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Decision–making on public facility location by using social choice rules with a deliberative suggestion

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Abstract

To decide a public facility location by considering both voters' preferences and a deliberative suggestion from the government (all of them are assumed to be complete preorders over a fixed finite location set), we propose a *democratic–deliberative preference update system* (DD system). The DD system updates the voters' preferences according to the government's preference as follows: if any two locations are indifferent for a voter, the updated preference of the voter is equivalent to that of the government, and if any two locations are ordered strictly by the voter, the updated preference is equal to the original one. We provide a simple axiomatic characterisation of the DD system and show that the system updates the voters' preferences in linear time. We then compare the set of winners by using (I) only the voters' preferences, (II) only the government's preference, and (III) the democratic–deliberative updated preferences based on the following five scoring rules: (adjusted) plurality, (adjusted) anti-plurality, and Borda rules. We run the simulation 10,000 times for each pair of the numbers of locations and voters and estimate the relationship between the set of winners, the numbers of locations and voters, and the voting rules. We find that the Borda rule has better performance than the others in our simulation and regression analyses.

Keywords: single–facility location, the democratic–deliberative preference update system, axiomatisation, time complexity, simulation analyses, regression analyses
JEL Classification: C63, D70, D71, D72, D83

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1. Introduction

We study single–facility location problems with a variable voter set and a fixed and finite location set by using voting systems and a deliberative suggestion from the government. Particularly, we associate an arbitrary NIMBY (Not In My Backyard) or YIMBY (Yes In My Backyard) public facility (e.g., public hospital, public school, library, refuse dump) in this study.

In the cross–sectional area of facility location problems and social choice theories, there are two major settings: facility location problems with (1) single–peaked preferences over points in a given area and (2) voters' preferences over a given finite location set, respectively.

In the setting (1), there is a given area (e.g., line, circle), and we assume that voters' preferences are single–peaked according to the duration between voters and points in the area. The setting (1) has been developed by Moulin (1980); Schummer and Vohra (2002); Procaccia and Tennenholtz (2009); Alon et al. (2010); Lu et al. (2010); Cheng et al. (2011); Todo et al. (2011); Cheng et al. (2013); Ye et al. (2015); Fong et al. (2018), and so forth.

In the setting (2), we give the finite location set in a metric space and use only voters' preferences. We do not use single–peaked preferences and the duration between voters and locations. The setting (2) has been developed by Boutilier et al. (2015); Feldman et al. (2016); Anshelevich and Postl (2017); Goel et al. (2017); Anshelevich et al. (2018); Anshelevich and Zhu (2018); Cheng et al. (2018), and so forth.

This study employs an analytical method locating between the settings (1) and (2) since there are conceptual problems in both settings when we study the public facility location problem.

We cannot say that the outcome is determined democratically in the setting (1) if 'assuming that every voter has a single–peaked preference' indicates that we cannot observe true voters' preferences. In the real world, several problems happen because of this assumption. For example, if a hospital is very close to our housing, some people will have a sleep disorder because of a siren in the deep of night. Furthermore, even if a resident needs to go to the hospital frequently, the duration from the resident and the hospital may be quite far by assuming that 'all' voters prefer closer locations. Furthermore, if the government tries to solve the problem, it will need enormous costs to collect information on voters' circumstances and attributes.

Should we then call an election to decide the facility location, such as the setting (2)? There must exist a reason why the government has not done that up to now in the real world. There are two main reasons. First, voters do not have enough specialised knowledge to represent their preferences over the locations. Second, each public facility has a specific role in society. Thus, if we use only voters' preferences over the locations, that is, personal considerations, it is possible that the facility does not work well. Hence, any prejudiced method is sometimes inappropriate to decide the public facility location.

This study shows a method to reflect both voters' preferences and a deliberative suggestion of the government (the government's preference) in the set of winners. Note that we do not assume that every voter has a single–peaked preference. Every voter and the government are assumed to have complete preorders over the finite location set. We then propose a *democratic–deliberative preference update system* (DD system) which updates the voters' preferences according to the government's preference as follows: if any two locations are indifferent for a voter, the updated preference of the voter is equivalent to that of the government, and if any two locations are ordered strictly by the voter, the updated preference is equal to the original one. We prove that every democratic–deliberative updated preference (DD updated preference) is also a complete preorder over the location set. Note that we should not add the government as a voter and call an election since the government's suggestion becomes meaningless if the number of voters is sufficiently large.

Our concept is similar to that of equilibriums in a market economy if we regard the voters' preferences and the deliberative suggestion of the government as the demand and the supply for feasible facility locations. We conceived the concept by referring to studies on the *judgement aggregation theory in social networks* and *diffusion of preferences* that have been developed by Brill et al. (2016); Botan et al. (2019) and Yildiz et al. (2010); Hassanzadeh et al. (2013); Bredereck and Elkind (2017), respectively. In the literature, we consider communications among all voters. If voter i changes his/her preference into that of voter j by communicating with j, we call j an *influencer* for i. After voters communicate with each other, society outputs the set of winners. We then give the role of partial influencer to the government, who is outside of the voter set, and do not use a social network structure. The partial influencer reflects its preference in the voters' preferences partially, such as the DD system.

From the above arguments, the major difference between our and previous studies is that we employ the combination of the settings (1) and (2) and consider the government as the partial influencer for all voters according to the DD system.

We characterise the DD system by *respectfulness*, *synchrony*, and *affinity*. *Respectfulness* requires that if a voter and the government have reversal preference orders of any two locations, the updated preference order of the two locations is the same as the voter's one. *Synchrony* requires that, for any two locations a and b, if either a voter or the government strictly prefers a to b and the other considers that a and b are indifferent, the updated preference order shows that a is strictly better than b. *Affinity* requires that, for any two locations, if both a voter and the government have the same preference orders of the two locations, the updated preference order is the same as their original orders. Furthermore, we prove that the DD system updates the voters' preferences in linear time by providing an algorithm.

