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A Model of Bank-Note Runs

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Abstract

This paper presents a three-period model to endogenize the need for bank notes given the availability of trade credit. The model shows that banks can improve risk sharing in the economy by discounting commercial bills to issue bank notes, because bank notes can serve as payment instruments backed by a diversified pool of commercial bills issued by payers. This characteristic of bank notes, however, can cause a self-fulfilling mass refusal of bank notes by payees due to endogenous default on commercial bills. This result holds even if bank notes are not redeemable on demand before maturity. The model shows that a capital requirement is not sufficient for preventing a self-fulfilling mass refusal of bank notes, while a reserve requirement is.

JEL classification: E42, G21.

Keywords: Bank notes; Trade credit; Commercial bills; Bank run; Reserves; Payment system.

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1 Introduction

This paper analyzes the role of banks in the supply of payment instruments in the economy. Historically, trade credit was developed earlier than commercial banking. Then, commercial banks issued bank notes by discounting commercial bills until the central bank monopolized bank-note issues in the country.[1] The standard theory of money does not fully capture this feature of bank notes, as it explains the need for money by lack of credit between buyers and sellers.[2] Motivated by this observation, this paper presents a parsimonious model to analyze the essentiality of bank notes given the availability of trade credit.

The key feature of the model is the imperfect synchronization of transactions in different markets. In the model, if a producer buys an input from a supplier by issuing a commercial bill, then there occurs an inevitable time lag before the supplier can access a financial market to share idiosyncratic default risk in commercial bills with other suppliers. Because idiosyncratic default risk in commercial bills can be updated any time, the presence of the time lag makes risk sharing among suppliers imperfect. As a result, suppliers charge a risk premium on commercial bills paid by producers.

This problem can be remedied if there exist commercial banks issuing bank notes in exchange for commercial bills before producers make payments to suppliers. Because bank notes are backed by a diversified pool of commercial bills through commercial banks' balance sheets, an update on idiosyncratic default risk in commercial bills does not affect the return on bank notes, given the law of large numbers. Thus, suppliers do not charge a risk premium on bank notes paid by producers.

The model, however, also shows that this characteristic of bank notes is fragile, because if suppliers refuse to sell inputs for bank notes paid by producers, then producers cannot produce goods, and hence must default on commercial bills held by commercial banks. In

[1] For example, see Quinn (2004) for payments in Britain before the 19th century.

[2] See Samuelson (1958), Townsend (1980), Kiyotaki and Wright (1989), and Lagos and Wright (2005) for the examples of seminal papers in the literature.

this case, commercial banks must default on bank notes, justifying suppliers' refusals of bank notes.

This is a run on bank notes by payees. This result is related to Diamond and Dybvig's (1983) model of a run on demand deposits, in a sense that both types of runs stem from underlying reasons for the essentiality of banking. Nonetheless, a run on bank notes is different from a run on demand deposits, as it occurs due to endogenous default on bank assets, rather than the costly liquidation of bank assets. Thus, it can occur even if bank notes are not redeemable on demand. This result is related to the result of Jacklin's (1987) model, which shows that banks can avoid a self-fulfilling run on demand deposits by issuing shares instead of demand deposits.[3] This paper demonstrates that banks issuing payment instruments can still suffer a self-fulfilling run even if they do not issue demand deposits.

The model further shows that banks cannot avoid a self-fulfilling mass refusal of bank notes just by financing part of commercial bills by their own capital, because endogenous default on commercial bills can happen regardless of the liability structure of banks. Instead, banks must hold as reserves a sufficiently large amount of assets whose returns are independent of commercial bills. This result is consistent with the historical fact that state laws required note-issuing commercial banks to deposit state bonds at state governments during the free banking era in the U.S., and also the observation that occasional declines in the value of state bonds caused banking crises during the period (Rolnick and Weber 1984).[4]

With these results, this paper adds to the literature on the fragility of private bank notes. For example, Calomiris and Gorton (1991) highlight the role of asymmetric information in banking crises during the free banking era and the national banking era in the U.S. Also, Monnet and Sanches (2015) and Sanches (2016a) analyze the equilibrium supply of private bank notes with limited commitment by banks. They show that a perpetual decline in the

[3]See Ennis and Keister (2010) for a survey of other models on the fragility of demand deposits, including those ruling out a self-fulfilling run on demand deposits.

[4]Another cause of a banking crisis during the free banking era in the U.S. was a fraud called wildcat banking. See Dwyer (1996) for more details.

value of bank notes is possible due to a self-fulfilling expectation of a declining continuation value of bank-note issues for banks. This paper contributes to this literature by showing the fragility of bank notes in the absence of asymmetric information or limited commitment.

From a wider perspective, there is a search-theoretic literature on money backed by illiquid collateral, such as Shi (1996), Ferraris and Watanabe (2008), and Andolfatto et al. (2016). Also, Kiyotaki and Moore (2012) investigate the effect of money supply that replaces illiquid private securities in a business cycle model. In contrast to these papers, this paper analyzes the substitution between bank notes and trade credit as well as its fragility without assuming a search friction or a resellability constraint that prevents a transfer of private securities between agents.

In addition, Kobayashi and Nakajima (2014) analyze a self-fulfilling dry-up of short-term lending to banks due to endogenous default by borrowers, given a constraint that borrowers must obtain working capital from lenders through banks.[5] This paper differs from their work in analyzing the fragility of payment instruments rather than short-term funding, and showing that the fragility stems from the underlying reason for the essentiality of payment instruments.

The remainder of this paper is organized as follows. The baseline model is defined in section 2. Bank notes are introduced in section 3. A self-fulfilling mass refusal of bank notes is analyzed in section 4. Section 5 presents a sensitivity analysis. The effects of a capital requirement and a reserve requirement are compared in section 6. Section 7 incorporates a maturity transformation by banks. Section 8 concludes this paper.

