

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} L_t.$$

The first order condition with respect to $W_t(j)$ yields

$$W_t(j) = \frac{\eta\theta}{(\theta-1)(1+\xi)} \frac{\mathrm{E_{t-1}}\left[L_t(j)^{1+\varphi}\right]}{\mathrm{E_{t-1}}\left[\frac{L_t(j)}{P_t C_t(j)}\right]}. \tag{6}$$

Households set the wage so that the expected marginal cost of supplying additional labor services equals the expected marginal revenue.[7] Along with the wage setting, household also choose their share holdings. The first order condition yields

$$\widetilde{V}_t = E_t\left[Q_{t,t+1}(j)\,\widetilde{D}_{t+1}\right],$$

where $Q_{t,t+1}$ is stochastic discount factor defied as $Q_{t,t+1}(j) = E_t\left[\frac{\beta P_t C_t(j)}{P_{t+1}C_{t+1}(j)}\right]$.

The first order condition with respect to bond holdings is given by

$$1 = (1+i_t)E_t\left[Q_{t,t+1}(j)\right].$$

Finally, the household maximizes its consumption and real money holdings. As a result, we have

$$P_t C_t(j) = \frac{M_t}{\chi}\left(\frac{i_t}{1+i_t}\right). \tag{7}$$

Nominal spending $P_t C_t(j)$ is tight down to the money supply M_t.

[6]While there is a strand of literature that models a downward wage rigidity (see for instance, Schmitt-Grohé and Uribe (2016), our setup employs a Calvo wage stickiness hence wages are rigid both upward and downward.

[7]The marginal cost is $\eta\theta W_t(j)^{-1}\mathrm{E_{t-1}}\left[L_t(j)^{1+\varphi}\right]$ and the marginal revenue is $(\theta-1)(1+\xi)\mathrm{E_{t-1}}\left[\frac{L_t(j)}{P_t C_t(j)}\right]$.

2.7 Balanced Trade and Labor Market Clearings

In equilibrium, there is a symmetry across households so that $C_t(j) = C_t$, $L_t(j) = L_t$, $M_t(j) = M_t$ and $W_t(j) = W_t$. Furthermore, we follow Corsetti et al. (2010) and Bergin and Corsetti (2020) and define monetary stance as

$$\mu_t \equiv P_t C_t.$$

Monetary stance is proportional to nominal expenditure.[8] Trade is assumed to be balanced, thus the value of Home exports is equal to the value of Home imports once they are converted to the same unit of currency: $\varepsilon_t P^*_{H,t} C^*_{H,t} = P_{F,t} C_{F,t}$. Combined with the demand of goods found previously, this implies

$$\varepsilon_t = \frac{\alpha_t^*}{\alpha_t} \frac{\mu_t}{\mu_t^*}.$$

Note that the terms of trade (defined as the price of average Foreign exported goods in average Home exported goods) are expressed as

$$TOT \equiv \frac{\widetilde{p}^*_{X,t}}{\varepsilon_t \widetilde{p}_{X,t}} = \frac{\alpha_t^*}{\alpha_t} \frac{\mu_t}{\varepsilon_t \mu_t^*} \frac{N_{X,t} \widetilde{y}_{X,t}}{N^*_{X,t} \widetilde{y}^*_{X,t}}.$$

The above is a general expression independent of the monetary policy rule. It is assumed that the government has no power to directly control private lending and borrowing. The balanced budget rule is assumed as

$$M_t - M_{t-1} = T_t^f + \xi W_t L_t.$$

Under nominal wage rigidity, the aggregate labor supply L_t adjusts to its demand and the labor market clears as

[8] When combining the monetary stance with the Euler equation on bond holdings, one gets

$$\frac{1}{\mu_t} = \mathrm{E}_t \lim_{s \to \infty} \beta^s \frac{1}{\mu_{t+s}} \prod_{\tau=0}^{s-1} (1 + i_{t+\tau}).$$

This shows that monetary stance μ_t may be expressed as a function of future expected path of interest rates or as a rule concerning money supply M_t as in equation (7).

$$L_t = N_{D,t}\frac{\widetilde{y}_{D,t}}{\widetilde{z}_D} + N_{X,t}\left(\frac{\widetilde{y}_{X,t}}{\widetilde{z}_{X,t}} + f_{X,t}\right) + N_{D,t+1}f_{E,t}. \tag{8}$$

In the above expression, $\widetilde{y}_{D,t}$ and $\widetilde{y}_{X,t}$ stand for production scale of each average domestic firms and average exporters.[9] The labor demand comes from producers selling their goods in the domestic and export markets (including export fixed costs), and from resources used for the creation of new firms. A similar expression holds for the Foreign country.

Using the general equilibrium conditions stated above, the equilibrium wage under the balanced trade for a given monetary stance is[10]

$$W_t = \Gamma\left\{\frac{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\}^{\frac{1}{1+\varphi}}.$$

Finally we assume the following process for the preference shift:

$$\alpha_t = \frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t, \quad \alpha_t^* = \frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^*, \tag{9}$$

with $\alpha_0 = \alpha_0^* = 1$, $E_{t-1}\left[\upsilon_t\right] = E_{t-1}\left[\upsilon_t^*\right] = 1$, $\upsilon_t + \upsilon_t^* = 2$ and $0 \leq \rho \leq 1$. Indeed, υ_t and υ_t^* are defined as the i.i.d. shocks. Thus shocks are asymmetric.[11]

We can derive the closed form solution of the model without relying on any approximation methods. The solution of the model is reported in Table 1 for a given monetary rule. We refer to the Appendix for the derivation of all the endogenous variables.

3 Exchange Rate Policy

In this section we evaluate the ability of alternative exchange rate policies to reproduce the allocation of our economy with *complete* financial markets and without nominal rigidities,

[9]See Appendix for more details.

[10]Note that $A_t \equiv \frac{\sigma-1}{\sigma}\left\{\alpha_t + \left(1+\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\alpha_t^* + \beta\left[\frac{1}{\sigma-1}\alpha_{t+1} + \frac{1}{\kappa}\alpha_{t+1}^*\right]\right\}$ captures the labor demand in the right hand side of equation (8), and $\Gamma \equiv \left[\frac{\eta\theta}{(\theta-1)(1+\xi)}\right]^{\frac{1}{1+\varphi}}$.

[11]See Appendix for the expression of A_t as a function of fundamental shocks for a symmetric steady state across countries, i.e. $\alpha_{t-1} = \alpha_{t-1}^*$.

which is close to the first best allocation by the social planner.[12] We solve our model for two polar exchange rate policies that emerge as a consequence of the monetary policy. First, we consider a constant monetary rule such that $\mu_t = \mu_t^* = \mu_0$ for all time periods. Under such rule, the exchange rate is free to float in response to demand shock. Second, we assume that the monetary stance automatically responds to demand shocks offsetting their impact on the exchange rate. This monetary rule corresponds to a cooperative peg system where $\mu_t = 2\mu_0\alpha_t$ and $\mu_t^* = 2\mu_0\alpha_t^*$. Table 2 shows how the equilibrium outcome diverges under these polar exchange rate policies. The table reports the equilibrium wages and selected variables under these polar cases as well as under incomplete financial markets and flexible wages.

3.1 Flexible Exchange Rate

We first consider the impact of a preference shock in our setting with incomplete markets and wage rigidities under a flexible exchange rate. As reported in the second column of Table 2, a relative demand shift for Home produced goods (a decrease in α_t^*/α_t) implies the appreciation of the nominal exchange rate ($\varepsilon_t = \frac{\alpha_t^*}{\alpha_t}$) which keeps trade balanced. The adjustment does not only involve a change in the intensive margin but also in the extensive margin of trade. Under our assumption of producer currency pricing, the nominal appreciation of Home currency improves the average profitability of Foreign exporters relative to Home exporters (a decrease in $\widetilde{D}_{X,t}^{FL}/\widetilde{D}_{X,t}^{*FL}$ on impact). In turn, this induces a higher number of Foreign exporters relative to Home exporters (a decrease in $N_{X,t}^{FL}/N_{X,t}^{*FL}$). Note that the higher number of imported varieties available for Home households does not immediately translates in a welfare gain because of the relative lower preference attached to goods produced in Foreign. At the same time, the change in the exporter cutoff productivity level lowers the relative average productivity of Foreign exporters compared to Home exporters (a rise in $\widetilde{z}_{X,t}^{FL}/\widetilde{z}_{X,t}^{*FL}$).

Given the constant monetary rule, the equilibrium wage under flexible exchange rate

[12] In the Appendix, we derive the allocation under complete financial markets and flexible wages and the allocation implied by the social planner. We show that these two solutions are very close.

depends upon the uncertainty about the *future* demand shock. As A_t captures the labor demand, it includes the uncertainty arising from the selection of Home exporters in the future. Both the equilibrium wage and the relative number of exporters (and their production scale) are a function of demand shocks, which generate substantial trade adjustments. We may therefore conclude that the nominal exchange rate is not a "shock absorber" (as in Friedman (1953) and Mundell (1961)) in our setting with heterogeneous firms and selection in the export market.[13]

The expression of the terms of trade under a flexible exchange rate explicitly shows the key features of our model with selection in the export market. Following a positive demand shock for Home produced goods, the terms of trade appreciate. However, the fall in terms of trade is dampened by the higher relative average productivity of Home exporters (a rise in $\widetilde{z}_{X,t}^{FL}/\widetilde{z}_{X,t}^{*FL}$).[14]

The positive demand shock for Home goods requires an adjustment in the domestic market as well. In the domestic market, the increase in α_t/α_t^* raises the production scale of average domestic firms compared to Foreign ($\widetilde{y}_{D,t}^{FL}/\widetilde{y}_{D,t}^{*FL}$ goes up). The relative investment, that is the creation of new firms ($N_{D,t+1}/N_{D,t+1}^*$) is relatively stable and depends upon the future expected demand shocks rather than the current demand shocks.

To sum up, a relative positive demand shift for Home goods induces a simultaneous adjustment in the nominal exchange rate ε_t. The demand shock is only partially absorbed by exchange rate movements under a flexible exchange rate i) because of the response of the intensive and extensive margin of trade, and ii) because of the adjustment in the domestic market, whose impact is however relatively modest.

We now compare the allocation under a flexible exchange rate to the flexible wage allocation. As shown in the first column of Table 2, following a positive demand shock for Home produced goods (a decrease in α_t^*/α_t), the relative number of exporters $N_{X,t}^{FW}/N_{X,t}^{*FW}$

[13]This is in contrast with existing models without firm heterogeneity - e.g. Devereux (2004) and Hamano and Picard (2017)) - where the nominal exchange rate works indeed as a shock absorber.

[14]Because of the selection into the exporting markets, a nominal appreciation of Home currency coexists with a higher average export price for the Home country. This result is similar to what Rodriguez-Lopez (2011) dubs a "negative expenditure switching effect".

decreases but they are on average more productive as $\widetilde{z}_{X,t}^{FW}/\widetilde{z}_{X,t}^{*FW}$ increases. Despite the selection effect the terms of trade appreciate as they do under a flexible exchange rate. Finally, the average production $\widetilde{y}_{X,t}^{FW}/\widetilde{y}_{X,t}^{*FW}$ increases while firm entry $N_{D,t+1}^{FW}/N_{D,t+1}^{*FL}$ is relatively stable in the domestic economy. Therefore, the allocation under flexible exchange rate mimics well the allocation of flexible wages.

In the peculiar case of infinite elasticity of labor supply, that is when $\varphi = 0$, the allocation with flexible wages (and incomplete financial markets) is exactly the same as under a flexible exchange rate. When labor supply is infinitely elastic, the flexible exchange rate can therefore compensate for the wage rigidity. However, this does not imply that a flexible exchange rate is the dominant one.[15] The adjustments at the extensive and intensive margins are inefficient even when wages are flexible in our setting with incomplete financial markets. This also implies that the fluctuations in the terms of trade under a flexible exchange rate do not reproduce the complete markets allocation.[16] The comparison between the flexible exchange rate and the flexible wage therefore highlights the role of financial market incompleteness for the exchange rate policy decision. In the next section, we solve the model under a fixed exchange rate in order to highlight the differences with the allocation under a flexible exchange rate.

3.2 Fixed Exchange Rate

Under a fixed exchange rate, the allocation in the economy dramatically changes. The monetary stance counteracts the impact of a demand shock such that $\mu_t = 2\mu_0\alpha_t$ and $\mu_t = 2\mu_0(1-\alpha_t)$ in both countries. This sets the exchange rate $\varepsilon_t = 1$ and mitigates the profit fluctuations in the export market. As reported in the third column of Table 2, the equilibrium number of exporters as well as their production scales and prices remain

[15] This allocation is indeed far from the first best allocation. As shown in the Appendix, following a positive demand shift for Home goods, the relative number of Home exporters decreases, whereas it would increase increases in the planner solution.

