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Nominal Contracts and the Payment System

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Abstract

This paper introduces into an overlapping generations model the court's inability to distinguish different qualities of goods of the same kind. Given the recognizability of fiat money for the court, this friction leads to the use of nominal debt contracts as well as the use of fiat money as a means of payment in the goods market. This result holds without dynamic inefficiency or lack of double coincidence of wants. Instead, money is necessary because it is essential for credit. However, there can occur a shortage of real money balances for liability settlements, even if the money supply follows a Friedman rule. This problem can be resolved if the central bank can lend fiat money to agents elastically at a zero intraday interest rate within each period. Given the economy being dynamically efficient, this policy makes the money supply cease to be the nominal anchor for the price level. In this case, the monetary steady state becomes compatible with other nominal anchors than the money supply.

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Keywords: Nominal contract; Discount window; Trade credit; Cashless economy; Payment system; Legal tender.

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1 Introduction

Credit contracts are usually nominal. This is true even for trade credit, in which producers buy inputs from suppliers on credit, and pay money to suppliers at a later date. This use of money is different from the standard theory on the need for money, in which buyers have to pay money to sellers because they cannot be committed to repaying credit. See Samuelson (1958), Townsend (1980), Kiyotaki and Wright (1989), and Lagos and Wright (2005) for examples of seminal papers in the literature.

A possible explanation for this observation is that sellers require buyers to repay money even if buyers can be committed to repaying credit, because sellers need to buy goods from third parties due to lack of double coincidence of wants between buyers and sellers. Freeman (1996) presents such a model to generate nominal debt endogenously. This explanation must assume that sellers cannot resell the products of buyers to third parties.

This paper presents an alternative friction that leads to the use of nominal debt contracts as well as the use of fiat money as a means of payment in the goods market. This result does not require lack of double coincidence of wants between buyers and sellers, or the inability of sellers to resell the products of buyers. This paper shows that money is necessary even without these frictions, because it is essential for credit.

More specifically, this paper introduces into an overlapping generations model the court's inability to distinguish different qualities of goods of the same kind. The underlying assumption is that, unlike quantities, the evaluation of qualities is inevitably subjective, and thus difficult to verify in court. As a result, the court cannot recognize it as a breach of contract, if producers acquire inputs from suppliers by promising to deliver high-quality goods later, but then deliver low-quality goods instead to save the production cost. Ex ante, the expectation of this moral hazard problem discourages suppliers from entering into a real credit contract to provide trade credit to producers. Otherwise, the model is a standard overlapping generations model with dynamic efficiency, in which money is not essential.

In contrast to goods, fiat money is assumed to be recognizable by the court. The underlying assumption is that the nominal value of fiat money is easier to recognize than the quantities and qualities of real assets, including a homogeneous commodity like gold. This assumption reflects the fact that the nominal value of fiat money is just a number. It is also consistent with the fact that even though the law in each country usually permits the use of any legal goods for a medium of exchange, it often designates fiat money issued by the central bank as legal tender in the country.[1] The definition of legal tender is such that "a debtor cannot successfully be sued for non-payment if he pays into court in legal tender," given debt being denominated in the currency unit in the country.[2] Thus, fiat money is characterized as a debt-settlement instrument recognizable by the court in legal terms.

This feature of the model is related to the paper by Lester, Postlewaite, and Wright (2012), which endogenizes the recognizability of assets for trading parties and analyzes the liquidity of assets. For simplicity, this paper abstracts from this issue, and assumes that fiat money is the most recognizable asset for the court. This paper also abstracts from counterfeiting, because existing technology can make it a minor problem, as observed in developed countries.

Given this environment, the model shows that suppliers can provide trade credit to producers by making producers liable to repay fiat money, if there is a competitive market for exchanges between goods and fiat money. On one hand, suppliers do not have to pay fiat money for producers' products in the goods market, if producers deliver low-quality goods. On the other hand, if producers fail to acquire fiat money to fulfill their nominal liabilities, then the court can seize their belongings, because the court can recognize fiat money repaid by them, if any. Moreover, the court can sell the seized belongings for fiat money in the

[1]For the U.S. case, see Section 31 U.S.C. 5103. This statute provides a legal tender status for U.S. coins and currency, including Federal reserve notes. There is no federal statute mandating that a person or a private entity must accept U.S. coins and currency as payment for goods or services. For more explanation, see `https://www.federalreserve.gov/faqs/currency_12772.htm` (accessed on October 05, 2019).

[2]This definition is provided by the U.K. Royal Mint. See `https://www.royalmint.com/help/trm-faqs/legal-tender-amounts/` (accessed on April 6, 2019).

competitive goods market without determining which part of the belongings are high-quality goods, because buyers in the market can distinguish the different qualities of goods. This assumption is consistent with the fact that courts across countries sell foreclosed properties for cash through auctions. Thus, the court can recover fiat money owed by producers in the goods market. Given this result, producers do not gain anything by delivering low-quality goods to the goods market or defaulting on nominal debt contracts. Therefore, producers can be committed to delivering high-quality goods to suppliers in exchange for fiat money, and repaying their nominal debt by the fiat money acquired in the goods market.

This result is related to the literature on incomplete contracts and collateral, such as Hart and Moore (1994, 1998) and Kiyotaki and Moore (1997). While these papers analyze the private enforcement of a contract without the power of the court, this paper highlights the role of fiat money as a remedy to the limited ability of the court, which makes a credit contract enforceable without collateral.

A challenge to this effect of fiat money, however, is that suppliers must pay fiat money for goods sold by producers before they receive the repayments of fiat money from producers. This endogenous flow-of-funds constraint can cause a shortage of real money balances for liability settlements, even if the money supply follows a Friedman rule. This problem in turn causes underproduction by producers.

This problem can be resolved if the central bank can lend fiat money to suppliers through a discount window at a zero intraday interest rate within each period. This result is similar to the finding in the literature on elastic money supplies and the payment system, such as Freeman (1996, 1999), Fujiki (2003, 2006), Martin (2004), Mills (2006), Gu et al. (2011), and Chapman and Martin (2013). This paper contributes to this literature by showing that a flow-of-funds constraint that leads to the need for an elastic money supply can be derived from a much simpler environment than complex traveling constraints in a spatial economy utilized in the literature.

Given the economy being dynamically efficient, the optimal intraday lending of fiat money makes it unnecessary for agents to hold fiat money overnight. As fiat money corresponds to base money issued by the central bank, this result is consistent with the current trend toward a cashless economy across countries, and also the adoption of the channel, or corridor, system by central banks in several developed countries.[3] This system can function without any overnight supply of bank reserves, as demonstrated in Canada for 2006-7.[4]

With the optimal intraday lending of fiat money, the money supply is not pre-determined, and thus cannot anchor the nominal price level. This result is similar to the finding of Smith (2002) in a spatial-economy model with an intraday discount window offered by the central bank.[5] Even though Smith interprets price indeterminacy as instability, this result also implies that a monetary economy can be compatible with nominal anchors for the long-run price level other than the money supply. This feature of the model contrasts with the feature of standard monetary models, in which the money supply solely determines the nominal price level in the steady state, regardless of whether money is introduced through a cash-in-advance constraint, a money-in-the-utility function, or a search friction.[6] This is true even in a cashless new-Keynesian model, because this type of model features a money-in-the-utility function to determine the nominal price level in the steady state by the money supply, as clarified by Nelson (2008). This feature of standard monetary models makes it impossible to introduce other nominal anchors proposed in the literature, such as nominal public debt.[7] In contrast, this paper presents an alternative environment in which the essentiality of money does not preclude other nominal anchors than the money supply.

This paper is also related to the literature on the optimality of nominal contracts for risk sharing, such as Jovanovic and Ueda (1997), Freeman and Tabellini (1998), and Doepke

[3]For example, Australia, Canada, and Sweden are adopting the channel system as of 2019.

[4]See section 5.1 for more details.

[5]See Antinolfi and Keister (2006) for a near-optimal monetary policy that eliminates price indeterminacy in Smith's (2002) model.

[6]See Walsh (2017) for more details on standard monetary models in the literature.

[7]See Sims (2016) for an example of recent discussion on the fiscal theory of price level.

and Schneider (2017). These papers highlight the role of money as a unit of account, which does not require money to be a means of payment. In contrast, this paper focuses on an alternative friction that leads to the use of nominal debt contracts as well as the use of fiat money as a means of payment in the goods market.

The remainder of this paper is organized as follows. A baseline model without money is described in section 2. Fiat money and the central bank's discount window are introduced in sections 3 and 4, respectively. Related issues are discussed in section 5. Section 6 concludes this paper.

2 Baseline model without money

Time is discrete with an infinite horizon. Agents are born in each period, and live for three periods. The set of agents born in period t is referred to as "cohort t" for $t = 0, 1, 2, ...$ Also, agents in their first period and those in their second period are referred to as "young" and "old", respectively. Agents in their third period do nothing in the baseline model.

In each cohort, agents are split into two types: producers and suppliers, each of which are on a $[0, 1]$ continuum. For each type, the measure of agents is defined by the Lebesgue measure over $[0, 1]$. Each young supplier is born with a unit of goods, and can use the goods to produce an amount ρ of goods when old.