[5]They assume that households' lending to firms must be intermediated by banks. They also assume that firms need to obtain from banks long-term loans to install capital stock, and then short-term loans to finance their use of factors of production, i.e., working capital. As a result, no provision of short-term lending to banks results in firms' default on long-term loans due to lack of working capital.

2 Baseline model

Time is discrete and indexed by 0, 1, and 2. There exist two types of agents, producers and suppliers, each of which are a unit continuum. Each agent is indexed by $(i, j) \in \{P, S\} \times [0, 1]$, where $i = P$ if the agent is a producer and $i = S$ if the agent is a supplier, and j is the subindex among agents of the same type. For each type, the measure of agents is defined by the Lebesgue measure of the agents' subindices on $[0, 1]$. A supplier is risk-averse, maximizing expected utility, $Eu(c_{S,j})$, where E is the expectation operator in period 0; u is a strictly increasing, concave, and twice differentiable function; and $c_{S,j}$ denotes supplier j's (i.e., agent (S, j)'s) consumption of goods in period 2 for $j \in [0, 1]$. A producer is risk-neutral, maximizing the expected value of consumption of goods in period 2.[6]

Each supplier is endowed with a unit of inputs in period 0, and can produce an amount $\underline{\alpha}$ of goods in period 2 by using a unit of inputs. In contrast to suppliers, each producer receives no endowment of inputs, but can produce an amount $\bar{\alpha}$ of goods with probability $\mu_{P,j}$, and no goods otherwise, in period 2 from each unit of inputs used in period 0. The value of $\mu_{P,j}$ is specific to producer j (i.e., agent (P, j)) for $j \in [0, 1]$. For all $j \in [0, 1]$, $\mu_{P,j}$ is an i.i.d. random variable whose value is revealed publicly in period 1, and the probability distribution of $\mu_{P,j}$ is uniform over $[0, 1]$. Assume that

$$\bar{\alpha} > 4\underline{\alpha} > 0 \tag{1}$$

This assumption ensures that it is socially optimal for producers to use suppliers' inputs for their production in any case considered below.

In period 0, each producer is randomly matched with a supplier, and vice versa. Thus, every agent has one match. In each match, a producer can make a take-it-or-leave-it offer to buy inputs from a supplier with an IOU that promises to deliver goods in period 2. A producer can be committed to delivering any share of goods produced in period 2 upon

[6] This assumption simplifies the model by obviating any need for risk sharing among producers.

successful production, and involuntarily defaults on its IOU if it fails to produce goods in the period. Hereafter, a producer's IOU is referred to as a "commercial bill".

In period 1, there exists a bill market, in which suppliers can swap commercial bills received from producers for bill dealers' IOUs. There are N symmetric bill dealers in the market, where N is an integer not less than 2. Bill dealers offer suppliers an exchange rate between their IOUs and commercial bills through Bertrand competition.

Assume that suppliers can arrive at the bill market only after the realization of $\mu_{P,j}$ for $j \in [0,1]$ in period 1. The underlying assumption is that agents cannot synchronize different market transactions perfectly. Thus, there exists an inevitable time lag between a goods market transaction and a bill market transaction. Given this time lag, it is natural to assume that there can be a revision of the idiosyncratic default probability for each commercial bill any time while the holders of commercial bills transit from the goods market to the bill market. The realization of $\mu_{P,j}$ for $j \in [0,1]$ in period 1 is a stylized assumption to incorporate such a continuous update on idiosyncratic default risk in a discrete-time set-up.

An equilibrium is characterized by a zero profit condition for each bill dealer in the bill market in period 1 due to Bertrand competition; a take-it-or-leave-it-offer of a commercial bill by a producer to a supplier in each pairwise meeting in period 0; and rational expectations held by all agents, including bill dealers. See Table 1 for a summary of the model.

Table 1: Chronological order of events in the baseline model

Period 0	Each producer can make a take-it-or-leave-it offer of its commercial bill for a supplier's inputs in a pairwise meeting.
Period 1	The idiosyncratic default probability for each commercial bill is revealed publicly. Suppliers can swap commercial bills for bill dealers' IOUs in a bill market with Bertrand competition among bill dealers.
Period 2	Producers repay their commercial bills upon successful production of goods. Bill dealers repay their IOUs by the repayments on commercial bills they hold.

2.1 Equilibrium

The zero profit condition for bill dealers implies that each commercial bill is priced fairly in the bill market in period 1:

$$p_{P,j} = \mu_{P,j} b_{TC} \tag{2}$$

for $j \in [0, 1]$, where $p_{P,j}$ denotes the face value of a bill dealer's IOU that is exchanged for producer j's commercial bill in period 1, given the realized value of $\mu_{P,j}$; and b_{TC} denotes the face value of a commercial bill issued by each producer to a supplier in a pairwise meeting in period 0.[7] Both face values, $p_{P,j}$ and b_{TC}, are in terms of the amounts of goods repayable in period 2. The value of b_{TC} is the same across producers, given the symmetry among producers in period 0. There is no risk premium in $p_{P,j}$, as each bill dealer can diversify idiosyncratic default risk by holding commercial bills issued by a positive measure of producers.

Given (2), each producer chooses the value of b_{TC} that just satisfies a supplier's participation constraint for selling inputs in a pairwise meeting in period 0:

$$\int_0^1 u(\mu b_{TC})\, d\mu = u(\underline{\alpha}) \tag{3}$$

where μ denotes the realized success probability for a producer's production in period 2. The left-hand side and the right-hand side of (3) are a supplier's expected utilities when selling inputs for a commercial bill and when retaining inputs for the supplier's own production, respectively.

In the equilibrium, each producer can buy inputs from a supplier by a commercial bill, if and only if the amount of goods that a producer can produce upon successful production, $\bar{\alpha}$, is not less than the value of b_{TC} that satisfies (3). Otherwise, suppliers retain inputs for their own production.

