WINPEC Working Paper Series No.E2003
May 2020

Growing Consideration

Yuta Inoue

Waseda INstitute of Political EConomy
Waseda University
Tokyo, Japan

Growing Consideration

Yuta Inoue*

May 14, 2020

Abstract

This paper proposes a behavioral model which we refer to as a Growing Consideration model, and derive observable implications using a revealed preference approach. This model inherits the main idea of Limited Consideration models, and adds to it an assumption that an agent's consideration grows over time. An agent makes choices over multiple time periods, while she may not pay attention to all available alternatives at all times. In addition, we require that she pays attention to alternatives that she chose in the past, which property we refer to as Growing Consideration. Revealed preference tests, as well as conditions under which we can robustly infer agent's preference, consideration, and non-consideration are given for two types of Growing Consideration models.

KEYWORDS: Choice; Behavior; Time; Revealed preference; Robust inference; Limited Consideration; Limited Attention; Overwhelming Choice; Growing Consideration

JEL CLASSIFICATION NUMBERS: D1 D8

---

* Graduate School of Economics, Waseda University. Email address: y.inoue@toki.waseda.jp
For helpful discussions and comments, the author is grateful to conference participants at the 2018 International Conference on Economic Theory and Applications (Chengdu).

1 Introduction

Let X be the grand set of alternatives, which we assume to be finite. Then, under the standard rational choice model, given any set of feasible alternatives $A \subseteq X$, an agent chooses her most preferred alternative, which is commonly assumed to be a strict preference. In testing whether an agent's behavior is consistent with this standard framework, the theory of revealed preference is one of the most powerful methods for economists. Typically, a choice function f is observed, where $\mathcal{D} \subseteq 2^X \backslash \varnothing$ is a collection of nonempty feasible sets, and for every feasible set $A \in \mathcal{D}$, $f(A) \in A$ is the chosen alternative from A.[1] It is well known that a choice function is consistent with the rational choice model, if and only if it obeys the *strong axiom of revealed preference (SARP)*. SARP requires acyclicity of the direct revealed preference relation $\succ^*$, which is defined as $x'' \succ^* x'$ if there exists a feasible set A such that $f(A) = x''$, $x' \in A$, and $x' \neq x''$.

However, it is reported in a number of experimental studies that agents are often inconsistent with this rational choice framework. In order to deal with such seemingly irrational behavior under a choice theoretic framework, various models of bounded rationality have been proposed. Amongst these models, in this paper, we focus on Limited Consideration models. Limited Consideration models allow that the agent does not pay attention to all feasible alternatives due to cognitive capacity. In particular, it is assumed that an agent has a strict preference $\succ$, but when facing any feasible set A, she pays attention to only a subset of what is available: $\Gamma(A) \subseteq A$. Then, the agent chooses the $\succ$-best alternative within $\Gamma(A)$. This subset $\Gamma(A)$ is called the *consideration set* of feasible set A, and the mapping $\Gamma : 2^X \backslash \varnothing \rightarrow 2^X \backslash \varnothing$ is referred to as the *consideration mapping* of the agent. Each Limited Consideration model differs depending on the restriction casted on the consideration mapping Γ. For example, the *Limited Attention* model in Masatlioglu, Nakajima, and Ozbay (2012) assumes that Γ obeys the *attention filter property*, which requires that removal of an ignored alternative does not change the consideration set; and the *Overwhelming Choice* model in Lleras, Masatlioglu, Nakajima, and Ozbay (2017) assumes that Γ obeys the *competition filter property*, which requires that an alternative considered in a larger set must be considered in a smaller set.[2] In fact, other

[1] Throughout the paper, let us abuse notation and abbreviate the braces, and write $2^X \backslash \varnothing$ instead of $2^X \backslash \{\varnothing\}$. Similar abbreviation of braces will be used whenever there is no fear of confusion.

[2] These properties are in fact well studied as properties on choice correspondences, which is summarized in a survey by Moulin (1985). Mastlioglu, Nakajima, and Ozbay (2012) and Lleras, Masatlioglu, Nakajima, and Ozbay (2017) are pioneering works to apply them to the context of Limited Consideration, as properties on consideration mappings.

important theories of non-standard decision making are theoretically special cases of Limited Consideration models introduced above. The *Rational Shortlisting Method* by Manzini and Mariotti (2007), the *Categorize-Then-Choose* model by Manzini and Mariotti (2012), and the *Order Rationalization* model by Cherepanov, Feddersen, and Sandroni (2013) are special cases of the Overwhelming Choice model, and the *Transitive Rational Shortlisting Method* by Au and Kawai (2011) is a special case of both Limited Attention model and Overwhelming Choice model.

In this paper, we adopt the main idea of Limited Consideration, and supplement it by adding "time" into the model: we propose a model where an agent makes decisions at multiple time periods, while she may not be aware of all available alternatives at all times. In particular, we assume that the agent has a time-invariant strict preference $\succ$, and letting $\mathcal{T} = \{1, 2, \ldots, T\}$ be a set of time periods, for every time period $t \in \mathcal{T}$, there exists a consideration mapping $\Gamma_t : 2^X \backslash \varnothing \rightarrow 2^X \backslash \varnothing$. Then, given any feasible set A at time period t, the agent chooses her $\succ$-best alternative within $\Gamma_t(A)$. In addition, we assume that the agent's consideration depends on past choices. Specifically, we assume that any alternative chosen in past periods must attract attention of the agent, which property we refer to as *Growing Consideration*. We believe that such assumption is plausible, since, for example, any commodity that the agent consumed in the past should be familiar to her and thus be easier to spot; or particular websites tell us what choices we made previously.

We formalize the model of Growing Consideration, and provide a necessary and sufficient condition for agent's choices to be consistent with the model. Furthermore, we derive conditions under which we can robustly infer the agent's preference, consideration, and non-consideration, provided that the observed choices are consistent with Growing Consideration model. By conditions for robust inference, we mean conditions under which we can surely say that (i) some alternative is preferred to another; (ii) some alternative is considered at some feasible set and time period; and (iii) some alternative is not considered at some set and period. Such inferences are useful from the viewpoint of welfare analysis, since it is not possible to pin down the agent's strict preference or consideration mappings even when choices are rationalizable. Moreover, under the Limited Consideration assumption, an alternative x'' being chosen over x' does not directly imply that the agent prefers x'' over x': we must also take

Rigorously, the attention filter property requires: for every $A \subseteq X$ and $x \in A$, if $x \notin \Gamma(A)$, then $\Gamma(A) = \Gamma(A \backslash x)$; and the competition filter property requires: for $x \in A \subseteq A'$, if $x \in \Gamma(A')$, then $x \in \Gamma(A)$.

into account the possibility that x' is preferred to x'', but x' was overlooked. In this paper, two cases of Growing Consideration models are dealt with. One is a baseline model where we cast no restriction (other than Growing Consideration) on the consideration mappings. The other is a model where we require that consideration mapping Γ_t obeys the competition filter property for every time period t.[3]

A paper closely related to ours is Ferreira and Gravel (2017), in which they explicitly analyze a situation where choices are observed across multiple time periods. They provide revealed preference tests for choice models with (i) changing preference; (ii) preference formation by trial and error; (iii) endogenous status-quo bias. One difference between Ferreira and Gravel (2017) and our paper, apart from the models analyzed, is the structure of the data set assumed to be observed. While Ferreira and Gravel (2017) assume that only one feasible set is observed for every time period, we adopt a more general assumption and deal with the case where a choice function is observed for each time period. Other related papers are Bernheim and Rangel (2007, 2009) and Salant and Rubinstein (2008): the framework we adopt have some similarity with theirs. These papers analyze a situation where each feasible set is supplemented with an additional condition, referred to as an *ancillary condition* by the former and as a *frame* by the latter. Such conditions represent either "some characteristic of the choice environment that is consequently irrelevant to outside observer" or "how alternatives are framed".[4] In principle, "time of choice" can be regarded as such condition, and in fact Bernheim and Rangel give it as an example of an ancillary condition.[5] In this regard, our framework itself can be seen as a special case of the framework dealt with in these papers. However, there are substantial differences that distinguish our model from those of Bernheim and Rangel's and Salant and Rubinstein's. Firstly, the Limited Consideration/Growing Consideration assumption is a feature not covered by Bernheim and Rangel (2007, 2009) or Salant and Rubinstein (2008).[6] As a technical issue, Bernheim and Rangel/Salant and Rubinstein

[3] In Appendix, we derive a revealed preference test for Growing Consideration model where Γ_t obeys the attention filter property for every period t.