In contrast, each young producer is born with no goods, but can produce an amount $f(x)$ of goods when old by investing an amount x of goods when young. The function $f : \mathbf{R}_+ \to \mathbf{R}_+$ is strictly increasing and concave, and satisfies the Inada condition (i.e., $f(0) = 0$ and $\lim_{x\downarrow 0} f'(x) = \infty$). Assume that the marginal return on investment in a producer's production diminishes sufficiently fast that it is never efficient to invest all the goods endowments of young suppliers in producers' production:

$$f'(1) < \rho \tag{1}$$

Alternatively, each producer can produce an amount $g(x)$ of low-quality goods when old by using an amount x of goods when young. Hereafter, low-quality goods are referred to as "wastes" to distinguish them from regular goods described above. Assume that the function $g : \mathbf{R}_+ \to \mathbf{R}_+$ satisfies

$$g(x) \begin{cases} = 0 & \text{if } x = 0 \\ > f(x) & \text{if } x > 0 \end{cases} \tag{2}$$

For simplicity, assume that wastes do not generate any utility for any agent. Both producers and suppliers can distinguish goods and wastes. Thus, there is no asymmetric information among agents.

Each agent maximizes the expected value of consumption of goods when old. Also assume that

$$\rho > 1 \tag{3}$$

This assumption implies that it is never Pareto-improving for young suppliers to transfer their goods endowments to old agents in the previous cohort. Thus, the economy is dynamically efficient. Throughout the paper, agents have perfect foresight, given no aggregate shock in the model.

2.1 Autarky due to imperfect contract enforcement

If the court can distinguish goods and wastes, then producers and suppliers in each cohort can arrange pledgeable real credit contracts such that suppliers give goods as inputs to producers when young, while producers are liable to deliver goods to suppliers when old. This is because if an old producer reneges on its liability to deliver goods to an old supplier, then the court can seize any goods held by the producer to enforce the repayment of goods to the supplier. Thus, old producers cannot increase their consumption of goods even if they renege on real credit contracts with old suppliers.

Hereafter, introduce incomplete enforcement of real credit contracts by the court:

Assumption 1. The difference between goods and wastes is unverifiable in court.

The underlying assumption is that goods and wastes are similar enough that the court cannot tell the difference between the two. This assumption can be interpreted as reflecting the difficulty in defining different qualities of goods in legal terms if the goods have the same physical features, because, in contrast to quantities, the evaluation of qualities is inevitably subjective. Assumption 1 is a stylized assumption to incorporate this kind of difficulty for the court. For simplicity, producers and suppliers are assumed to have the same tastes. They can distinguish goods and wastes for their subjective tastes, but cannot verity the difference in an objective manner.

Given Assumption 1, if a producer and a supplier enter into a real credit contract when young, then the producer repays only wastes to the supplier when old, because it is cheaper to produce wastes than goods, as implied by (2). This way, the producer can use more goods for the production of goods for its own consumption. In this case, the supplier cannot claim the producer's default in court, because the producer can successfully claim that the wastes delivered by the producer fall into the same category of substance that the producer is obliged to deliver in the contract. Expecting this moral hazard problem, no young supplier participates in a real credit market with a young producer; thus agents live in autarky if Assumption 1 holds in the baseline model.

3 Introducing fiat money and nominal debt contracts

3.1 Introducing fiat money

Let us introduce fiat money into the baseline model. Assume that there are a unit continuum of the initial old in period 0, each of which maximizes the consumption of goods in the period. The initial old are endowed with an amount M_0 (≥ 0) of fiat money for each in period 0. The central bank imposes a lump-sum transfer of fiat money to, or from, each old supplier from period 1 onward so that

$$M_t = \gamma M_{t-1} \tag{4}$$

for $t = 1, 2, 3, ...$, where M_t is the supply of fiat money per old supplier at the beginning of period t, and $\gamma > 0$. Assume that both agents and the court can identify the amount of fiat money correctly. This assumption reflects the fact that the nominal value of fiat money is just a number. Also assume that it is too costly to counterfeit fiat money. The model abstracts from counterfeiting, as existing technology can make it a minor problem, as observed in developed countries.

3.2 Competitive goods market and nominal debt contracts

With the introduction of fiat money, assume that a competitive market for exchanges between goods and fiat money is organized in each period. Given this environment, let us guess and verify that agents can write a pledgeable nominal debt contract such that a young supplier transfers goods to a young producer in exchange for the producer's promise to repay fiat money to the supplier in the next period. Assume that there exists a competitive market for nominal debt contracts between young producers and suppliers in each period.

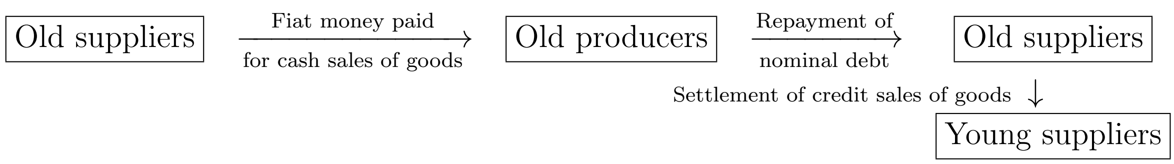
Old producers entering into nominal debt contracts have to acquire fiat money to repay their nominal debt by selling their output in the goods market. Thus, if young producers and suppliers enter into nominal debt contracts, then the sales of goods by old producers must be cash transactions, and take place before the maturity of nominal debt contracts in each period.

Therefore, old suppliers cannot receive the repayments of nominal debt by old producers until the end of the goods market in each period. They can still buy goods from young suppliers in the goods market on credit, because young suppliers are not liable to repay nominal debt, and thus can wait for the payments of fiat money for their goods until old suppliers receive the repayments of nominal debt by old producers. Young suppliers are willing to sell their goods for fiat money paid by old suppliers, because they need fiat money to buy goods produced by old producers in the next period. See Table 1 for the summary of events in each period, and Figure 1 for an illustration of flows of fiat money in each period.

Table 1: Summary of events in each period in the presence of fiat money

Birth of a new cohort	Young producers and suppliers are born. Suppliers are born with a unit of goods for each.
Returns on investments	Old producers and suppliers obtain goods from their investments of goods in the previous period.
Lump-sum money transfer	The central bank imposes a lump-sum transfer of fiat money from, or to, old suppliers.
Goods market	Old producers sell goods for fiat money, and young suppliers sell goods on credit.
Maturity of nominal debt contracts for old producers	Old producers repay fiat money to old suppliers to fulfill their liabilities in nominal debt contracts written in the previous period.
Settlement of credit sales of goods by young suppliers	Old suppliers pay fiat money to young suppliers to settle their credit purchases of goods in the goods market.
Credit market	Young producers can arrange nominal debt contracts to obtain goods from young suppliers in the same cohort.
Investments	Young producers and suppliers invest goods in their production.
Consumption	Old producers and suppliers consume goods.

Figure 1: Flows of fiat money in each period



3.3 The court's action in case of default on a nominal debt contract

If an old producer defaults on a maturing nominal debt contract with an old supplier, then the court can seize the producer's belongings on behalf of the supplier. The court can take this action because it can recognize fiat money repaid by an old producer, if any.[8]

[8]For example, the court can ask a producer to present a receipt received from a supplier if the supplier claims the producer's default on a nominal debt contract. Such a receipt can be certified by a third party that intermediates the payments of fiat money, such as a commercial bank providing a bank transfer service. If the court still cannot detect a forged receipt, then it can accept the repayment of fiat money by the producer, and pass on the fiat money to the supplier to fulfill the producer's nominal liability. Such an arrangement is feasible because the court can identify fiat money correctly, as assumed above. This arrangement can be interpreted as a deposit of a payment in court in practice.

The court can sell the seized belongings for fiat money only in the goods market in the next period, because the goods market in the current period is already over when nominal debt contracts mature in the period, as described above. Accordingly, assume that the court can store goods and wastes until the next period.[9]

When selling the seized belongings in the goods market, the court does not have to determine which part of the belongings are goods, because agents in the market can distinguish goods and wastes, and compete with each other until arbitrage opportunities disappear.[10] Therefore, the court can receive the competitive nominal price of goods among the defaulting producer's belongings in the market. The price of these goods equals the price of goods sold by old producers in the market, as both the court and old producers sell goods for immediate payments of fiat money.

After selling the seized belongings in the goods market, the court can pass on the acquired fiat money to the supplier in the defaulted contract up to the defaulting producer's nominal liability, and then return any residual of fiat money to the producer. Note that the producer and the supplier are in the third period of their lives when this happens.

3.4 Pledgeability of a nominal debt contract

Given this environment, an old producer cannot obtain any fiat money by selling wastes, because agents in the goods market can distinguish goods and wastes. This is the same even if an old producer defaults on a nominal debt contract to make the court sell its belongings on its behalf. Thus, young producers have no incentive to invest goods to produce wastes.

[9] Given (3), this storage technology never dominates the investment of goods by either a young producer or a young supplier. Also, it is implicitly assumed that there is no time to open another session in the goods market after the maturity of nominal debt contracts, even if there occurs default on a nominal debt contract. It is natural to set this assumption, because each old producer would like to sell its goods after the other old producers repay nominal debt, so that old suppliers buying its goods have more fiat money to spend. Therefore, old producers prefer to delay the maturity of nominal debt contracts as much as possible until they reach the limit within each period.