[16] Note that the unit price elasticity implied by the Cobb-Douglas specification between Home and Foreign basket of goods does not reestablish the allocation of complete markets in our setting with demand shocks.

constant following a relative demand shift.

The fixed exchange rate sterilizes the movements in the extensive and intensive margins of trade. Both the relative number of exporters and their average productivity are not a function of the demand shock. Moreover, there are no fluctuations in the terms of trade. However, a fixed exchange rate induces a drastic change in the domestic market. In order to prevent movements in the nominal exchange rate, the monetary authority intervenes and aggregate demand fluctuates in response to demand shocks. For instance, following a positive demand shock for Home produced goods (a rise in α_t/α_t^*), both the relative Home production scale of domestic firms and the investment in future product varieties rise.

Comparing these expressions with those under a flexible exchange rate, one may notice that the response of the domestic average production and the number of entrants in the domestic economy to a given demand shock are higher than under a flexible exchange rate. The monetary intervention and the resulting fixed exchange rate insulates the trade sector and shifts entirely the impact of the demand shock on firms operating in the domestic market. This highlights the trade-off between fluctuations in the trade sector and the domestic economy.

Finally, the equilibrium wage under a fixed exchange rate depends on the expected interaction between labor demand fluctuations and the monetary response to the demand shock captured by $(A_t\alpha_t)^{1+\varphi}$. The wage under a fixed exchange rate W_t^{FX} therefore depends on the level of each component (A_t and α_t) and the covariance $\mathrm{Cov}(A_t, \alpha_t)$ augmented by the elasticity of labor supply, φ.

There are two effects of monetary intervention on the wage W_t^{FX}. First, it increases wages in level because of an expected higher aggregate demand, captured by α_t. Second, a monetary policy that aims at fixing the exchange rate simultaneously dampens the fluctuations in labor demand and hence uncertainty in the export market namely. This implies that the labor demand and the monetary policy can be *negatively* correlated, i.e. $\mathrm{Cov}(A_t, \alpha_t) < 0$.

Intuitively, under a fixed exchange rate, the profitability of exporters remains constant, whereas the domestic production of incumbents and firm entry rise more abruptly than under a flexible exchange rate. The negative correlation between labor demand and demand shift is a function of the fundamentals of the economy, including the firm productivity distribution. As we will describe later in detail, the larger is the negative correlation between labor demand and demand shift, the more appealing is the stabilizing role played by a fixed exchange rate.

4 Welfare Analysis

In this section, we perform a welfare analysis of different exchange rate policies. As shown previously, a flexible exchange rate is not able to reproduce the benchmark allocation of our economy. In turn, this implies that the allocation under incomplete financial markets and flexible wage is inefficient. We therefore exclude that a flexible exchange rate is the dominant one in our framework with incomplete financial markets. As a consequence, this opens the way for a welfare comparison with an alternative exchange rate policy. In what follows, we first derive the optimal monetary policy in a Nash equilibrium as the monetary authority maximizes the welfare of domestic households. We then compare the welfare in the two polar cases of fixed and flexible exchange rate.

4.1 Optimal monetary policy in a Nash equilibrium

The policy commitment of the monetary authority is to maximize the expected utility of domestic households while taking as given the monetary stance abroad. As previously discussed, the expected utility is a function of utility at time t and $t+1$ uniquely. Given the symmetry across countries, we therefore analyze the problem of Home monetary authority which maximizes $\mathrm{E}_{t-1}[\mathcal{U}]$, where

$$\begin{aligned}\mathrm{E}_{t-1}[\mathcal{U}] = \mathrm{E}_{t-1}\left[\alpha_t\left(\ln N_{D,t}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t}\right) + \alpha_t^*\left(\ln N_{X,t}^{*\frac{\sigma}{\sigma-1}}\frac{\widetilde{y}_{X,t}^*}{\tau}\right)\right] \\ + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\left(\ln N_{D,t+1}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t+1}\right) + \alpha_{t+1}^*\left(\ln N_{X,t+1}^{*\frac{\sigma}{\sigma-1}}\frac{\widetilde{y}_{X,t+1}^*}{\tau_t}\right)\right]. \quad (10)\end{aligned}$$

Equation (10) shows that the expected utility is a function of the expected logarithm of extensive and intensive margins, as well as their covariance with demand shocks.[17] A large *level* of extensive ($N_{D,t}$) and intensive ($\widetilde{y}_{D,t}$) margins improve welfare, but a large *volatility* of these margins is detrimental for welfare. On the other hand, positive covariances between demand shifts and the extensive and intensive margins also improve welfare. This is the case both for domestic and imported goods.

The trade-offs faced by policymakers in an open economy are fundamentally similar to the one discussed in Obstfeld and Rogoff (1998) and Corsetti and Pesenti (2001). As in the related literature, the price of imported goods is indeed sensitive to the fluctuations in nominal exchange rate stemming from the conduct of monetary policy. In addition, in our setting policymakers do not only consider the domestic intensive and extensive margins (i.e. domestic output gap stabilization), but also the selection of importers and their prices. The conduct of monetary policy indeed affects the import prices because of the endogenous entry of heterogeneous firms. In this respect, the impact of firm selection on import prices represents a new dimension of the terms of trade externalities.

We can further arrange the expected utility by plugging the equilibrium expressions in Table 1 and the shock process of equation (9). The expected utility can therefore be expressed as a function of shocks and monetary stances in Home and Foreign. The monetary authority maximizes (10) with respect to μ_t and takes μ_t^* as given. The first order condition with respect to μ_t is

$$\frac{1}{2}\left\{\frac{\upsilon_t}{\mu_t}-\frac{1}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}\frac{(A_t\mu_t)^{1+\varphi}}{\mu_t}\right\} + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\frac{\upsilon_t^{\rho}}{\mu_t}-\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}\frac{(A_t\mu_t)^{1+\varphi}}{\mu_t}\right\}=0. \quad (11)$$

A similar condition applies for Foreign monetary authority (see the Appendix for derivation). Equation (11) shows that the monetary stance comoves with the preference shock

[17]For instance, the first argument in $\mathrm{E}_{t-1}[\mathcal{U}]$ can be written as $\mathrm{E}_{t-1}\left[\alpha_t\left(\ln N_{D,t}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t}\right)\right] = \mathrm{E}_{t-1}[\alpha_t]\left\{\mathrm{E}_{t-1}[\ln N_{D,t}]+\mathrm{E}_{t-1}[\ln\widetilde{y}_{D,t}]\right\}+\left(1+\frac{1}{\sigma-1}\right)cov\left(\alpha_t,\ln N_{D,t}\right)+cov\left(\alpha_t,\ln\widetilde{y}_{D,t}\right)$. The same decomposition applies for the other terms in $\mathrm{E}_{t-1}[\mathcal{U}]$.

hence limiting the fluctuations in the nominal exchange rate. Under the above optimal monetary policy, the nominal exchange rate is expressed as

$$\varepsilon_t^{opt} = \frac{\alpha_t^*}{\alpha_t}\frac{\mu_t}{\mu_t^*} = \frac{\upsilon_t^*}{\upsilon_t}\frac{A_t^*}{A_t}\left[\frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\upsilon_t^* + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{*\rho}}\right]^{\frac{1}{1+\varphi}}.$$

The magnitude of the response of the optimal monetary policy to the demand shock depends upon the fundamentals of our economy. In the following section, we develop the expected utility in the two polar cases of flexible and fixed exchange rate. This will highlight the role of firm heterogeneity on the exchange rate policy.

4.2 Fixed vs. Flexible Exchange Rate

As explained in Section 3, the equilibrium level and the variability of the intensive and extensive margins following the demand shock depend on the exchange rate policy. Once we replace the equilibrium number of firms and their production scales under the two polar exchange rate policies (see Table 2), the welfare difference between a fixed and a flexible exchange rate boils down to

$$\begin{aligned}\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right] &= \frac{1}{2}\left(\frac{1}{\sigma-1} + 2 - \frac{1}{\kappa}\right)\left\{E_{t-1}\left[\upsilon_t \ln \upsilon_t\right] - \Delta \ln W_t\right\} \\ &+ \left(\frac{1}{2}\right)^{1+\rho}\beta\left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right)\left\{E_{t-1}\left[\upsilon_t^{\rho} \ln \upsilon_t\right] - E_{t-1}\left[\upsilon_t^{\rho}\right]\Delta \ln W_t\right\} \quad (12)\end{aligned}$$

where $\Delta \ln W_t \equiv \ln W_t^{FX} - \ln W_t^{FL}$ represents the wage difference between the fixed and flexible exchange rate:[18]

$$\Delta \ln W_t \;\equiv\; \ln W_t^{FX} - \ln W_t^{FL} \;=\; \frac{1}{1+\varphi}\left[\ln E_{t-1}\left[(A_t \upsilon_t)^{1+\varphi}\right] - \ln E_{t-1}\left[A_t^{1+\varphi}\right]\right] \quad (13)$$

In the expression of welfare ranking (12), both $E_{t-1}\left[\upsilon_t \ln \upsilon_t\right]$ and $E_{t-1}\left[\upsilon_t^{\rho} \ln \upsilon_t\right]$ are greater than 0. These terms capture the welfare gain stemming from the better congruence between the preference shock and the amount of both domestic and imported goods under a fixed exchange rate. However, the fluctuations of monetary stance in response to the

[18] See Appendix for more details.

stochastic preference shocks under a fixed exchange rate may be costly and detrimental to welfare when $\Delta \ln W_t$ is positive.

We now discuss the role played by the fundamentals of our economy in the welfare comparison (12). First, we explore the role of firm heterogeneity on the sign and magnitude of the wage difference $\Delta \ln W_t$. Second, we discuss the welfare implications arising from current and future number of varieties.

4.2.1 The effects of a fixed exchange rate on labor demand fluctuations

In determining the welfare ranking in (12), in addition to the variety effect, the sign of $\Delta \ln W_t$ is crucial. In turn, the sign of $\Delta \ln W_t$ depends on the covariance terms in the equilibrium wage under a fixed exchange rate W_t^{FX}. Under a fixed exchange rate, the monetary intervention increases the expected labor demand (*direct effect*), but at the same time it dampens labor demand fluctuations due to the selection in the export market (*indirect effect*). The intuition can be best described by setting $\varphi = 0$ (the case of infinitely elastic labor supply). In such a case, the wage difference equation (13) is expressed as

$$\Delta \ln W_t \mid_{\varphi=0} = \left[\ln E_{t-1}\left[(A_t \upsilon_t)\right] - \ln E_{t-1}\left[A_t\right]\right] = \ln \left[1 + \frac{E_{t-1}\left[\upsilon_t\right] + Cov\left(A_t, \upsilon_t\right)}{E_{t-1}\left[A_t\right]}\right].$$

The term $E_{t-1}\left[\upsilon_t\right]$ is the *direct effect*, that is the positive first order effect of the monetary intervention under a fixed exchange rate. $Cov\left(A_t, \upsilon_t\right)$ represents instead the *indirect effect,* which is the second order effect stemming from the covariance between the labor demand, captured by A_t as in equation (8), and the monetary stance which responds to the demand shock υ_t.

The sign of the *indirect* effect is given by the derivative of A_t with respect to the monetary stance of a fixed exchange rate. Assuming a symmetric steady state across countries, i.e. $\alpha_{t-1} = \alpha_{t-1}^*$, we have

$$\frac{\partial A_t}{\partial \upsilon_t} = -\frac{1}{2\sigma}\left(1 - \frac{\sigma - 1}{\kappa}\right)\left[1 - \left(\frac{1}{2}\right)^{\rho} \beta \rho \upsilon_t^{\rho-1}\right] < 0. \tag{14}$$

The expression is strictly negative indicating a negative covariance between labor demand and monetary intervention. Importantly, the extent of the negative covariance depends on the degree of firm productivity dispersion. When κ is high, firms are less dispersed and less productive, and the relative number of exporter over domestic firms is higher. Labor demand is higher because there are more exporter firms potentially affected by the demand shock (larger extensive margin of trade). In such a situation, the active monetary policy ensures a fixed exchange rate which stabilizes the firm selection in the export market and hence the volatile labor demand, improving welfare. In this case, the *indirect* covariance effect mitigates the *direct* level effect on higher labor demand. As a consequence, the equilibrium wage difference $\Delta \ln W_t$ therefore decreases with a lower firm dispersion. When firms are small and less dispersed, the monetary intervention can be a powerful stabilization tool as the suboptimal wage markup under a fixed exchange rate is relatively lower.

