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International Risk Sharing with Heterogeneous Firms

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Abstract

Little is known about the consequence of firm heterogeneity and its resulting reallocation effect on international consumption risk sharing. This paper explores international risk sharing in a theoretical model with firm heterogeneity and shows that firm heterogeneity changes the nature of international risk sharing, thus driving a wedge between relative consumption growth and real exchange rate fluctuations. A correlation is found to be conditional on the fluctuations in the number of product varieties and their qualities arising from the reallocation effect induced by heterogeneous firms; the conventional unconditional correlation can be thus biased. Using world trade data covering more than two decades, I note the existence of bias and find that the extent of international risk sharing is underestimated. The analysis indicates a larger welfare gain from international trade than we have been measuring.

Keywords: *exchange rate, international risk sharing, product quality, firm heterogeneity*

JEL classification: *F12, F41, F43*

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Table 1: The Kollmann-Backus-Smith Correlations

Australia (AUS)	0.15	Finland (FIN)	-0.05	Luxembourg (LUX)	0.15
Austria (AUT)	0.19	France (FRA)	0.54	Netherlands (NLD)	0.38
Belgium (BEL)	0.70	United Kingdom (GBR)	0.09	Norway (NOR)	0.15
Canada (CAN)	0.21	Greece (GRC)	-0.01	New Zealand (NZL)	-0.13
Switzerland (CHE)	0.23	Hungary (HUN)	-0.13	Poland (POL)	-0.33
Chile (CHL)	-0.36	Ireland (IRL)	0.22	Portugal (PRT)	-0.12
Czech Republic (CZE)	0.36	Iceland (ISL)	-0.56	Slovakia (SVK)	-0.04
Germany (DEU)	0.09	Israel (ISR)	-0.30	Slovenia (SVN)	-0.13
Denmark (DNK)	0.26	Italy (ITA)	0.21	Sweden (SWE)	0.26
Spain (ESP)	0.18	Japan (JPN)	0.05	Mean	0.06
Estonia (EST)	-0.16	Korea (KOR)	-0.35	Median	0.09

Note: The table shows the KBS correlations of each OECD country against the United States from 1984 to 2011. Data on per capital consumptions and the real exchange rates are taken from Penn World Table, version 9.0.

1 Introduction

When one country becomes richer than others, a positive transfer of wealth is organized through international financial markets so that the level of consumption of the country decreases and that of the rest of the world increases. Higher consumption growth must thus be associated with the real exchange rate depreciation under well-organized financial markets. However, that outcome predicted under such a complete financial market is not widely observed in data (Backus and Smith, 1993 and Kollmann, 1995). Table 1 provides correlations between the growth rate of per capita consumption and bilateral real exchange rate growth for each pair of OECD countries. The table shows correlations that are close to zero or even negative. The mean correlation is 0.06.

I revisit this well-established lack of international consumption risk sharing in the theoretical model with heterogeneous firms. In the model, wealth transfer stems from the reallocation of heterogeneous firms that produce a variety of differentiated goods as well as the terms of trade fluctuations. I find that the nature of international risk sharing changes dramatically with the presence of heterogeneous firms; the extent to which it can be biased can be understood by simply looking at the correlation between consumption growth and real exchange rate fluctuations across countries. The Kollmann-Backus-Smith correlation between consumption growth and real exchange rate fluctuations across countries is thus conditional on changes in the number of product varieties and their qualities produced by heterogeneous firms.

As economic growth is "missing" (Aghion et al., 2017) and the gain from international trade is underestimated due to imperfectly observed expansion in the number of product varieties and their qualities (Broda and Weinstein, 2004 and Broda and Weinstein, 2006), international risk sharing can be also subject to measurement errors. In the theoretical model, I show that the above conditional link is robust with or without any imperfection in assessing the number of product varieties and their qualities. This finding implies that the accuracy of price indices cannot solve the bias of unconditional KBS correlation. The bias is more fundamental and relates to the reallocation induced by the presence of heterogeneous firms.

I next turn to test the prediction of the theoretical model with data. Using bilateral world trade data for almost two decades, I find a systematic *negative* bias in the observed unconditional KBS coefficients. Once they are conditioned with changes in the number product varieties and their qualities, the correlation between consumption growth and the real exchange rate fluctuations shows more positive signs, which are closer to the allocation under complete financial markets. The results indicate that the world is embedding better international consumption risk sharing owing to the reallocation arising from firm heterogeneity.