[10] This feature of the competitive goods market implies that a producer and a supplier can write an enforceable real credit contract such that the producer is obliged to give up all belongings to the supplier when old. Assume that there is an infinitesimally small utility cost for a producer to engage in the production of goods, so that a young producer does not have incentive to enter into such a contract.

An old producer has no incentive to strategically default on a nominal debt contract either, because it would have no use of fiat money if it received fiat money from the court in the third period of its life. For this result, note that a producer can gain utility only from consuming goods when old, as assumed in the baseline model.

Even though there remains a possibility that a defaulting old producer buys goods from young suppliers by promising to deliver fiat money returned by the court in the next period, assume a standard feature of bankruptcy law in practice that prohibits a defaulter from issuing new debt until the end of the bankruptcy process:

Assumption 2. If an old producer defaults on a nominal debt contract, then it cannot incur new nominal debt until the end of sales of its belongings by the court.

Such a restriction is necessary in practice, because otherwise a defaulter would be able to gain money from new lenders at the expense of incumbent creditors, who would lose their shares of the bankruptcy estate to new lenders. This assumption prohibits a defaulting old producer from buying goods with a promise to deliver fiat money returned by the court in the next period, because such a promise is nominal debt. This effect of Assumption 2 can be interpreted as representing the cost of default in practice due to the lack of liquidity for a defaulter in the bankruptcy process.[11]

Even without Assumption 2, it is possible to show that there is no net gain for an old producer and an old supplier from strategic default on a nominal debt contract in a monetary equilibrium defined below. This is because the court must store goods held by the producer for one period before selling the goods in the next period. This storage technology is inferior

[11] Like a defaulting old producer, an old supplier in a defaulted nominal debt contract must wait until the next period to receive fiat money from the court. The old supplier, however, can still buy goods from young suppliers in the current period by promising to pay fiat money delivered by the court after the goods market in the next period. Such a promise is enforceable, as the court can seize fiat money retained by the old supplier if the old supplier defaults. Even though young suppliers accepting this promise cannot receive fiat money during the goods market in the next period, they can buy goods from young suppliers in the next period by paying the promise, because young suppliers in the next period can wait until the end of the next period to receive fiat money. See Appendix A for a visual illustration of the flows of goods and fiat money described here.

to a young supplier's production technology, given (3). Thus, the total amount of goods available for an old producer and an old supplier never increases by strategic default. Given this result, it is optimal for a young producer and a young supplier to write a nominal debt contract such that the producer must repay a sufficiently large amount of fiat money as a penalty if it fails to repay the face value of nominal debt fully at the end of the goods market in the next period. Such a clause is sufficient to eliminate an old producer's incentive to strategic default. See Appendix A for a formal proof for this result. For simplicity, Assumption 2 is maintained throughout the paper.

3.5 Characterization of a monetary equilibrium

Given the competitive markets for goods and nominal debt contracts, the utility maximization problem for a young supplier in cohort t is specified as follows:

$$\begin{aligned}
\max_{\{x_{S,t},b_{S,t},m_{S,t},m'_{S,t+1}\}} \quad & c_{S,t+1} \\
\text{s.t.} \quad & x_{S,t} + q_t b_{S,t} + p_{2,t} m_{S,t} = 1 \\
& c_{S,t+1} = \rho x_{S,t} + p_{1,t+1} m'_{S,t+1} + p_{2,t+1}(b_{S,t} + m_{S,t} + \tau_{S,t+1} - m'_{S,t+1}) \\
& m'_{S,t+1} \in [0,\ m_{S,t} + \tau_{S,t+1}] \\
& b_{S,t} + m_{S,t} + \tau_{S,t+1} - m'_{S,t+1} \geq 0 \\
& x_{S,t}, m_{S,t} \geq 0
\end{aligned} \tag{5}$$

where $c_{S,t+1}$ is the amount of goods consumed by an old supplier in period $t+1$; $x_{S,t}$ is the amount of goods invested in a young supplier's production in period t; $b_{S,t}$ is the face value of nominal debt issued by young producers to a young supplier in period t; q_t is the competitive real discount price of young producers' nominal debt in terms of goods held by a young supplier in period t; $m_{S,t}$ is the amount of fiat money held by a young supplier at the end of period t; $m'_{S,t+1}$ is the amount of fiat money spent by an old supplier on goods sold for immediate money payments in period $t+1$; $\tau_{S,t+1}$ is a lump-sum transfer of fiat money from, or to, an old supplier in period $t+1$, which is a tax if it is negative and a subsidy if it is positive; and $p_{1,t}$ and $p_{2,t}$ are the competitive real values of a unit of fiat money in terms of goods sold for immediate money payments and goods sold on credit, respectively, in period t. Each supplier takes as given the values of q_t, $p_{2,t}$, $p_{2,t+1}$, and $p_{1,t+1}$.

The first and second constraint in (5) are the flow-of-funds constraints for a young supplier in period t and an old supplier in period $t+1$, respectively. The right-hand side of the first constraint is the amount of a goods endowment for a young supplier at the beginning of period t. The third constraint is the feasibility constraint on $m'_{S,t+1}$, which implies that an old supplier can spend fiat money on goods sold for immediate money payments up to the supplier's money holding at the beginning of the period. The fourth constraint is the non-negativity constraint on the amount of fiat money that an old supplier pays to settle its credit purchase of goods from young suppliers. The last constraint implies that the values of $x_{S,t}$ and $m_{S,t}$ must be non-negative by definition.

The utility maximization problem for a young producer in cohort t is specified as follows:

$$\begin{aligned} \max_{\{x_{P,t},b_{P,t}\}} \quad & c_{P,t+1} \\ \text{s.t.} \quad & x_{P,t} = q_t b_{P,t} \\ & c_{P,t+1} = f(x_{P,t}) - p_{1,t+1} b_{P,t} \\ & x_{P,t} \geq 0 \end{aligned} \tag{6}$$

where $c_{P,t+1}$ is the amount of goods consumed by an old producer in period $t+1$; $x_{P,t}$ is the amount of goods invested in a young producer's production in period t; and $b_{P,t}$ is the face value of nominal debt issued by a young producer to young suppliers in period t. The first and second constraint are the flow-of-funds constraints for a young producer in period t and an old producer in period $t+1$, respectively. Note that $p_{1,t+1}b_{P,t}$ on the right-hand side of the second constraint is the amount of goods that an old producer must sell for immediate money payments to repay the face value of its nominal debt, $b_{P,t}$. The last constraint implies that $x_{P,t}$ is non-negative by definition. Each producer takes as given the values of q_t and $p_{1,t+1}$.

Given each type of agent in each cohort having a unit measure, the market clearing conditions are specified as

$$
\begin{aligned}
b_{P,t} &= m'_{S,t+1} && (7)\\
m_{S,t} &= \begin{cases} M_0 & \text{if } t = 0 \\ b_{S,t-1} + m_{S,t-1} + \tau_{S,t} - m'_{S,t} & \text{if } t = 1, 2, 3, ... \end{cases} && (8)\\
b_{S,t} &= b_{P,t} && (9)
\end{aligned}
$$

for $t = 0, 1, 2, ...$, where (7) implies that the amount of fiat money that old producers repay in nominal debt contracts must equal the amount of fiat money that old suppliers pay for goods sold by old producers for immediate money payments; (8) for $t = 1, 2, 3, ...$ implies that the amount of fiat money acquired by young suppliers must equal the amount of fiat money that old suppliers pay to settle their credit purchases of goods from young suppliers; and (9) implies that supply and demand must be equal in the credit market.

A monetary equilibrium is defined as follows:[12]

Definition 1. A monetary equilibrium is characterized by the solutions to (5) and (6), given q_t, $p_{2,t}$, $p_{2,t+1}$, and $p_{1,t+1}$, and the values of q_t, $p_{2,t}$, and $p_{1,t+1}$ that satisfy (7)-(9) for $t = 0, 1, 2, ...$

Substituting (7) and (9) into (8) yields

$$m_{S,t} = m_{S,t-1} + \tau_{S,t} \tag{10}$$

for $t = 1, 2, 3, ...$ Set the following assumption:

Assumption 3. $\tau_{S,t} = (\gamma - 1)M_{t-1}$ for $t = 1, 2, 3, ...$

This assumption ensures (4) and

$$m_{S,t} = M_t \tag{11}$$

for $t = 1, 2, 3, ...$, given $m_{S,0} = M_0$ in a monetary equilibrium.

[12]Note that $p_{1,0}$ does not exist in the model, as there are no old producers selling goods in period 0.

3.6 Monetary steady state

Hereafter, let us focus on the monetary steady state:

Definition 2. A monetary steady state is a monetary equilibrium such that $x_{S,t}$ and $x_{P,t}$ are constant, and

$$\frac{p_{2,t+1}}{p_{2,t}} = \frac{p_{1,t+1}}{p_{1,t}} = \frac{q_{t+1}}{q_t} = \frac{1}{\gamma} \tag{12}$$

in each period.

In addition, let us define social welfare by aggregate consumption in each period in the monetary steady state.[13] To maximize the aggregate production of goods in each period, the gross marginal rate of return on investment by each young producer, $f'(x_{P,t})$, must equal that on investment by each young supplier, ρ, at the social optimum. Such an allocation of goods is feasible, given the assumption that each young supplier is endowed with a sufficiently large amount of goods to achieve $f'(x_{P,t}) = \rho$, as implied by (1).[14] Given this property of the social optimum, it is referred to as underinvestment if $f'(x_{P,t}) > \rho$.