[4] Bernheim and Rangel (2007, 2009) put weight on normative aspects of such model, while Salant and Rubinstein (2008) put weight on positive aspects.

[5] Salant and Rubinstein, in their paper, do not refer to time as a frame.

[6] In fact, Salant and Rubinstein (2008) deal with a specific type of limited consideration, where they apply the "number of alternatives that the agent can pay attention to" as the frame. Such behavioral assumption is different from the limited consideration that we consider. Moreover, if we try to illustrate our model in terms of Salant and Rubinstein (2008), we must apply both "time of choice" *and* "structure of limited consideration" as the frame. Such case is not considered by Salant and Rubinstein (2008).

assume that for every ancillary condition/frame, a choice function (or correspondence) defined on an exhaustive domain is observed, while we assume that choice functions observed for each time period are defined on arbitrary collections of feasible sets.[7] Therefore, results derived by Bernheim and Rangel/Salant and Rubinstein are not directly applicable to our context. Furthermore, Bernheim and Rangel (2007, 2009) propose conditions under which welfare judgements can be made, but we show that our results regarding robust inference of preference may lead to completely opposite welfare implications.

Organization of the paper: In Section 2, we introduce the model of Growing Consideration, and define the concept of rationalizability and robust inference of preference/consideration/non-consideration. Section 3 is devoted to analysis of the baseline model, where there is no intra-temporal restriction on consideration mappings. In particular, we provide a revealed preference test in Section 3.1, and then derive conditions for robust inference of preference/consideration/non-consideration in Section 3.2. In Section 4, we analyze a Growing Consideration model where consideration mapping of each time period obeys the competition filter property. A revealed preference test for Growing Consideration model with the attention filter property is given in Appendix.

2 The model

Let X be a finite set, which is the grand set of alternatives, and consider a choice model where an agent makes choices over multiple time periods. We assume that the agent has a time-invariant, complete, transitive, and asymmetric preference $\succ$, which we refer to as a *strict preference*.[8] In addition, we assume that the agent exhibits Limited Consideration. Letting $\mathcal{T} = \{1, 2, \ldots, T\}$ be the set of time periods, for every period $t \in \mathcal{T}$, there is a consideration mapping $\Gamma_t : 2^X \backslash \varnothing \to 2^X \backslash \varnothing$ such that $\Gamma_t(A) \subseteq A$ for every $A \subseteq X$. Then, given any feasible set $A \subseteq X$ and period $t \in \mathcal{T}$, the agent chooses the $\succ$-best alternative within $\Gamma_t(A)$. Regarding the structure of agent's consideration, assume that alternatives chosen in past periods must be considered, which property we refer to as *Growing Consideration.*

We assume that a choice function f_t is observed for every time period $t \in \mathcal{T}$. In particular,

[7] By an exhaustive domain of a choice function, we mean that the choice function is defined on all nonempty subsets of X, i.e., $f : 2^X \backslash \varnothing \to X$.

[8] A binary relation $\succ$ is *complete* if for every $x', x'' \in X$, we have $x' \succ x''$ or $x'' \succ x'$; it is *transitive* if $x'' \succ x'$ and $x' \succ x$ implies $x'' \succ x$; it is *asymmetric* if $x'' \succ x'$ implies $x' \nsucc x''$.

for every time period $t \in \mathcal{T}$, let $\mathcal{D}_t \subseteq 2^X \backslash \varnothing$ be the collection of feasible sets observed at period t. Then the profile of choice functions $(f_t)_{t\in\mathcal{T}}$ is such that, for every $t \in \mathcal{T}$, $f_t : \mathcal{D}_t \to X$ and $f_t(A) \in A$ for every $A \in \mathcal{D}_t$. Note that $f_t(A)$ is the alternative chosen by the agent from feasible set A at time period t. Then, Growing Consideration is formally defined as below:

Definition 1. Given a profile of choice functions $(f_t)_{t\in\mathcal{T}}$, a profile of consideration mappings $(\Gamma_t)_{t\in\mathcal{T}}$ exhibits *Growing Consideration*, if alternatives chosen in the past are included in the consideration set, i.e., for every $t' \in \mathcal{T}$, $A' \subseteq X$, and $x \in A'$,

$$x \in \Gamma_{t'}(A') \text{ whenever } x = f_t(A) \text{ for some } t < t', A \in \mathcal{D}_t. \tag{1}$$

We refer to a pair of strict preference and profile of consideration mappings $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ as a *Growing Consideration model*, or in short a *GC model*, whenever $(\Gamma_t)_{t\in\mathcal{T}}$ exhibits Growing Consideration.

In this paper, we derive a necessary and sufficient condition that a profile of choice functions $(f_t)_{t\in\mathcal{T}}$ must obey, in order for it to be rationalizable by a GC model. A formal definition of rationalizablity is as follows:

Definition 2. A profile of choice functions $(f_t)_{t\in\mathcal{T}}$ is *rationalizable by a Growing Consideration model*, if there exists a Growing Consideration model $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ such that, for every $t' \in \mathcal{T}$ and $A' \in \mathcal{D}_{t'}$, $f_{t'}(A')$ is the $\succ$-best alternative within $\Gamma_{t'}(A')$.

Under Growing Consideration models, an alternative x'' observed to be chosen over x' does not necessarily imply that the agent prefers x'' to x': it may be the case that x' is preferred to x'', but x' was overlooked by the agent. Moreover, even when an agent's choices are consistent with GC model, the GC model that rationalizes choices is not uniquely determined in general. Nevertheless, it is possible to pin down the relative ranking between particular alternatives, or robustly infer that some alternative attracts attention/is ignored at some feasible set and time period. These conditions are derived in Sections 3.2 and 4.2 for the baseline GC model and the GC model with competition filter property respectively. Henceforth, for notational simplicity, let us use the expression (t, A) when dealing with feasible set A at time period t.

Definition 3. Let $(f_t)_{t\in\mathcal{T}}$ be rationalizable by a Growing Consideration model. Then

- *x'' is robustly preferred to x'* if $x'' \succ x'$ holds under every $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ that rationalizes $(f_t)_{t\in\mathcal{T}}$;

- x' *is robustly considered at* (t', A') if $x' \in \Gamma_{t'}(A')$ holds under every $\langle \succ, (\Gamma_t)_{t \in \mathcal{T}} \rangle$ that rationalizes $(f_t)_{t \in \mathcal{T}}$;
- x' *is robustly* **not** *considered at* (t', A') if $x' \notin \Gamma_{t'}(A')$ holds under every $\langle \succ, (\Gamma_t)_{t \in \mathcal{T}} \rangle$ that rationalizes $(f_t)_{t \in \mathcal{T}}$.

In the following sections, we derive observable implications of Growing Consideration models, namely conditions for rationalizability and robust inference. A baseline model, where no intra-temporal restriction is casted on $(\Gamma_t)_{t \in \mathcal{T}}$, is dealt with first, followed by a GC model where we require Γ_t to obey the competition filter property for every $t \in \mathcal{T}$.

3 Baseline Growing Consideration model

In this section, we deal with the baseline Growing Consideration model, where we cast no intra-temporal restriction on consideration mappings $(\Gamma_t)_{t \in \mathcal{T}}$. It is worth noting that when $T = 1$, which corresponds to standard Limited Consideration model with no "time", rationalizability becomes vacuous when Γ has no restriction. Given a choice function f, we can simply set $\Gamma(A) = \{f(A)\}$ for every $A \in \mathcal{D}$, and set $\Gamma(A) \subseteq A$ arbitrarily for $A \notin \mathcal{D}$. Then, any strict preference $\succ$ accompanied with this consideration mapping Γ would rationalize the choice function. When $T \geqslant 2$, rationalizability would have a bite, which is shown in the following subsection.