Finally, we compare the set of winners by using (I) only the voters' preferences, (II) only the government's preference, and (III) the DD updated preferences. We apply the (adjusted) plurality, (adjusted) anti-plurality, and Borda rules for the comparative analyses. We prepare 294 pairs of the numbers of locations and voters: three to eight locations and two–50 voters. We apply the five voting rules for each pair. Thus, there are 1,470 situations which are identified based on the numbers of locations and voters and a kind of voting rules, such as 'the Borda rule with five locations and 14 voters'. We assign a preference to each voter and the government at random and calculate the outcomes for 10,000 times for each situation. After that, we create data set whose number of observations is 1,470 and use the following three dependent variables for the log–link–log regressor in this study: the first is the number of times that the outcomes using (III) includes only one location per 10,000 times. The second is the ratio that the outcomes using (I) and (III) are the same and thoes using (II) and (III) are different per 10,000 times. The third is the ratio that the outcomes using (I) and (III) are different and thoes using (II) and (III) are the same per 10,000 times. The last two variables are related to the weak points of settings (1) and (2) since if the first ratio increases, it indicates that the DD system cannot avoid democratic and non–deliberative results even more, and if the second ratio increases, it indicates that the DD system cannot avoid deliberative and non–democratic results even more. We then estimate the relationship between the three dependent variables, the numbers of locations and voters, and the voting rules. As a result, the Borda rule has better performance than the others in our analyses.

The remainder of this paper is structured as follows. Section 2 reports our notations and definitions. Section 3 characterises the DD system and proves that the DD system updates the voters' preferences in linear time. Section 4 introduces our analytical methods for the comparative analyses of the set of winners by using the preference profiles (I)–(III) for each voting rule and reports the results. Finally, Section 5 provides concluding remarks.

2. Preliminaries

Let $V \subseteq \mathbb{Z}_+$ be a finite voter (e.g., resident, or household) set, where $\mathbb{Z}_+$ is the set of positive integers and $|V| \geq 1$. The finite location set is denoted by X, and each location is denoted by $a, b, c, ...$ Furthermore, let $g \in \mathbb{Z}_+ \setminus V$ be the government of the society giving the citizenship for all voters in V. We suppose that locations in X are feasible locations for a public facility, and the government has an arbitrary principle to decide facility location and a deliberative preference ranking of all locations in X as a supplier.

A complete preorder over X is denoted by $R_i \in \mathscr{R}$ for each $i \in V$, where $\mathscr{R}$ is the set of all complete preorders over X. The asymmetric and symmetric components are denoted by P_i and I_i, respectively. Let $\mathcal{R} = (R_i)_{i \in V} \in \mathscr{R}^{|V|}$ be a profile of all R_i. Thereby, we assume *unrestricted domain* in this paper. Additionally, g has a complete preorder over X denoted by $R_g \in \mathscr{R}$. We then regard R_g as the *deliberative suggestion* from the government.

Let $C : \mathscr{R}^{|V'|} \rightrightarrows X$ be a social choice correspondence over X, which outputs a non–empty subset of X, for any $V' \subset \mathbb{Z}_+$ (both V and $\{g\}$ are allowed to be V').

2.1. Deliberative preference update system

We assume that if the society decides to update $\mathcal{R}$ by using R_g for getting the deliberative and democratic collective choice, the society employs an arbitrary update system $s(R_i, R_g)$ whose mapping is $s : \mathscr{R}^2 \to \mathscr{R}$.

In the beginning, we consider two specific update systems called *democratic–deliberative* and *moderate–deliberative preference update systems*.

The democratic–deliberative preference update system (DD system), denoted by s_{dd}, updates R_i with R_g 'only if' there exist indifferent locations for $i \in V$. We define the DD system as follows:

Definition 1. *democratic–deliberative preference update system*:

$$s = s_{dd} \Leftrightarrow s(R_i, R_g) = R_i^{dd} \text{ s.t. } \forall a, b \in X, [aI_ib \Rightarrow [aR_i^{dd}b \Leftrightarrow aR_gb]] \wedge [aP_ib \Rightarrow aP_i^{dd}b].$$

From Definition 1, if all locations are indifferent for g, or i has a linear order over X, $R_i^{dd} = R_i$, and if all locations are indifferent for i, $R_i^{dd} = R_g$.

Proposition 1 shows that R_i^{dd} is a complete preorder over X for every $i \in V$.

Proposition 1. R_i^{dd} satisfies *reflexivity*, *completeness*, and *transitivity*.

Proof. It is trivial that R_i^{dd} satisfies *reflexivity* and *completeness*. We then prove that R_i^{dd} satisfies *transitivity*.

From Definition 1, we obtain that, for any $a, b \in X$,

$$aP_i^{dd}b \Leftrightarrow [aP_ib \vee [aI_ib \wedge aP_gb]], \quad aI_i^{dd}b \Leftrightarrow [aI_ib \wedge aI_gb].$$

Take any three locations $a, b, c \in X$. From the DD system and *transitivity* of R_i and R_g,

$$\begin{aligned}
[aP_i^{dd}b \wedge bP_i^{dd}c] &\Leftrightarrow [aP_ib \vee [aI_ib \wedge aP_gb]] \wedge [bP_ic \vee [bI_ic \wedge bP_gc]] \\
&\Leftrightarrow [aP_ibP_ic \vee [aP_ibI_ic \wedge bP_gc] \vee [aI_ibP_ic \wedge aP_gb] \vee [aI_ibI_ic \wedge aP_gbP_gc]] \\
&\Rightarrow [aP_ic \vee [aP_ic \wedge bP_gc] \vee [aP_ic \wedge aP_gb] \vee [aI_ic \wedge aP_gc]] \\
&\Leftrightarrow [aP_ic \vee [aI_ic \wedge aP_gc]] \\
&\Leftrightarrow aP_i^{dd}c,
\end{aligned}$$

and

$$\begin{aligned}
[aP_i^{dd}b \wedge bI_i^{dd}c] &\Leftrightarrow [[aP_ib \vee [aI_ib \wedge aP_gb]] \wedge [bI_ic \wedge bI_gc]] \\
&\Leftrightarrow [[aP_ibI_ic \wedge bI_gc] \vee [aI_ibI_ic \wedge aP_gbI_gc]] \\
&\Rightarrow [[aP_ic \wedge bI_gc] \vee [aI_ic \wedge aP_gc]] \\
&\Leftrightarrow [aP_ic \vee [aI_ic \wedge aP_gc]] \\
&\Leftrightarrow aP_i^{dd}c.
\end{aligned}$$