In addition to the variety effect, the elasticity of substitution among goods σ also determines the size of the negative covariance hence the wage difference across different exchange rate policies. As σ increases, goods are more substitutable and relative demand shifts require less movements in labor demand. As a consequence, there is less need of a monetary intervention to stabilize the trade sector.[19]

4.2.2 Variety effect with selection in the export market

In the expression (12), the term $\frac{1}{\sigma-1} - \frac{1}{\kappa} > 0$ captures the balance between preference for the *current* number of imported varieties and the price of those varieties. Other things equal, the gain under a fixed exchange rate - due to the improved congruence between preferences and imported number of varieties - increases with a higher preference for variety (lower value of σ) and decreases with a higher firm dispersion (lower value of κ). This is because when the number of imported varieties goes up, these varieties are produced by less efficient firms that charge expensive prices on average. Given the love for variety, the welfare gain in consuming a higher number imported varieties is fully realized

[19]To be precise, the derivative of (14) with respect to σ gives

$$\frac{\partial A_t/\partial \upsilon_t}{\partial \sigma} = \frac{1}{2\sigma}\left[\frac{1}{\kappa} + \frac{1}{\sigma}\left(1 - -\frac{\sigma-1}{\kappa}\right)\right]\left[1 - \left(\frac{1}{2}\right)^{\rho} \beta\rho\upsilon_t^{\rho-1}\right] < 0.$$

when exporters are homogeneous ($\kappa = \infty$) and hence there is no increase in import prices.

In a similar way, the term $\left(\frac{1}{2}\right)^{1+\rho} \beta \left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right)$ in the expression (12) determines the welfare impact on the number of *future* domestic varieties and the *future* cutoff level of imported goods. Other things equal, a rise in the future number of domestic products provides a higher utility gain when the love for variety is high (lower value of σ). Note that the impact is amplified by a lower discount factor (a higher value of β). A higher persistence of the shock (a higher value of ρ) also amplifies this effect, as a current positive shock will result in a higher number of future varieties. Furthermore, a rise in the future number of varieties toughens the selection in the export market in the next period. As a result, the future cutoff level increases due to selection and the price of imported varieties become cheaper. From this channel, the higher the value of κ, the lower is the welfare gain because of the survival of less efficient producers that charge higher prices in the future.

As previously discussed, under incomplete financial markets a fixed exchange rate allows a larger consumption of the good which is relatively more demanded. This finding is similar to Devereux (2004) where the number of producers is fixed. As Hamano and Picard (2017) emphasize in a model with firm entry but without firm heterogeneity, a fixed exchange rate can further amplify this welfare improvement by realizing a better congruence between preference shift and extensive margins. In our framework, the welfare improving variety effect is also active but it is mitigated by the entry of less productive producers.

4.2.3 Numerical illustration

We now provide a numerical illustration of our analytical results. First, we document the variability of the nominal exchange rate under the optimal monetary policy with respect to different values of κ and σ. Figure 1 shows that the variability of the nominal exchange rate is decreasing with κ and increasing with σ. As the firm productivity dispersion decreases (κ increases), there is a higher number of homogeneous firms subject to uncertainty about their export profits. As a consequence, wages are higher and the monetary intervention is beneficial in stabilizing the export turnover. Accordingly, the optimal fluctuations in the nominal exchange rate decrease with κ. With respect to the elasticity of substitution among goods, the covariance is decreasing with a higher value of σ and the welfare improving impact of monetary intervention is thus weaker. As shown in the right panel of Figure 1, the optimal volatility of the nominal exchange rate indeed increases with σ.

The variability of the nominal exchange rate is determined by the optimal monetary policy. As such, this may be interpreted as an intermediate solution of the polar cases of flexible and fixed exchange rate. Panel a) of Figure 2 reports the difference in terms of utility between the fixed and flexible exchange rate for different values of κ. As previously discussed, when κ is high, the wage difference between the two polar exchange rate policies is reduced and a fixed exchange rate provides a higher welfare. Similarly, with a higher value of σ, labor demand is low in export sector due to a tougher competition and the welfare improving effect of monetary intervention is lower. As shown in panel b) of Figure 2, the wage gap between the two exchange rate policies is increasing with σ, and a fixed exchange rate is less supported.

Finally, when the shock persistence ρ is lower or the elasticity of labor supply $1/\varphi$ is higher, the covariance decreases giving more support to a fixed exchange rate.[20] Indeed, the robustness analysis in Figure 3 shows that the welfare implications of the *current* monetary intervention are dampened when shocks have a lower persistence.[21]

5 Conclusion

This paper examines the exchange rate policy in a model with endogenous firm entry and selection in the export market. We show that the response of monetary policy to external demand shocks determines exchange rate fluctuations in presence of nominal rigidities. The flexible price allocation is not efficient under incomplete financial markets, raising a

[20]This is in line with Devereux (2004) and Hamano and Picard (2017).

[21]The same mechanism applies for a higher discount factor β. In this case, workers put a lower weight on the monetary intervention which does not have a persistent impact on their future welfare.

discussion about the desired exchange rate policy.

In our model, a flexible exchange rate implies a high volatility of the extensive margin of trade especially when firms are small and less heterogeneous. On the other hand, a fixed exchange rate limits the fluctuations of profits for exporters and shuts down the fluctuations of firm selection in the export market. Despite generating a substantially high volatility in the domestic market, a fixed exchange rate helps reducing the uncertainty about future labor demand in the export market. A sub-optimally high wage markup under a fixed exchange rate is thus reduced when the turnover of exporters is relatively important. For this reason, a fixed exchange rate may dominate when firms are small and less heterogeneous.

Our paper therefore provides a rationale for the observed "currency manipulation" policies aimed at protecting exporter firms from excessive volatility in their export market profits. While we consider a two country framework, our findings would hold true in a small open economy framework adding a new dimension to the *fear of floating* that often hits emerging markets. In the meantime, our paper suggests that the presence of large and efficient exporters is a rationale for a flexible exchange rate since the selection in the export markets is less sensitive to external demand shocks. Finally, we focus uniquely on the impact of monetary intervention in response to external demand shocks from a qualitative standpoint, leaving for further research the quantitative implications under alternative shocks and nominal frictions.

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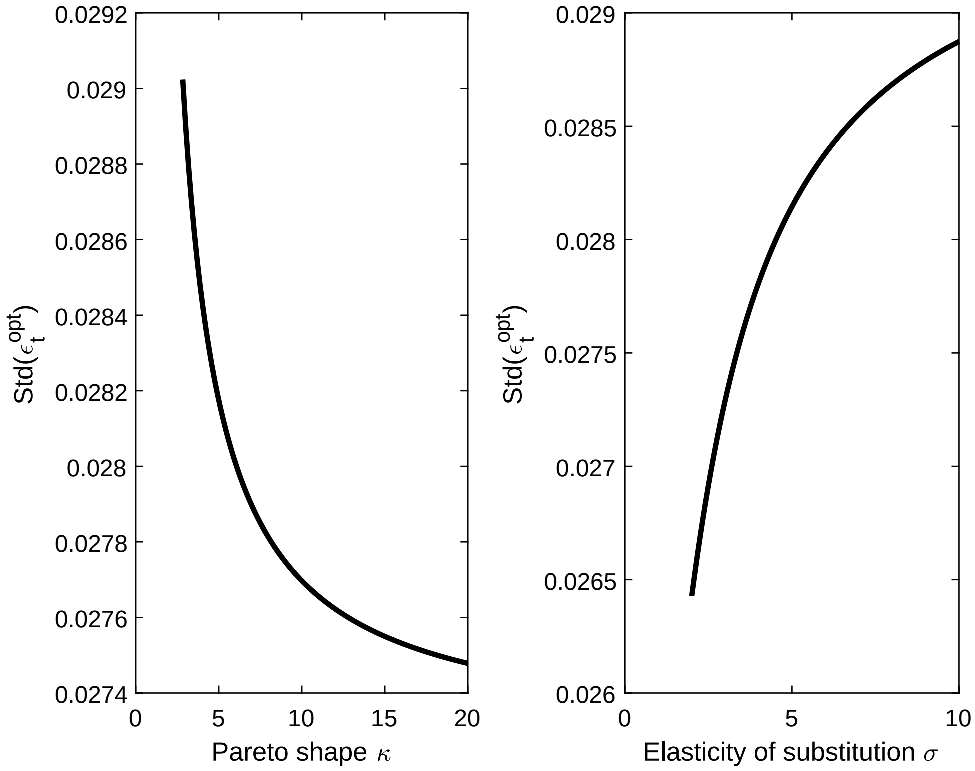
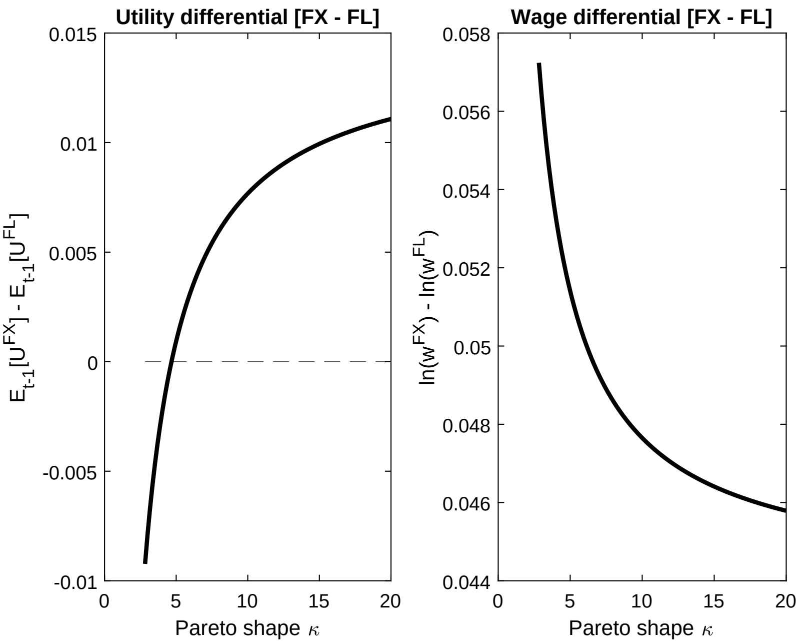
Figures and tables

Figure 1: Optimal monetary policy: nominal exchange rate variability.

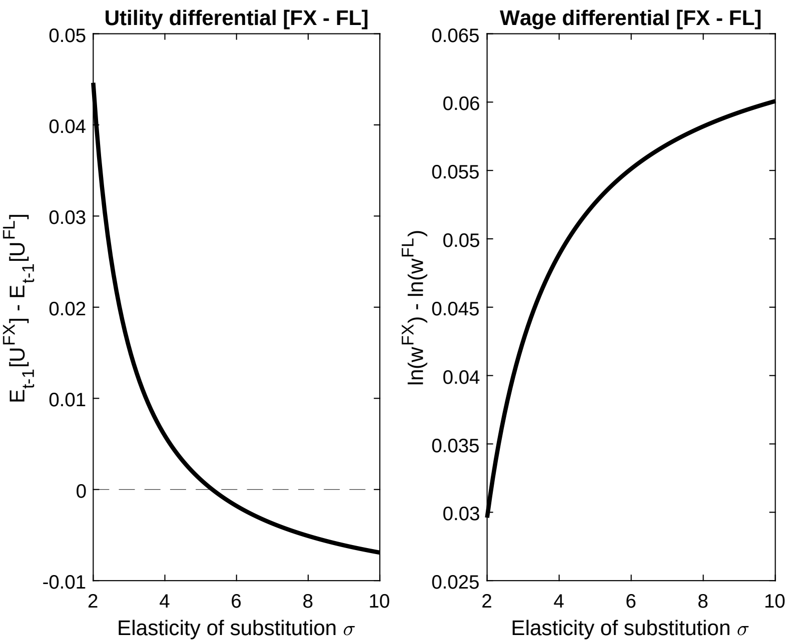


Notes. In the benchmark calibration, we set the value of $\sigma = 3.8$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$. In panel a) the variance of the nominal exchange rate is shown for different values of κ. In panel b) it is shown for different values of σ, while keeping $\kappa = (10 - 1) * 1.05$.

Figure 2: Welfare ranking and wage differential: fixed vs. flexible exchange rate.