The recent trade literature discusses the welfare implication of the reallocation effect due to firm heterogeneity (Arkolakis et al., 2012, Melitz and Redding, 2015). Although transmission through the terms of trade fluctuations has been extensively explored in the literature on open economy macroeconomics (Cole and Obstfeld, 1991, Acemoglu and Ventura, 2002), little is known about the consequence of firm heterogeneity and the resulting reallocation for international risk sharing. A realistic Kollmann-Backus-Smith correlation can typically be obtained through a wealth effect that simultaneously brings the real exchange rate into appreciation and achieves higher consumption. Assuming a weak role of financial assets in hedging consumption risk, Corsetti et al. (2008) emphasize such a wealth effect due to a lower value of elasticity of substitution between local and imported goods and/or a high persistence of productivity shock. Hamano (2013) shows that a similar wealth effect is obtained with entry of new product varieties. In the theoretical model here, the wealth effect can be further driven by a higher quality of products produced by heterogeneous firms. The mechanism of the real exchange rate appreciation hinges on the Harrod-Balassa-Samuelson effect based on heterogeneous firms, as discussed in Ghironi and Melitz (2005).[1] Finally, consistent with my empirical results, Fitzgerald (2012), relying on the gravity equation and thus a welfare-consistent measure of price indices, finds biased estimates that emerge from imperfectly measured real exchange rates in international consumption risk sharing for OECD countries.

The remainder of this paper is organized as follows. In the next section, I present the model. In Section 3, I analytically investigate the nature of international risk sharing and transmission with a linearized system of equations. I next calibrate the model and document its quantitative implications. An empirical investigation is conducted in Section 5. In the last section, I conclude.

2 The model

2.1 Household Preferences and Intratemporal Choices

The world consists of two countries, Home and Foreign. Foreign variables are denoted with an asterisk (*). Each country is populated by one unit mass of atomic households.

[1] In Benigno and Thoenissen (2008), with exogenously determined traded and nontraded sectors, the appreciation in the real exchange rate is driven by the well-known standard Harrod-Balassa-Samuelson effect and, thus, the wealth effect due to the presence of the nontraded sector.

I discuss the representative household in Home. In what follows, similar expressions hold for the representative household in Foreign.

The Home representative household maximizes expected intertemporal utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t$, where β $(0 < \beta < 1)$ is the exogenous discount factor. The utility at time t depends on consumption and the labor supply as follows:

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}},$$

In the above expression, γ (≥ 1) denotes risk aversion. χ (> 0) represents the degree of non-satisfaction from supplying labor L_t, and φ (≥ 0) denotes Frisch elasticity of the labor supply.

The basket of goods C_t is defined as

$$C_t = \left[C_{H,t}^{1-\frac{1}{\omega}} + C_{F,t}^{1-\frac{1}{\omega}} \right]^{\frac{1}{1-\frac{1}{\omega}}},$$

where ω (> 0) denotes the elasticity of substitution between local $(C_{H,t})$ and imported goods $(C_{F,t})$. $C_{H,t}$ and $C_{F,t}$ are defined over a continuum of goods Ω:

$$C_{H,t} = V_{H,t} \left(\int_{\zeta \in \Omega} q_D(\zeta) c_{D,t}(\zeta)^{1-\frac{1}{\sigma}} d\zeta \right)^{\frac{1}{1-\frac{1}{\sigma}}}, \quad C_{F,t} = V_{F,t}^{*} \left(\int_{\vartheta \in \Omega} q_X^{*}(\vartheta) c_{X,t}(\vartheta)^{1-\frac{1}{\sigma}} d\vartheta \right)^{\frac{v1}{1-\frac{1}{\sigma}}},$$

where $V_{H,t} \equiv N_{D,t}^{\psi - \frac{1}{\sigma-1}}$ and $V_{F,t}^{*} \equiv N_{X,t}^{*\psi - \frac{1}{\sigma-1}}$. $N_{D,t}$ and $N_{X,t}^{*}$ represent the number of domestic and imported product varieties. ψ (≥ 0) represents the marginal utility that stems from one additional increase in the number of varieties in each basket (Benassy, 1996). Specifically, the preference becomes Dixit and Stiglitz (1977) when $\psi = \frac{1}{\sigma-1}$. At any given time t, only a subset of goods $\Omega_t \in \Omega$ is available. $c_{D,t}(\zeta)$ and $c_{X,t}(\vartheta)$ represent the demand addressed for individual product variety ζ and ϑ, which are produced domestically and imported, respectively. $q_D(\zeta)$ and $q_X(\vartheta)$ denote the quality of these product varieties. σ (> 1) denotes the elasticity of substitution among varieties. I assume conventionally that $\sigma \geq \omega$.

Optimal consumption for each domestic, imported basket and individual product variety is found to be

$$C_{H,t} = \left(\frac{P_{H,t}}{P_t} \right)^{-\omega} C_t, \qquad C_{F,t} = \left(\frac{P_{F,t}}{P_t} \right)^{-\omega} C_t.$$

$$c_{D,t}(\zeta) = (V_{H,t}q_D(\zeta))^{\sigma-1}\left(\frac{p_{D,t}(\zeta)}{P_{H,t}}\right)^{-\sigma} C_{H,t}, \qquad c_{X,t}(\vartheta) = \left(V^*_{F,t}q^*_X(\vartheta)\right)^{\sigma-1}\left(\frac{p^*_{X,t}(\vartheta)}{P_{F,t}}\right)^{-\sigma} C_{F,t}.$$

In particular, $p^*_{X,t}(\vartheta)$ denotes the price of exported goods from Foreign.