Similarly, from the DD system and *transitivity* of R_i and R_g, $aI_i^{dd}b$ and $bP_i^{dd}c$ imply that $aP_i^{dd}c$. Finally, from the DD system and *transitivity* of R_i and R_g,

$$\begin{aligned}
[aI_i^{dd}b \wedge bI_i^{dd}c] &\Leftrightarrow [[aI_ib \wedge aI_gb] \wedge [bI_ic \wedge bI_gc]] \\
&\Leftrightarrow [aI_ibI_ic \wedge bI_gc \wedge bI_ic \wedge aI_gbI_gc] \\
&\Rightarrow [aI_ic \wedge aI_gc] \\
&\Leftrightarrow aI_i^{dd}c.
\end{aligned}$$

From these results, R_i^{dd} satisfies *transitivity*. □

Next, the moderate–deliberative preference update system (MD system), denoted by s_{md}, updates R_i with R_g if there exist indifferent locations for $i \in V$. Additionally, if i and g have reverse strict preference orders of any two locations for each other, the system updates i's strict order with an indifference order for the two locations. We define the MD system as follows:

Definition 2. *Moderate–deliberative update system*:

$$s = s_{md} \Leftrightarrow s(R_i, R_g) = R_i^{md} \text{ s.t. } \forall a, b \in X, [aI_ib \Rightarrow [aR_i^{md}b \Leftrightarrow aR_gb]]$$

$$\wedge[aP_ib \wedge aR_gb \Rightarrow aP_i^{md}b] \wedge [aP_ib \wedge bP_ga \Rightarrow aI_i^{md}b].$$

However, we obtain a negative result of the MD system, Proposition 2, which shows that R_i^{md} is not a complete preorder over X. For example, if aP_ibP_ic and bP_gcP_ga, $aI_i^{md}b$ and $aI_i^{md}c$, but $bP_i^{md}c$ from Definition 2.

Proposition 2. R_i^{md} violates *transitivity*.

We then treat only the DD system in this study. We denote the DD updated preference profiles $\mathcal{R}^{dd} = (R_i^{dd})_{i \in V} \in \mathscr{R}^{|V|}$. If we interpret that $\mathcal{R}$ and R_g indicate the demand and the supply for each location, respectively, $\mathcal{R}^{dd}$ indicates an equilibrium on the $|X|$–location market.

This study considers the following three cases: the society determines the outcomes by using (I) only R_g, (II) only $\mathcal{R}$, and (III) $\mathcal{R}^{dd}$ hereafter.

2.2. Preference converters of the government

Next, we describe the construction processes of R_g as a background of this study. As said earlier, we assume that the government has an arbitrary principle to decide facility location and a deliberative preference ranking of all locations in X as a supplier.

Let $\boldsymbol{y}_g \in \mathbb{R}^w$ be the information vectors used to construct R_g. Suppose that $f^g : \mathbb{R}^w \rightarrow \mathscr{R}$ is a *preference converter* of the government such that $f^g(\boldsymbol{y}_g) = R_g$. By using this setting, we can interpret that the government uses exogenous factors (e.g. the duration between voters and locations) or provisional voters' preferences to construct R_g instead of assuming that the voter's preferences are single–peaked.

2.2.1. Examples

There are two fundamental preference converters in this area. Suppose that $\boldsymbol{y}_g = (t_{ia})_{i \in V, a \in X}$, where $t_{ia} \in \mathbb{R}_{>0}$ is the duration between voter $i \in V$ and location $a \in X$.

The first is the *minisum location preference converter*, which is appropriate to decide YIMBY (yes in my backyard) facility location. Let the total duration be denoted by $t_a = \Sigma_{i \in V} t_{ia} \in \mathbb{R}_{>0}$ for each $a \in X$. We define the minisum location preference converter denoted by f^g_{min} as follows:

Definition 3. *Minisum location preference converter*:

$$f^g = f^g_{min} \Leftrightarrow f^g((t_{ia})_{i \in V, a \in X}) = R_g \text{ s.t. } aR_gb \Leftrightarrow t_a \leq t_b \; \forall a, b \in X.$$

The second is the *maxisum location preference converter*, which is appropriate to decide NIMBY (not in my backyard) facility location. We then define the maxisum location preference converter denoted by f^g_{max} as follows:

Definition 4. *Maxisum location preference converter*:

$$f^g = f^g_{max} \Leftrightarrow f^g((t_{ia})_{i\in V, a\in X}) = R_g \text{ s.t. } aR_gb \Leftrightarrow t_a \geq t_b \ \forall a, b \in X.$$

In the following analyses, we do not use a specific f^g in our comparative analyses in Section 4. We assume that the government employs an arbitrary preference converter f^g, such as f^g_{min} and f^g_{max}, and do not construct R_g based on their own benefit.

3. Characterisations of the DD system

3.1. Axiomatic analysis

We introduce the following axioms to characterise the DD system, s_{dd}.

Anti–enforcement: For each $i \in V$, $s(R_i, R_g) = R^*_i$ such that aP_ib does not imply bP^*_ia for any $a, b \in X$. This requires that if each voter has a strict preference order for any two locations, the order must not be reversed after updating R_i by an arbitrary update system s.

Respectfulness: For each $i \in V$, $s(R_i, R_g) = R^*_i$ such that aP_ib and bP_ga imply that aP^*_ib for any $a, b \in X$. This requires that if i's and g's preference orders are reverse, i's updated preference order of the two locations is the same as i's original one.