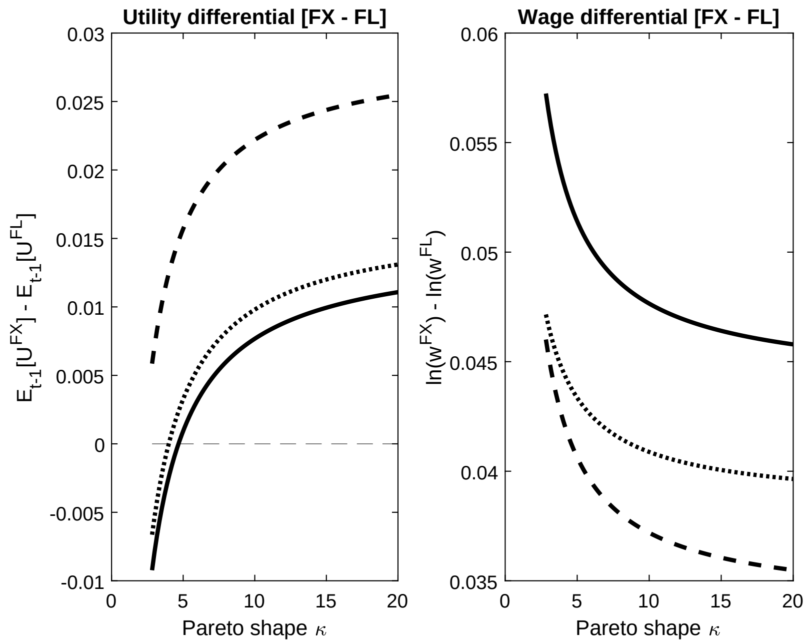
(a) Firm productivity distribution



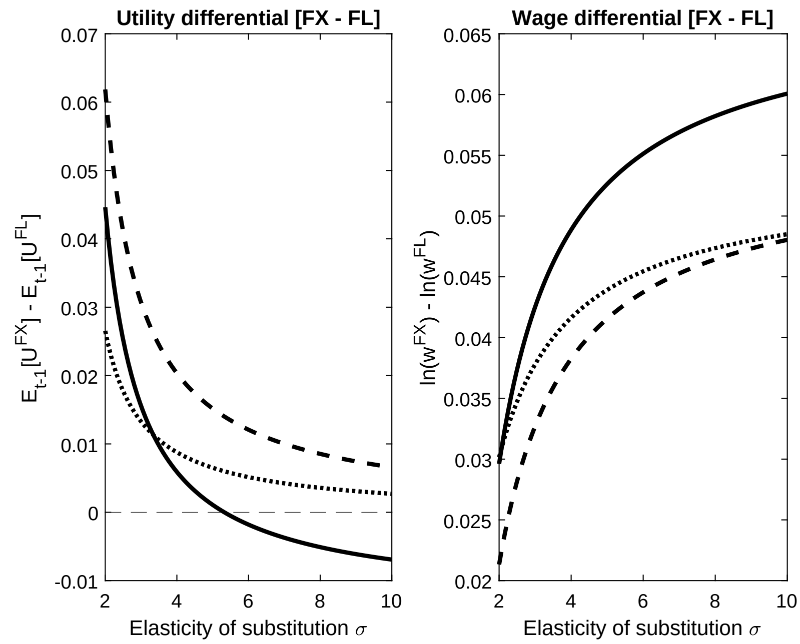
(b) Elasticity of substitution

Notes. In the benchmark calibration, we set the value of $\sigma = 3.8$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$. In panel a) $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ and $\ln(W^{FX}) - \ln(W^{FL})$ are shown for different values of κ. In panel b) they are shown for different values of σ, while keeping $\kappa = (10-1) * 1.05$.

Figure 3: Welfare ranking and wage differential - robustness.



(a) Firm productivity distribution



(b) Elasticity of substitution

Notes. With respect to the benchmark calibration (solid line), the dashed line refers to $\varphi^{-1} = 1$, and the dotted line to $\rho = 0.5$. In panel a) $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ and $\ln(W^{FX}) - \ln(W^{FL})$ are shown for different values of κ. In panel b) they are shown for different values of σ, while keeping $\kappa = (10-1) \cdot 1.05$.

Table 1: The Model's Closed Form Solution

Nb of Entrants	$N_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}} E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right]$	$N^*_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu^*_t}{W^*_t f^*_{E,t}} E_t\left[\alpha^*_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]$
Nb of Exporters	$N_{X,t} = \frac{1}{\sigma}(1-\frac{\sigma-1}{\kappa})\frac{\alpha^*_t\mu_t}{W_t f_{X,t}}$	$N^*_{X,t} = \frac{1}{\sigma}(1-\frac{\sigma-1}{\kappa})\frac{\alpha_t\mu^*_t}{W^*_t f^*_{X,t}}$
Av. Exporters	$\widetilde{z}_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$	$\widetilde{z}^*_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N^*_{X,t}}{N^*_{D,t}}\right)^{-\frac{1}{\kappa}}$
Production	$\widetilde{y}_{D,t} = \frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t\widetilde{z}_D}{N_{D,t}W_t}, \quad \widetilde{y}_{X,t} = \frac{\sigma-1}{\sigma}\frac{\alpha^*_t\mu_t\widetilde{z}_{X,t}}{N_{X,t}W_t}$	$\widetilde{y}^*_{D,t} = \frac{\sigma-1}{\sigma}\frac{\alpha^*_t\mu^*_t\widetilde{z}^*_D}{N^*_{D,t}W^*_t}, \quad \widetilde{y}^*_{X,t} = \frac{\sigma-1}{\sigma}\frac{\alpha_t\mu^*_t\widetilde{z}^*_{X,t}}{N^*_{X,t}W^*_t}$
Average Price	$\widetilde{p}_{D,t} = \frac{\sigma}{\sigma-1}\frac{W_t}{\widetilde{z}_D}, \quad \widetilde{p}_{X,t} = \frac{\sigma}{\sigma-1}\frac{\tau_t\varepsilon_t^{-1}W_t}{\widetilde{z}_{X,t}}$	$\widetilde{p}^*_{D,t} = \frac{\sigma}{\sigma-1}\frac{W^*_t}{\widetilde{z}^*_D}, \quad \widetilde{p}^*_{X,t} = \frac{\sigma}{\sigma-1}\frac{\tau_t\varepsilon_t W^*_t}{\widetilde{z}^*_{X,t}}$
Price Indices	$P_{H,t} = N_{D,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{D,t}, \quad P_{F,t} = N_{X,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^*_{X,t}, \quad P_t = P_{H,t}^{\alpha_t}P_{F,t}^{\alpha^*_t}$	$P^*_{F,t} = N_{D,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^*_{D,t}, \quad P^*_{H,t} = N_{X,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{X,t}, \quad P^*_t = P_{F,t}^{*\alpha^*_t}P_{H,t}^{*\alpha_t}$
Consumption	$C_t = \left(\frac{C_{H,t}}{\alpha_t}\right)^{\alpha_t}\left(\frac{C_{F,t}}{\alpha^*_t}\right)^{\alpha^*_t}$	$C^*_t = \left(\frac{C^*_{F,t}}{\alpha^*_t}\right)^{\alpha^*_t}\left(\frac{C^*_{H,t}}{\alpha_t}\right)^{\alpha_t}$
Profits	$\widetilde{D}_{D,t} = \frac{\alpha_t}{\sigma}\frac{\mu_t}{N_{D,t}}, \quad \widetilde{D}_{X,t} = \frac{\sigma-1}{\kappa}\frac{\alpha_t}{\sigma}\frac{\varepsilon_t\mu^*_t}{N_{X,t}}, \quad \widetilde{D}_t = \widetilde{D}_{D,t} + \frac{N_{X,t}}{N_{D,t}}\widetilde{D}_{X,t}$	$\widetilde{D}^*_{D,t} = \frac{\alpha^*_t}{\sigma}\frac{\mu^*_t}{N^*_{D,t}}, \quad \widetilde{D}^*_{X,t} = \frac{\sigma-1}{\kappa}\frac{\alpha^*_t}{\sigma}\frac{\varepsilon_t^{-1}\mu_t}{N^*_{X,t}}, \quad \widetilde{D}^*_t = \widetilde{D}^*_{D,t} + \frac{N^*_{X,t}}{N^*_{D,t}}\widetilde{D}^*_{X,t}$
ZPC	$\widetilde{D}_{X,t} = W_t f_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$	$\widetilde{D}^*_{X,t} = W^*_t f^*_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$
Share Price	$\widetilde{V}_t = f_{E,t}W_t$	$\widetilde{V}^*_t = f^*_{E,t}W^*_t$
Labor Supply	$L_t = (\sigma-1)\frac{N_{D,t}\widetilde{D}_t}{W_t} + \sigma N_{X,t}f_{X,t} + N_{D,t+1}f_{E,t}$	$L^*_t = (\sigma-1)\frac{N^*_{D,t}\widetilde{D}^*_t}{W^*_t} + \sigma N^*_{X,t}f^*_{X,t} + N^*_{D,t+1}f^*_{E,t}$
Monetary Stance	$\mu_t = P_t C_t$	$\mu^*_t = P^*_t C^*_t$
Wages	$W_t = \Gamma\left\{\frac{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}{E_{t-1}[A_t]}\right\}^{\frac{1}{1+\varphi}}$	$W^*_t = \Gamma\left\{\frac{E_{t-1}\left[(A^*_t\mu^*_t)^{1+\varphi}\right]}{E_{t-1}[A^*_t]}\right\}^{\frac{1}{1+\varphi}}$
Exchange Rate	$\varepsilon_t = \frac{\alpha^*_t}{\alpha_t}\frac{\mu_t}{\mu^*_t}$	
Definition of A_t	$A_t = \frac{\sigma-1}{\sigma}\alpha_t + \left(1-\frac{\sigma-1}{\sigma\kappa}\right)\alpha^*_t + \frac{\beta}{\sigma}E_{t-1}\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right]$	$A^*_t = \frac{\sigma-1}{\sigma}\alpha^*_t + \left(1-\frac{\sigma-1}{\sigma\kappa}\right)\alpha_t + \frac{\beta}{\sigma}E_{t-1}\left[\alpha^*_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]$
Shock Process	$\alpha_t = \frac{1}{2}\alpha^{\rho}_{t-1}\upsilon_t, \quad \alpha^*_t = \frac{1}{2}\alpha^{*\rho}_{t-1}\upsilon^*_t,\ \alpha_0 = \alpha^*_0 = 1, \quad E_{t-1}[\upsilon_t] = E_{t-1}[\upsilon^*_t] = 1, \quad E_{t-1}[\upsilon_t\upsilon_{t+1}] = 1, \quad \upsilon_t + \upsilon^*_t = 2, \quad 0 < \rho < 1$	

Table 2: Selected variables under alternative exchange rate policies

	Without nominal rigidities	Flexible exchange rate	Fixed exchange rate
Wages (Home)	$W_t^{FW} = A_t^{\frac{\varphi}{1+\varphi}}$	$W_t^{FL} = \Gamma\mu_0 \left\{ \frac{E_{t-1}\left[A_t^{1+\varphi}\right]}{E_{t-1}[A_t]} \right\}^{\frac{1}{1+\varphi}}$	$W_t^{FX} = 2\Gamma\mu_0 \left\{ \frac{E_{t-1}\left[(A_t\alpha_t)^{1+\varphi}\right]}{E_{t-1}[A_t]} \right\}^{\frac{1}{1+\varphi}}$
Relative average production	$\frac{\widetilde{y}_{D,t}^{FW}}{\widetilde{y}_{D,t}^{*FW}} = \frac{\alpha_t \widetilde{z}_D}{\alpha_t^* \widetilde{z}_D^*} \left(\frac{L_t^{*FW}}{L_t^{FW}}\right)^{\varphi} \frac{N_{D,t}^{*FW}}{N_{D,t}^{FW}}$	$\frac{\widetilde{y}_{D,t}^{FL}}{\widetilde{y}_{D,t}^{*FL}} = \frac{\alpha_t \widetilde{z}_D}{\alpha_t^* \widetilde{z}_D^*} \frac{N_{D,t}^{*FL} W_t^{*FL}}{N_{D,t}^{FL} W_t^{FL}}$	$\frac{\widetilde{y}_{D,t}^{FX}}{\widetilde{y}_{D,t}^{*FX}} = \frac{\alpha_t^2 \widetilde{z}_D}{\alpha_t^{*2} \widetilde{z}_D^*} \frac{N_{D,t}^{*FX} W_t^{*FX}}{N_{D,t}^{FX} W_t^{FX}}$
Relative number of exporters	$\frac{N_{X,t}^{FW}}{N_{X,t}^{*FW}} = \frac{\alpha_t^*}{\alpha_t} \left(\frac{L_t^{*FW}}{L_t^{FW}}\right)^{\varphi} \frac{f_{X,t}^*}{f_{X,t}}$	$\frac{N_{X,t}^{FL}}{N_{X,t}^{*FL}} = \frac{\alpha_t^*}{\alpha_t} \frac{W_t^{*FL} f_X^*}{W_t^{FL} f_X}$	$\frac{N_{X,t}^{FX}}{N_{X,t}^{*FX}} = \frac{W_t^{*FX} f_X^*}{W_t^{FX} f_X}$
Relative average productivity	$\frac{\widetilde{z}_{X,t}^{FW}}{\widetilde{z}_{X,t}^{*FW}} = \left(\frac{N_{X,t}^{FW}}{N_{X,t}^{*FW}} \frac{N_{D,t}^{*FW}}{N_{D,t}^{FW}}\right)^{-\frac{1}{\kappa}}$	$\frac{\widetilde{z}_{X,t}^{FL}}{\widetilde{z}_{X,t}^{*FL}} = \left(\frac{N_{X,t}^{FL}}{N_{X,t}^{*FL}} \frac{N_{D,t}^{*FL}}{N_{D,t}^{FL}}\right)^{-\frac{1}{\kappa}}$	$\frac{\widetilde{z}_{X,t}^{FX}}{\widetilde{z}_{X,t}^{*FX}} = \left(\frac{N_{X,t}^{FX}}{N_{X,t}^{*FX}} \frac{N_{D,t}^{*FX}}{N_{D,t}^{FX}}\right)^{-\frac{1}{\kappa}}$
Relative number of entrants	$\frac{N_{D,t+1}^{FW}}{N_{D,t+1}^{*FW}} = \left(\frac{L_t^{*FW}}{L_t^{FW}}\right)^{\varphi} \frac{f_E^* E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]}{f_E E_t\left[\alpha_{t+1}^* + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]}$	$\frac{N_{D,t+1}^{FL}}{N_{D,t+1}^{*FL}} = \frac{W_t^{*FL} f_E^* E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]}{W_t^{FL} f_E E_t\left[\alpha_{t+1}^* + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]}$	$\frac{N_{D,t+1}^{FX}}{N_{D,t+1}^{*FX}} = \frac{\alpha_t}{\alpha_t^*} \frac{W_t^{*FX} f_E^* E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]}{W_t^{FX} f_E E_t\left[\alpha_{t+1}^* + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]}$
Terms of trade	$TOT_t^{FW} = \frac{\alpha_t^*}{\alpha_t} \left(\frac{L_t^{*FW}}{L_t^{FW}}\right)^{\varphi} \frac{\widetilde{z}_{X,t}^{FW}}{\widetilde{z}_{X,t}^{*FW}}$	$TOT_t^{FL} = \frac{\alpha_t^*}{\alpha_t} \frac{W_t^{*FL}}{W_t^{FL}} \frac{\widetilde{z}_{X,t}^{FL}}{\widetilde{z}_{X,t}^{*FL}}$	$TOT_t^{FX} = \frac{W_t^{*FX}}{W_t^{FX}} \frac{\widetilde{z}_{X,t}^{FX}}{\widetilde{z}_{X,t}^{*FX}} = 1$