3.7 Underinvestment due to a shortage of real money balances for liability settlements

In the monetary steady state, suppliers must acquire fiat money when young, and spend the fiat money on goods sold by old producers for immediate money payments when old, because otherwise old producers would not be able to acquire fiat money to repay their nominal debt, as indicated by Table 1. In the following, it is shown that this endogenous requirement for intertemporal money holdings by young suppliers can cause a shortage of real money balances for liability settlements, and underinvestment.

[13]This assumption is equivalent to giving an equal weight to the utility of each cohort in the definition of social welfare. As a result, the relative utility weight for the initial old becomes zero compared with the sum of weights for subsequent cohorts.

[14]The economy is dynamically efficient as implied by (3). Thus, an inter-generational transfer of goods from young agents to old agents is never socially optimal.

To remove any artificial bias from the monetary policy, suppose that the central bank implements a Friedman rule:

$$\frac{1}{\gamma} = \rho \tag{13}$$

Note that $1/\gamma$ equals the gross rate of return on fiat money in the monetary steady state, as implied by (12), whereas ρ is the gross marginal rate of return on investment by a young supplier, as assumed in the baseline model. Thus, (13) eliminates the opportunity cost of an intertemporal holding of fiat money for a young supplier in the monetary steady state.

Despite a Friedman rule, underinvestment can occur in the monetary steady state if the first-best investment in a producer's production, i.e., $f'^{-1}(\rho)$, is too large. On one hand, consider the flow-of-funds constraint for a young supplier:

$$1 - q_t b_{S,t} - p_{2,t} m_{S,t} = x_{S,t} \geq 0 \tag{14}$$

which can be derived from the first and last constraint in (5). This constraint implies that a young supplier cannot allocate more than a unit of goods to young producers through the purchase of nominal debt, $q_t b_{S,t}$, and to old suppliers through the purchase of fiat money, $p_{2,t} m_{S,t}$, because each young supplier is endowed with only a unit of goods.

On the other hand, the market clearing conditions for the goods markets and the credit market, (7)-(9), imply that the face value of nominal debt issued by a young producer, $b_{S,t}$, must equal the amount of fiat money that an old producer can acquire by selling goods to old suppliers in the next period. Therefore, the value of $b_{S,t}$ is capped by the amount of fiat money that each old supplier holds at the beginning of the next period, $m_{S,t+1}$, which equals $\gamma m_{S,t}$, given (10) and Assumption 3. Thus,

$$b_{S,t} \leq \gamma m_{S,t} \tag{15}$$

in the monetary steady state.

A young producer's utility maximization problem, (6), implies that

$$f'(x_{P,t}) = \frac{p_{1,t+1}}{q_t} \tag{16}$$

in which $q_t > 0$ in a monetary equilibrium. It can be shown that if the first-best investment in each producer's production is implemented in the monetary steady state, then the real value of a unit of fiat money must be the same regardless of whether fiat money is spent on goods sold for immediate money payments or goods sold on credit, given no shortage of fiat money available for payments. Thus,

$$p_{1,t+1} = p_{2,t+1} \tag{17}$$

in the monetary steady state if $f'(x_{P,t}) = \rho$. In this case, (16) and (17) imply

$$q_t = p_{2,t} \tag{18}$$

in the monetary steady state, given $f'(x_{P,t}) = \rho$ and $\rho\gamma = 1$ as implied by (13).[15] Substituting (15) and (18) into (14) yields

$$q_t b_{S,t} \leq \frac{1}{1+\rho} \tag{19}$$

as a necessary condition for the first-best investment in each producer's production in the monetary steady state.

The real discount price of nominal debt issued by young producers to a young supplier, $q_t b_{S,t}$, must equal the amount of goods supplied to a young producer in exchange for nominal debt, $x_{P,t}$, in a monetary equilibrium:[16]

$$q_t b_{S,t} = x_{P,t} \tag{20}$$

Thus, if the first-best investment in a producer's production, i.e., $x_{P,t} = f'^{-1}(\rho)$, exceeds $(1+\rho)^{-1}$, then (19) must be violated. In this case, underinvestment must occur in the monetary steady state.

[15] Note that $p_{1,t+1}/p_{1,t} = 1/\gamma$ in the monetary steady state, as implied by (12). Thus, (16) implies that $\gamma\rho = 1 = p_{1,t}/q_t$. Therefore, (17) implies (18) in the monetary steady state.

[16] See the market clearing condition for the credit market, (9), and the first constraint in (6).

This result holds because old producers can repay the face value of nominal debt only up to the amount of fiat money supplied at the beginning of each period. Given the competitive real value of fiat money in the goods market, young producers cannot issue a sufficiently large face value of nominal debt to finance the first-best investment in their production, if the first-best investment is too large. In this case, a shortage of real money balances for liability settlements causes underinvestment in the monetary steady state.

If underinvestment occurs in the monetary steady state, then it is accompanied by $p_{1,t} > p_{2,t}$—that is, the nominal price of goods sold by old suppliers for immediate money payments, $1/p_{1,t}$, is cheaper than the nominal price of goods sold on credit by young suppliers, $1/p_{2,t}$. In this case, old suppliers spend their entire holdings of fiat money on goods sold by old suppliers, and then settle their credit purchases of goods sold by young suppliers after receiving the repayments of nominal debt from old producers. Because they can pay no more fiat money on goods sold for immediate money payments, there remains a difference between $p_{1,t}$ and $p_{2,t}$ in the monetary steady state. See Appendix B for the formal characterization of the monetary steady state.[17]

4 Introducing the central bank's discount window

A shortage of real money balances for liability settlements can be resolved if the central bank supplies fiat money on demand within each period. To see this result, assume that each old supplier can borrow fiat money from the central bank at a zero intraday interest rate up to the face value of nominal debt that the supplier holds at the beginning of the period. This facility can be interpreted as a discount window, or a standing liquidity facility, offered by the central bank in practice. The maturity of the central bank's lending comes at the end of the same period. Thus, the intraday supply of fiat money through the central bank's discount window is separated from the overnight supply of fiat money, M_t. This feature of

[17] Appendix B also covers the cases with $\rho \leq 1$.

the model is consistent with the fact that the central bank in each country usually offers intraday overdrafts of bank reserves to commercial banks in order to facilitate the interbank settlement of bank transfers.

Note that an old supplier's liability to the central bank in this lending is nominal debt. Thus, the court can identify fiat money repaid by an old supplier to the central bank, if any, and seize the repayment of old producers' nominal debt to an old supplier if the old supplier defaults. Therefore, old suppliers can be committed to repaying fiat money to the central bank up to their holdings of nominal debt issued by old producers.[18]

Given the availability of the central bank's discount window, old suppliers can spend a greater amount of fiat money on goods sold for immediate money payments than they hold at the beginning of the period. In the utility maximization problem for a young supplier, (5), this assumption implies that there disappears the upper bound on the amount of fiat money that an old supplier can spend on goods sold for immediate money payments, i.e., $m'_{S,t+1} \leq m_{S,t} + \tau_{S,t+1}$. The other part of the model remains the same.[19] Accordingly, the definition of a monetary equilibrium is revised as follows:

Definition 3. A monetary equilibrium with the central bank's discount window is characterized as in Definition 1, except that $m'_{S,t+1} \in [0,\ m_{S,t} + \tau_{S,t+1}]$ in the utility maximization problem for a young supplier, (5), is replaced with $m'_{S,t+1} \geq 0$.

4.1 No underinvestment with the central bank's discount window

Introducing the central bank's discount window makes it possible to achieve the first-best investment in each producer's production, i.e., $x_{P,t} = f'^{-1}(\rho)$, in the monetary steady state,

[18]In (5), the non-negativity constraint on the amount of fiat money that an old supplier pays to settle its credit purchase of goods from young suppliers, $b_{S,t} + m_{S,t} + \tau_{S,t+1} - m'_{S,t+1} \geq 0$, ensures that the amount of fiat money that an old supplier borrows from the central bank within the period, $m'_{S,t+1} - m_{S,t} - \tau_{S,t+1}$, cannot exceed the face value of nominal debt held by the supplier, $b_{S,t}$.

[19]There is no need for an explicit market clearing condition for the central bank's discount window lending, as the central bank accommodates demand for this lending from old suppliers passively at a zero intraday interest rate.

regardless of the size of $f'^{-1}(\rho)$. In this case, old suppliers can spend fiat money on goods sold by old producers for immediate money payments up to the face value of nominal debt they hold, because they can borrow fiat money up to this value through the central bank's discount window within each period. Therefore, young producers can promise to repay any face value of nominal debt, i.e., $b_{S,t}$, in order to finance the first-best investment in their production. See Appendix C for the formal proposition on this result.

4.2 Optimality of no overnight money supply with the central bank's discount window

As assumed by (1), young suppliers are endowed with a sufficiently large amount of goods to allow the first-best investment in each producer's production. Thus, young suppliers still hold some amount of goods to sell for fiat money, or invest in their own production, after transferring their goods to young producers in exchange for nominal debt in each period. Given the Friedman rule set by (13), young suppliers are indifferent to the two saving options.