Synchrony: For each $i \in V$, $s(R_i, R_g) = R^*_i$ such that aP^*_ib if (i) aP_ib and aI_gb or (ii) aI_ib and aP_gb for any $a, b \in X$. This requires that, for any $a, b \in X$, if one of i and g strictly prefers a to b and the other considers that a and b are indifferent, i's updated preference order shows that a is strictly better than b.

Affinity: For each $i \in V$, $s(R_i, R_g) = R^*_i$ such that (i)' aP_ib and aP_gb imply that aP^*_ib, and (ii)' aI_ib and aI_gb imply that aI^*_ib. This requires that, for any $a, b \in X$, if both i and g have the same preference order of a and b, i's updated preference order of a and b is the same as the original one.

Proposition 3 shows that *respectfulness* implies *anti–enforcement*. We do not propose proofs of Proposition 3 since it is trivial from the statements of the axioms.

Proposition 3. If s satisfies *respectfulness*, it also satisfies *anti–enforcement*.

Theorem 1 shows the characterisation of the DD system.

Theorem 1. $s = s_{dd}$ if and only if s satisfies *respectfulness*, *synchrony*, and *affinity*.

Proof. It is trivial that s_{dd} satisfies *respectfulness*, *synchrony*, and *affinity*. We prove that $s = s_{dd}$ if s satisfies *respectfulness*, *synchrony*, and *affinity*.

Suppose that $s(R_i, R_g) = R_i^*$ for each $i \in V$, and s satisfies *respectfulness*, *synchrony*, and *affinity*.

From *respectfulness*, aP_ib and bP_ga imply that aP_i^*b for any $a, b \in X$. From the part (i) of *synchrony*, aP_ib and aI_gb imply that aP_i^*b. From the part (i)' of *affinity*, aP_ib and aP_gb imply that aP_i^*b. From these assumptions, we obtain that aP_ib implies that aP_i^*b.

Next, from the part (ii) of *synchrony*, aI_ib and aP_gb imply that aP_i^*b. From the part (ii)' of *affinity*, aI_ib and aI_gb imply that aI_i^*b. From these assumptions, we obtain that aI_ib implies that $R_i^* = R_g$.

We then find that $R_i^* = R_i^{dd}$. Thus, $s = s_{dd}$ if and only if s satisfies *respectfulness*, *synchrony*, and *affinity*. □

We show the independence of *respectfulness*, *synchrony*, and *affinity* in the following manner. First, the *non–democratic preference update system* s_{ndem} is defined as follows: $s_{ndem}(R_i, R_g) = R_i^{ndem}$ such that $aP_i^{ndem}b$ if 'aP_ib and aR_gb' or 'aR_ib and aP_gb', $aI_i^{ndem}b$ if aI_ib and aI_gb, and $bP_i^{ndem}a$ if aP_ib and bP_ga. Then, s_{ndem} violates only *respectfulness*. Second, suppose that the *non–deliberative preference update system*, s_{ndel}, is defined as follows: $s_{ndel}(R_i, R_g) = R_i$. Then, s_{ndel} violates only *synchrony*. Third, the *contrarian preference update system*, s_c is defined as follows: $s_c(R_i, R_g) = R_i^c$ such that aP_i^cb if 'aP_ib and bR_ga' or 'aR_ib and aP_gb', aI_i^cb if aI_ib and aI_gb, and bP_i^ca if aP_ib and aP_gb. Then, s_c violates only *affinity*.

From Proposition 3 and Theorem 1, we obtain Corollary 1.

Corollary 1. s_{dd} satisfies *anti–enforcement*.

3.2. *Algorithm and time complexity*

Theorem 2 shows the time complexity of the DD system.

Theorem 2. We can compute the DD updated preferences in linear time.

Proof. Rename $a, b, c, ...(\in X)$ to $a_1, a_2, a_3, ..., a_{|X|}$. We introduce a function $r : X^2 \times \mathscr{R} \to \{1, 0, -1\}$ such that $r(a_k, a_l, R) = 1$ if and only if a_kPa_l, $r(a_k, a_l, R) = 0$ if and only if a_kIa_l, and $r(a_k, a_l, R) = -1$ if and only if a_lPa_k for all $k, l \in \{1, ..., |X|\}$ and for any $R \in \mathscr{R}$. We then consider Algorithm 1.

Algorithm 1: the DD system

Input $(r(a_k, a_l, R_{i'}))_{k,l\in\{1,...,|X|\},k<l,i'\in V\cup\{g\}}$
for each $i \in V$ **do**
  for each $k, l \in \{1, ..., |X|\}, k < l$ **do**
    if $r(a_k, a_l, R_i) = 0$ **then**
      set $r(a_k, a_l, R_i) \leftarrow r(a_k, a_l, R_g)$
    end if
  end for

end for
return $(r(a_k, a_l, R_i))_{k,l\in\{1,...,|X|\},k<l,i\in V}$ ▷ The output is $(r(a_k, a_l, R_i^{dd}))_{k,l\in\{1,...,|X|\},k<l,i\in V}$

Since $r(a_l, a_k, R_{i'}) = -r(a_k, a_l, R_{i'})$, we do not have to consider $r(a_l, a_k, R_{i'})$ for all $k, l \in \{1, ..., |X|\}$, $k < l$, for all $i' \in V \cup \{g\}$. Then, the input size is $n = {}_{|X|}\mathrm{C}_2 \cdot (|V| + 1)$. Furthermore, since we do not update $r(a_k, a_l, R_g)$, the calculation amount is $2 \cdot {}_{|X|}\mathrm{C}_2 \cdot |V| = 2n(1-1/(|V|+1))$. Finally, $|V| \geq 1$ implies that $n \leq 2n(1 - 1/(|V| + 1)) < 2n$, therefore the time complexity is $O(n)$. □

4. Simulations and regression analyses

4.1. Examples of simple scoring rules

We use five scoring rules to simulate elections based on different preference profiles ($\mathcal{R}$, R_g, and $\mathcal{R}^{dd}$).