[7] The subscript "TC" stands for "trade credit".

2.2 Interpretation of the baseline model

The baseline model can be interpreted as an economy relying on trade credit without a banking service. This interpretation is consistent with the fact that trade credit from the wholesalers of inputs to producers had become common in various industries in England by the late 17th century, preceding the development of commercial banking in the country.[8] In addition, the goods market is assumed to be decentralized, which makes it impossible for a supplier to diversify idiosyncratic default risk in commercial bills by selling inputs to a continuum of producers simultaneously. This assumption can be interpreted as the need for each producer to use a specific supplier's input. Only the modern feature of the baseline model is the presence of bill dealers for asset pooling, which is the function performed by mutual funds today. In the next section, let us analyze what if bill dealers issue payment instruments to become commercial banks.

3 Introducing bank notes

In the baseline model, (3) implies that the face value of a commercial bill that a producer must issue to buy inputs, b_{TC}, involves a risk premium, because a supplier faces uncertainty over the resale price of a commercial bill in period 1, $p_{P,j}$ for $j \in [0, 1]$, when selling inputs for a commercial bill in period 0. This incomplete risk sharing due to a time lag between a goods market transaction and a bill market transaction can be remedied if bill dealers can pool commercial bills in advance, so that producers can pay suppliers bill dealers' IOUs backed by a diversified pool of commercial bills. Hereafter, such bill dealers are referred to as "banks", and their IOUs are referred to as "bank notes", given their use as payment instruments. See Table 2 for the classification of IOUs when banks are introduced into the model.

[8]For example, producers in the 18th-century West Riding textile industry often got wool from wholesalers on credit. Big wholesalers in other industries like linen, iron, and groceries were also major sources of credit. See Quinn (2004) for more details.

Table 2: Classification of IOUs in the model with banks

Issuers	Name of IOUs
Producers	Commercial bills
Banks	Bank notes

3.1 Introducing note-issuing banks

Suppose that there exist N symmetric banks, where N is an integer not less than 2. At the beginning of period 0, producers and banks can swap commercial bills and bank notes with each other in a competitive market, where banks offer an exchange rate between commercial bills and bank notes through Bertrand competition. This market opens before producers and suppliers are matched pairwise in period 0. The subsequent events are as same as in the baseline model. The definition of an equilibrium is also as same as in the baseline model, except that it includes a zero profit condition for each bank due to Bertrand competition. See Table 3 for a summary of events in the model with banks. The bill market in period 1 is omitted in Table 3 and thereafter, as it does not play any role in the presence of banks.

Table 3: Chronological order of events in the model with banks

Period 0	Producers can swap their commercial bills for bank notes in a competitive market with Bertrand competition among banks. Each producer can make a take-it-or-leave-it offer of its commercial bills and bank notes for a supplier's inputs in a pairwise meeting.
Period 1	The idiosyncratic default probability for each commercial bill is revealed publicly.
Period 2	Producers repay commercial bills upon successful production of goods. Banks repay bank notes by the repayments on commercial bills they hold.

3.2 Equilibrium with bank notes

In an equilibrium, the zero profit condition for each bank implies that commercial bills are exchanged for bank notes at a fair price in period 0:

$$q = E[\mu_i b_{BK}] = \frac{b_{BK}}{2} \tag{4}$$

where b_{BK} denotes the face value of a commercial bill issued by each producer to a bank; and q denotes the face value of bank notes that are exchanged for each commercial bill.[9] Both face values, b_{BK} and q, are in terms of the amount of goods repayable in period 2. As is the case with bill dealers in the baseline model, there is no risk premium in q because each bank can diversify idiosyncratic default risk by holding commercial bills issued by a positive measure of producers.

Given (4), each producer chooses the value of b_{BK} that just satisfies a supplier's participation constraint for selling inputs in a pairwise meeting in period 0:

$$u\left(\frac{b_{BK}}{2}\right) = u(\underline{\alpha}) \tag{5}$$

The left-hand side and the right-hand side of (5) are a supplier's expected utilities when selling inputs for bank notes paid by a producer and when retaining inputs, respectively. On the left-hand side, note that the face value of bank notes, $b_{BK}/2$, is safe regardless of the realized distribution of $\mu_{P,j}$ over $j \in [0,1]$ in period 1, because the distribution of $\mu_{P,j}$ does not affect the total return on bank notes by the law of large numbers. Thus, $b_{BK} = 2\underline{\alpha}$, given $u' > 0$. Because $\bar{\alpha} > b_{BK}$ as implied by (1), it is feasible for each producer to issue a commercial bill whose face value equals b_{BK}.

Given $u'' < 0$, applying Jensen's inequality to (3) and (5) implies that

$$b_{BK} < b_{TC} \tag{6}$$

This difference is due to a risk premium on an individual commercial bill paid to a supplier. Therefore, producers have incentive to swap their commercial bills for bank notes in period 0

[9] The subscript "BK" stands for "bank".

to avoid incurring a risk premium.[10] Thus, there exists an equilibrium in which banks issue bank notes to producers in period 0.

Note that producers receive the entire social surplus, as they keep suppliers indifferent by making take-it-or-leave-it offers for inputs, while bill dealers and banks earn no profit in an equilibrium. Hence, a higher expected profit for each producer implies a Pareto improvement.