If young suppliers acquire fiat money in the monetary steady state, however, they transfer a constant amount of goods to old suppliers in each period, because the real value of a unit of fiat money is increasing at a rate γ, while the overnight supply of fiat money is declining at the same rate, as indicated by (10)-(12). Thus, the gross social rate of return on fiat money is unity for each cohort in this case. Therefore, it is socially optimal if young suppliers invest all the residual of goods in their own production after financing the first-best investment in each producer's production, as the gross rate of return on their production, ρ, is greater than unity—that is, the economy is dynamically efficient—as assumed by (3).

If the central bank supplies a positive amount of fiat money overnight in the monetary steady state, i.e., $M_t > 0$, then old suppliers accommodate the supply of fiat money by holding a positive amount of fiat money overnight, i.e., $m_{S,t} > 0$, in a monetary equilibrium, as implied by (11). Thus, no overnight money supply is socially optimal. See Appendix C for the formal description of this result.[20]

5 Discussion

5.1 Interpretation of the economy with no overnight money supply

Fiat money in the model corresponds to base money issued by the central bank, which consists of cash and bank reserves. Thus, the economy with no overnight money supply in the model can be interpreted as a cashless economy, which has become a realistic possibility with the advancement of information technology.[21]

Also, supplying a zero amount of bank reserves overnight has been experimented in Canada for 2006-7. This experiment was possible because the Bank of Canada adopted the channel (or corridor) system. In this system, the central bank sets a narrow band between the interest rate on the overnight lending of bank reserves to commercial banks, and the interest rate on the overnight deposits of bank reserves by commercial banks, so that the overnight interbank interest rate comes in the middle of the two rates automatically without a change in the supply of bank reserves. The central bank adopting this system usually maintains only a small overnight supply of bank reserves to commercial banks. Nonetheless, commercial banks in this system can settle bank transfers among depositors smoothly by paying bank reserves to each other, because they are allowed to run up overdrafts of bank reserves at a zero intraday interest rate if they run short of bank reserves during the day. At the end of the day, they can borrow and lend bank reserves among them to repay the overdrafts to the

[20] There is no transaction between old suppliers and young suppliers in the monetary steady state, if the overnight money supply is set to zero. This is because old suppliers do not hold any amount of fiat money after repaying discount window lending from the central bank in each period, while young suppliers do not need to hold fiat money overnight, given the availability of fiat money through the central bank's discount window within each period.

[21] For example, Bagnall et al. (2016) report that the cash share of payments in a 2012 consumer diary survey in the U.S. was 23%. Also, Wang and Wolman (2016) analyze a nation-wide U.S. discount retailer's data between 2010 and 2013, and estimate that the overall cash fraction of transactions declined by 2.46% per year in the three-year sample period due to an increase in the use of debit cards.

central bank. The Bank of Canada run this system targeting at a zero overnight supply of bank reserves for March 2006 to May 2007, and was successful in hitting the target in several months, as shown in Table 2. Thus, the economy with no overnight supply of fiat money in the model can be interpreted as a cashless economy adopting the channel system with a zero overnight supply of bank reserves. This interpretation of the model is consistent with the role of intraday overdrafts in the channel system, which correspond to the central bank's intraday discount window in the model.[22]

5.2 Nominal anchor for the long-run price level in an economy with no overnight money supply

No overnight supply of fiat money with the central bank's intraday discount window implies no pre-determined supply of fiat money in each period, as fiat money is supplied to old suppliers on demand within each period. Therefore, there is no nominal anchor that pins down the nominal price level, i.e., the inverse of the real value of a unit of fiat money, in the monetary steady state in such a case.[23]

In this case, the model can be compatible with other nominal anchors than the money supply proposed in the literature, such as the central bank's inflation target as argued by Woodford (2008), or nominal public debt in the fiscal theory of price level (see Sims (2016) for example).[24] This result holds because the values of real variables at the social optimum

[22]Bank deposits can be introduced into the model without any contradiction to no overnight supply of fiat money. For example, suppose that young suppliers swap nominal debt issued by young producers for bank deposits issued by commercial banks in each period. This transaction corresponds to bill discounting by commercial banks in practice. Old suppliers can pay deposit balances for old producers' output, while old producers can repay nominal debt by received deposit balances. In addition, assume that commercial banks pay fiat money to each other to settle the transfers of deposit balances from old suppliers to old producers. Banks can borrow fiat money from the central bank within each period by asking the central bank to discount their holdings of nominal debt owed by old producers at a zero intraday interest rate. These transactions can be intact even if there is no overnight supply of fiat money.

[23]Price indeterminacy remains even if the central bank fixes the intraday supply of fiat money through a discount window, given a zero intraday interest rate. See Appendix D for more details. In addition, a positive overnight supply of fiat money is optimal if the economy is dynamically inefficient, as an inter-generational transfer of goods is Pareto-improving. In this case, the overnight supply of fiat money pins down the nominal price level in the monetary steady state. See Appendix C for more details.

[24]Woodford (2008) argues that the central bank can determine the long-run inflation rate only by its infla-

Table 2: Zero overnight supply of bank reserves in Canada for 2006-7 (Billion CAD)

Month	Daily average transfer of bank reserves	Average overnight supply of bank reserves	
		Target	Actual
Apr 2006	164.38	0	0.00
May 2006	157.87	0	-0.06
Jun 2006	170.19	0	0
Jul 2006	168.95	0	0
Aug 2006	156.98	0	-0.03
Sep 2006	181.51	0	0.03
Oct 2006	170.71	0.00	0.08
Nov 2006	159.64	0	0
Dec 2006	181.58	0	0.07
Jan 2007	163.60	0	0
Feb 2007	166.18	0	-0.03
Mar 2007	178.87	0	-0.05
Apr 2007	169.42	0	0.05

Source: The Bank of Canada.
Notes: "0.00" is a rounded number which is not absolutely equal to zero. "0" means the figure is precisely zero. The policy to target at a zero overnight supply of bank reserves started in the middle of March 2006 and ended in the middle of May 2007. Thus, the first and last month of this policy are excluded from the table. The daily target for the overnight supply of bank reserves was positive on October 31, 2006, which is why the average target value for Oct 2006 was not exactly zero.

with no overnight money supply in the monetary steady state are as same as those in the model without Assumption 1, which is a standard overlapping generations model without money. Thus, given the optimal intraday supply of fiat money through the central bank's discount window, the rest of the model can be treated as a real model without money. Hence, it is possible to introduce a nominal anchor into the model in the same way as into a real model.

This feature of the model contrasts with the standard feature of monetary models in the literature, in which the quantity theory of money holds for the long-run price level—that is, the exogenous money supply solely determines the nominal price level in the steady state.[25] Thus, these models do not have any room for other nominal anchors than the money supply. This inflexibility, however, is inconvenient to explain the recent observation that central banks in developed countries cannot raise the current inflation rate simply by promising to continue monetary easing for the future.[26]

The compatibility between the model and nominal anchors other than the money supply is not redundant for existing models that incorporate such nominal anchors. Without incorporating money, a model cannot explain why money is essential, given the nominal price level being a rate of exchange between goods and money. If a model incorporates money by a standard assumption, however, it cannot have a nominal anchor other than the money supply. The model in this paper is free of this conundrum.

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tion target, anchoring people's inflation expectations independently of monetary aggregates. It is trivial to introduce a similar assumption into the model by assuming that the central bank can determine the nominal price level in the monetary steady state by navigating agents' inflation expectations through communications. Also, see Katagiri et al. (forthcoming) for an example of an overlapping generations model with the fiscal theory of the price level.

[25]See Walsh (2017) for more details on standard monetary models in the literature.

[26]A notable example of such a central bank is the Bank of Japan, which has been fighting against a low inflation rate since the mid-1990s. Note that even if a zero nominal interest rate separates the price level and the money supply at present, the central bank's commitment to a long-run money growth should be able to raise the expected long-run inflation rate, which should in turn feed back into the current inflation rate, if the quantity theory of money holds for the long-run price level.

6 Conclusions

This paper shows in an overlapping generations model that if the court cannot distinguish different qualities of goods of the same kind, then it leads to the use of nominal debt contracts as well as the use of fiat money as a means of payment in the goods market. The model further shows that underinvestment can occur due to a shortage of real money balances for liability settlements, even if the money supply follows a Friedman rule. This problem can be resolved if the central bank lends fiat money through a discount window elastically at a zero intraday interest rate within each period. In this case, supplying no fiat money overnight is socially optimal, given the economy being dynamically efficient. This economy can be interpreted as a cashless economy adopting the channel system with a zero overnight supply of bank reserves.

No overnight money supply with the central bank's intraday discount window causes price indeterminacy in the monetary steady state. This result implies that a nominal anchor for the long-run price level other than the money supply can be compatible with the essentiality of fiat money. Further investigation into the possibility of other nominal anchors than those proposed in the literature is left for future research.

Also, the model implies that if there is a friction in the intraday supply of fiat money through the central bank's discount window, then it can cause a real disturbance to the economy. Further analysis of this issue is left for future research.