Suppose that $pl_a(\mathcal{R}')$ is the number of voters whose best location is $a \in X$, where $\mathcal{R}' = \mathcal{R}$, R_g, or $\mathcal{R}^{dd}$. We then define the *plurality rule* as follows:

Definition 5. *Plurality rule*:

$$C_{pl}(\mathcal{R}') = \{a \in X \mid pl_a(\mathcal{R}') \geq pl_{a'}(\mathcal{R}') \ \forall a' \in X\}.$$

Each voter gives one point to the best locations. Thus, if a voter has more than two best locations in X, all of them are assigned one point. It indicates that the point allotted to each voter is changed according to his/her preference.

We then define the *adjusted* plurality rule. Suppose that $\hat{A}_i = \{a \in X \mid aR_ib \ \forall b \in X\}$ includes the best locations for i. Suppose that

$$a \in \hat{A}_i \Rightarrow apl_{ia}(R_i) = 1/|\hat{A}_i|;$$

$$a \in X \setminus \hat{A}_i \Rightarrow apl_{ia}(R_i) = 0$$

for each $a \in X$ and for each $i \in V \cup \{g\}$. Then, the point of each $a \in X$ becomes

$$apl_a(\mathcal{R}) = \Sigma_{i\in V} apl_{ia}(R_i);$$

$$apl_a(R_g) = apl_{ga}(R_g);$$

$$apl_a(\mathcal{R}^{dd}) = \Sigma_{i\in V} apl_{ia}(R_i^{dd}).$$

We then define the adjusted plurality rule as follows:

Definition 6. *Adjusted plurality rule*:

$$C_{apl}(\mathcal{R}') = \{a \in X \mid apl_a(\mathcal{R}') \geq apl_{a'}(\mathcal{R}') \ \forall a' \in X\}.$$

Next, suppose that $ap_a(\mathcal{R}')$ is the number of voters whose worst location is a for any $a \in X$. We then define the *anti–plurality* rule as follows:

Definition 7. *Anti–Plurality rule*:

$$C_{ap}(\mathcal{R}') = \{a \in X \mid ap_a(\mathcal{R}') \le ap_{a'}(\mathcal{R}')\ \forall a' \in X\}.$$

As with C_{pl}, the point allotted to each voter is changed according to his/her preference if we apply the anti–plurality rule. We then define the *adjusted* anti–plurality rule. Suppose that $\check{A}_i = \{a \in X \mid bR_ia\ \forall b \in X\}$ includes the worst locations of i. Suppose that

$$a \in \check{A}_i \Rightarrow aap_{ia}(R_i) = 1/|\check{A}_i|;$$

$$a \in X \setminus \check{A}_i \Rightarrow aap_{ia}(R_i) = 0$$

for each $a \in X$ and for each $i \in V \cup \{g\}$. Then, the point of each $a \in X$ becomes

$$aap_a(\mathcal{R}) = \Sigma_{i\in V} aap_{ia}(R_i);$$

$$aap_a(R_g) = aap_{ga}(R_g);$$

$$aap_a(\mathcal{R}^{dd}) = \Sigma_{i\in V} aap_{ia}(R_i^{dd}).$$

We then define the adjusted anti–plurality rule as follows:

Definition 8. *Adjusted anti–plurality rule*:

$$C_{aap}(\mathcal{R}') = \{a \in X \mid aap_a(\mathcal{R}') \le aap_{a'}(\mathcal{R}')\ \forall a' \in X\}.$$

Suppose that $n_{ab}(\mathcal{R}')$ is the number of voters who strictly prefer a to b for any $a, b \in X$. We then define the Borda score of $a \in X$ as follows:

$$br_a(\mathcal{R}') = \sum_{b\in X\setminus\{a\}} n_{ab}(\mathcal{R}') - n_{ba}(\mathcal{R}'),$$

and the Borda rule is defined as follows:

Definition 9. *Borda rule*:

$$C_{br}(\mathcal{R}') = \{a \in X \mid br_a(\mathcal{R}') \ge br_{a'}(\mathcal{R}')\ \forall a' \in X\}.$$

From Definition 1, we do not analyse the approval voting rule in our simulation. If every voter has a dichotomous complete preorder over X, the approval voting rule is Pareto efficient, but if not, the approval voting rule violates (*weak*) *Pareto efficiency*.[1] In the case of dichotomous complete preorders, X is divided into two groups A and N such that all locations in A (or N) are indifferent and every location in A is strictly better than that in N. Consider that every voter and g has dichotomous complete preorders over X. If we update voters' dichotomous complete preorders by the DD system, it is possible that the updated

[1] Suppose that there are three locations $a, b, c \in X$, and all locations are approved for all voters. Then, the social choice becomes X even if all voters strictly prefer a to b and c.

preference relations over X become complete preorders or linear orders. For example, there are three locations $a, b, c \in X$, and i's preference is aI_ibP_ic. In this case, i approves a and b. If g's preference is aI_gcP_gb (it is also a dichotomous complete preorder), i's DD updated preference becomes $aP_i^{dd}bP_i^{dd}c$. This implies that the dichotomous complete preorder R_i is updated to the linear order P_i^{dd}. From this argument, we cannot analyse the approval voting rule satisfying (*weak*) *Pareto efficiency* by using the DD system.

Note that if the society determines the set of winners according to only R_g, $C_{pl}(R_g) = C_{br}(R_g)$ includes the best location(s) for the government. Thus, we normally focus on g's preference converter, f^g, when the society uses only R_g to decide the outcome. If $f^g = f^g_{min}$ (alternatively, $f^g = f^g_{max}$), we can call the plurality and the Borda rules the *minisum* (alternatively, *maxisum*) *location rule*.

4.2. Analytical methods

4.2.1. Simulations

We prepare 294 pairs of $|X|$ and $|V|$ such that $|X| \in \{3, ..., 8\}$ and $|V| \in \{2, ..., 50\}$. We calculate the set of winners of all voting rules 10,000 times for each pair $(|X|, |V|)$ by using our program written in Python.[2] Note that we apply the same 10,000 examples to the five voting rules for each pair $(|X|, |V|)$.