3.3 Discussion

To clarify, the result of the model does not change even if there is an update on the idiosyncratic default probability for each commercial bill, $1 - \mu_{P,j}$ for $j \in [0, 1]$, during a time lag between the supply of bank notes and goods market transactions in period 0. This is because such an update has no effect on the return on a pool of commercial bills held by each bank, or the return on bank notes, given the law of large numbers. Thus, the model is robust to any timing of an update on idiosyncratic default risk in commercial bills after banks issue bank notes at the beginning of period 0.[11]

The exchange between commercial bills and bank notes in period 0 can be interpreted as banks' lending of bank notes through discounting commercial bills. This interpretation is consistent with the historical fact that banks issued bank notes mainly by discounting promissory notes or bills presented by customers during the free banking era in the U.S.[12]

In the following, let us analyze the fragility of note-issuing banks. Without loss of generality, the remainder of this paper focuses on an equilibrium in which N banks are symmetric

[10] Note that banks can diversify idiosyncratic default risk in commercial bills, even if they issue bank notes to part of producers in period 0, and then buy commercial bills issued by the rest of producers to suppliers in the bill market in period 1. Thus, suppliers receiving commercial bills from the rest of producers in period 0 can sell each commercial bill to a bank at a fair price in the bill market in period 1, as shown in (2). Therefore, the minimum face value of a commercial bill that a producer must issue to buy inputs from a supplier in period 0 remains b_{TC}, as shown in (3).

[11] It is possible to assume that suppliers cannot observe an update on the idiosyncratic default probability for each commercial bill held by banks. Even in such a case, the result of the model does not change if each bank can be committed to not unwinding the pool of commercial bills acquired at the beginning of period 0, so that adverse selection does not occur. See Dang et al. (2017) and Tomura (2014) for further analysis of this type of banking service.

[12] See Weber (2015a). Even today, banks create bank deposits through loan provisions and bond purchases. See McLeay, Radia, and Thomas (2014) for more details.

in each period; thus, each bank deals with one-Nth of each type of other agent, i.e., producers and suppliers.

4 Self-fulfilling mass refusal of bank notes due to endogenous default on commercial bills

There exists another equilibrium with a self-fulfilling mass refusal of bank notes by suppliers. To confirm this result, suppose that producers swap their commercial bills for bank notes in period 0, and then a supplier expects the other suppliers' refusals to sell inputs to producers in pairwise meetings in the period.

If this expectation is correct, then the producers meeting the other suppliers must default on their commercial bills in period 2, because they cannot produce any goods without inputs. Following the standard bankruptcy rule in reality, assume a pro rata distribution of default losses to the holders of bank notes at each bank.

Assumption 1. *If the returns on assets held by a bank in period 2 are less than the total face value of liabilities issued by the bank, then the returns are distributed to the creditors of the bank on a pro rata basis in period 2.*

Given this assumption, if the supplier with the aforementioned expectation sells inputs to the producer that it meets in period 0, then it expects that only this producer can repay commercial bills held by banks upon successful production in period 2. In this case, Assumption 1 implies that the producer's repayment to a bank is allocated equally among the holders of bank notes issued by the bank. At each bank, there are a measure $1/N$ of producers holding refused bank notes, given the symmetry among N banks. The measure of bank-note holders remains the same even if the supplier with the aforementioned expectation accepts bank notes paid by the producer, because the supplier just replaces the producer as a bank-note holder. Because the measure of a producer, and thus a producer's repayment to a bank, is zero while the measure of bank-note holders is positive at each bank, the recovery value of bank notes per holder is zero at any bank. Hence, the supplier refuses to sell inputs for bank notes paid by the producer.

The supplier still sells inputs to the producer if it can receive an additionally issued commercial bill that satisfies its participation constraint for selling inputs in period 0:

$$\int_0^1 \mu u(\hat{b}) + (1-\mu)u(0)\, d\mu = u(\underline{\alpha}) \tag{7}$$

where μ denotes the realized success probability for the producer's production in period 2, and $\hat{b}$ denotes the minimum face value of an additional commercial bill that the producer must issue to buy inputs from the supplier. The left-hand side and the right-hand side of (7) are the values of the supplier's expected utilities when selling inputs and when retaining inputs, respectively. The left-hand side reflects the supplier's expectation that if the supplier sells inputs for an additional commercial bill issued by the producer, then it cannot share idiosyncratic default risk in the commercial bill with the other suppliers, because the other suppliers do not hold any commercial bills, given their refusals to sell inputs to producers. Thus, the supplier expects to receive the direct repayment of the commercial bill, $\hat{b}$, upon successful production, and no repayment otherwise, as indicated by (7).[13]

To issue an additional commercial bill whose face value equals $\hat{b}$, the producer must be able to repay all the commercial bills it owes upon successful production in period 2.[14] Thus, the producer can issue an additional commercial bill whose face value equals $\hat{b}$ if and only if

$$\bar{\alpha} \geq b_{BK} + \hat{b} \tag{8}$$

where b_{BK} is the face value of a commercial bill that the producer issues to a bank in period 0, and $\bar{\alpha}$ is the amount of goods that the producer can produce upon successful production in period 2.

[13]The left-hand side of (7) integrates the supplier's expected utility over the possible realizations of the idiosyncratic probability of successful production for the producer, which is denoted by μ. If default occurs, then the supplier receives no goods, as indicated by $u(0)$ on the left-hand side of (7).

[14]Assume that law prohibits a borrower from issuing debt to a new lender in order to be bankrupt and reduce incumbent lenders' shares of the borrower's bankruptcy estate. It is also assumed that the bankruptcy process is so slow that a producer cannot reorganize commercial bills issued to banks before issuing an additional commercial bill to a supplier.

Here, refused bank notes held by the producer do not offset the commercial bill that the producer issues to a bank in period 0, because the recovery value of bank notes is zero. This assumption will be relaxed in the next section by allowing producers to return refused bank notes to banks as the early repayments of commercial bills.

Because the difference between the left-hand side and the right-hand side of (8) equals the producer's profit upon successful production, while the producer's profit is zero if production fails, it is incentive-compatible for the producer to issue an additional commercial bill if (8) is satisfied. In this case, a mass refusal of bank notes by suppliers cannot be a self-fulfilling rational expectation, as each supplier sells inputs to a producer even if the other suppliers do not.