3.1 Rationalizability

In dealing with rationalizability, we first consider a necessary condition. Suppose that choice function $(f_t)_{t \in \mathcal{T}}$ is generated by an agent obeying a Growing Consideration model $\langle \succ, (\Gamma_t)_{t \in \mathcal{T}} \rangle$. Consider any $t \geqslant 2$, and fix any $A \in \mathcal{D}_t$. Then, it follows by Growing Consideration that whenever there exist $t' < t$ and $A' \in \mathcal{D}_{t'}$ with $f_{t'}(A') \in A$, then we have $f_{t'}(A') \in \Gamma_t(A)$. Then, for every period $t \geqslant 2$, we can define a binary relation P_t as follows: $x'' P_t x'$ if there exists $A \in \mathcal{D}_t$ such that $x'' = f_t(A)$, and

$$\text{there exist } t' < t \text{ and } A' \in \mathcal{D}_{t'} \text{ such that } x' = f_{t'}(A') \in A \backslash x''. \tag{2}$$

Note that whenever $x'' P_t x'$ holds, then we have $x'' \succ x'$. Now let us define a binary relation P as a union of all P_t's, i.e., $P = \bigcup_{t=2}^{T} P_t$. Then under GC model, $x'' P x'$ implies $x'' \succ x'$,

$A \in \mathcal{D}_1$	$\{x_1, x_2\}$	$\{x_1, x_2, x_3\}$
$f_1(A)$	x_1	x_2

$A \in \mathcal{D}_2$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2, x_3, x_4\}$
$f_2(A)$	x_2	x_1

Table 1: Choice function $(f_t)_{t\in\mathcal{T}}$ of Example 1.

and therefore, acyclicity of P is a necessary condition for $(f_t)_{t\in\mathcal{T}}$ to be rationalizable by a GC model.[9] In fact, the opposite direction is true as well.

PROPOSITION 1 (GC model rationalizability). *A profile of choice functions $(f_t)_{t\in\mathcal{T}}$ is rationalizable by a Growing Consideration model if and only if binary relation P is acyclic.*

Proof. We show sufficiency here by constructing a GC model that rationalizes $(f_t)_{t\in\mathcal{T}}$. Since binary relation P is acyclic, by Szpilrajn's Theorem, there exists a complete, transitive, and asymmetric extension of P, which we denote by $\succ$.[10] We define consideration mapping Γ_t for every $t \in \mathcal{T}$ as follows:

$$\text{if } A \in \mathcal{D}_t, \quad \Gamma_t(A) = \{f_t(A)\} \cup \{x \in A : f_t(A) \succ x\}; \tag{3}$$

$$\text{if } A \notin \mathcal{D}_t, \quad \Gamma_t(A) = \{x \in A : x = f_{t'}(A') \text{ for some } t' < t \text{ and } A' \in \mathcal{D}_{t'}\}. \tag{4}$$

It is clear by construction that $f_t(A)$ is the $\succ$-best alternative within $\Gamma_t(A)$ for every $t \in \mathcal{T}$ and every $A \in \mathcal{D}_t$. It remains to show that $(\Gamma_t)_{t\in\mathcal{T}}$ obeys Growing Consideration. Fix any $t \geqslant 2$ and $A \subseteq X$, and take any $x' \in A$ such that $x' = f_{t'}(A')$ for some $t' < t$ and $A' \in \mathcal{D}_{t'}$. If $A \in \mathcal{D}_t$, then we have either $f_t(A) = x'$ or $f_t(A) P_t x'$, and the latter case in turn implies $f_t(A) \succ x'$. In both cases, by (3), we have $x' \in \Gamma_t(A)$. If $A \notin \mathcal{D}_t$, then it follows immediately from (4) that we have $x' \in \Gamma_t(A)$. We conclude that $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ is a GC model that rationalizes $(f_t)_{t\in\mathcal{T}}$. □

EXAMPLE 1. We give here a profile of choice functions that is not rationalizable by a GC model. Let $X = \{x_1, x_2, x_3, x_4\}$, $\mathcal{T} = \{1, 2\}$, and consider (f_1, f_2) summarized in Table 1. Since x_2 is chosen at $t = 2$ from feasible set $\{x_1, x_2, x_3\}$, and x_1 is an alternative chosen in the past, it follows that $x_2 P x_1$. Similarly, we have $x_1 P x_2$. Since binary relation P has a cycle,

[9]A binary relation P is *acyclic*, if for any $x^1, x^2, \ldots, x^K \in X$, $x^1 P x^2 P \cdots P x^{K-1} P x^K$ implies that $x^K P x^1$ does not hold.

[10]Note that binary relation P' is an extension of binary relation P, if $x'' P x'$ implies $x'' P' x'$. A comprehensive summary of Szpilrajn's Theorem and related extension theorems is given in Andrikopoulos (2009).

$(f_t)_{t \in \mathcal{T}}$ is not rationalizable by a GC model. Indeed, if we attempt to find a pair of preference and consideration mappings that is a GC model, it must be the case that x_1, x_2 are considered at $t = 2$ at both $\{x_1, x_2, x_3\}$ and $\{x_1, x_2, x_3, x_4\}$. Then, since x_2 and x_1 are chosen alternatives at $t = 2$ from $\{x_1, x_2, x_3\}$ and $\{x_1, x_2, x_3, x_4\}$ respectively, it must follow that x_2 is strictly preferred to x_1 and vice versa, which is impossible.

3.2 Robust inference

Suppose that a profile of choice functions $(f_t)_{t \in \mathcal{T}}$ is rationalizable by a Growing Consideration model. Even when $(f_t)_{t \in \mathcal{T}}$ is rationalizable, the GC model that rationalizes it is not uniquely determined in general. Nevertheless, there are cases where we can robustly infer the agent's preference, consideration, and non-consideration. Here we derive conditions under which such inferences are possible. Let us denote by P^{TC} the transitive closure of P: that is, $x'' P^{TC} x'$, if there exist $y^0, y^1, \ldots, y^K \in X$ such that $x'' = y^0, x' = y^K$, and $y^{k-1} P y^k$ for every $k \in \{1, 2, \ldots, K\}$. Henceforth, we use superscript "TC" to denote the transitive closure of any binary relation.

Proposition 2. *Suppose that a profile of choice functions $(f_t)_{t \in \mathcal{T}}$ is rationalizable by a Growing Consideration model. Then,*

1. *x'' is robustly preferred to x' if and only if $x'' P^{TC} x'$;*
2. *$x' \in A'$ is robustly considered at (t', A') if and only if*
   (a) *$x' = f_{t'}(A')$, or*
   (b) *there exist $t'' < t'$ and $A'' \in \mathcal{D}_{t''}$ such that $x' = f_{t''}(A'')$;*
3. *$x' \in A'$ is robustly not considered at (t', A') if and only if $x' P^{TC} f_{t'}(A')$.*[11]

Proof. We first show 1. Since sufficiency of 1 is clear, we prove necessity by showing the contrapositive. Suppose that $x'' P^{TC} x'$ does not hold. Then, it is known that there exists an extension $\succ$ of P^{TC} such that (i) $\succ$ is complete, transitive, and asymmetric; and (ii) $x' \succ x''$.[12] Applying the construction of $(\Gamma_t)_{t \in \mathcal{T}}$ in the proof of Proposition 1, we have a GC model $\langle \succ, (\Gamma_t)_{t \in \mathcal{T}} \rangle$ with $x' \succ x''$ that rationalizes $(f_t)_{t \in \mathcal{T}}$.