APPENDIX

A Solution of the Model

We derive here the closed form solution of the theoretical model presented in Table 1. Similar expressions hold for Foreign. First, note using average prices and the expressions of price indices, we have $P_{H,t} = N_{D,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{D,t}$ and $P_{F,t} = N_{X,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^{*}_{X,t}$. Plugging these expressions in the expression of domestic profits, profits from exporting and total profits on average, we have $\widetilde{D}_{D,t} = \frac{\alpha_t}{\sigma}\frac{\mu_t}{N_{D,t}}$, $\widetilde{D}_{X,t} = \frac{\alpha_t}{\sigma}\frac{\varepsilon_t\mu^*_t}{N_{X,t}} - f_{X,t}W_t$ and $\widetilde{D}_t = \widetilde{D}_{D,t} + \frac{N_{X,t}}{N_{D,t}}\widetilde{D}_{X,t}$. With zero cutoff profits (ZCP) condition, we have $\widetilde{D}_{X,t} = W_t f_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$. Note that by combining these two expressions of $\widetilde{D}_{X,t}$ we have $\widetilde{D}_{X,t} = \frac{\sigma-1}{\kappa}\frac{\alpha_t}{\sigma}\frac{\varepsilon_t\mu^*_t}{N_{X,t}}$. Also with ZCP and the expression of $\widetilde{D}_{X,t}$ previously found and the exchange rate implied under the balanced trade, $\varepsilon_t = \frac{\alpha^*_t}{\alpha_t}\frac{\mu_t}{\mu^*_t}$, we have $N_{X,t} = \frac{1}{\sigma}(1 - \frac{\sigma-1}{\kappa})\frac{\alpha^*_t\mu_t}{W_t f_{X,t}}$. With the Pareto distribution as in the paper, it implies that $\widetilde{z}_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$.

We are now ready to derive the number of new entrant, $N_{D,t+1}$. Free entry implies that $\widetilde{V}_t = f_{E,t}W_t$. Combined with the expression of $\widetilde{D}_{t+1}$, the Euler equation about the share holdings, $\widetilde{V}_t = E_t\left[Q_{t,t+1}\widetilde{D}_{t+1}\right]$, is expressed as

$$E_t\left[\frac{\beta P_t C_t}{P_{t+1}C_{t+1}}\left(\widetilde{D}_{D,t+1} + \frac{N_{X,t+1}}{N_{D,t+1}}\widetilde{D}_{X,t+1}\right)\right] = f_{E,t}W_t.$$

Plugging the expression of $\widetilde{D}_{D,t+1}$, $\widetilde{D}_{X,t+1}$ and using the definition of monetary stance, it is rewritten as

$$E_t\left[\frac{\beta\mu_t}{\mu_{t+1}}\left(\frac{\alpha_{t+1}}{\sigma}\frac{\mu_{t+1}}{N_{D,t+1}} + \frac{N_{X,t+1}}{N_{D,t+1}}\frac{\sigma-1}{\kappa}\frac{\alpha_{t+1}}{\sigma}\frac{\varepsilon_{t+1}\mu^*_{t+1}}{N_{X,t+1}}\right)\right] = f_{E,t}W_t$$

Further, by plugging the expression of the equilibrium exchange rate $\varepsilon_t = \frac{\alpha^*_t}{\alpha_t}\frac{\mu_t}{\mu^*_t}$ and rearranging the terms, we have

$$\frac{\beta}{\sigma}\frac{\mu_t}{N_{D,t+1}}E_t\left[\left(\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right)\right] = f_{E,t}W_t$$

which gives $N_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}}E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right]$.

Next we derive the labor demand in general equilibrium. Note that $\widetilde{D}_{X,t} = \frac{1}{\sigma}\frac{\varepsilon_t \widetilde{p}_{X,t}}{\tau}\widetilde{y}_{X,t} - f_{X,t}W_t$ and $\widetilde{D}_{D,t} = \frac{1}{\sigma}\widetilde{p}_{D,t}\widetilde{y}_{D,t}$. Also plugging the expression of prices into these profits, we have $\widetilde{y}_{D,t} = (\sigma-1)\frac{\widetilde{D}_{D,t}\widetilde{z}_D}{W_t}$ and $\widetilde{y}_{X,t} = (\sigma-1)\frac{\left(\widetilde{D}_{X,t}+f_{X,t}W_t\right)\widetilde{z}_{X,t}}{W_t}$. Putting these expression of intensive margins of average domestic and exporter firms in the labor market clearings (8), we have

$$L_t = N_{D,t}(\sigma-1)\frac{\widetilde{D}_{D,t}}{W_t} + N_{X,t}\left((\sigma-1)\frac{\widetilde{D}_{X,t}+f_{X,t}W_t}{W_t} + f_{X,t}\right) + N_{D,t+1}f_{E,t}$$

Plugging the expression of $\widetilde{D}_{D,t}$ and $\widetilde{D}_{X,t}$ found previously, the above expression becomes

$$L_t = \frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t}{W_t} + \frac{(\sigma-1)^2}{\sigma\kappa}\frac{\alpha_t\varepsilon_t\mu_t^*}{W_t} + \sigma N_{X,t}f_{X,t} + N_{D,t+1}f_{E,t}$$

Further, plugging $N_{D,t+1}$, $N_{X,t}$ and the exchange rate found previously, we have

$$L_t = \frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t}{W_t} + \frac{(\sigma-1)^2}{\sigma\kappa}\frac{\alpha_t^*\mu_t}{W_t} + (1-\frac{\sigma-1}{\kappa})\frac{\alpha_t^*\mu_t}{W_t} + \frac{\beta}{\sigma}\frac{\mu_t}{W_t}E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]$$

which can be further rewritten as

$$L_t = \frac{\mu_t}{W_t}\left[\frac{\sigma-1}{\sigma}\alpha_t + (1-\frac{\sigma-1}{\sigma\kappa})\alpha_t^* + \frac{\beta}{\sigma}E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]\right]$$

Finally, plugging the expression found in wage setting equation (6), we have

$$W_t = \Gamma\left\{\frac{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\}^{\frac{1}{1+\varphi}}.$$

where

$$A_t \equiv \frac{\sigma-1}{\sigma}\left\{\alpha_t + \left(1+\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\alpha_t^* + \beta\left[\frac{1}{\sigma-1}\alpha_{t+1} + \frac{1}{\kappa}\alpha_{t+1}^*\right]\right\}$$

captures the labor demand in the right hand side of equation (8), and $\Gamma \equiv \left[\frac{\eta\theta}{(\theta-1)(1+\xi)}\right]^{\frac{1}{1+\varphi}}$. Note that assuming a symmetric steady state across countries, i.e. $\alpha_{t-1} = \alpha_{t-1}^*$, we can express A_t as a function of fundamental shocks as follows

$$A_t = \frac{\sigma-1}{\sigma}\left\{\alpha_t + \left(1+\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\alpha_t^* + \beta\left[\frac{1}{\sigma-1}\alpha_{t+1} + \frac{1}{\kappa}\alpha_{t+1}^*\right]\right\}$$
$$= \frac{1}{2}\frac{\sigma-1}{\sigma}\left\{\alpha_{t-1}^{\rho}\upsilon_t + \left(1+\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\alpha_{t-1}^{*\rho}\upsilon_t^* + \beta\left[\frac{1}{\sigma-1}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t\right)^{\rho}\upsilon_{t+1} + \frac{1}{\kappa}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t\right)^{\rho}\upsilon_{t+1}\right]\right\}$$
$$= \frac{1}{2}\frac{\sigma-1}{\sigma}\left\{\upsilon_t + \left(1+\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\upsilon_t^* + \beta\left(\frac{1}{2}\right)^{\rho}\left[\frac{1}{\sigma-1}\upsilon_t^{\rho}\upsilon_{t+1} + \frac{1}{\kappa}\upsilon_t^{*\rho}\upsilon_{t+1}\right]\right\}.$$

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A.1 Producer Currency Pricing versus Local Currency Pricing

In the literature, it has been widely discussed the importance of producer currency pricing versus local currency pricing known as a pricing to market. Let us depart from our modeling of producer currency pricing and consider instead the case of local currency pricing. We assume that the price is set one period in advance at the same timing as wage setting. Without stochastic productivity shock, the equilibrium price of exported goods under the local currency pricing is given by

$$p_{X,t}(z) = \frac{\sigma\tau}{\sigma-1}\frac{W_t}{z}E_{t-1}\left[\varepsilon_t^{-1}\right]$$

Intuitively, the price of export should take into account the future level of exchange rate as a result of direct pricing in local currency. Because of this additional distortion on top of the nominal wage rigidity, the equilibrium outcome (including W_t) is different from the solution of the benchmark model. We have chosen to introduce distortions in the wage setting rather than in the price setting behavior of firms for the simplicity and the clarity of the analysis. However, considering a more realistic model with local currency pricing and, for instance, uncertainty in aggregate productivity, would be an interesting extension at least from a quantitative standpoint.

B Planner's Solution

In this section, we highlight the role of incomplete financial markets. In order to do so, we start by deriving the first best allocation implied by the social planner. We show that the planner solution is close to the solution in our model when we allow for complete markets (CM) and flexible wages (FW). The only difference stems from the monopolistic distortions in both goods and labor markets that are absent in the planner allocation. We can therefore consider our framework with $CM+FW$ as the benchmark allocation against which we measure the performance of different exchange rate policies in our setting with nominal rigidities and incomplete financial markets.