If (8) is violated, then a mass refusal of bank notes by suppliers can be a self-fulfilling rational expectation, because each supplier follows suit if the other suppliers do not sell inputs to producers. In the equilibrium with this rational expectation, banks do not issue bank notes in period 0. Thus, the equilibrium is as same as in the baseline model.

4.1 Implication of the model

Note that bank notes in the model are not redeemable into goods before the maturity in period 2. Thus, the self-fulfilling collapse of note-issuing banks described above does not require bank notes to be demand notes.

This result can be related to the fact that banking crises often occurred among note-issuing banks in Britain and the U.S. in the 19th century. Given that bank notes were usually convertible into specie on demand during that period, one of the standard responses to a crisis was the suspension of payments.[15] As banking crises were recurrent in the 19th century, however, this observation raises a question as to whether bank-note holders rationally expected the suspension of payments conditional on the occurrence of a crisis. Such an expectation would have prevented a banking crisis if the crisis had been a run on demand

[15] See Dwyer (1996) and Quinn (2004) for more details.

notes due to the costly liquidation of bank assets, as implied by Diamond and Dybvig's (1983) model. A possible explanation for this conundrum is that bank-note holders were rational, but expected a delay in the suspension of payments due to a slow information transmission at that time. The result described above offers an alternative explanation, as it implies that a self-fulfilling collapse of note-issuing banks can occur even if bank notes are not demand notes.

5 Sensitivity analysis

5.1 Allowing the early repayment of refused bank notes

In the previous section, producers are not allowed to retire commercial bills before maturity by returning bank notes refused by suppliers to banks in period 0. Because there is no gain for a producer and a bank to maintain cross liabilities between them until period 2, hereafter assume that a producer can cancel the exchange between its commercial bill and bank notes if its payment of bank notes is refused by a supplier.

Assumption 2. *A producer can cancel an exchange between its commercial bill and bank notes issued by a bank in a period by returning the bank notes to the bank in the same period.*

This assumption can be interpreted as allowing an early repayment of a commercial bill before maturity.[16] It is also a conservative assumption for the analysis of fragility of banks, as it minimizes the size of each bank's balance sheet.

It can be shown that a self-fulfilling mass refusal of bank notes can occur even with Assumption 2. In this case, banks do not default on bank notes, as producers return all refused bank notes to banks to retire commercial bills. Each supplier still refuses bank notes paid by a producer, as bank notes are not backed by a diversified pool of commercial bills if the other suppliers refuse to sell inputs to producers. Thus, a self-fulfilling mass refusal

[16] The difference between the face value of bank notes lent by a bank, $b_{BK}/2$, and the face value of commercial bills issued by a producer, b_{BK}, can be interpreted as interest payable at maturity. A producer does not have to incur this interest expense if it returns borrowed bank notes to a bank before maturity.

of bank notes occurs due to an endogenous disappearance of risk diversification, rather than endogenous default on commercial bills. See Appendix A for more details.

Assumption 2, however, does not preclude endogenous default on commercial bills if there are sunk costs of production. In the following, let us extend the model to confirm this result.

5.2 Introducing sunk costs of production

Consider the following modification of the model. There are two sets of suppliers, early suppliers and late suppliers, each of which are a $[0, 1]$ continuum. Early suppliers and late suppliers are endowed with inputs in periods 0 and 1, respectively. Both types of suppliers can produce an amount $\underline{\alpha}$ of goods in period 2 by using a unit of inputs, as in the baseline model. They also maximize the same concave utility function as a supplier in the baseline model.

In contrast, a producer needs to use not only a unit of inputs in period 0, but also a unit of inputs in period 1, to complete its production. Once used, inputs cannot be retrieved for any alternative use. The other features of a producer's production technology, i.e., the probability of successful production, and the amount of goods produced in period 2 upon successful production, remain the same as in the baseline model.

The markets in periods 0 and 1 are organized in the same way as described in section 3.1: producers can swap commercial bills for bank notes in a competitive market at the beginning of each of periods 0 and 1, whereby banks offer an exchange rate between commercial bills and bank notes through Bertrand competition; and producers are matched pairwise with early suppliers and late suppliers in periods 0 and 1, respectively.[17] Assume that the value of $\mu_{P,j}$ for $j \in [0, 1]$ is realized after the pairwise meetings in period 1. This assumption represents a continuous update on idiosyncratic default risk in commercial bills, as described in section 2. The other part of the model remains the same as assumed in section 3.1. See

[17]For this set-up, assume that late suppliers cannot be committed to forward contracts arranged with producers in period 0. Renegotiation-proof forward contracts would be equivalent to requiring producers to buy inputs in pairwise meetings in period 1 when late suppliers are endowed with inputs.

Table 4 for a summary of the model.

Table 4: Chronological order of events in the model with sunk costs of production

Period 0	(∗) Producers can swap their commercial bills for bank notes in a competitive market with Bertrand competition among banks. (∗∗) Each producer can make a take-it-or-leave-it offer of its commercial bills and bank notes for an early supplier's inputs in a pairwise meeting.
Period 1	The same events as (∗) and (∗∗) in period 0 take place, except that producers are matched with late suppliers, rather than early suppliers. The idiosyncratic default probability for each commercial bill is revealed publicly.
Period 2	Producers repay commercial bills upon successful production of goods. Banks repay bank notes by the repayments on commercial bills they hold.

5.3 Equilibrium without a mass refusal of bank notes

There exists an equilibrium in which each producer repeats twice the same set of transactions described in section 3.2. In this equilibrium, each producer issues to a bank a commercial bill whose face value equals b_{BK} in exchange for bank notes whose face value equals $b_{BK}/2$ in each of periods 0 and 1. Then, each producer pays the bank notes to buy inputs from an early supplier and a late supplier in periods 0 and 1, respectively. These transactions satisfy the zero profit condition for banks as well as participation constraints for an early supplier and a late supplier to sell their inputs, as implied by (4) and (5). Also, (1) ensures that each producer can repay the two commercial bills issued in periods 0 and 1, the face values of which sum up to $2b_{BK}$, or $4\underline{\alpha}$ as implied by (5), upon successful production in period 2. Table 5 summarizes the evolution of each bank's balance sheet in periods 0 and 1 in this equilibrium.