To show sufficiency of 2, take any GC model $\langle \succ, (\Gamma_t)_{t \in \mathcal{T}} \rangle$ that rationalizes $(f_t)_{t \in \mathcal{T}}$ and consider (t', A'). Whenever (a) or (b) holds, $x' \in \Gamma_{t'}(A')$ follows, by definition of consideration

[11]Note that 2-(a) and 3 have a bite only if $A' \in \mathcal{D}_{t'}$.
[12]See Andrikopoulos (2009) for details.

mappings in the case of (a), and by definition of Growing Consideration in the case of (b). We prove necessity of 2 by showing the contrapositive. Suppose that neither (a) nor (b) holds. Letting $\succ$ be a complete, transitive, and asymmetric extension of P, define $(\Gamma_t)_{t\in\mathcal{T}}$ as follows: for every $(t, A) \neq (t', A')$,

$$\text{if } A \in \mathcal{D}_t, \quad \Gamma_t(A) = \{f_t(A)\} \cup \{x \in A : x = f_{t''}(A'') \text{ for some } t'' < t \text{ and } A'' \in \mathcal{D}_{t''}\};$$
$$\text{if } A \notin \mathcal{D}_t, \quad \Gamma_t(A) = \{x \in A : x = f_{t''}(A'') \text{ for some } t'' < t \text{ and } A'' \in \mathcal{D}_{t''}\};$$

and for (t', A'),

$$\text{if } A' \in \mathcal{D}_{t'}, \quad \Gamma_{t'}(A') = \{f_{t'}(A')\} \cup \{x \in A' \backslash x' : x = f_{t''}(A'') \text{ for some } t'' < t' \text{ and } A'' \in \mathcal{D}_{t''}\};$$
$$\text{if } A' \notin \mathcal{D}_{t'}, \quad \Gamma_{t'}(A') = \{x \in A' \backslash x' : x = f_{t''}(A'') \text{ for some } t'' < t' \text{ and } A'' \in \mathcal{D}_{t''}\}.$$

Then, for every $t \in \mathcal{T}$ and every $A \in \mathcal{D}_t$, since $f_t(A) P_t x$ holds for every $x \in \Gamma_t(A) \backslash f_t(A)$, $f_t(A)$ is the $\succ$-best alternative within $\Gamma_t(A)$. By construction of $(\Gamma_t)_{t\in\mathcal{T}}$, Growing Consideration holds, and we have $x' \notin \Gamma_{t'}(A')$. Thus, $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ is a GC model that rationalizes $(f_t)_{t\in\mathcal{T}}$, but x' is not considered at (t', A').

To show sufficiency of 3, take any GC model $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ that rationalizes $(f_t)_{t\in\mathcal{T}}$, and suppose that $x' P^{TC} f_{t'}(A')$. It follows from 1 that $x' P^{TC} f_{t'}(A')$ implies $x' \succ f_{t'}(A')$. Meanwhile, since $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ rationalizes $(f_t)_{t\in\mathcal{T}}$, $f_{t'}(A')$ must be the $\succ$-best element in $\Gamma_{t'}(A')$. Hence $x' \notin \Gamma_{t'}(A')$ must follow. We prove necessity of 3 by showing the contrapositive. Suppose that $x' P^{TC} f_{t'}(A')$ does not hold. Then there exists an extension $\succ$ of P^{TC} such that (i) $\succ$ is complete, transitive, and asymmetric; and (ii) $f_{t'}(A') \succ x'$. Defining $(\Gamma_t)_{t\in\mathcal{T}}$ as in the proof of Proposition 1, we have a GC model $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ that rationalizes $(f_t)_{t\in\mathcal{T}}$ with $x' \in \Gamma_{t'}(A')$. $\square$

Below we give an example of a choice function that is rationalizable by a GC model, and demonstrate how robust inference can be conducted.

Example 2. Let $X = \{x_1, x_2, x_3, x_4\}$ and $\mathcal{T} = \{1, 2\}$, and consider choice function $(f_t)_{t\in\mathcal{T}}$ summarized in Table 2. We first show that $(f_t)_{t\in\mathcal{T}}$ is rationalizable by a GC model. By definition of binary relation P, it follows that $x_1 P x_2$ and $x_2 P x_3$. Since P is acyclic, $(f_t)_{t\in\mathcal{T}}$ is rationalizable by a GC model. Regarding robust inference, it follows that x_1 is robustly preferred to x_2, and x_2 is robustly preferred to x_3, i.e., we can surely say that $x_1 \succ x_2 \succ x_3$. Looking at 2-(a) of Proposition 2, we see that, for example, x_2 is robustly considered at $(t' =$

$A \in \mathcal{D}_1$	$\{x_1, x_2\}$	$\{x_1, x_3\}$	$\{x_1, x_3, x_4\}$
$f_1(A)$	x_2	x_3	x_1

$A \in \mathcal{D}_2$	$\{x_1, x_2\}$	$\{x_2, x_3\}$
$f_2(A)$	x_1	x_2

Table 2: Choice function $(f_t)_{t\in\mathcal{T}}$ of Example 2.

$1, A' = \{x_1, x_2\}$). Focusing on 2-(b), first note that x_1, x_2, x_3 are chosen at $t = 1$. Therefore, for $i \in \{1, 2, 3\}$, x_i is robustly considered at $A \subseteq X$ at $t = 2$ whenever $x_i \in A$. Finally, since $x_1 P^{TC} x_3$ and $x_3 = f_1(x_1, x_3)$, x_1 is robustly not considered at $(t' = 1, A' = \{x_1, x_3\})$.

4 Growing Consideration model with competition filter property

In this section, we consider a Crowing Consideration model, where we cast an intra-temporal restriction on consideration mapping of each time period. In particular, for every time period $t \in \mathcal{T}$, we require Γ_t to obey the *competition filter property* (henceforth *CFP*), which is defined as follows:

Definition 4. A consideration mapping $\Gamma : 2^X\backslash\varnothing \to 2^X\backslash\varnothing$ obeys the *competition filter property (CFP)* if

$$[x \in A \subseteq A' \text{ and } x \in \Gamma(A')] \Longrightarrow x \in \Gamma(A). \tag{5}$$

The competition filter property requires that an alternative considered at a larger feasible set must be considered at a smaller feasible set. This intuitive property was applied to limited consideration models by Lleras, Masatlioglu, Nakajima, and Ozbay (2017), and is in line with many real-world examples, such as situations where an agent pays attention to: (a) $n \in \mathbb{N}$ most advertised commodities; (b) all goods of a specific brand, and if there are none, then all goods of another brand; (c) $n \in \mathbb{N}$ top candidates in each field in a job market.[13] Moreover, CFP is a "general" requirement, in that it nests many behavioral models. Under models dealt in Manzini and Mariotti (2007, 2012), Au and Kawai (2011), and Cherepanov, Feddersen, and

[13]These examples are given in Lleras, Masatlioglu, Nakajima, and Ozbay (2017).

Sandroni (2013), facing a feasible set, an agent creates a "shortlist" prior to making a final decision. In fact, the "shortlists" created in these models obey CFP, and thus these models can be seen as special cases of Limited Consideration model with CFP.

Below, we first provide a necessary and sufficient condition under which $(f_t)_{t\in\mathcal{T}}$ is rationalizable by a Growing Consideration model, where Γ_t obeys CFP for every $t \in \mathcal{T}$. We refer to such GC model as *Growing Consideration model with CFP*, or in short *GC(CFP) model*. Then, we derive conditions for robust inference of preference/consideration/non-consideration, provided that $(f_t)_{t\in\mathcal{T}}$ is rationalizable by a GC(CFP) model.

4.1 Rationalizability

Suppose that choice function $(f_t)_{t\in\mathcal{T}}$ is generated by an agent obeying a Growing Consideration model $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ with CFP. We derive observable implications of GC(CFP) model, by inferring the agent's consideration and preference from the choice function $(f_t)_{t\in\mathcal{T}}$. To begin with, consider period $t = 1$. For distinct alternatives $x', x'' \in X$, $x'' \succ x'$ can be inferred by the following logic: if there exist $A, A' \in \mathcal{D}_t$ such that $x'' = f_t(A)$,

$$x', x'' \in A \subseteq A', \text{ and } x' = f_t(A'), \tag{6}$$

then by CFP, we must have $x' \in \Gamma_t(A)$, which in turn implies $f_t(A) = x'' \succ x'$. This motivates us to define a binary relation Q_1 as follows: for distinct alternatives $x', x'' \in X$, $x''Q_1x'$ if there exist $A, A' \in \mathcal{D}_1$ such that $x'' = f_1(A)$, and (6) holds for $t = 1$. Now fix any period $t \geqslant 2$ and any $A \subseteq X$. We can infer $x' \in \Gamma_t(A)$ for some $x' \in A$, if (i) there exists $A' \supseteq A$ such that $x' = f_t(A')$; or (ii) there exist $t' < t$ and $A' \in \mathcal{D}_{t'}$ with $x' = f_{t'}(A')$. Then let us define binary relation Q_t as follows: for distinct $x', x'' \in X$, $x''Q_tx'$ if there exists $A \in \mathcal{D}_t$ such that $x'' = f_t(A)$ and (a) there exists $A' \in \mathcal{D}_t$ that obeys (6), or (b) there exist $t' < t$ and $A' \in \mathcal{D}_{t'}$ as in (2). Note that letting $Q = \bigcup_{t\in\mathcal{T}} Q_t$, $x''Qx'$ implies $x'' \succ x'$. Thus, binary relation Q is acyclic under a GC(CFP) model. In fact, acyclicity of Q is not only necessary but also sufficient for rationalizability.