We choose preference profile at random. For example, if $|X| = 3$, there are 13 preference rankings of a, b, and c in the case of complete preorders. We then choose preference rankings of five voters and the government at random form the 13 preference rankings. We conduct the following five comparative analyses:

- Plurality rule: $C_{pl}(\mathcal{R})$ vs $C_{pl}(R_g)$ vs $C_{pl}(\mathcal{R}^{dd})$
- Adjusted plurality rule: $C_{apl}(\mathcal{R})$ vs $C_{apl}(R_g)$ vs $C_{apl}(\mathcal{R}^{dd})$
- Anti–plurality rule: $C_{ap}(\mathcal{R})$ vs $C_{ap}(R_g)$ vs $C_{ap}(\mathcal{R}^{dd})$
- Adjusted anti–plurality rule: $C_{aap}(\mathcal{R})$ vs $C_{aap}(R_g)$ vs $C_{aap}(\mathcal{R}^{dd})$
- Borda rule: $C_{br}(\mathcal{R})$ vs $C_{br}(R_g)$ vs $C_{br}(\mathcal{R}^{dd})$

4.2.2. Regression models

After $1,470 \times 10,000$ times simulations, we analyse performance of the voting rules by using a log–link–log model.[3] We create a data set, including three dependent variables and nine independent variables. We estimate all regressors by using STATA 15.1.

Before introducing the definitions of all variables, we introduce the log–link–log model. Suppose that y is a dependent variable, x_1, ..., x_k are k independent variables, c_1, ..., c_l are l

[2]If we input the real data of preference profile, a 1.99 GHz personal laptop takes less than one second to output the result at least for any $|V| \leq 50$, for each run. However, the laptop does not work when we prepare all weak orders over X for any $|X| \geq 9$.

[3]This is one of generalised linear models. For further discussions, see Venables and Dichmont (2004).

control variables, and ε is an error term. Then, we apply the following log–link–log regressor:[4]

$$Model1 : \ln(y - \varepsilon) = \beta_0 + \beta_1 \ln x_1 + \cdots + \beta_k \ln x_k + \gamma_1 c_1 + \cdots + \gamma_l c_l$$
$$\Leftrightarrow y = x_1^{\beta_1} \cdots x_k^{\beta_k} e^{\beta_0 + \gamma_1 c_1 + \cdots + \gamma_l c_l} + \varepsilon.$$

For each $i \in \{1, ..., k\}$ and for any $x_i > 0$, if $x_j > 0$ for every $j \in \{1, ..., k\} \setminus \{i\}, \beta_i > 1$ implies that $\partial E(y)/\partial x_i, \partial^2 y/\partial x_i^2 > 0$, $\beta_i = 1$ implies that $\partial y/\partial x_i > 0$ and $\partial^2 y/\partial x_i^2 = 0$, $1 > \beta_i > 0$ implies that $\partial y/\partial x_i > 0$ and $\partial^2 y/\partial x_i^2 < 0$, $\beta_i = 0$ implies that $\partial y/\partial x_i = 0$, and $0 > \beta_i$ implies that $\partial y/\partial x_i < 0$ and $\partial^2 y/\partial x_i^2 > 0$. We thus find signs of partial first and second derivatives for all independent variables.

The following log–log model is also usable:

$$\ln y = \beta_0 + \beta_1 \ln x_1 + \cdots + \beta_k \ln x_k + \gamma_1 c_1 + \cdots + \gamma_l c_l + \varepsilon'$$
$$\Leftrightarrow y = x_1^{\beta_1} \cdots x_k^{\beta_k} e^{\beta_0 + \gamma_1 c_1 + \cdots + \gamma_l c_l + \varepsilon'}.$$

Which model should we use according to our data? We derive an answer by estimating the following level–log model and checking the distribution of residuals:

$$y = \beta_0 + \beta_1 \ln x_1 + \cdots + \beta_k \ln x_k + \gamma_1 c1 + \cdots + \gamma_l c_l + u,$$

where u is an error term of the level–log model. If the distribution of $\hat{u}$ is close to the normal distribution, the log–link–log model is better than the log–log model since the logarithmic transformation of $(y - \varepsilon) = E(y)$ does not affect to the distribution of $\hat{u}$, where $\hat{u}$ is a residual. From this verification process, we employ the log–link–log model in this study.[5]

Next, we consider using cross–terms of one x_i and dummy variables d_1, ..., d_l in the log–link–log model as follows:

$$Model2 : \ln(y - \varepsilon'') = \beta'_0 + \beta'_1 \ln x_i + \delta_1 \ln x_i * d_1 + \cdots + \delta_l \ln x_i * d_l$$
$$\Leftrightarrow y = x_i^{\beta'_1 + \delta_1 d_1 + \cdots + \delta_l d_l} e^{\beta_0} + \varepsilon''.$$

From the last equation, we can obtain the second derivative, which depends on each dummy variable. In general, we add dummy variables independently. However, a correlation coefficient of every x and every d our data is more than 0.9. We thus eliminate all isolated terms of dummy variables from Model 2.

[4] We call the model 'log–link–log' since we combine the log–link and log–log models.

[5] By comparing with both results, AICs of the log–link–log model are larger than the log–log model (the log–link–log model has worse performance than the log–log model). However, robust standard errors of coefficients based on the log–link–log model are smaller than those based on the log–log model. Thus, we can obtain more significant results for independent variables by using the log–link–log model than the log–log model in our data.

4.3. Data

Each sample is identified based on values of $(|X|, |V|)$ and a kind of voting rules such as 'the Borda rule with $(|X|, |V|) = (5, 14)$'. Thus, the number of observations is 1,470.