Table 5: Evolution of each bank's balance sheet with sunk costs of production in case of no mass refusal of bank notes

	Events	Each bank's balance sheet	
Period 0	Producers and banks swap IOUs (commercial bills and bank notes) with each other.	Commercial bills issued in period 0	Bank notes supplied to producers
	Producers pay bank notes for early suppliers' inputs in pairwise meetings.	Commercial bills issued in period 0	Bank notes paid to early suppliers in period 0
Period 1	Producers and banks swap IOUs with each other.	Commercial bills issued in period 0 Commercial bills issued in period 1	Bank notes paid to early suppliers in period 0 Bank notes supplied to producers
	Producers pay bank notes for late suppliers' inputs in pairwise meetings.	Commercial bills issued in period 0 Commercial bills issued in period 1	Bank notes paid to early suppliers in period 0 Bank notes paid to late suppliers in period 1

5.4 Self-fulfilling mass refusal of bank notes with sunk costs of production

Given the same actions as described in Table 5 before pairwise meetings between producers and late suppliers in period 1, suppose that a late supplier expects the other late suppliers' refusals to sell inputs to producers in those meetings. If this expectation is correct, then the producers meeting the other late suppliers in period 1 return refused bank notes to banks to retire their commercial bills issued in the period, given Assumption 2. These producers, however, must default on commercial bills issued in period 0, because they cannot produce any goods in period 2, or retrieve any inputs used in period 0, while bank notes issued in period 0 are already paid to early suppliers.

Given this circumstance, consider the pairwise meeting between the late supplier with the aforementioned expectation and a producer in period 1. If the late supplier sells inputs to the producer, then it expects that only this producer can repay commercial bills held by banks upon successful production in period 2. In this case, the producer's repayment to a bank is shared equally among the holders of bank notes issued by the bank, given Assumption 1. At each bank, a measure $1/N$ of early suppliers hold bank notes received in period 0, given the symmetry among N banks. If the late supplier accepts bank notes paid by the producer in period 1, then it becomes an additional bank-note holder of a measure zero for the bank issuing the bank notes. Given the measure of the producer being zero, Assumption 1 implies that the recovery value of bank notes per holder is zero at each bank in any case.[18] Therefore, the late supplier refuses bank notes paid by the producer, and the producer returns refused bank notes to the bank issuing the bank notes in period 1, given Assumption 2. See Table 6 for each bank's balance sheet in period 1 in this case.

The participation constraint for the late supplier to sell inputs to the producer in period

[18]This result does not depend on whether a producer's commercial bills issued in periods 0 and 1 are held by two banks or one bank. In either case, if a late supplier sells inputs to a producer in period 1, then the producer's repayment of a commercial bill upon successful production in period 2 is shared with a measure $1/N$ of early suppliers at the bank holding the commercial bill.

Table 6: Each bank's balance sheet in case of a self-fulfilling mass refusal of bank notes in period 1

Commercial bills issued in period 0 ($0 \cdot (1/N)$)	Bank notes paid to early suppliers in period 0 ($b_{BK}/2 \cdot (1/N)$)
Commercial bills issued in period 1 (canceled)	Bank notes supplied to producers in period 1 (canceled)

Notes: The table shows each bank's balance sheet after producers repay refused bank notes to banks in period 1 to retire their commercial bills issued to the banks in the same period. In parentheses are the expected returns on commercial bills if they are on the asset side, and the face values of bank notes if they are on the liability side. The first term of each product in parentheses is the value of securities (i.e., the expected return or the face value) per agent, and the second term is the measure of the agents included in the item. "canceled" in a parenthesis indicates that the balance-sheet item is canceled within period 1.

1 is the same as (7), as it receives no payment of bank notes. The necessary and sufficient condition for no self-fulfilling mass refusal of bank notes in period 1 also remains the same as (8), because the producer must repay both a commercial bill issued to a bank in period 0 and an additional commercial bill paid to the late supplier in period 1. Thus, it remains the same as in section 4 that a mass refusal of bank notes can be a self-fulfilling rational expectation if and only if (8) is violated. This result is due to endogenous default on commercial bills issued in period 0.

6 Comparison between a capital requirement and a reserve requirement

Given the existence of sunk costs of production, let us introduce bank capital into the model to analyze a measure to prevent a self-fulfilling mass refusal of bank notes. It will be shown that a reserve requirement is a effective measure, while a capital requirement is not.

6.1 Introducing bank capital

Suppose that each bank is endowed with an amount g (> 0) of goods in period 0, which can be interpreted as shareholders' capital. Banks can store the goods until period 2 without depreciation. The storage technology can be interpreted as a safe asset that can be held as reserves.

Alternatively, producers can use banks' goods as inputs for their production in period 0. Accordingly, modify each producer's production technology as follows:

$$y_{P,j,2} = \bar{\alpha} x_{P,j,1}^{\sigma} x_{P,j,0}^{1-\sigma} \tag{9}$$

where $\sigma \in (0,1)$; (P,j) is the index of each producer, as assumed in the baseline model; $y_{P,j,2}$ denotes the output of goods in period 2; and $x_{P,j,t}$ denotes the amount of inputs used in period t for $t = 0, 1$. This function nests the production technology assumed in section 5.2, as each producer can still produce an amount $\bar{\alpha}$ of goods by using a unit of inputs in each of periods 0 and 1. It also allows producers to increase the amount of inputs in period 0 by using banks' goods. As is the case with inputs supplied by early suppliers, banks' goods are not retrievable once they are used for producers' production.[19]

Assume that banks can offer producers an exchange rate between their goods and commercial bills through Bertrand competition, along with an exchange rate between bank notes and commercial bills in period 0. The other parts of the model are as same as assumed in section 5.2.