Proposition 3 (GC(CFP) rationalizability). *A profile of choice functions $(f_t)_{t\in\mathcal{T}}$ is rationalizable by a Growing Consideration model with CFP, if and only if binary relation Q is acyclic.*

Proof. We show sufficiency here. Let $\succ$ be a complete, transitive, and asymmetric extension of Q, and define consideration mapping $(\Gamma_t)_{t \in \mathcal{T}}$ as follows: for every $t \in \mathcal{T}$ and every $A \subseteq X$,

$$\Gamma_t(A) = \{x \in A : x = f_t(A''), \exists A'' \supseteq A\} \cup \{x \in A : x = f_{t''}(A''), \exists t'' < t, \exists A'' \in \mathcal{D}_{t''}\}. \qquad (7)$$

Note that $(\Gamma_t)_{t \in \mathcal{T}}$ obeys Growing Consideration by construction. Now fix any $t \in \mathcal{T}$ and $A \in \mathcal{D}_t$, and take any $x \in \Gamma_t(A) \backslash f_t(A)$. This means that (i) there exists $A'' \supset A$ such that $x = f_t(A'')$; or (ii) there exist $t'' < t$ and $A'' \in \mathcal{D}_{t''}$ such that $x = f_{t('}(A'')$. In either case we have $f_t(A) Q_t x$, which in turn implies $f_t(A) \succ x$. This shows that $f_t(A)$ is the $\succ$-best alternative within $\Gamma_t(A)$, for every $t \in \mathcal{T}$ and every $A \in \mathcal{D}_t$. It remains to show that for every $t \in \mathcal{T}$, Γ_t obeys CFP. Fix any $A, A' \subseteq X$ such that $A \subseteq A'$, and take any $x \in \Gamma_t(A') \cap A$. Note that $x \in \Gamma_t(A')$ implies: (a) $x = f_t(A')$; (b) there exists $A'' \supset A'$ with $x = f_t(A'')$; or (c) there exist $t'' < t$ and $A'' \in \mathcal{D}_{t''}$ with $x = f_{t''}(A'')$. Taking a look at (7), we have $x \in \Gamma_t(A)$ in all of these cases. □

Example 2 (continued). We show that $(f_t)_{t \in \mathcal{T}}$ in Table 2 is not rationalizable by a GC(CFP) model. Note that we have $x_1 Q x_2 Q x_3 Q x_1$: $x_1 Q_2 x_2$ holds because x_1 is chosen from $\{x_1, x_2\}$ at $t = 2$ and x_2 is a chosen alternative at $t = 1$; $x_2 Q_2 x_3$ holds following an analogous logic; and $x_3 Q_1 x_1$ holds because, at $t = 1$, $x_1, x_3 \in \{x_1, x_3\} \subset \{x_1, x_3, x_4\}$, x_1 is chosen from $\{x_1, x_3, x_4\}$, and x_3 is chosen from $\{x_1, x_3\}$. Since Q has a cycle, $(f_t)_{t \in \mathcal{T}}$ is not rationalizable by a GC(CFP) model.

4.2 Robust inference

Suppose that a profile of choice functions $(f_t)_{t \in \mathcal{T}}$ is rationalizable by a Growing Consideration model with CFP. As stated before, the GC(CFP) model that rationalizes $(f_t)_{t \in \mathcal{T}}$ is not uniquely determined. Nevertheless, it is possible to infer the agent's preference and consideration, which is shown in the proposition below.

Proposition 4. *Suppose that a profile of choice functions $(f_t)_{t \in \mathcal{T}}$ is rationalizable by a Growing Consideration model with CFP. Then:*

1. *x'' is robustly preferred to x' if and only if $x'' Q^{TC} x'$;*
2. *$x' \in A'$ is robustly considered at (t', A') if and only if*
   (a) *there exists $A'' \supseteq A'$ such that $x' = f_{t'}(A'')$, or*

(b) *there exist* $t'' < t'$ *and* $A'' \in \mathcal{D}_{t''}$ *such that* $x' = f_{t''}(A'')$;

3. $x' \in A'$ *is robustly not considered at* (t', A') *if and only if* $x' Q^{TC} f_{t'}(A'')$ *for some* A'' *such that* $x' \in A'' \subseteq A'$.[14]

Proof. To show sufficiency of 1, take any GC(CFP) model $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ that rationalizes $(f_t)_{t\in\mathcal{T}}$. Suppose $x'' Q^{TC} x'$. In fact, it suffices to show that $x'' Q x'$ implies $x'' \succ x'$. Whenever we have $x'' Q x'$, one of the following holds: (i) there exist $t \in \mathcal{T}$ and $A, A' \in \mathcal{D}_t$ such that $x', x'' \in A \subset A'$, $x'' = f_t(A)$, and $x' = f_t(A')$; or (ii) there exist t, t' with $t' < t$ and $A \in \mathcal{D}_t, A' \in \mathcal{D}_{t'}$ such that $x'' = f_t(A)$, $x' = f_{t'}(A')$, and $x' \in A$. In both cases, we have $x' \in \Gamma_t(A)$, via CFP in the case of (i) and via Growing Consideration in the case of (ii), which in turn implies $x'' \succ x'$. We prove necessity of 1 by showing the contrapositive. Suppose that $x'' Q^{TC} x'$ does not hold. Then there exists and extension $\succ$ of Q^{TC} such that (i) $\succ$ is complete, transitive, and asymmetric; and (ii) $x' \succ x''$. Defining $(\Gamma_t)_{t\in\mathcal{T}}$ as (7), $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ is a GC(CFP) model that rationalizes $(f_t)_{t\in\mathcal{T}}$, and we have $x' \succ x''$.

To show sufficiency of 2, take any GC(CFP) model $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ that rationalizes $(f_t)_{t\in\mathcal{T}}$. If (a) holds, then by CFP we must have $x' \in \Gamma_{t'}(A')$; and if (b) holds, then by Growing Consideration we must have $x' \in \Gamma_{t'}(A')$. We prove necessity of 2 by showing the contrapositive. Suppose that neither (a) nor (b) holds. Then by letting $\succ$ be a complete, transitive, and asymmetric extension of Q^{TC} and defining $(\Gamma_t)_{t\in\mathcal{T}}$ as (7), we have a GC(CFP) model $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ that rationalizes $(f_t)_{t\in\mathcal{T}}$ and $x' \notin \Gamma_{t'}(A')$.

To show sufficiency of 3, take any GC(CFP) model $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ that rationalizes $(f_t)_{t\in\mathcal{T}}$. By robust inference of preference in 1, $x' Q^{TC} f_{t'}(A'')$ implies $x' \succ f_{t'}(A'')$, which means that $x' \notin \Gamma_{t'}(A'')$ holds. Then, since $A'' \subseteq A'$, it follows by CFP that $x' \notin \Gamma_{t'}(A')$.[15] Necessity of 3 will be proved by showing the contrapositive. Suppose that there does not exist $A'' \in \mathcal{D}_{t'}$ such that $x' \in A'' \subseteq A'$ and $x' Q^{TC} f_{t'}(A'')$. Now define $(\Gamma_t)_{t\in\mathcal{T}}$ as in (7), and additionally define a profile of mappings $(\tilde{\Gamma}_t)_{t\in\mathcal{T}}$ such that $\tilde{\Gamma}_t = \Gamma_t$ for every $t \neq t'$ and for period t':

$$\begin{aligned} \tilde{\Gamma}_{t'}(A) &= \Gamma_{t'}(A) \cup \{x'\} \quad \text{if } x' \in A \subseteq A', \\ &= \Gamma_{t'}(A) \quad \text{otherwise.} \end{aligned}$$

Then, define a binary relation $\tilde{Q}$ as follows: for distinct elements $x, y \in X$, $x\tilde{Q}y$ if (i) $xQ^{TC}y$;

[14]Note that Q^{TC} is the transitive closure of Q.
[15]An equivalent expression of (5) is: $[x \in A \subseteq A' \text{ and } x \notin \Gamma(A)] \Longrightarrow x \notin \Gamma(A')$.

or (ii) $y = x'$ and $\neg[x' Q^{TC} x]$. This binary relation ranks x' as low as possible, as long as it does not contradict binary relation Q^{TC}.