We first show the solution of a benevolent social planner. By definition, the social planner is not subject to the nominal wage rigidity, but faces technological and resource constraints. The expected discounted sum of utility is defined over an infinite horizon of time, but the intervention of the social planner at time t has an impact only for two consecutive time periods. This is due to the assumption of one period to build and the full depreciation of firms after one period of production. In deriving the welfare metrics, we thus express the expected utility only for two consecutive periods without loss of generality as

$$\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] &\equiv \mathrm{E}_{t-1}[U_t] + \beta \mathrm{E}_{t-1}[U_{t+1}] \\
&= \mathrm{E}_{t-1}[\ln C_t] - \frac{\eta}{1+\varphi}\mathrm{E}_{t-1}\left[L_t^{1+\varphi}\right] + \beta\left\{\mathrm{E}_{t-1}[\ln C_{t+1}] - \frac{\eta}{1+\varphi}\mathrm{E}_{t-1}\left[L_{t+1}^{1+\varphi}\right]\right\} \\
&= \mathrm{E}_{t-1}\left[\alpha_t\left(1+\frac{1}{\sigma-1}\right)\ln N_{D,t} + \alpha_t \ln\widetilde{y}_{D,t} + \alpha_t^*\left(1+\frac{1}{\sigma-1}\right)\ln N_{X,t}^* + \alpha_t^*\ln\widetilde{y}_{X,t}^*\right] \\
&\qquad - \frac{\eta}{1+\varphi}\mathrm{E}_{t-1}\left[L_t^{1+\varphi}\right] \\
&+\beta \mathrm{E}_{t-1}\left[\alpha_{t+1}\left(1+\frac{1}{\sigma-1}\right)\ln N_{D,t+1} + \alpha_{t+1}\ln\widetilde{y}_{D,t+1} + \alpha_{t+1}^*\left(1+\frac{1}{\sigma-1}\right)\ln N_{X,t+1}^* + \alpha_{t+1}^*\ln\widetilde{y}_{X,t+1}^*\right] \\
&\qquad - \frac{\beta\eta}{1+\varphi}\mathrm{E}_{t-1}\left[L_{t+1}^{1+\varphi}\right]
\end{aligned}$$

From the second to the third line, we have used good market clearing: $\widetilde{c}_{D,t} = \widetilde{y}_{D,t}, \widetilde{c}_{X,t} = \widetilde{y}_{X,t}^* \widetilde{c}_{D,t}^* = \widetilde{y}_{D,t}^*, \widetilde{c}_{X,t}^* = \widetilde{y}_{X,t}$. As argued in the text, the planner maximizes $\mathrm{E}_{t-1}[\mathcal{U}] + \mathrm{E}_{t-1}[\mathcal{U}^*]$ with respect to $\widetilde{y}_{D,t}, \widetilde{y}_{D,t}^*, N_{D,t+1}, N_{D,t+1}^*, \widetilde{y}_{X,t}, \widetilde{y}_{X,t}^*, N_{X,t}, N_{X,t}^*$ by plugging two types of technological constraints, namely (5) and (8) for each country. The solution is given by Table B1.

The planner solution corresponds to an optimal cooperative policy, as she maximizes the sum of equally weighted utility in Home and Foreign, $\mathrm{E}_{t-1}[\mathcal{U}]+\mathrm{E}_{t-1}[\mathcal{U}^*]$, by choosing directly labor supply, the average scale of production and the number of firms in both domestic and export sector ($L_t, L_t^*, \widetilde{y}_{D,t}, \widetilde{y}_{D,t}^*, N_{D,t+1}, N_{D,t+1}^*, \widetilde{y}_{X,t}, \widetilde{y}_{X,t}^*, N_{X,t}$ and $N_{X,t}^*$) subject to the labor market clearing condition (8) in both countries, and taking as given the Pareto distribution of productivity.

The optimal labor supply in Home is given by

$$L_t^{PL} = \left\{\frac{1}{\eta}\left[\left(2+\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\alpha_t + \beta(\frac{1}{\sigma-1}+\frac{1}{\kappa})E_t\left[\alpha_{t+1}\right]\right]\right\}^{\frac{1}{1+\varphi}}, \tag{15}$$

where "PL" stands for planner. For our purpose, it is very informative to express the solution in relative terms across countries. The relative labor supply is given by

$$\frac{L_t^{PL}}{L_t^{*PL}} = \left[\frac{\left(2+\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\alpha_t + \beta(\frac{1}{\sigma-1}+\frac{1}{\kappa})E_t\left[\alpha_{t+1}\right]}{\left(2+\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\alpha_t^* + \beta(\frac{1}{\sigma-1}+\frac{1}{\kappa})E_t\left[\alpha_{t+1}^*\right]}\right]^{\frac{1}{1+\varphi}}.$$

Following a positive relative demand shock for Home produced goods (an increase in α_t/α_t^*), the planner let Home households work more, the extent of which depends on the elasticity of labor supply, $1/\varphi$. When the labor supply is completely inelastic, there are no fluctuations in labor supply. An expected future increase in relative demand (a rise in $E_t\left[\alpha_{t+1}\right]/E_t\left[\alpha_{t+1}^*\right]$) also increases the relative labor supply for Home. Given the above solution, the relative number of exporters and the cutoff levels of productivity are expressed as

$$\frac{N_{X,t}^{PL}}{N_{X,t}^{*PL}} = \frac{\alpha_t}{\alpha_t^*}\left(\frac{L_t^{*PL}}{L_t^{PL}}\right)^{\varphi}\frac{f_{X,t}^*}{f_{X,t}}, \qquad \frac{\widetilde{z}_{X,t}^{PL}}{\widetilde{z}_{X,t}^{*PL}} = \left(\frac{N_{X,t}^{PL}}{N_{X,t}^{*PL}}\frac{N_{D,t}^{*PL}}{N_{D,t}^{PL}}\right)^{-\frac{1}{\kappa}}.$$

In response to a higher demand attached to Home produced goods, it is optimal to increase the relative number of Home exporters. However, the rise in the relative number of exporters is mitigated when φ is low, that is for a larger marginal dis-utility in labor supply. Because of the one time to build assumption, the relative number of domestic producers ($N_{D,t}^{*PL}/N_{D,t}^{PL}$) is constant following a demand shock at time t. Accordingly, Home exporters become on average less productive than Foreign exporters (a decrease in

$\widetilde{z}^{PL}_{X,t}/\widetilde{z}^{*PL}_{X,t}$), and the scale of production of Home exporters decreases relative to Foreign exporters (a decrease in $\widetilde{y}^{PL}_{X,t}/\widetilde{y}^{*PL}_{X,t}$).

In contrast, the solution for domestic variables is

$$\frac{\widetilde{y}^{PL}_{D,t}}{\widetilde{y}^{*PL}_{D,t}} = \frac{\alpha_t \widetilde{z}_D}{\alpha^*_t \widetilde{z}^*_D} \left(\frac{L^{*PL}_t}{L^{PL}_t}\right)^{\varphi} \frac{N^{*PL}_{D,t}}{N^{PL}_{D,t}}, \qquad \frac{N^{PL}_{D,t+1}}{N^{*PL}_{D,t+1}} = \left(\frac{L^{*PL}_t}{L^{PL}_t}\right)^{\varphi} \frac{f^*_{E,t} E_t\left[\alpha_{t+1}\right]}{f_{E,t} E_t\left[\alpha^*_{t+1}\right]}.$$

Alike the exporter firms, the planner increases the average production of domestic goods in Home more than in Foreign following a positive relative demand shock. The impact of an *expected* positive demand shocks for Home produced goods may also be observed in the number of future domestic varieties (increase in $N^{PL}_{D,t+1}/N^{*PL}_{D,t+1}$).

In the end, the social planner increases the relative number of Home exporters after a positive demand shock for Home produced goods. However, the entry of less productive Home exporters decreases their average productivity and hence their scale of production. In the domestic market instead, Home firms expand the production of their goods which face a higher demand.

B.1 Complete Financial Markets and Flexible Wages

In this subsection, we show that the planner allocation is very close to the one in our framework once we allow for complete financial markets and flexible wages. Indeed, the monopolistic distortions in the goods and labor markets represent the only difference between the two allocations, making the planner's solution a superior one. To begin with, we characterize the equilibrium exchange rate. Under complete asset markets, the marginal utility stemming from one additional unit of nominal wealth is equal across countries. Given our preferences defined in equation (1), this implies

$$\varepsilon_t = \frac{\mu_t}{\mu^*_t}.$$

Note that complete markets allow households to perfectly ensure against demand shocks, and as a consequence, the exchange rate is also independent from demand shocks. Table B2 reports the solution of other variables under complete asset markets (with wage rigidity).

To what extent the allocation under complete asset markets differs from that implied by the social planner? Without wage rigidities, equation (6) implies that the equilibrium wage is $W_t = \Gamma^{1+\varphi} \mu_t L_t^{\varphi}$ and $W_t^* = \Gamma^{1+\varphi} \mu_t^* L_t^{*\varphi}$, where monetary stances serve just as the "nominal anchors" which determine the wage level in each country. As a result, the real variables are independent from monetary stances. In particular, the equilibrium labor supply under complete asset markets and flexible wages is

$$L_t^{CM+FW} = \left\{ \frac{\sigma - 1}{\sigma} \frac{(\theta - 1)(1 + \xi)}{\eta\theta} \left[\left(2 + \frac{1}{\sigma - 1} - \frac{1}{\kappa} \right) \alpha_t + \beta\left(\frac{1}{\sigma - 1} + \frac{1}{\kappa}\right) E_t \left[\alpha_{t+1} \right] \right] \right\}^{\frac{1}{1+\varphi}}.$$

In the above expression, "CM+FW" stands for complete markets and flexible wages. Comparing the above solution with (15), we can state that the equilibrium allocation under complete financial markets and flexible wages is identical to the one implied by the social planner once monopolistic distortions both in goods and labor markets are removed. Indeed, by setting $\frac{\sigma-1}{\sigma} = 1$ and $\frac{(\theta-1)(1+\xi)}{\theta} = 1$, the labor supply is equal to the one in the planner problem, that is $L_t^{CM+FW} = L_t^{PL}$. As a result, the relative number of exporters and domestic producers, as well as their production and the relative labor supply are identical to those implied by the social planner. We can therefore write the terms of trade under complete markets and flexible wages as

$$TOT_t^{CM+FW} = \left(\frac{L_t^{*PL}}{L_t^{PL}} \right)^{\varphi} \frac{\tilde{z}_{X,t}^{PL}}{\tilde{z}_{X,t}^{*PL}}.$$

Following a positive demand shock for Home produced goods, the Home terms of trade appreciate because L_t^{PL}/L_t^{*PL} increases and $\tilde{z}_{X,t}^{PL}/\tilde{z}_{X,t}^{*PL}$ decreases. The extent of the appreciation is higher for a lower elasticity of labor supply, $1/\varphi$. This expression is considered as the desired terms of trade by the social planner. By shutting down monopolistic power and firm heterogeneity, i.e., without variation in the cutoff level of productivity, the expression of the desired terms of trade by the social planner collapses indeed to the one found in Devereux (2004).

C Optimal monetary policy in a Nash equilibrium

In competitive equilibrium, $\mathrm{E}_{t-1}\left[L_{t+1}^{1+\varphi}\right]$ is constant, thus the expected utility of Home representative household for any consecutive time period is given by

$$\begin{aligned}\mathrm{E}_{t-1}[\mathcal{U}] &\equiv \mathrm{E}_{t-1}[U_t] + \beta\mathrm{E}_{t-1}[U_{t+1}] \\ &= \mathrm{E}_{t-1}\left[\alpha_t\left(\ln N_{D,t}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t}\right) + \alpha_t^*\left(\ln N_{X,t}^{*\frac{\sigma}{\sigma-1}}\frac{\widetilde{y}_{X,t}^*}{\tau}\right)\right] \\ &\quad + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\left(\ln N_{D,t+1}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t+1}\right) + \alpha_{t+1}^*\left(\ln N_{X,t+1}^{*\frac{\sigma}{\sigma-1}}\frac{\widetilde{y}_{X,t+1}^*}{\tau_t}\right)\right]\end{aligned}$$

Plugging the equilibrium expression of $\widetilde{y}_{D,t}$, $\widetilde{y}_{X,t}^*$, $\widetilde{y}_{D,t+1}$ and $\widetilde{y}_{X,t+1}^*$,

$$\begin{aligned}\mathrm{E}_{t-1}[\mathcal{U}] &= \mathrm{E}_{t-1}\left[\alpha_t\left(\ln N_{D,t}^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t\widetilde{z}_D}{W_t}\right) + \alpha_t^*\left(\ln N_{X,t}^{*\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t^*\widetilde{z}_{X,t}^*}{W_t^*\tau}\right)\right] \\ &+ \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\left(\ln N_{D,t+1}^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_{t+1}\mu_{t+1}\widetilde{z}_D}{W_{t+1}}\right) + \alpha_{t+1}^*\left(\ln N_{X,t+1}^{*\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_{t+1}\mu_{t+1}^*\widetilde{z}_{X,t+1}^*}{W_{t+1}^*\tau}\right)\right]\end{aligned}$$

Developing the expression, we have

$$\begin{aligned}\mathrm{E}_{t-1}[\mathcal{U}] &= \frac{1}{\sigma-1}\mathrm{E}_{t-1}[\alpha_t\ln N_{D,t}] + \mathrm{E}_{t-1}[\alpha_t\ln\alpha_t] + \mathrm{E}_{t-1}[\alpha_t\ln\mu_t] - \mathrm{E}_{t-1}[\alpha_t\ln W_t] \\ &+ \frac{1}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_t^*\ln N_{X,t}^*\right] + \mathrm{E}_{t-1}[\alpha_t^*\ln\alpha_t] \\ &+ \mathrm{E}_{t-1}[\alpha_t^*\ln\mu_t^*] + \mathrm{E}_{t-1}\left[\alpha_t^*\ln\widetilde{z}_{X,t}^*\right] - \mathrm{E}_{t-1}[\alpha_t^*\ln W_t^*] \\ &+ \frac{\beta}{\sigma-1}\mathrm{E}_{t-1}[\alpha_{t+1}\ln N_{D,t+1}] + \beta\mathrm{E}_{t-1}[\alpha_{t+1}\ln\alpha_{t+1}] \\ &+ \beta\mathrm{E}_{t-1}[\alpha_{t+1}\ln\mu_{t+1}] - \beta\mathrm{E}_{t-1}[\alpha_{t+1}\ln W_{t+1}] \\ &+ \frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln N_{X,t+1}^*\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\alpha_{t+1}\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\mu_{t+1}^*\right] \\ &+ \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\widetilde{z}_{X,t+1}^*\right] - \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln W_{t+1}^*\right] + \text{cst}\end{aligned}$$