6.2 Insufficiency of a capital requirement

Suppose that banks transfer all of their goods to producers in exchange for commercial bills in period 0, in addition to supplying bank notes as described in section 5.3. In this case,

[19] To make it profitable for a producer to acquire goods from a bank, assume $\bar{\alpha}(1+g)^{(1-\sigma)} - \bar{\alpha} \geq 2g$. The left-hand side is an inclement of output upon successful production if a producer uses goods supplied by banks, whereas the right-hand side is the required increase in the face value of commercial bills issued by the producer, given the mean of the idiosyncratic probability of a producer's successful production is 0.5, as assumed in the baseline model. Here, note that g is the maximum amount of goods that a bank can supply per producer, given the measure of producers being one. Also, the expected gain from an increase in the face value of commercial bills for a bank is $0.5 \cdot 2g$, which equals g. Banks are only concerned with the expected gain from each commercial bill, as they can diversify idiosyncratic default risk by offering goods to a positive measure of producers. Because of the strict concavity of $(1+g)^{(1-\sigma)}$ with respect to g given $\sigma \in (0,1)$, the marginal profit for a bank is positive if the bank supplies a smaller amount of goods than g per producer.

banks hold only commercial bills in period 2. Because all commercial bills are defaulted if late suppliers refuse to sell inputs to producers in period 0, it remains the same that a mass refusal of bank notes is a self-fulfilling rational expectation if and only if (8) is violated. This result implies that even if banks are required to finance part of their assets by their own capital, it does not necessarily prevent a self-fulfilling mass refusal of bank notes.

6.3 Effectiveness of a reserve requirement

Next, suppose that banks store their goods until period 2. Then, given the same actions as described in Table 5 before pairwise meetings between producers and late suppliers in period 1, suppose that a late supplier expects the other late suppliers' refusals to sell inputs to producers in those meetings. If this expectation is correct, then the producers meeting the other late suppliers in period 1 must default on their commercial bills issued in period 0, and return refused bank notes to banks to retire commercial bills issued in period 1, as described in section 5.4.

Now consider the pairwise meeting between the late supplier with the aforementioned expectation and a producer in period 1. As described in section 5.4, the recovery value of bank notes per holder would be zero if banks had no goods in period 2. In the current case, however, the goods held by each bank in period 2, g, are distributed equally among the holders of bank notes if banks default. Given the symmetry among N banks, each bank has a measure $1/N$ of agents as bank-note holders, as described in section 5.4. Thus, given Assumption 1, the late supplier expects that the recovery value of bank notes paid by the producer is $g/(1/N)$, or gN, unless gN is greater than the face value of the bank notes, $b_{BK}/2$.

Given this circumstance, the producer has two options: to pay bank notes to the late supplier; or to return bank notes to a bank to retire its commercial bill issued in period 1. If the producer chooses the latter option, then the late supplier's participation constraint for selling inputs remains (7), because the late supplier receives no bank note in this case, as in the case described in section 5.4. As a result, the total face value of commercial bills issued by the producer is $b_{BK} + \hat{b}$ in this case, as implied by (8).

Instead, suppose that the producer chooses the former option, given $gN < b_{BK}/2$, which is equivalent to $gN < \underline{\alpha}$ as implied by (5). In this case, the participation constraint for the late supplier to sell inputs to the producer changes from (7) to

$$\int_0^1 \mu u(gN + \hat{b}') + (1 - \mu) u(gN) \, d\mu = u(\underline{\alpha}) \tag{10}$$

where μ is the realized success probability for the producer's production in period 2, and $\hat{b}'$ denotes the minimum face value of an additional commercial bill that the producer must issue to buy inputs from the late supplier in period 1. In contrast to (7), the recovery value of bank notes, gN, enters into the late supplier's utility functions on the left-hand side of (10) as a safe return. The feasibility condition for the producer to issue an additional commercial bill that satisfies (10) is

$$\bar{\alpha} \geq 2b_{BK} + \hat{b}' \tag{11}$$

where $2b_{BK}$ equals the total face value of two commercial bills that the producer issues to banks in periods 0 and 1.

There exists a threshold, $\bar{g} \in (0, \underline{\alpha}/N)$, such that $2b_{BK} + \hat{b}' < b_{BK} + \hat{b}$ if and only if $g > \bar{g}$.[20] Thus, to minimize the total face value of commercial bills, the producer pays bank notes to the late supplier in period 1 if $g > \bar{g}$, and returns bank notes to a bank in the period otherwise. The condition for no self-fulfilling mass refusal of bank notes remains (8) in the latter case, whereas it becomes (11) in the former case. Hence, banks can shrink the parameter space in which a self-fulfilling mass refusal of bank notes can occur, if they store goods until period 2.

[20]Note that $\hat{b}'$ equals $\hat{b}$ if $gN = 0$, and that $\hat{b}'$ is decreasing in gN, and converges to 0 as gN approaches to $\underline{\alpha}$ from below, as implied by (10). Also, $u(\hat{b}) - u(0) = 2(u(\underline{\alpha}) - u(0))$, as implied by (7). Given $\underline{\alpha} > 0$ and $u'' < 0$, this equality implies $\hat{b} > 2\underline{\alpha}$. Because $2\underline{\alpha} = b_{BK}$ as implied by (5), the sign of $b_{BK} + \hat{b} - (2b_{BK} + \hat{b}')$ is the same as the sign of g minus the threshold.