Lemma 1. *Binary relation $\tilde{Q}$ is acyclic.*

Proof. Suppose by way of contradiction that $\tilde{Q}$ has a cycle. Since Q^{TC} is acyclic (or asymmetric) and transitive, this cycle must involve x', and it must be in the form: $x' Q^{TC} x \tilde{Q} x'$, and $x \tilde{Q} x'$ must be defined via (ii) in the construction of $\tilde{Q}$. However, this is impossible when $x' Q^{TC} x$, which shows that $\tilde{Q}$ is acyclic. □

Now let $\succ$ be a complete, transitive, and asymmetric extension of $\tilde{Q}$.[16] It remains to show that $\langle \succ, (\tilde{\Gamma}_t)_{t \in \mathcal{T}} \rangle$ is a GC(CFP) model that rationalizes $(f_t)_{t \in \mathcal{T}}$, that is, $(\tilde{\Gamma}_t)_{t \in \mathcal{T}}$ obeys Growing Consideration; $\tilde{\Gamma}_t$ obeys CFP for every $t \in \mathcal{T}$; and $f_t(A)$ is the $\succ$-best element in $\tilde{\Gamma}_t(A)$ for every $t \in \mathcal{T}$ and every $A \in \mathcal{D}_t$. Note that $(\Gamma_t)_{t \in \mathcal{T}}$ defined as (7) obeys Growing Consideration, and since $\Gamma_t(A) \subseteq \tilde{\Gamma}_t(A)$ for every (t, A), $(\tilde{\Gamma}_t)_{t \in \mathcal{T}}$ obeys Growing Consideration as well. Since Γ_t obeys CFP for every t, and $\tilde{\Gamma}_t = \Gamma_t$ for every $t \neq t'$, CFP is satisfied by $\tilde{\Gamma}_t$ for every $t \neq t'$. Now focusing on period t', take any $A, A'' \subseteq X$ such that $A \subseteq A''$ and any $x \in \tilde{\Gamma}_{t'}(A'') \cap A$. If $x \in \Gamma_{t'}(A'')$, then since $\Gamma_{t'}$ obeys CFP, we have $x \in \Gamma_{t'}(A)$, and thus $x \in \tilde{\Gamma}_{t'}(A)$. If $x \notin \Gamma_{t'}(A'')$, this means that $x = x'$ and $x' \in A \subseteq A'' \subset A'$. Then, by construction of $\tilde{\Gamma}_{t'}$, it follows that $x \in \tilde{\Gamma}_{t'}(A)$. Finally, we show that $f_t(A)$ is the $\succ$-best element in $\tilde{\Gamma}_t(A)$ for every $t \in \mathcal{T}$ and every $A \in \mathcal{D}_t$. We already know that this holds at every $t \neq t'$, so fix period t', and consider any $A \in \mathcal{D}_{t'}$ and any $x \in \tilde{\Gamma}_{t'}(A) \backslash f_{t'}(A)$. If $x \in \Gamma_{t'}(A)$, then $f_{t'}(A) \succ x$ follows. If $x \notin \Gamma_{t'}(A)$, this means that $x = x'$ and $x' \in A \subseteq A'$. By assumption, we have $\neg[x' Q^{TC} f_{t'}(A)]$, and thus $f_{t'}(A) \tilde{Q} x'$ holds. Hence $f_{t'}(A) \succ x$ follows. Summarizing, $\langle \succ, (\tilde{\Gamma}_t)_{t \in \mathcal{T}} \rangle$ is a GC(CFP) model that rationalizes $(f_t)_{t \in \mathcal{T}}$, and we have $x' \in \tilde{\Gamma}_{t'}(A')$. □

The following example gives a choice function $(f_t)_{t \in \mathcal{T}}$ that is rationalizable by a GC(CFP) model, and we demonstrate how robust inference can be conducted.

Example 3. Let $X = \{x_1, x_2, x_3, x_4\}$ and $\mathcal{T} = \{1, 2\}$, and consider a choice function $(f_t)_{t \in \mathcal{T}}$ as in Table 3. We first show that $(f_t)_{t \in \mathcal{T}}$ is rationalizable by a GC(CFP) model. From $(f_t)_{t \in \mathcal{T}}$, it follows that $x_1 Q x_2$ and $x_2 Q_3$: we have $x_1 Q_1 x_2$ by (6); and $x_2 Q_2 x_3$ since x_2 is chosen from $\{x_2, x_3\}$ at $t = 2$ and x_3 is a chosen alternative at $t = 1$. Acyclicity of Q implies that $(f_t)_{t \in \mathcal{T}}$ is rationalizable. Considering robust inference, we see that x_1 is robustly preferred to x_2,

[16] Note that $Q^{TC} \subseteq \tilde{Q}$, so $\succ$ is an extension of Q^{TC} as well.

$A \in \mathcal{D}_1$	$\{x_1, x_2\}$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2, x_4\}$
$f_1(A)$	x_1	x_3	x_2

$A \in \mathcal{D}_2$	$\{x_2, x_3\}$
$f_2(A)$	x_2

Table 3: Choice function $(f_t)_{t\in\mathcal{T}}$ of Example 3.

$A \in \mathcal{D}_1$	$\{x_1, x_2\}$	$\{x_1, x_3\}$	$\{x_2, x_3\}$	$\{x_1, x_2, x_3\}$
$f_1(A)$	x_2	x_3	x_2	x_2

$A \in \mathcal{D}_2$	$\{x_1, x_2\}$	$\{x_1, x_3\}$	$\{x_2, x_3\}$	$\{x_1, x_2, x_3\}$
$f_2(A)$	x_1	x_3	x_2	x_2

Table 4: Choice function $(f_t)_{t\in\mathcal{T}}$ of Example 4.

and that x_2 is robustly preferred to x_3. Therefore, we can surely say that $x_1 \succ x_2 \succ x_3$. Regarding consideration, x_2 is robustly considered at $(t' = 1, A' = \{x_1, x_2\})$ following 2-(a) in Proposition 4, and x_2, x_3 are robustly considered at $(t' = 2, A' = \{x_2, x_3\})$ following 2-(b). Finally, since $x_i Q^{TC} x_3$ for $i \in \{1, 2\}$, it holds that x_1, x_2 are robustly not considered at $(t' = 1, A' = \{x_1, x_2, x_3\})$.

We show below an example where our robust inference of preference and the welfare criterion by Bernheim and Rangel (2007, 2009) lead to opposite welfare implications. The welfare criterion proposed by Bernheim and Rangel, which is referred to as an *unambiguous choice* relation, ranks alternative x'' over x', if x' is never chosen when x'' is available. Note that the choice function of Example 4 is defined on an exhaustive domain, so the discussion regarding Bernheim and Rangel's unambiguous choice relation is well-defined.

Example 4. Let $X = \{x_1, x_2, x_3\}$ and $\mathcal{T} = \{1, 2\}$, and consider a choice function $(f_t)_{t\in\mathcal{T}}$ as in Table 4. Note that we have $x_1 Q x_2 Q x_3$. Since Q is acyclic, $(f_t)_{t\in\mathcal{T}}$ is rationalizable by a GC(CFP) model, and applying Proposition 4, we can surely say that x_1 is preferred to x_3. On the other hand, unambiguous choice relation by Bernheim and Rangel concludes that x_3 is welfare-improving over x_1, since x_1 is never chosen when x_3 is available.