In our analyses, we use the following three dependent variables: *unique*, *nondelib*, and *nondemoc*. First, *unique* is the number of times that the the set of winners includes only one location per 10,000 times. Second, *nondelib* is the rate of number of times that $C(\mathcal{R}^{dd}) = C(\mathcal{R})$ and $C(\mathcal{R}^{dd}) \neq C(R_g)$ per 10,000 times. Third, *nondemoc* is the rate of number of times that $C(\mathcal{R}^{dd}) \neq C(\mathcal{R})$ and $C(\mathcal{R}^{dd}) = C(R_g)$ per 10,000 times. We interpret that *unique* is an appropriate evaluation criterion for single–winner voting systems, and *nondelib* or *nondelib* is directly related to the weak point for deciding the set of winners based only on $\mathcal{R}$ or R_g. The higher the values of *unique*, the better. Contrary, the lower the values of *nondelib* and *nondemoc*, the better.

Next, we use nine independent variables $|X|$, $|V|$, *pl apl*, *ap*, *aap*, *br*, *gl*, and *vl*. $|X|$ and $|V|$ are the numbers of locations and voters, respectively, and *pl apl*, *ap*, *aap*, *br* are dummy variables. For example, $pl = 1$ if we apply the plurality rule, and otherwise $pl = 0$. The same applies to *apl* (adjusted plurality), *ap* (anti–plurality), *aap* (adjusted anti–plurality), and *br* (Borda). Finally, *gl* and *vl* are characteristics of preference profiles for 10,000 times simulation. The rate of linear orders over X (%) is calculated as follows:

$$rliner(|X|) = 100|X|!(\Sigma_{m=0}^{|X|} m!S(|X|, m))^{-1},$$

$$S(|X|, m) = (m!)^{-1}\Sigma_{h=0}^{m}(-1)^h \binom{m}{h}(m-h)^{|X|}.$$

Additionally, let $rglinear(|X|)$ be the rate that R_g is a linear order over X per 10,000 times. Then, *gl* takes the value of $rglinear(|X|) - rliner(|X|)$ for each $|X| \in \{3, ..., 8\}$. Similarly, suppose that $rvlinear(|X|)$ is the average rate that R_i is a linear order over X per 10,000 times. Then, *vl* takes the value of $rvlinear(|X|) - rliner(|X|)$ for each $|X| \in \{3, ..., 8\}$. If R_g is a linear order, the impact to R_i^{dd} from R_g becomes large, and if R_i is a linear order, $R_i^{dd} = R_i$. We thus control the effect of preference profiles to the three dependent variables by using *gl* and *vl*. Table 1 shows the descriptive statistics of all variables.

4.4. Results

Tables 2–4 report results of regression analyses based on Models 1 and 2. If there is no asterisk mark, the result is not significant at the 10% level.

Note that every AIC in Tables 2–4 is calculated by using the following equation: AIC$= (-2\ln L + 2k')/N$, where $\ln L$ is the overall likelihood reported by the command 'glm' in STATA, k' is the number of parameters of the model, N is the number of observations. The command 'estat ic' in STATA outputs a different AIC, that is, AIC$= -2\ln L + 2k'$. (See Akaike, 1973, 1974.)

Table 1: The descriptive statistics

variable	mean	std. dev.	min	max
unique	7872.783	1897.413	.000	9776.000
nondelib	87.258	11.891	54.627	100.000
nondemoc	60.372	20.518	12.782	100.000
$\lvert X\rvert$	5.500	1.708	3.000	8.000
$\lvert V\rvert$	26.000	14.147	2.000	50.000
pl	.200	.400	.000	1.000
apl	.200	.400	.000	1.000
ap	.200	.400	.000	1.000
aap	.200	.400	.000	1.000
br	.200	.400	.000	1.000
gl	.011	.380	-.974	1.520
vl	-.013	.109	-.490	.640

4.4.1. Effects of $|X|$ *and* $|V|$

From Table 2, $|X|$ is negatively correlated to *unique*, and is positively correlated to *nondelib* and *nondemoc* at 0.1% significance level. Additionally, $\partial^2 unique/\partial|X|^2 > 0$ and $\partial^2 y/\partial|X|^2 < 0$ for $y = nondelib$ or *nondemoc* since $|V| > 0$. Next, $|V|$ is positively correlated to *unique* and *nondemoc*, and is negatively correlated to *nondelib* at 0.1% significance level. Additionally, $\partial^2 y/\partial|V|^2 < 0$ for $y = unique$ or *nondemoc* and $\partial^2 nondelib/\partial|V|^2 > 0$ since $|X| > 0$.

From Table 3, if we do not employ the Borda rule, $|X|$ is more negatively correlated to *unique*, but $\partial^2 unique/\partial|X|^2 > 0$ still holds. If we do not employ the Borda or the adjusted plurality rule, $|X|$ is more positively correlated to *nondelib*, but $\partial^2 nondelib/\partial|X|^2 < 0$. Furthermore, if we employ the Borda or (adjusted) plurality rule, $|X|$ is negatively correlated to *nondemoc*, and $\partial^2 nondemoc/\partial|X|^2 > 0$.

From Table 4, if we do not employ the Borda rule, $|V|$ is more negatively correlated to *unique*, but $\partial^2 unique/\partial|V|^2 > 0$ still holds. If we employ the Borda or (adjusted) plurality rule, $|V|$ is negatively correlated to *nondelib*, and $\partial^2 nondelib/\partial|V|^2 > 0$. According to the coefficients of $\ln|V| * pl$, $\ln|V| * apl$, and $\ln|V| * br$, the adjusted plurality has the best performance to reduce *nondelib* when $|V|$ increases (, however the differences of the coefficients are almost zero). Additionally, if we employ the Borda rule, $|V|$ is negatively correlated to *nondemoc*, and $\partial^2 nondemoc/\partial|X|^2 > 0$.

From the above results, if we apply the Borda rule, the effect of increasing $|X|$ to each dependent variable becomes the best. If we apply the Borda (alternatively, adjusted plurality) rule, the effect of increasing $|V|$ to *unique* or *nondemoc* (alternatively, *nondelib*) becomes the best.