The key to this result is that the return on goods held as reserves is independent of the returns on commercial bills, rather than being safe. Even if the return on the storage of goods were random, it would prevent the stochastic distribution of the recovery value of bank notes from being degenerated to zero in case of endogenous default on commercial bills. This effect of reserves reduces each late supplier's incentive to refuse bank notes in period 1, and thus shrinks the parameter space in which a self-fulfilling mass refusal of bank notes can happen.

Also, if banks could exchange their goods for commercial bills issued by a positive measure of producers in both periods 0 and 1, then these producers would be able to repay their commercial bills upon successful production in period 2, regardless of late suppliers' refusals of bank notes in period 1. These actions by banks, however, are equivalent to holding safe reserves, as banks can expect a safe return on these producers' commercial bills by the law of large numbers. Thus, the storage of goods described in this section represents a general reserve asset whose return is immune from endogenous default on commercial bills.

This result is consistent with the historical fact that note-issuing banks were required to acquire state or federal government bonds by shareholders' capital, and then deposit the bonds at the corresponding government as reserves during the free banking era and the national banking era in the U.S.[21] The model implies that such a regulation helps to prevent the collapse of a note-issuing bank due to a self-fulfilling mass refusal of bank notes. This result is also consistent with the observation that occasional declines in the value of state bonds caused banking crises during the free banking era in the U.S.[22]

[21] See Weber (2015a, b) and Sanches (2016b) for more details.

[22] See Rolnick and Weber (1984).

7 Incorporating a maturity mismatch between bank liabilities and bank assets

While the maturities of commercial bills and bank notes coincide in the model, a large proportion of bank liabilities have shorter maturities than bank assets in reality. It is possible to incorporate such a maturity mismatch into the model without changing the main results of the model.

Suppose there are another type of agents called sellers in period 1 that are endowed with an arbitrarily large amount of goods in the period, and are indifferent to whether to consume goods in period 1 or 2. Also suppose that early suppliers deposit bank notes received in period 0 at banks to acquire demand deposits, and consume goods in period 1 rather than in period 2. In this case, if there is a market for exchanges between goods and bank notes in period 1, then early suppliers can buy goods from sellers by withdrawing bank notes from banks and paying them to sellers.[23] Thus, there is no other change than a change in the identities of bank-note holders in period 1.

On this result, note that the value of commercial bills held by banks is not affected even if early suppliers fail to buy goods from sellers. Thus, each seller accepts the payment of bank notes from an early supplier even if the other sellers refuse to do so. Hence, there is no self-fulfilling mass refusal of bank notes by sellers.

This property of the model hinges on the assumption that bank notes are not redeemable into goods in period 1. If bank notes must be demand notes, then it is possible to consider an environment in which bank notes are subject to a self-fulfilling run due to the costly liquidation of bank assets, as shown by Diamond and Dybvig (1983). The results of the model does not require such a feature of bank liabilities.

[23]If the market is perfectly competitive, then early suppliers can buy the amount of goods worth the face value of bank notes they pay. If the market is bilateral and the result of each transaction is determined by Nash bargaining, then the amount of goods that early suppliers receive from sellers is less than the face value of bank notes they pay, unless early suppliers have all the bargaining power.

8 Conclusions

This paper presents a three-period model to analyze an endogenous need for bank notes given the availability of trade credit. The key environment in the model is the imperfect synchronization of transactions in different markets, which prevents the suppliers of inputs from perfectly diversifying idiosyncratic default risk in commercial bills paid by producers. To circumvent this friction, banks can issue bank notes to producers in exchange for commercial bills in advance, so that producers can pay suppliers IOUs that are already backed by a diversified pool of commercial bills. Bank notes, however, are subject to a self-fulfilling mass refusal by suppliers due to endogenous default on commercial bills. Banks cannot prevent a self-fulfilling mass refusal of bank notes just by financing part of commercial bills by their own capital. Instead, they need to hold a sufficiently large amount of independent assets as reserves.

This result has an implication for a recent increase in electronic retail payment instruments, or e-monies, issued by private institutions, as these instruments are akin to private bank notes. The model implies that private e-monies can be fragile even if they are not convertible into traditional currencies, or any other assets, on demand. The model also demonstrates that a reserve requirement is effective in addressing this fragility, while a capital requirement is not. To draw further implications from the model, it is necessary to incorporate more features of private e-monies at present. Addressing this issue is left for future research.

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Appendices

A Self-fulfilling mass refusal of bank notes due to an endogenous disappearance of risk diversification

Suppose Assumption 2 holds in the model described in section 3. Then, suppose that producers swap their commercial bills for bank notes in period 0, and that a supplier expects the other suppliers' refusals to sell inputs to producers in pairwise meetings in the period. Given Assumption 2, the supplier in turn expects that the producers meeting the other suppliers return bank notes to banks to retire their commercial bills. In this case, the producer that the supplier meets in period 0 becomes the sole holder of remaining bank notes, while banks hold only the commercial bill issued by the producer. Thus, bank notes issued by the producer are solely backed by the producer's commercial bill.

Because the repayment on the bank notes held by the producer never exceeds that on the commercial bill issued by the producer, it is optimal for the producer to retire the commercial bill by returning the bank notes to the bank issuing them. Thus, banks do not default, as all bank notes are returned to them. In this case, the participation constraint for the supplier to sell inputs to the producer remains the same as (7), because the producer pays an additionally issued commercial bill to the supplier without paying bank notes, as in section 4.

As the producer retires the commercial bill that it issues to a bank in period 0, it can issue an additional commercial bill whose face value equals $\hat{b}$ if and only if

$$\bar{\alpha} \geq \hat{b} \tag{A.1}$$

Applying Jensen's inequality to (3) and (7) implies

$$\hat{b} > b_{TC} > b_{BK} \tag{A.2}$$

given $u'' < 0$ and (6). Thus, the violation of (A.1) can be compatible with (1), or $\bar{\alpha} > b_{BK}$. If (A.1) is violated, then a mass refusal of bank notes can be a self-fulfilling rational expectation.