Appendix: Growing Consideration model with attention filter property

Here we derive revealed preference tests for Growing Consideration model where we require consideration mapping Γ_t to obey the *attention filter property* (henceforth *AFP*) for every $t \in \mathcal{T}$. Let us refer to such model as *Growing Consideration model with AFP*, or in short GC(AFP) model. Below is a formal definition of the AFP.

Definition 5. A consideration mapping $\Gamma : 2^X \backslash \varnothing \to 2^X \backslash \varnothing$ obeys the *attention filter property (AFP)* if for every $A \subseteq X$ and $x \in A$,

$$x \notin \Gamma(A) \Longrightarrow \Gamma(A) = \Gamma(A \backslash x); \tag{8}$$

or equivalently, for every $A', A'' \subseteq X$,

$$\Gamma(A'') \subseteq A' \subseteq A'' \Longrightarrow \Gamma(A') = \Gamma(A''). \tag{9}$$

In words, AFP requires that removal of an ignored alternative does not alter the agent's consideration: if she ignores alternative x at set A, then the set $A \backslash x$ should be treated the same as A. This intuitive requirement was applied to the choice theoretical framework by Masatlioglu, Nakajima, and Ozbay (2012).[17] In fact, AFP is one of the fundamental properties regarding Limited Consideration models, in that it is a property independent from CFP, and together with CFP it nests most Limited-Consideration-related models. A revealed preference characterization, as well as conditions for robust inference, for Limited Consideration model with AFP (i.e., the Limited Attention model) are given by Masatlioglu, Nakajima, and Ozbay (2012). However, their approach is not directly applicable to ours, since they assume that the choice function is defined on an exhaustive domain.

Revealed preference tests under the assumption that choices are defined on a non-exhaustive domain are given by De Clippel and Rozen (2018) and Inoue and Shirai (2018). These two papers derive revealed preference tests through different approaches. In De Clippel and Rozen (2018) they derive a property that any Limited-Attention-consistent preference relation must obey, and then express their revealed preference test in terms of existence of a binary relation

[17] The three real-world examples given at the beginning of Section 4 are in line with AFP as well.

obeying that property. On the other hand, Inoue and Shirai (2018) focus on the fact that we must take into consideration Limited Consideration models only when there are revealed preference cycles, and derive a revealed preference test for Limited Attention models in terms of a condition on the structure of revealed preference cycles.[18] While both approaches are applicable to deriving a revealed preference test for Growing Consideration model with AFP, here we adopt De Clippel and Rozen's, since it is closer to the approaches that we used in the baseline GC model and GC(CFP) model.

It is worth noting here that revealed preference tests for the Limited Attention model provided by De Clippel and Rozen (2018) and Inoue and Shirai (2018) are computationally challenging, since both tests require combinatorial search.[19] Moreover, under the non-exhaustive domain assumption on the observed data set, conditions for robust inference are yet to be discovered. This is partly due to sparsity of data, which makes it relatively difficult to make deterministic statements regarding behavior. In fact, the reason why these revealed preference test requires combinatorial search is because it is not possible to pin down the agent's consideration in general. Since robust inference in Limited Consideration model with AFP is a challenge even in the static case, in this appendix, we focus on showing that there exists a revealed preference test for Growing Consideration model with AFP, and postpone the issue of robust inference for future research.

Now let us derive a revealed preference test for GC(AFP) model. Suppose that choice function $(f_t)_{t\in\mathcal{T}}$ is generated by an agent obeying a GC(AFP) model $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}}\rangle$. To see observable restrictions of AFP, let us first fix any period $t \in \mathcal{T}$. Suppose that there exist feasible sets $A', A'' \in \mathcal{D}_t$ such that

$$f_t(A'), f_t(A'') \in A' \cap A'' \text{ and } f_t(A') \neq f_t(A''). \tag{10}$$

In this case, it must follow that

$$[y \in \Gamma_t(A') \text{ for some } y \in A' \backslash A''] \textbf{ or } [z \in \Gamma_t(A'') \text{ for some } z \in A'' \backslash A']. \tag{11}$$

[18]When a choice function exhibits no revealed preference cycles, it can be accounted for by the rational choice model, so we need not deal with Limited Consideration models.

[19]Nevertheless, both papers propose tractable methods to conduct their revealed preference tests, provided that the data set is not too "large". Revealed preference test by De Clippel and Rozen (2018) can be conducted by a method called *enumeration*, and Inoue and Shirai (2018)'s test can be conducted by applying a search method called *backtracking*. See these papers for details.

To see this, suppose not. In this case, we have $\Gamma_t(A') \subseteq (A' \cap A'') \subseteq A'$ and $\Gamma_t(A'') \subseteq (A' \cap A'') \subseteq A''$. Then, by AFP, it must follow that $\Gamma_t(A') = \Gamma_t(A' \cap A'') = \Gamma_t(A'')$. However, this is a contradiction, since we have $f_t(A') \neq f_t(A'')$. Whenever (11) holds, this means that $[f_t(A') \succ y$ for some $y \in A' \backslash A'']$ or $[f_t(A'') \succ z$ for some $z \in A'' \backslash A']$. Therefore, any binary relation $\mathcal{P}$ that reflects the agent's preference must obey the following: for every $A', A'' \in \mathcal{D}_t$ such that (10) holds,

$$[f_t(A')\mathcal{P}y \text{ for some } y \in A' \backslash A''] \textbf{ or } [f_t(A'')\mathcal{P}z \text{ for some } z \in A'' \backslash A']. \tag{12}$$

In addition to this, there are observable restrictions of Growing Consideration. Any binary relation $\mathcal{P}$ that reflects the agent's preference must obey the following: for distinct $x', x'' \in X$, $x''\mathcal{P}x'$ if there exist $t', t'' \in \mathcal{T}$ with $t' < t''$, and $A' \in \mathcal{D}_{t'}$, $A'' \in \mathcal{D}_{t''}$ such that

$$f_{t'}(A') = x', f_{t''}(A'') = x'', \text{ and } x' \in A''. \tag{13}$$

Summarizing, we have a condition for rationalizability by a GC(AFP) model. Lemmas used in proof of the following proposition are proved at the end of the Appendix.

Proposition 5 (GC(AFP) rationalizability). *A profile of choice functions $(f_t)_{t \in \mathcal{T}}$ is rationalizable by a Growing Consideration model with AFP, if and only if there exists an acyclic binary relation $\mathcal{P}$ that obeys (12) and (13).*

Proof. Since necessity in shown in the discussion preceding this proposition, we show sufficiency here. Let $\succ$ be a complete, transitive, and asymmetric extension of $\mathcal{P}$. Then, for every $t \in \mathcal{T}$, define consideration mapping Γ_t as follows:

$$A \in \mathcal{D}_t \Longrightarrow \Gamma_t(A) = \{f_t(A)\} \cup \{x \in A : f_t(A) \succ x\}; \tag{14}$$

$$A \notin \mathcal{D}_t \Longrightarrow \Gamma_t(A) = \begin{cases} \Gamma(A'') \text{ if } \Gamma(A'') \subseteq A \subseteq A'' \text{ for some } A'' \in \mathcal{D}_t; \\ A \text{ otherwise.} \end{cases} \tag{15}$$

Lemma 2. *Consideration mapping Γ_t is well-defined for every $t \in \mathcal{T}$.*

It remains to show that (i) $f_t(A)$ is the $\succ$-best alternative within $\Gamma_t(A)$ for every $t \in \mathcal{T}$ and every $A \in \mathcal{D}_t$; (ii) Γ_t obeys AFP for every $t \in \mathcal{T}$; and (iii) $(\Gamma_t)_{t \in \mathcal{T}}$ obeys Growing Consideration. By constriction of $(\Gamma_t)_{t \in \mathcal{T}}$, it follows immediately that $f_t(A)$ is the $\succ$-best alternative within

$\Gamma_t(A)$ for every $t \in \mathcal{T}$ and every $A \in \mathcal{D}_t$.