Table 2: Results of Model 1 (without cross terms)

y:	*unique*	*nondelib*	*nondelib*	*nondemoc*
$\ln\|X\|$	-.212***	.248***	.248***	.104***
	[.009]	[.005]	[.005]	[.016]
$\ln\|V\|$	.185***	-.015***	-.015***	.100***
	[.008]	[.002]	[.002]	[.006]
pl	-.177***	.006	-.176***	.177***
	[.008]	[.004]	[.004]	[.017]
apl	-.119***	.001	-.181***	.082***
	[.008]	[.004]	[.004]	[.018]
ap	-.299***	.181***	-.002	.661***
	[.011]	[.004]	[.004]	[.017]
aap	-.233***	.182***		.594***
	[.010]	[.004]		[.019]
br			-.182***	
			[.004]	
gl	.002	.007	007	-.005
	[.007]	[.004]	[.004]	[.012]
vl	.006	-.016	-.016	.029
	[.052]	[.014]	[.014]	[.045]
const.	8.911***	4.025***	4.207***	3.283***
	[.027]	[.010]	[.009]	[.036]
obs.	1,470	1,470	1,470	1,470
AIC	16.830	5.987	5.987	7.582

Robust standard errors are reported in brackets.
*: $p < 0.05$, **: $p < 0.01$, and ***: $p < 0.001$.

Table 3: Results of Model 2 (with cross terms of $\ln|X|$)

y:	*unique*	*nondelib*	*nondelib*	*nondemoc*
$\ln\|X\|$	-.117***	.206***	.300***	-.180***
	[.008]	[.007]	[.007]	[.011]
$\ln\|X\| * pl$	-.116***	.003	-.091***	.112***
	[.004]	[.002]	[.003]	[.006]
$\ln\|X\| * apl$	-.078***	.000	-.094***	.053***
	[.004]	[.002]	[.003]	[.007]
$\ln\|X\| * ap$	-.199***	.094***	-.001	.412***
	[.006]	[.003]	[.003]	[.007]
$\ln\|X\| * aap$	-.156***	.094***		.374***
	[.006]	[.003]		[.009]
$\ln\|X\| * br$			-.094***	
			[.003]	
$\ln\|V\|$	.183***	-.016***	-.016***	.099***
	[.008]	[.002]	[.002]	[.006]
gl	.002	.007	.007	-.005
	[.007]	[.005]	[.005]	[.010]
vl	.009	-.017	-.017	.033
	[.049]	[.015]	[.015]	[.040]
const.	8.775***	4.107***	4.107***	3.742***
	[.025]	[.012]	[.012]	[.023]
obs.	1,470	1,470	1,470	1,470
AIC	16.707	6.395	6.395	7.170

Robust standard errors are reported in brackets.
*: $p < 0.05$, **: $p < 0.01$, and ***: $p < 0.001$.

Table 4: Results of Model 2 (with cross terms of $\ln|V|$)

y:	*unique*	*nondelib*	*nondelib*	*nondemoc*
$\ln\|V\|$	.230***	-.043***	.015***	.159***
	[.009]	[.002]	[.002]	[.009]
$\ln\|V\| * pl$	-.047***	.001	-.057***	-.118***
	[.002]	[.001]	[.001]	[.005]
$\ln\|V\| * apl$	-.030***	-.001	-.059***	-.147***
	[.002]	[.002]	[.001]	[.005]
$\ln\|V\| * ap$	-.076***	.058***	-.001	.022***
	[.003]	[.001]	[.001]	[.005]
$\ln\|V\| * aap$	-.056***	.058***		
	[.003]	[.001]		
$\ln\|V\| * br$			-.058***	-.171***
			[.001]	[.007]
$\ln\|X\|$	-.217***	.247***	.247***	.095***
	[.009]	[.005]	[.005]	[.018]
gl	.002	.007	.007	-.005
	[.008]	[.004]	[.004]	[.013]
vl	.006	-.015	-.015	.027
	[.058]	[.011]	[.011]	[.059]
const.	8.746***	4.113***	4.113***	3.680***
	[.030]	[.010]	[.010]	[.032]
obs.	1,470	1,470	1,470	1,470
AIC	16.993	5.944	5.944	7.872

Robust standard errors are reported in brackets.
*: $p < 0.05$, **: $p < 0.01$, and ***: $p < 0.001$.

4.4.2. performance of voting rules

From Tables 2–4, we check the performance of voting rules for each evaluation criteria (*unique*, *nondelib*, and *nondemoc*).

From Table 2, the ranking of voting rules based on their performance for *unique* or *nondemoc* is as follows: (1) Borda, (2) adjusted plurality, (3) plurality, (4) adjusted anti-plurality, and (5) anti-plurality rules. Next, the ranking of voting rules based on their performance for *nondelib* is as follows: (1) Borda, (2) adjusted plurality, (3) plurality, (4) anti-plurality, and (5) adjusted anti-plurality rules. Table 8 shows the summary of the above results.

Table 5: performance of voting rules

Voting rules	*unique*	*nondelib*	*nondemoc*
Plurality	3	3	3
Adjusted plurality	2	2	2
Anti–plurality	5	4	5
Adjusted anti–plurality	4	5	4
Borda	1	1	1

We thus find that the Borda rule has the best performance between the five voting rules. Furthermore, if we apply the Borda rule, the effects of increasing $|X|$ and $|V|$ will be improved from Section 4.4.1. From all results, the Borda rule with the DD system has relatively a good performance based on the evaluation criteria, that is, *unique*, *nondelib*, and *nondemoc*.

5. Conclusions

We propose a method to decide a single–public facility location based on voters' preferences and a suggestion from the government over the finite location set, that is, the DD system. This system considers communication between voters and the government, not among the voters. The setting of this study shows a new possible research direction in the cross-sectional field of facility location problems, social choice theories, and democratic deliberation theories.

We obtain the following three results: First, we characterise the DD system by *respectfulness*, *synchrony*, and *affinity*. Second, we show that the DD updated preferences are computed in linear time. Third, we find that the Borda rule based on the DD updated preferences has better performance than other rules by simulation analyses using our program in Python 3 and regression analyses using the log–link–log model.

Finally, the major remained study is to propose and characterise a preference update system which is suitable to the approval voting rule and dichotomous preference relations over the finite location set. Futheremore, we can consider social networks and the communication among the voters as an extended framework.

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