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Appendix

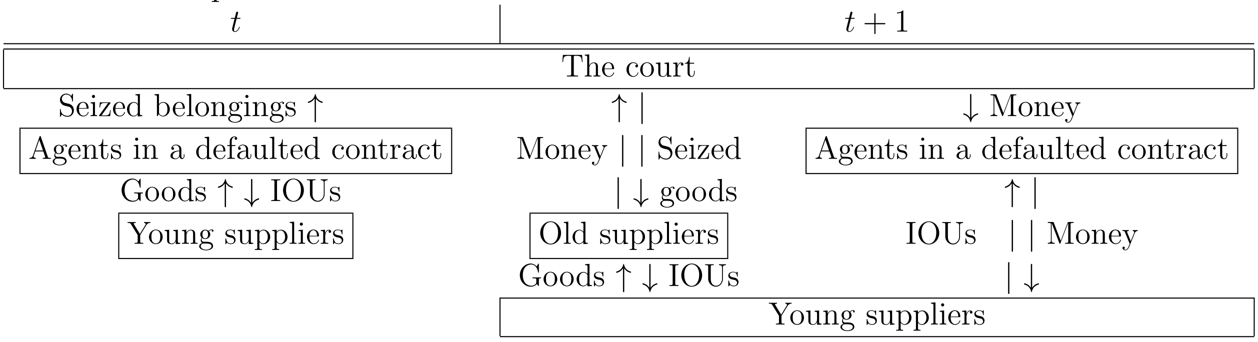
A Proof for no strategic default on a nominal debt contract in a monetary equilibrium without Assumption 2

Suppose that Assumption 2 does not hold. Then, suppose that an old producer defaults on a nominal debt contract with an old supplier. In this case, the court sells the producer's belongings for fiat money in the goods market in the next period, and then delivers the acquired fiat money to the producer and the supplier in the period, as described in section 3.3.

Without Assumption 2, both an old producer and an old supplier entering into a defaulted nominal debt contract can buy goods from young suppliers in the current period by promising to deliver fiat money received from the court after the goods market in the next period. Their promises are enforceable, as the court can simply seize fiat money retained by them if they default. Even though young suppliers accepting these promises cannot receive fiat money during the goods market in the next period, they can buy goods from young suppliers in the next period by paying the promises, because young suppliers in the next period can wait until the end of the next period to receive fiat money. The flows of goods and fiat money in this case is illustrated in Figure A.1.

Now let us compute the total amount of goods available for an old producer and an old supplier in a defaulted nominal debt contract. Denote by y_t the amount of goods produced by the old producer in period t. In period t, the court seizes these goods along with any other belongings of the producer, and sell them for fiat money paid by old suppliers in period $t+1$ at $p_{1,t+1}$, which is the competitive real value of a unit of fiat money in terms of goods sold for immediate money payments. Therefore, the producer and the supplier in the defaulted contract receive an amount $y_t/p_{1,t+1}$ of fiat money from the court in total after the goods

Figure A.1: Flows of goods and fiat money in case of default on a nominal debt contract without Assumption 2



Notes: Agents in the same row are in the same cohort. "t" and "$t+1$" denote the time periods. "Agents in a default contract" in period t are an old producer and an old supplier in a defaulted nominal debt contract, and "Agents in a default contract" in period $t+1$ are the same agents in the third period of their lives. "Seized belongings" are the defaulting old producer's belongings seized by the court. "Money" denotes fiat money, and "IOUs" are promises made by agents in a defaulted contract to deliver fiat money after the goods market in period $t+1$. Each allow represents the flow of an item in the label.

market in period $t+1$. Because they can promise to deliver this amount of fiat money after the goods market in period $t+1$, old suppliers accepting these promises in period $t+1$ can acquire an amount $p_{2,t+1}y_t/p_{1,t+1}$ of goods from young suppliers by paying the promises in the same period, where $p_{2,t+1}$ is the competitive real value of a unit of fiat money in terms of goods sold on credit. Expecting this consequence, young suppliers in period t are willing to sell an amount $p_{2,t+1}y_t/(\rho p_{1,t+1})$ of goods to the producer and the supplier in the defaulted contract in exchange for the promises to deliver fiat money after the goods market in period $t+1$. This is because they discount the future receipts of goods by ρ, i.e., the gross marginal rate of return on the investment of goods in their production, given that young suppliers never invest all of their endowments of goods into young producers' production, as implied by (1).

The second constraint in a young supplier's utility maximization problem, (5), implies $m'_{S,t+1} = 0$ if $p_{1,t+1} < p_{2,t+1}$. In such a case, there would be no purchase of goods sold for immediate payments of fiat money in the goods market; thus, it would be impossible for old producers to acquire fiat money to fulfill their liabilities in nominal debt contracts.

Therefore, the real value of a unit of fiat money in terms of goods sold for immediate money payments must be weakly higher than that in terms of goods sold on credit. Thus,

$$p_{1,t+1} \geq p_{2,t+1} \tag{A.1}$$

for all t in a monetary equilibrium. This result implies that

$$y_t > \frac{p_{2,t+1} y_t}{\rho p_{1,t+1}} \tag{A.2}$$

given (3). Hence, an old producer and an old supplier in a nominal debt contract can never increase the total amount of goods that they can consume by making the producer default on the contract in an monetary equilibrium.

B Monetary steady state without the central bank's discount window

The following assumption is a single-crossing condition on a producer's production function, f, that ensures the uniqueness of a monetary steady state:

Assumption 4. The function f satisfies the following properties:

$$\lim_{x\to 0} \frac{\gamma(x^{-1}-1)^2}{f'(x)} > 1 \tag{A.3}$$

$$\frac{d}{dx}\left\{\gamma\left(\frac{1}{x}-1\right)^2\right\} < f''(x) \quad \text{for all } x \in \left(0, \frac{\gamma}{1+\gamma}\right) \tag{A.4}$$

Assumption 4 is a single-crossing condition between $\gamma(x^{-1}-1)^2$ and $f'(x)$ for $x \in \left(0, \frac{\gamma}{1+\gamma}\right)$. For example, if $f(x) = Ax^{\sigma}$ where $\sigma \in (0,1)$, then (A.3) is satisfied for all $A > 0$, and (A.4) is satisfied if $A \in (0, 2^{1+\sigma}/[\sigma(1-\sigma)])$.

Given no discount window offered by the central bank, the monetary steady state can be characterized as follows:

Proposition 1. Suppose $M_0 > 0$ and Assumptions 1-4 hold. Given Definitions 1 and 2, there exists a monetary steady state such that

$$x_{P,t} = x_b^* \equiv f'^{-1}\left(\frac{1}{\gamma}\right) \tag{A.5}$$

$$q_t = p_{2,t} = p_{1,t} = \begin{cases} \frac{1-x_b^*}{M_t} & \text{if } \frac{1}{\gamma} > \rho \\ \text{any real number in } \left[\frac{x_b^*}{\gamma M_t},\ \frac{1-x_b^*}{M_t}\right] & \text{if } \frac{1}{\gamma} = \rho \end{cases} \tag{A.6}$$

$$x_{S,t} = \begin{cases} 0 & \text{if } \frac{1}{\gamma} > \rho \\ 1 - x_b^* - p_{2,t}M_t & \text{if } \frac{1}{\gamma} = \rho \end{cases} \tag{A.7}$$

if $\rho\gamma \leq 1$ and $x_b^* \leq \gamma(1+\gamma)^{-1}$;

$$x_{P,t} = x_b^{**} \text{ such that } f'(x_b^{**}) = \gamma\left(\frac{1}{x_b^{**}} - 1\right)^2 \tag{A.8}$$

$$(q_t,\ p_{2,t},\ p_{1,t}) = \left(\frac{x_b^{**}}{\gamma M_t},\ \gamma\left(\frac{1}{x_b^{**}} - 1\right) q_t,\ \gamma^2\left(\frac{1}{x_b^{**}} - 1\right)^2 q_t\right) \tag{A.9}$$

$$x_{S,t} = 0 \tag{A.10}$$

where $x_b^{**} \in (0, \gamma(1+\gamma)^{-1})$, if $\rho\gamma \leq 1$ and $x_b^* > \gamma(1+\gamma)^{-1}$;

$$x_{P,t} = x_b^{***} \equiv f'^{-1}\left(\rho^2\gamma\right) \tag{A.11}$$

$$(q_t,\ p_{2,t},\ p_{1,t}) = \left(\frac{x_b^{***}}{\gamma M_t},\ \rho\gamma q_t,\ \rho^2\gamma^2 q_t\right) \tag{A.12}$$

$$x_{S,t} = 1 - x_b^{***} - p_{2,t}M_t \tag{A.13}$$

if $\rho\gamma > 1$ and $x_b^{***} \leq (1+\rho)^{-1}$; and (A.8)-(A.10) hold where $x_b^{**} \in (0, (1+\rho)^{-1})$, if $\rho\gamma > 1$ and $x_b^{***} > (1+\rho)^{-1}$. In all four cases,

$$b_{S,t} = b_{P,t} = m'_{S,t+1} = \frac{x_{P,t}}{q_t} \tag{A.14}$$

There is no other monetary steady state.

Proof. First of all, (12) implies $q_t > 0$. Given $q_t > 0$, the two market clearing conditions, (7) and (9), and the first constraint in (6), i.e., $x_{P,t} = q_t b_{P,t}$, imply (A.14) immediately. Given $M_0 > 0$ and (3), $m_{S,t} = M_t = \gamma M_{t-1}$ for $t = 1, 2, 3, \ldots$ Thus, (12) holds in a monetary steady state, as otherwise $p_{2,t}m_{S,t} = 1 - x_{P,t} - x_{S,t}$, which is implied by (A.14) and the first constraint in (5), would not be constant.