We show here that $(\Gamma_t)_{t\in\mathcal{T}}$ obeys Growing Consideration. Take any $t \geqslant 2$, $A \subseteq X$, and any $x' \in A$ such that $x' = f_{t'}(A')$ for some $t' < t$ and $A' \in \mathcal{D}_{t'}$. We show that $x' \in \Gamma_t(A)$ for three cases. If $A \in \mathcal{D}_t$, then it must be the case that $f_t(A)\mathcal{P}x'$ or $f_t(A) = x'$. By (14), we have $x' \in \Gamma_t(A)$ in either case. Now consider the case where $A \notin \mathcal{D}_t$ and $\Gamma(A'') \subseteq A \subseteq A''$ for some $A'' \in \mathcal{D}_t$. Note that we have $x' \in \Gamma_t(A'')$, as shown right above. Then, by construction of Γ_t, $x' \in \Gamma_t(A)$ follows. Finally, if $A \notin \mathcal{D}_t$ and $\Gamma(A'') \subseteq A \subseteq A''$ for no $A'' \in \mathcal{D}_t$, then since $\Gamma_t(A) = A$, we have $x' \in \Gamma_t(A)$. Summarizing, $(\Gamma_t)_{t\in\mathcal{T}}$ obeys Growing Consideration.

As the final step of the proof, we show that Γ_t obeys AFP for every $t \in \mathcal{T}$. Fix any $t \in \mathcal{T}$ and $A \subseteq X$ such that $x' \in A$ and $x' \notin \Gamma_t(A)$ hold. Denoting $A' = A \backslash x'$, it suffices to show that $\Gamma_t(A') = \Gamma_t(A)$. Note that $x' \notin \Gamma_t(A)$ implies that either (a) $A \in \mathcal{D}_t$ or (b) $A \notin \mathcal{D}_t$ and $\Gamma_t(A'') \subseteq A \subseteq A''$ for some $A'' \in \mathcal{D}_t$, which are the two cases that we must deal with.

Case (a): $A \in \mathcal{D}_t$. Note that $x' \notin \Gamma_t(A)$ means that $x' \succ f_t(A)$, and that $\Gamma_t(A) \subseteq A' \subseteq A$ holds. Therefore, if $A' \notin \mathcal{D}_t$, then by (15), it follows that $\Gamma_t(A') = \Gamma_t(A)$. When $A' \in \mathcal{D}_t$, we shall apply the following lemma.

Lemma 3. *Suppose that $B, B' \in \mathcal{D}_t$ obeys $\Gamma_t(B) \subseteq B' \subseteq B$. Then $f_t(B') = f_t(B)$, and moreover, we have $\Gamma_t(B') = \Gamma_t(B)$.*

Then, since $A, A' \in \mathcal{D}_t$ and $\Gamma_t(A) \subseteq A' \subseteq A$, it follows that $\Gamma_t(A') = \Gamma_t(A)$.

Case (b): $A \notin \mathcal{D}_t$ and $\Gamma_t(A'') \subseteq A \subseteq A''$ for some $A'' \in \mathcal{D}_t$. Note that in this case, we have $\Gamma_t(A) = \Gamma_t(A'')$. Since we have $x \notin \Gamma_t(A)$, we have $x \notin \Gamma_t(A'')$ as well. This in turn implies $\Gamma_t(A'') \subseteq A' \subseteq A''$. If $A' \in \mathcal{D}_t$, then it follows from Lemma 3 that $\Gamma_t(A'') = \Gamma_t(A')$. Therefore, $\Gamma_t(A') = \Gamma_t(A)$ holds.

Summarizing, we have a Growing Consideration model $\langle \succ, (\Gamma_t)_{t\in\mathcal{T}} \rangle$ with AFP that rationalizes choice function $(f_t)_{t\in\mathcal{T}}$. □

Proof of Lemma 2

Fixing any $t \in \mathcal{T}$, the substantial case that we must consider is when there exists $A \notin \mathcal{D}_t$ such that for some $A', A'' \in \mathcal{D}_t$,

$$[\Gamma_t(A') \subseteq A \subseteq A'] \text{ and } [\Gamma_t(A'') \subseteq A \subseteq A'']. \tag{16}$$

In this case, it suffices to show that $\Gamma_t(A') = \Gamma_t(A'')$. Note that (16) implies that $\Gamma_t(A') \subseteq (A' \cap A'')$ and $\Gamma_t(A'') \subseteq (A' \cap A'')$. Then by construction of Γ_t, it follows that [$y \succ f_t(A')$ for every $y \in A' \backslash A''$] and [$z \succ f_t(A'')$ for every $z \in A'' \backslash A'$]. This in turn implies that [$f_t(A')\mathcal{P}y$ for no $y \in A' \backslash A''$] and [$f_t(A'')\mathcal{P}z$ for no $z \in A'' \backslash A'$]. Since $\mathcal{P}$ obeys (12), it must be the case that $f_t(A') = f_t(A'') =: x''$. Then, $\Gamma_t(A') \subseteq A$ means that $\Gamma_t(A') = \{x''\} \cup \{x \in A : x'' \succ x\}$. Analogously, we have $\Gamma_t(A'') = \{x''\} \cup \{x \in A : x'' \succ x\}$, and thus it follows that $\Gamma_t(A') = \Gamma_t(A'')$.

Proof of Lemma 3

Firstly, to see that $f_t(B') = f_t(B)$, suppose to the contrary. Note that we have $f_t(B), f_t(B') \in (B \cap B')$ and $B' \backslash B = \varnothing$. Moreover, $\Gamma_t(B) \subseteq B'$ implies that

$$x \succ f_t(B) \text{ for every } x \in B \backslash B'. \tag{17}$$

This in turn implies that $f_t(B)\mathcal{P}x$ holds for no $x \in B \backslash B'$. However, this contradicts that $\mathcal{P}$ obeys (12), and we conclude $f_t(B') = f_t(B)$. Letting $x'' = f_t(B') = f_t(B)$, since (17) holds, it follows that $\Gamma_t(B) = \{x''\} \cup \{x \in B : x'' \succ x\} = \{x''\} \cup \{x \in B' : x'' \succ x\} = \Gamma_t(B')$.

References

[1] Andrikopoulos, A. (2009): Szpilrajn-type theorems in economics. MPRA paper No. 14345.

[2] Au, P.H. and K. Kawai (2011): Sequentially rationalizable choice with transitive rationales. *Games and Economic Behavior*, 73, 608-614.

[3] Bernheim, B.D. and A. Rangel (2007): Toward choice-theoretic foundations for behavioral welfare economics. *American Economic Review: Papers and Proceedings*, 97(2), 464-470.

[4] Bernheim, B.D. and A. Rangel (2009): Beyond revealed preference: choice-theoretic foundations for behavioral welfare economics. *Quarterly Journal of Economics*, 124(1), 51-104.

[5] Cherepanov, V., T. Feddersen, and A. Sandroni (2013): Rationalization. *Theoretical Economics*, 8, 775-800.

[6] De CLippel, G. and K. Rozen (2018): Bounded Rationality and Limited Datasets. Mimeo

[7] Ferreira, J.V. and N. Gravel (2017): Choice with time. Mimeo.

[8] Inoue, Y. and K. Shirai (2018): Limited consideration and limited data: revealed preference and observable restrictions. Kwansei Gakuin University School of Economics Discussion paper No.176.

[9] Lleras, J.S., Y. Masatlioglu, D. Nakajima, and E. Ozbay (2017): When more is less: limited consideration. *Journal of Economic Theory*, 170, 70-85.

[10] Manzini, P. and M. Mariotti (2007): Sequentially rationalizable choice. *American Economic Review*, 97, 1824-1839.

[11] Manzini, P. and M. Mariotti (2012): Categorize then choose: boundedly rational choice and welfare. *Journal of the European Economic Association*, 10, 1141-1165.

[12] Masatlioglu, Y., D. Nakajima, and E.Y. Ozbay (2012): Revealed attention. *American Economic Review*, 102, 2183-2205.

[13] Moulin, H. (1985): Choice functions over a finite set: a summary. *Social Choice and Welfare*, 2, 147-160.

[14] Salant, Y. and A. Rubinstein (2008): (A, f): Choice with frames. *Review of Economic Studies*, 75, 1287-1297.