Plugging the equilibrium solution of $\widetilde{z}_{X,t}^*$ and $\widetilde{z}_{X,t+1}^*$ and relegating some terms as constant,

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] = \mathrm{E}_{t-1}\left[\alpha_t \ln \mu_t\right] - \mathrm{E}_{t-1}\left[\alpha_t \ln W_t\right] \\
+ \frac{1}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_t^* \ln N_{X,t}^*\right] + \mathrm{E}_{t-1}\left[\alpha_t^* \ln \mu_t^*\right] \\
+ \mathrm{E}_{t-1}\left[\alpha_t^* \ln \left(\frac{N_{X,t}^*}{N_{D,t}^*}\right)^{-\frac{1}{\kappa}}\right] - \mathrm{E}_{t-1}\left[\alpha_t^* \ln W_t^*\right] \\
+ \frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1} \ln N_{D,t+1}\right] + \beta \mathrm{E}_{t-1}\left[\alpha_{t+1} \ln \mu_{t+1}\right] - \beta \mathrm{E}_{t-1}\left[\alpha_{t+1} \ln W_{t+1}\right] \\
+ \frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1}^* \ln N_{X,t+1}^*\right] + \beta \mathrm{E}_{t-1}\left[\alpha_{t+1}^* \ln \alpha_{t+1}\right] \\
+ \beta \mathrm{E}_{t-1}\left[\alpha_{t+1}^* \ln \mu_{t+1}^*\right] + \beta \mathrm{E}_{t-1}\left[\alpha_{t+1}^* \ln \left(\frac{N_{X,t+1}^*}{N_{D,t+1}^*}\right)^{-\frac{1}{\kappa}}\right] - \beta \mathrm{E}_{t-1}\left[\alpha_{t+1}^* \ln W_{t+1}^*\right] + \mathrm{cst}
\end{aligned}
$$

Neglecting the terms for future policies, that is keeping constant μ_{t+1} and μ_{t+1}^* and the variables that depend on these policies W_{t+1}, W_{t+1}^* and $N_{X,t+1}^*$, and further rearranging,

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] = \mathrm{E}_{t-1}\left[\alpha_t \ln \mu_t\right] - \mathrm{E}_{t-1}\left[\alpha_t \ln W_t\right] \\
+ \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\alpha_t^* \ln N_{X,t}^*\right] + \mathrm{E}_{t-1}\left[\alpha_t^* \ln \mu_t^*\right] \\
- \mathrm{E}_{t-1}\left[\alpha_t^* \ln W_t^*\right] + \frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1} \ln N_{D,t+1}\right] \\
+ \frac{\beta}{\kappa}\mathrm{E}_{t-1}\left[\alpha_{t+1}^* \ln N_{D,t+1}^*\right] + \mathrm{cst.}
\end{aligned}
$$

Plugging the equilibrium solution of $N_{X,t}^*$, $N_{D,t+1}$and $N_{D,t+1}^*$, we have

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] = \mathrm{E}_{t-1}\left[\alpha_t \ln \mu_t\right] - \mathrm{E}_{t-1}\left[\alpha_t \ln W_t\right] \\
+ \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\alpha_t^* \ln \frac{\alpha_t \mu_t^*}{W_t^* f_{X,t}^*}\right] + \mathrm{E}_{t-1}\left[\alpha_t^* \ln \mu_t^*\right] \\
- \mathrm{E}_{t-1}\left[\alpha_t^* \ln W_t^*\right] + \frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1} \ln \frac{\mu_t}{W_t f_E}\right] \\
+ \frac{\beta}{\kappa}\mathrm{E}_{t-1}\left[\alpha_{t+1}^* \ln \frac{\mu_t^*}{W_t^* f_E^*}\right] + \mathrm{cst}
\end{aligned}
$$

Further rearranging,

$$
\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = {} & \mathrm{E}_{t-1}[\alpha_t \ln \mu_t] - \mathrm{E}_{t-1}[\alpha_t \ln W_t] \\
& + \left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\{\mathrm{E}_{t-1}[\alpha_t^* \ln \mu_t^*] - \mathrm{E}_{t-1}[\alpha_t^* \ln W_t^*]\} \\
& - \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\alpha_t^* \ln f_{X,t}^*\right] \\
& + \frac{\beta}{\sigma-1}\{\mathrm{E}_{t-1}[\alpha_{t+1} \ln \mu_t] - \mathrm{E}_{t-1}[\alpha_{t+1} \ln W_t]\} \\
& + \frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\alpha_{t+1}^* \ln \mu_t^*\right] - \mathrm{E}_{t-1}\left[\alpha_{t+1}^* \ln W_t^*\right]\right\} + \mathrm{cst}
\end{aligned}
$$

Rearranging and plugging shock process, the expression becomes

$$
\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = {} & \mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t \ln \mu_t\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t \ln W_t\right] \\
& + \left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^* \ln \mu_t^*\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^* \ln W_t^*\right]\right\} \\
& - \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^* \ln f_{X,t}^*\right] \\
& + \frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t\right)^{\rho}\upsilon_{t+1}\ln \mu_t\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t\right)^{\rho}\upsilon_{t+1}\ln W_t\right]\right\} \\
& + \frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^*\right)^{\rho}\upsilon_{t+1}\ln \mu_t^*\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^*\right)^{\rho}\ln W_t^*\right]\right\} + \mathrm{cst}
\end{aligned}
$$

The monetary authority aims at maximizing the expected utility by optimally setting μ_t which has impact on for any two consecutive periods. With a symmetric steady state across countries we assume that $\alpha_{t-1} = \alpha_{t-1}^* = 1$, and the expression becomes finally

$$
\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = {} & \frac{1}{2}\mathrm{E}_{t-1}[\upsilon_t \ln \mu_t] - \frac{1}{2}\mathrm{E}_{t-1}[\upsilon_t \ln W_t] \\
& + \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\{\mathrm{E}_{t-1}[\upsilon_t^* \ln \mu_t^*] - \mathrm{E}_{t-1}[\upsilon_t^* \ln W_t^*]\} \\
& - \frac{1}{2}\left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\upsilon_t^* \ln f_{X,t}^*\right] \\
& + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\{\mathrm{E}_{t-1}[\upsilon_t^{\rho}\upsilon_{t+1}\ln \mu_t] - \mathrm{E}_{t-1}[\upsilon_t^{\rho}\upsilon_{t+1}\ln W_t]\} \\
& + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\upsilon_{t+1}^* \ln \mu_t^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\upsilon_{t+1}^* \ln W_t^*\right]\right\} + \mathrm{cst}.
\end{aligned}
$$

With symmetry of shocks as $\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\upsilon_{t+1}\ln\upsilon_t\right] = \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\upsilon_{t+1}^{*}\ln\upsilon_t^{*}\right]$ and with no serial correlation across them such that $\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln\upsilon_t\right]\mathrm{E}_{t-1}\left[\upsilon_{t+1}\right] = \mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln\upsilon_t\right]$, we have

$$\begin{aligned}\mathrm{E}_{t-1}\left[\mathcal{U}\right] = \frac{1}{2}\mathrm{E}_{t-1}\left[\upsilon_t\ln\mu_t\right] - \frac{1}{2}\mathrm{E}_{t-1}\left[\upsilon_t\ln W_t\right] \\ + \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*}\ln\mu_t^{*}\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{*}\ln W_t^{*}\right]\right\} \\ - \frac{1}{2}\left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\upsilon_t^{*}\ln f_{X,t}^{*}\right] \\ + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln\mu_t\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln W_t\right]\right\} \\ + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\ln\mu_t^{*}\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\ln W_t^{*}\right]\right\} + \mathrm{cst}\end{aligned}$$

Shutting down the fluctuations of export fixed cost and plugging the expression of wages in equilibrium, the expression becomes:

$$\begin{aligned}\mathrm{E}_{t-1}\left[\mathcal{U}\right] = \frac{1}{2}\left\{\mathrm{E}_{t-1}\left[\upsilon_t\ln\mu_t\right] - \frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]\right\} \\ + \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*}\ln\mu_t^{*}\right] - \frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t^{*}\mu_t^{*}\right)^{1+\varphi}\right]\right\} \\ + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln\mu_t\right] - \frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{1+\varphi}\ln E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]\right\} \\ + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\ln\mu_t^{*}\right] - \frac{E_{t-1}\left[\upsilon_t^{*\rho}\right]}{1+\varphi}\ln E_{t-1}\left[\left(A_t^{*}\mu_t^{*}\right)^{1+\varphi}\right]\right\} + \mathrm{cst}\end{aligned}$$

The objective of monetary policy is:

$$\max_{\mu_t}\mathrm{E}_{t-1}\left[\mathcal{U}\right]$$

The first order condition with respect to μ_t is

$$\begin{aligned}\frac{1}{2}\left\{\frac{\upsilon_t}{\mu_t} - \frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\frac{\left(A_t\mu_t\right)^{1+\varphi}}{\mu_t}\right\} \\ + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\frac{\upsilon_t^{\rho}}{\mu_t} - \frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\frac{\left(A_t\mu_t\right)^{1+\varphi}}{\mu_t}\right\} = 0\end{aligned}$$

$$\left\{\upsilon_t - \frac{1}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}(A_t\mu_t)^{1+\varphi}\right\} + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\left\{\upsilon_t^{\rho} - \frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}(A_t\mu_t)^{1+\varphi}\right\} = 0$$

$$\left\{\frac{1}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]} + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}\right\}(A_t\mu_t)^{1+\varphi} = \upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}$$

$$(A_t\mu_t)^{1+\varphi} = \frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\frac{1}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]} + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}}$$

$$\mu_t = \frac{1}{A_t}\left\{\frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\text{cst}}\right\}^{\frac{1}{1+\varphi}}$$

$$\varepsilon_t = \frac{\upsilon_t^*}{\upsilon_t}\frac{\frac{1}{A_t}\left\{\frac{\upsilon_t+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\text{cst}}\right\}^{\frac{1}{1+\varphi}}}{\frac{1}{A_t^*}\left\{\frac{\upsilon_t^*+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{*\rho}}{\text{cst}}\right\}^{\frac{1}{1+\varphi}}}$$

$$\varepsilon_t = \frac{\upsilon_t^*}{\upsilon_t}\frac{A_t^*}{A_t}\left[\frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\upsilon_t^* + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{*\rho}}\right]^{\frac{1}{1+\varphi}}$$

Note that when $\beta = 0$ and $A_t = A_t^* = cst$, we have

$$\varepsilon_t = \frac{\upsilon_t^*}{\upsilon_t}\left[\frac{\upsilon_t}{\upsilon_t^*}\right]^{\frac{1}{1+\varphi}}$$

The above is the expression found in Devereux (2004). In the above expression, note further that when $\varphi = 0$, the optimal policy is a fixed exchange rate as $\varepsilon_t = 1$.