Given $q_t > 0$ and the Inada condition satisfied by the function f, the first-order condition for $b_{P,t}$ in (6) implies

$$f'(q_t b_{P,t}) = \frac{p_{1,t+1}}{q_t} \tag{A.15}$$

in a monetary equilibrium, which in turn implies $b_{P,t} > 0$ as $p_{1,t+1}$ is finite in a monetary equilibrium.

Given $q_t > 0$ and $b_{P,t} = b_{S,t} > 0$ as implied by (9), the first-order conditions for $x_{S,t}$, $b_{S,t}$, $m_{S,t}$, and $m'_{S,t+1}$ in (5) are respectively

$$-\eta_{1,t} + \rho + \underline{\lambda}_{x,\ell,t} = 0 \tag{A.16}$$

$$-\eta_{1,t}q_t + p_{2,t+1} + \eta_{4,t} = 0 \tag{A.17}$$

$$-\eta_{1,t}p_{2,t} + p_{2,t+1} + \bar{\lambda}_{m',\ell,t+1} + \eta_{4,t} + \underline{\lambda}_{m,\ell,t} = 0 \tag{A.18}$$

$$p_{1,t+1} - p_{2,t+1} + \underline{\lambda}_{m',\ell,t+1} - \bar{\lambda}_{m',\ell,t+1} + \eta_{4,t} = 0 \tag{A.19}$$

where $\eta_{1,t}$, $\bar{\lambda}_{m',\ell,t+1}$, $\underline{\lambda}_{m',\ell,t+1}$, $\eta_{4,t}$, $\underline{\lambda}_{x,\ell,t}$, and $\underline{\lambda}_{m,\ell,t}$ are Lagrange multipliers for $x_{S,t} + q_t b_{S,t} + p_{2,t}m_{S,t} = 1$, $m'_{S,t+1} \geq 0$, $m'_{S,t+1} \leq m_{S,t} + \tau_{S,t+1}$, $b_{S,t} + m_{S,t} - m'_{S,t+1} + \tau_{S,t+1} \geq 0$, $x_{S,t} \geq 0$, and $m_{S,t} \geq 0$.

Because Assumption (3) implies $m_{S,t} = M_t = \gamma M_{t-1}$, $m_{S,t} > 0$, and thus $\underline{\lambda}_{m,\ell,t} = 0$, given $M_0 > 0$. Also, (8) implies $m_{S,t} = b_{S,t} + m_{S,t} - m'_{S,t+1} + \tau_{S,t+1} > 0$, and (7) implies that $m'_{S,t+1} > 0$, given $b_{P,t} > 0$. Hence, $\eta_{4,t} = 0$ and $\underline{\lambda}_{m',\ell,t+1} = 0$. Substituting these results into (A.16)-(A.19) yields

$$\underline{\lambda}_{x,\ell,t} = \eta_{1,t} - \rho \geq 0 \tag{A.20}$$

$$\eta_{1,t} = \frac{p_{2,t+1}}{q_t} = \frac{p_{1,t+1}}{p_{2,t}} \tag{A.21}$$

$$\bar{\lambda}_{m',\ell,t+1} = p_{1,t+1} - p_{2,t+1} \geq 0 \tag{A.22}$$

Now split the parameter space into two regions: $\rho \leq \gamma^{-1}$ and $\rho > \gamma^{-1}$. Suppose $\rho \leq \gamma^{-1}$ and $p_{1,t+1} = p_{2,t+1}$. In this case,

$$\eta_{1,t} = \frac{p_{1,t+1}}{p_{2,t}} = \frac{p_{2,t+1}}{p_{2,t}} = \frac{1}{\gamma} \tag{A.23}$$

given (12). Thus,

$$\frac{p_{2,t}}{q_t} = \frac{p_{2,t+1}}{q_t} \cdot \frac{p_{2,t}}{p_{2,t+1}} = \eta_{1,t}\gamma = 1, \tag{A.24}$$

$$f'(q_t b_{P,t}) = \frac{p_{1,t+1}}{q_t} = \frac{p_{2,t+1}}{q_t} = \frac{1}{\gamma} \tag{A.25}$$

as implied by (12) and (A.15). Because $f'' < 0$, the inverse function of f', f'^{-1}, exists. Therefore, the steady state value of $q_t b_{P,t}$ is unique. Given $\underline{\lambda}_{x,\ell,t} \geq 0$ and $\bar{\lambda}_{m',\ell,t+1} = 0$, $x_{S,t} \geq 0$ and $b_{P,t} = m'_{S,t+1} \leq m_{S,t} + \tau_{S,t+1}$. These constraints are satisfied if and only if

$$q_t b_{P,t} = q_t m'_{S,t+1} \leq q_t(m_{S,t} + \tau_{S,t+1}) = \gamma q_t m_{S,t} \tag{A.26}$$

$$x_{S,t} = 1 - q_t b_{S,t} - p_{2,t} m_{S,t} = 1 - q_t b_{P,t} - q_t m_{S,t} \geq 0 \tag{A.27}$$

given Assumption 3 and (A.24). Denote $f'^{-1}(\gamma^{-1})$ by x_b^*. These conditions are equivalent to

$$q_t m_{S,t} \in \left[\frac{x_b^*}{\gamma}, 1 - x_b^*\right] \tag{A.28}$$

This range is non-empty if and only if $x_b^* \leq \gamma(1+\gamma)^{-1}$. If $\rho = \gamma^{-1}$, then $\underline{\lambda}_{x,\ell,t} \geq 0 = \eta_{1,t} - \rho = 0$. Thus, $x_{S,t}$ is indeterminate, and any value of $q_t m_{S,t}$ in this range can be a steady state value. If $\rho < \gamma^{-1}$, then $\eta_{1,t} > \rho$. Hence, $x_{S,t} = 0$, which implies $q_t m_{S,t} = q_t M_t = 1 - x_b^*$. The results described in this paragraph are sufficient for (A.5)-(A.7).

If $\rho \leq \gamma^{-1}$ and $p_{1,t+1} > p_{2,t+1}$, then $\bar{\lambda}_{m',\ell,t+1} > 0$. Thus, $b_{P,t} = m'_{S,t+1} = m_{S,t} + \tau_{S,t+1} = \gamma m_{S,t}$, given Assumption 3. Also,

$$\eta_{1,t} = \frac{p_{1,t+1}}{p_{2,t}} = \frac{p_{1,t+1}}{p_{2,t+1}} \frac{p_{2,t+1}}{p_{2,t}} > \frac{1}{\gamma} \geq \rho \tag{A.29}$$

Hence, $\underline{\lambda}_{x,\ell,t} > 0$ and $x_{S,t} = 1 - q_t b_{S,t} - p_{2,t} m_{S,t} = 0$. Therefore,

$$\eta_{1,t} = \frac{p_{2,t+1}}{q_t} = \frac{p_{2,t}}{q_t} \frac{p_{2,t+1}}{p_{2,t}} = \frac{1}{x_{P,t}} - 1 \tag{A.30}$$

given $b_{P,t} = \gamma m_{S,t}$. As a result, (A.15) implies that

$$f'(x_{P,t}) = \frac{p_{1,t+1}}{q_t} = \frac{p_{2,t+1}}{q_t} \cdot \frac{p_{2,t}}{p_{2,t+1}} \cdot \frac{p_{1,t+1}}{p_{2,t}} = \left(\frac{p_{2,t+1}}{q_t}\right)^2 \frac{p_{2,t}}{p_{2,t+1}} = \gamma\left(\frac{1}{x_{P,t}} - 1\right)^2 \quad \text{(A.31)}$$

given (12), (A.21), and (A.30). As implied by (A.29) and (A.30), the solution for this equation must satisfy $\frac{1}{x_{P,t}} - 1 > \frac{1}{\gamma}$, or $x_{P,t} < \gamma(1+\gamma)^{-1}$. Given Assumption 4, $f'(x_{P,t})$ and $\gamma(x_{P,t}^{-1} - 1)^2$ can have only one intersection for $x_{P,t} \in (0, \gamma(1+\gamma)^{-1})$ at most, and they do have one if and only if $f'(\gamma(1+\gamma)^{-1}) > \gamma[\gamma^{-1}(1+\gamma) - 1]^2 = \gamma^{-1}$, or $x_b^* \equiv f'^{-1}(\gamma^{-1}) > \gamma(1+\gamma)^{-1}$. Given $m_{S,t} = M_t$ and (12), the results described in this paragraph are sufficient for (A.8)-(A.10).

Suppose $\rho > \gamma^{-1}$. Because $\eta_{1,t} = p_{1,t+1}p_{2,t+1}^{-1}\gamma^{-1}$ as implied by (A.21), (A.20) requires $p_{1,t+1} \geq \rho\gamma p_{2,t+1}$. Thus, $\bar{\lambda}_{m',\ell,t+1} > 0$ and $m'_{S,t+1} = m_{S,t} + \tau_{S,t+1}$, given $\rho > \gamma^{-1}$ and (A.22). Hence,

$$b_{P,t} = b_{S,t} = \gamma m_{S,t} \quad \text{(A.32)}$$

given (7), (9), and Assumption 3 in this case.

If $\rho > \gamma^{-1}$ and $p_{1,t+1} = \rho\gamma p_{2,t+1}$, then $\eta_{1,t} = p_{1,t+1}p_{2,t}^{-1} = \rho > \gamma^{-1}$, given (12) and (A.21). Thus, $\underline{\lambda}_{x,\ell,t} = 0$ and $x_{S,t} = 1 - q_t b_{S,t} - p_{2,t}m_{S,t} \geq 0$, as implied by (A.20). In this case, (A.15) implies that

$$f'(x_{P,t}) = \frac{p_{1,t+1}}{q_t} = \frac{p_{2,t+1}}{q_t} \cdot \frac{p_{2,t}}{p_{2,t+1}} \cdot \frac{p_{1,t+1}}{p_{2,t}} = \rho^2\gamma \quad \text{(A.33)}$$

given (12) and (A.21). Because $\eta_{1,t} = p_{2,t+1}q_t^{-1} = \rho$ and (A.21) also imply that $p_{2,t} = \rho\gamma q_t$, $x_{S,t} = 1 - q_t b_{S,t} - p_{2,t}m_{S,t} \geq 0$ implies that $x_{P,t} \leq 1 - \rho\gamma q_t m_{S,t}$, given the first constraint in (6). Also, $x_{P,t} = q_t b_{P,t} = \gamma q_t m_{S,t}$ as implied by (A.32). Thus, the unique root for $x_{P,t}$ in (A.33) must be in $(0, (1+\rho)^{-1})$. The results described in this paragraph are sufficient for (A.11)-(A.13).

If $\rho > \gamma^{-1}$ and $p_{1,t+1} > \rho\gamma p_{2,t+1}$, then $\underline{\lambda}_{x,\ell,t} > 0$ and $x_{S,t} = 1 - q_t b_{S,t} - p_{2,t}m_{S,t} = 0$. Thus, (A.30) holds, given $b_{P,t} = b_{S,t} = \gamma m_{S,t}$. Hence, (A.31) holds. Also, $\eta_{1,t} = p_{1,t+1}p_{2,t+1}^{-1}\gamma^{-1} > \rho$

and (A.30) imply that (A.31) must have a root for $x_{P,t}$ in $(0,(1+\rho)^{-1})$ if there exists a monetary steady state in this case. Because $\rho > \gamma^{-1}$ implies that $(1+\rho)^{-1} < \gamma(1+\gamma)^{-1}$, Assumption 4 implies that $f'(x_{P,t})$ and $\gamma(x_{P,t}^{-1}-1)^2$ can have only one intersection for $x_{P,t} \in (0,(1+\rho)^{-1})$ at most, and that they do have one if and only if $f'((1+\rho)^{-1}) > \gamma[(1+\rho)-1]^2 = \rho^2\gamma$. The results described in this paragraph are sufficient for (A.8)-(A.10). □

C Monetary steady state with the central bank's discount window

Proposition 2. Suppose Assumptions 1-3 hold. Given Definitions 2 and 3, (A.14) holds in any monetary steady state. If $M_0 > 0$ and $\rho\gamma \leq 1$, then there exists a monetary steady state such that

$$f'(x_{P,t}) = \frac{1}{\gamma} \tag{A.34}$$

$$q_t = p_{2,t} = p_{1,t} = \frac{1 - x_{P,t} - x_{S,t}}{M_t} \tag{A.35}$$

$$x_{S,t} = \begin{cases} 0 & \text{if } \frac{1}{\gamma} > \rho \\ \text{any real number in } [0,\ 1 - x_{P,t}) & \text{if } \frac{1}{\gamma} = \rho \end{cases} \tag{A.36}$$

Alternatively, if $M_0 = 0$, then there exists a monetary steady state such that

$$f'(x_{P,t}) = \frac{p_{1,t+1}}{q_t} = \rho \tag{A.37}$$

$$x_{S,t} = 1 - x_{P,t} > 0 \tag{A.38}$$

where $p_{2,t}$ can be any real number in $[q_t,\ p_{1,t}]$ satisfying $p_{2,t+1}/p_{2,t} \leq \rho$, given (12); and the values of q_t, $b_{P,t}$, $b_{S,t}$, and $m'_{S,t+1}$ can be any set of positive real numbers satisfying (A.14). There is no other monetary steady state.

Proof. In any monetary steady state, $q_t > 0$, $x_{P,t} = q_t b_{P,t}$, and (A.14), given (7), (9), (12), and the first constraint in (6). Also, $q_t > 0$ and the Inada condition satisfied by the function f imply (A.15) and $b_{P,t} > 0$ in a monetary equilibrium, as described in Appendix B.

Given $q_t > 0$ and $b_{P,t} = b_{S,t} > 0$ as implied by (9), the first-order conditions for $x_{S,t}$, $b_{S,t}$, $m_{S,t}$, and $m'_{S,t+1}$ in (5) without $m'_{S,t+1} \in [0,\ m_{S,t} + \tau_{S,t+1}]$ in the constraint set are (A.16)-(A.19) with $\bar{\lambda}_{m',\ell,t+1} = 0$. Also, (7) implies that $m'_{S,t+1} > 0$, given $b_{P,t} > 0$, and thus $\underline{\lambda}_{m',\ell,t+1} = 0$. Substituting these equalities into (A.16)-(A.19) yields

$$\underline{\lambda}_{x,\ell,t} = \eta_{1,t} - \rho \geq 0 \tag{A.39}$$

$$\eta_{1,t} = \frac{p_{1,t+1}}{q_t} \tag{A.40}$$

$$\underline{\lambda}_{m,\ell,t} = p_{1,t+1} \left(\frac{p_{2,t}}{q_t} - 1 \right) \geq 0 \tag{A.41}$$

$$\eta_{4,t} = p_{1,t+1} - p_{2,t+1} \tag{A.42}$$

If $b_{S,t} + m_{S,t} + \tau_{S,t+1} - m'_{S,t+1} = 0$ in a monetary equilibrium, then it contradicts (A.14) unless $m_{S,t} + \tau_{S,t+1} = 0$. Suppose $m_{S,t} + \tau_{S,t+1} > 0$. In this case, $b_{S,t} + m_{S,t} + \tau_{S,t+1} - m'_{S,t+1} > 0$, and thus $\eta_{4,t} = p_{1,t+1} - p_{2,t+1} = 0$; (12) holds because this case implies $M_0 > 0$ and $m_{S,t} = M_t > 0$; and because (8) implies $m_{S,t+1} \geq 0$, $\underline{\lambda}_{m,\ell,t} = p_{1,t+1}(p_{2,t}/q_t - 1) = 0$ in a monetary steady state in this case. Hence,

$$\underline{\lambda}_{x,\ell,t} = \eta_{1,t} - \rho = 1/\gamma - \rho \geq 0 \tag{A.43}$$

Also, (A.34) and (A.36) holds, whereas $x_{S,t}$ and $p_{2,t}$ can be any pair of non-negative real numbers satisfying $x_{S,t} + p_{2,t}M_t = 1 - x_{P,t}$ if $\frac{1}{\gamma} = \rho$.

Next, suppose that $m_{S,t} + \tau_{S,t+1} = 0$. In this case, (8) and (A.14) imply that $m_{S,t+1} = b_{S,t} + m_{S,t} + \tau_{S,t+1} - m'_{S,t+1} = 0$. Thus,

$$x_{S,t} = 1 - q_t b_{S,t} - p_{2,t} m_{S,t} = 1 - x_{P,t} \tag{A.44}$$

given (A.14). Therefore, if $x_{S,t} = 0$, then it violates (1). Hence, $x_{S,t} > 0$ and $\underline{\lambda}_{x,\ell,t} = \eta_{1,t} - \rho = 0$. Substituting this result and (A.40) into (A.15) yields (A.37). Also, (12), (A.41), and (A.42) imply

$$p_{1,t+1} \geq p_{2,t+1} \tag{A.45}$$

$$p_{2,t} \geq q_t \tag{A.46}$$

Therefore, it must be the case that

$$\rho = \frac{p_{1,t+1}}{q_t} \geq \frac{p_{2,t+1}}{q_t} = \frac{p_{2,t+1}}{p_{2,t}} \cdot \frac{p_{2,t}}{q_t} \geq \frac{p_{2,t+1}}{p_{2,t}} \tag{A.47}$$

where the first equality is implied by (A.37). □

D Price indeterminacy with a fixed intraday supply of fiat money

Price indeterminacy cannot be prevented even if the central bank fixes the amount of fiat money supplied through a discount window. To see this result, denote by B_t the fixed supply of fiat money per old supplier through a discount window in period t, given a zero intraday interest rate. Given $M_0 = 0$, the market clearing condition for the central bank's discount window lending is $m'_{S,t} = B_t$, where $m'_{S,t}$ is the amount of fiat money paid by an old supplier for goods sold by old producers for immediate money payments in period t. Given (A.14), the face value of nominal debt issued by a producer, $b_{P,t}$, equals B_{t+1} in the monetary steady state. This condition pins down the real discount price of nominal debt in the current period, q_t, given the equilibrium value of $x_{P,t}$ and $q_t = x_{P,t}/B_{t+1}$, as implied by (A.14). Given $b_{P,t-1}$, however, the real value of fiat money in terms of goods sold for immediate money payments in the current period, $p_{1,t}$, can deviate from the expectation in the previous period, $E_{t-1}p_{1,t} = \rho q_{t-1}$, which is implied by (A.37). This is because once investment in each producer's production in the previous period, $x_{P,t-1}$, takes place, $p_{1,t}$ needs to satisfy only $p_{2,t} \in [q_t,\ p_{1,t}]$ in the current period, as implied by Proposition 2.