D Fixed vs. Flexible Exchange Rate

Again with symmetry at the steady state and with $\Delta \ln W_t \equiv \ln W_t^{FX} - \ln W_t^{FL}$, the difference of the expected utility between the two polar cases of exchange rate policies is

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FX}}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FL}}\right] &= \frac{1}{2}\mathrm{E}_{t-1}\left[\upsilon_t \mathrm{ln}\upsilon_t\right]-\frac{1}{2}\Delta\ln W_t \\
&+\frac{1}{2}\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^* \mathrm{ln}\upsilon_t^*\right]-\Delta\ln W_t\right\} \\
&+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^\rho \upsilon_{t+1}\mathrm{ln}\upsilon_t\right]-\mathrm{E}_{t-1}\left[\upsilon_t^\rho\upsilon_{t+1}\right]\Delta\ln W_t\right\} \\
&+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\upsilon_{t+1}^*\mathrm{ln}\upsilon_t^*\right]-\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\upsilon_{t+1}^*\right]\Delta\ln W_t\right\}
\end{aligned}
$$

With symmetry of shock $\mathrm{E}_{t-1}\left[\upsilon_t^\rho \upsilon_{t+1}\mathrm{ln}\upsilon_t\right]=\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\upsilon_{t+1}^*\mathrm{ln}\upsilon_t^*\right]$ and with no serial correlation of the shock such that $\mathrm{E}_{t-1}\left[\upsilon_t^\rho \mathrm{ln}\upsilon_t\right]\mathrm{E}_{t-1}\left[\upsilon_{t+1}\right]=\mathrm{E}_{t-1}\left[\upsilon_t^\rho \mathrm{ln}\upsilon_t\right]$, we have

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FX}}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FL}}\right] &= \frac{1}{2}\mathrm{E}_{t-1}\left[\upsilon_t \mathrm{ln}\upsilon_t\right]-\frac{1}{2}\Delta\ln W_t \\
&+\frac{1}{2}\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t \mathrm{ln}\upsilon_t\right]-\Delta\ln W_t\right\} \\
&+\beta\left(\frac{1}{2}\right)^{1+\rho}\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^\rho \mathrm{ln}\upsilon_t\right]-\mathrm{E}_{t-1}\left[\upsilon_t^\rho\right]\Delta\ln W_t\right\}.
\end{aligned}
$$

where $\Delta\ln W_t\equiv\ln W_t^{FX}-\ln W_t^{FL}$ represents the wage difference between a fixed and a flexible exchange rate:

$$
\begin{aligned}
\Delta\ln W_t\equiv\ln W_t^{FX}-\ln W_t^{FL} &= \ln\Gamma\left\{\frac{E_{t-1}\left[\left(A_t 2\mu_0\alpha_t\right)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\}^{\frac{1}{1+\varphi}}-\ln\Gamma\left\{\frac{E_{t-1}\left[\left(A_t\mu_0\right)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\}^{\frac{1}{1+\varphi}} \\
&= \frac{1}{1+\varphi}\ln\left\{\frac{E_{t-1}\left[\left(A_t 2\mu_0\alpha_t\right)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\}-\frac{1}{1+\varphi}\ln\left\{\frac{E_{t-1}\left[\left(A_t\mu_0\right)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\} \\
&= \frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t 2\mu_0\alpha_t\right)^{1+\varphi}\right]-\frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t\mu_0\right)^{1+\varphi}\right] \\
&= \frac{1}{1+\varphi}\left[\ln E_{t-1}\left[\left(A_t\upsilon_t\right)^{1+\varphi}\right]-\ln E_{t-1}\left[A_t^{1+\varphi}\right]\right].
\end{aligned}
$$

Table B1: The Planner's Solution

Nb of Entrants	$N_{D,t+1} = \beta(\frac{1}{\sigma-1} + \frac{1}{\kappa})\frac{E_t[\alpha_{t+1}]}{\eta L_t^{\varphi} f_{E,t}}$	$N^*_{D,t+1} = \beta(\frac{1}{\sigma-1} + \frac{1}{\kappa})\frac{E_t[\alpha^*_{t+1}]}{\eta L_t^{*\varphi} f^*_{E,t}}$
Nb of Exporters	$N_{X,t} = (\frac{1}{\sigma-1} - \frac{1}{\kappa})\frac{\alpha_t}{\eta L_t^{\varphi} f_{X,t}}$	$N^*_{X,t} = (\frac{1}{\sigma-1} - \frac{1}{\kappa})\frac{\alpha^*_t}{\eta L_t^{*\varphi} f^*_{X,t}}$
Av. Exporters	$\widetilde{z}_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$	$\widetilde{z}^*_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{N^*_{X,t}}{N^*_{D,t}}\right)^{-\frac{1}{\kappa}}$
Production	$\widetilde{y}_{D,t} = \frac{\alpha_t \widetilde{z}_D}{\eta L_t^{\varphi} N_{D,t}}, \quad \widetilde{y}_{X,t} = (\frac{1}{\sigma-1} - \frac{1}{\kappa})^{-1}\widetilde{z}_{X,t} f_{X,t}$	$\widetilde{y}^*_{D,t} = \frac{\alpha^*_t \widetilde{z}^*_D}{\eta L_t^{*\varphi} N^*_{D,t}}, \quad \widetilde{y}^*_{X,t} = (\frac{1}{\sigma-1} - \frac{1}{\kappa})^{-1}\widetilde{z}^*_{X,t} f^*_{X,t}$
Consumption	$C_t = N_{D,t}^{(1+\frac{1}{\sigma-1})\alpha_t} N_{X,t}^{*(1+\frac{1}{\sigma-1})\alpha^*_t} \left(\frac{\widetilde{y}_{D,t}}{\alpha_t}\right)^{\alpha_t} \left(\frac{\widetilde{y}^*_{X,t}/\tau_t}{\alpha^*_t}\right)^{\alpha^*_t}$	$C^*_t = N_{D,t}^{*(1+\frac{1}{\sigma-1})\alpha^*_t} N_{X,t}^{(1+\frac{1}{\sigma-1})\alpha_t} \left(\frac{\widetilde{y}^*_{D,t}}{\alpha^*_t}\right)^{\alpha^*_t} \left(\frac{\widetilde{y}^*_{X,t}/\tau_t}{\alpha_t}\right)^{\alpha_t}$
Labor Supply	$L_t = \left\{\frac{1}{\eta}\left[\left(2 + \frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\alpha_t + \beta(\frac{1}{\sigma-1} + \frac{1}{\kappa})E_t\left[\alpha_{t+1}\right]\right]\right\}^{\frac{1}{1+\varphi}}$	$L^*_t = \left\{\frac{1}{\eta}\left[\left(2 + \frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\alpha^*_t + \beta(\frac{1}{\sigma-1} + \frac{1}{\kappa})E_t\left[\alpha^*_{t+1}\right]\right]\right\}^{\frac{1}{1+\varphi}}$
Shock Process	$\alpha_t = \frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t, \quad \alpha^*_t = \frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon^*_t, \alpha_0 = \alpha^*_0 = 1, \quad E_{t-1}\left[\upsilon_t\right] = E_{t-1}\left[\upsilon^*_t\right] = 1, \quad E_{t-1}\left[\upsilon_t\upsilon_{t+1}\right] = 1, \quad \upsilon_t + \upsilon^*_t = 2, \quad 0 < \rho < 1$	

Table B2: The Model's Solution for a given monetary rule under complete financial markets

Nb of Entrants	$N_{D,t+1}=\beta\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\frac{\mu_t}{W_t f_{E,t}}E_t\left[\alpha_{t+1}\right]$	$N^*_{D,t+1}=\beta\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\frac{\mu^*_t}{W^*_t f^*_{E,t}}E_t\left[\alpha^*_{t+1}\right]$
Nb of Exporters	$N_{X,t}=\frac{\sigma-1}{\sigma}\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\frac{\alpha_t\mu_t}{W_t f_{X,t}}$	$N^*_{X,t}=\frac{\sigma-1}{\sigma}\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\frac{\alpha^*_t\mu^*_t}{W^*_t f^*_{X,t}}$
Av. Exporters	$\widetilde{z}_{X,t}=\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$	$\widetilde{z}^*_{X,t}=\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N^*_{X,t}}{N^*_{D,t}}\right)^{-\frac{1}{\kappa}}$
Production	$\widetilde{y}_{D,t}=\frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t\widetilde{z}_D}{N_{D,t}W_t},\quad \widetilde{y}_{X,t}=\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)^{-1}\widetilde{z}_{X,t}f_{X,t}$	$\widetilde{y}^*_{D,t}=\frac{\sigma-1}{\sigma}\frac{\alpha^*_t\mu^*_t\widetilde{z}^*_D}{N^*_{D,t}W^*_t},\quad \widetilde{y}^*_{X,t}=\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)^{-1}\widetilde{z}^*_{X,t}f^*_{X,t}$
Average Price	$\widetilde{p}_{D,t}=\frac{\sigma}{\sigma-1}\frac{W_t}{\widetilde{z}_D},\quad \widetilde{p}_{X,t}=\frac{\sigma}{\sigma-1}\frac{\tau\varepsilon_t^{-1}W_t}{\widetilde{z}_{X,t}}$	$\widetilde{p}^*_{D,t}=\frac{\sigma}{\sigma-1}\frac{W^*_t}{\widetilde{z}^*_D},\quad \widetilde{p}^*_{X,t}=\frac{\sigma}{\sigma-1}\frac{\tau\varepsilon_t W^*_t}{\widetilde{z}^*_{X,t}}$
Price Indices	$P_{H,t}=N_{D,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{D,t},\quad P_{F,t}=N_{X,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^*_{X,t},\quad P_t=P_{H,t}^{\alpha_t}P_{F,t}^{\alpha^*_t}$	$P^*_{F,t}=N_{D,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^*_{D,t},\quad P^*_{H,t}=N_{X,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{X,t},\quad P^*_t=P_{F,t}^{*\alpha^*_t}P_{H,t}^{*\alpha_t}$
Consumption	$C_t=N_{D,t}^{\left(1+\frac{1}{\sigma-1}\right)\alpha_t}N_{X,t}^{*\left(1+\frac{1}{\sigma-1}\right)\alpha^*_t}\left(\frac{\widetilde{y}_{D,t}}{\alpha_t}\right)^{\alpha_t}\left(\frac{\widetilde{y}^*_{X,t}/\tau_t}{\alpha^*_t}\right)^{\alpha^*_t}$	$C^*_t=N_{D,t}^{*\left(1+\frac{1}{\sigma-1}\right)\alpha^*_t}N_{X,t}^{\left(1+\frac{1}{\sigma-1}\right)\alpha_t}\left(\frac{\widetilde{y}^*_{D,t}}{\alpha^*_t}\right)^{\alpha^*_t}\left(\frac{\widetilde{y}^*_{X,t}/\tau_t}{\alpha_t}\right)^{\alpha_t}$
Profits	$\widetilde{D}_{D,t}=\frac{\alpha_t}{\sigma}\frac{\mu_t}{N_{D,t}},\quad \widetilde{D}_{X,t}=\frac{\sigma-1}{\sigma\kappa}\frac{\alpha_t\mu_t}{N_{X,t}},\quad \widetilde{D}_t=\widetilde{D}_{D,t}+\frac{N_{X,t}}{N_{D,t}}\widetilde{D}_{X,t}$	$\widetilde{D}^*_{D,t}=\frac{\alpha^*_t}{\sigma}\frac{\mu^*_t}{N^*_{D,t}},\quad \widetilde{D}^*_{X,t}=\frac{\sigma-1}{\sigma\kappa}\frac{\alpha^*_t\mu^*_t}{N^*_{X,t}},\quad \widetilde{D}^*_t=\widetilde{D}^*_{D,t}+\frac{N^*_{X,t}}{N^*_{D,t}}\widetilde{D}^*_{X,t}$
ZPC	$\widetilde{D}_{X,t}=W_t f_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$	$\widetilde{D}^*_{X,t}=W^*_t f^*_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$
Share Price	$\widetilde{V}_t=f_{E,t}W_t$	$\widetilde{V}^*_t=f^*_{E,t}W^*_t$
Labor Supply	$L_t=(\sigma-1)\frac{N_{D,t}\widetilde{D}_t}{W_t}+\sigma N_{X,t}f_{X,t}+N_{D,t+1}f_{E,t}$	$L^*_t=(\sigma-1)\frac{N^*_{D,t}\widetilde{D}^*_t}{W^*_t}+\sigma N^*_{X,t}f^*_{X,t}+N^*_{D,t+1}f^*_{E,t}$
Monetary Stance	$\mu_t=P_tC_t$	$\mu^*_t=P^*_tC^*_t$
Wages	$W_t=\Gamma\left\{\frac{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}{E_{t-1}[A_t]}\right\}^{\frac{1}{1+\varphi}}$	$W^*_t=\Gamma\left\{\frac{E_{t-1}\left[(A^*_t\mu^*_t)^{1+\varphi}\right]}{E_{t-1}[A^*_t]}\right\}^{\frac{1}{1+\varphi}}$
Exchange Rate	$\varepsilon_t=\frac{\mu_t}{\mu^*_t}$	
Definition of A_t	$A_t=\frac{\sigma-1}{\sigma}\left[\left(2+\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\alpha_t+\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\alpha_{t+1}\right]$	$A^*_t=\frac{\sigma-1}{\sigma}\left[\left(2+\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\alpha^*_t+\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\alpha^*_{t+1}\right]$
Shock Process	$\alpha_t=\frac{1}{2}\alpha^{\rho}_{t-1}\upsilon_t,\quad \alpha^*_t=\frac{1}{2}\alpha^{*\rho}_{t-1}\upsilon^*_t,\ \alpha_0=\alpha^*_0=1,\quad E_{t-1}\left[\upsilon_t\right]=E_{t-1}\left[\upsilon^*_t\right]=1,\quad E_{t-1}\left[\upsilon_t\upsilon_{t+1}\right]=1,\quad \upsilon_t+\upsilon^*_t=2,\quad 0<\rho<1$	