Price indices that minimize expenditure on each consumption basket are given by

$$P_t = \left[P_{H,t}^{1-\omega} + P_{F,t}^{1-\omega}\right]^{\frac{1}{1-\omega}},$$

$$P_{H,t} = \frac{1}{V_{H,t}}\left(\int_{\zeta\in\Omega_t}\left(\frac{p_{D,t}(\zeta)}{q_D(\zeta)}\right)^{1-\sigma} d\zeta\right)^{\frac{1}{1-\sigma}}, \qquad P_{F.t} = \frac{1}{V^*_{F.t}}\left(\int_{\vartheta\in\Omega_t}\left(\frac{p^*_{X,t}(\vartheta)}{q^*_X(\vartheta)}\right)^{1-\sigma} d\vartheta\right)^{\frac{1}{1-\sigma}}.$$

Observe that the price indices defined so that they fluctuate with changes in the number of varieties and product qualities. Finally, I choose the welfare-based consumer price index in Home, P_t, as a numéraire and define real prices as $\rho_{H,t} \equiv \frac{P_{H,t}}{P_t}$, $\rho_{F,t} \equiv \frac{P_{F,t}}{P_t}$, $\rho_{D,t}(\zeta) \equiv \frac{p_{D,t}(\zeta)}{P_t}$ and $\rho^*_{X,t}(\vartheta) \equiv \frac{p^*_{X,t}(\vartheta)}{P_t}$.

Similar expressions hold in Foreign. Crucially, the subset of goods available in Foreign during period t, $\Omega^*_t \in \Omega$, can be different from the subset of goods available in Home.

2.2 Production, Pricing and the Export Decision

In every period, there is a mass of $N_{E,t}$ entrants. Prior to entry, these new entrants are identical and face a sunk entry cost of f_E, which is defined as follows:

$$f_E = Z_t l_{E,t},$$

where Z_t denotes the labor productivity level, which is common for all firms. $l_{E,t}$ is the demand for labor in the firm setup. Upon entry, each firm draws her productivity level z from a distribution $G(z)$ with support on $[z_{\min}, \infty)$. Since there are no fixed production costs, all firms produce unless they are hit by exogenous depreciation shock, which occurs with probability $\delta \in (0,1)$. This exit-inducing shock is independent of the firm-specific productivity level and assumed to take place at the end of every period.

Exporting requires fixed operational costs f_X in every period. Specifically,

$$f_X = Z_t l_{f_X,t},$$

where $l_{f_X,t}$ is the demand for labor required to produce f_X amount of fixed costs. Only a subset of firms with a productivity level z that is above the cutoff level $z_{X,t}$ exports by charging sufficiently lower quality-adjusted prices and earning positive profits despite the existence of fixed export costs f_X. Thus, non-tradeness in the economy arises endogenously with changes in the cutoff productivity level.

Each firm faces a residual demand curve with constant elasticity σ. The production scale is thus determined by the demand addressed to the firm. Profit maximization of the firm with productivity z yields the following optimal real prices:

$$\rho_{D,t}(z) = \frac{\sigma}{\sigma - 1} mc_t(z),$$

where $mc_t(z)$ is the real marginal cost of production. I assume that producing a high-quality goods requires higher marginal costs $mc_t(z)$ such that

$$mc_t(z) = \left(1 + \frac{q(z)^{\frac{1}{\phi}}}{z}\right) \frac{w_t}{Z_t z}, \tag{1}$$

where ϕ $(0 \leq \phi < 1)$ is a parameter that determines quality ladder and w_t denotes real wage. Provided a firm-specific productivity level z, the firm endogenously chooses its specific quality level $q(z)$. Specifically, the firm minimizes the quality-adjusted marginal cost $mc_t(z)/q(z)$. As a result, optimal quality of the firm with productivity z is given by

$$q(z) = \left(\frac{\phi}{1 - \phi} z\right)^{\phi}.$$

Provided $\phi > 0$, as is consistent with empirical findings, firms with high productivities produce product varieties of high quality.[2] Observe that when there is no quality ladder ($\phi = 0$), all firms produce a similar quality of goods, irrespective of their specific productivity levels as $q(z) = q = 1$. In such a case, the model is isomorphic to the one in Ghironi and Melitz (2005).

Due to the fixed operational export costs f_X, the firm with productivity z may not export. If the firm exports, its export price is $\rho_{X,t}(z) = \tau_t \rho_{D,t}(z) Q_t^{-1}$. In the expression,

[2] Whatever the type of endogenization of product quality, we can have some mapping between firm-specific productivity and its specific quality. See, for instance, Verhoogen (2008).

τ_t stands for iceberg trade costs. Q_t is the real exchange rate defined as the price of the foreign consumption basket in terms of home consumption basket as $Q_t \equiv P_t^*/P_t$. $\rho_{X,t}(z)$ is thus denominated in the Foreign consumption basket.

Total profits of the firm with productivity z, $d_t(z)$, can be decomposed into those from domestic sales $d_{D,t}(z)$ and from exporting sales $d_{X,t}(z)$ as $d_t(z) = d_{D,t}(z) + d_{X,t}(z)$. Using the demand functions found previously, we can write profits in each market as

$$d_{D,t}(z) = \frac{1}{\sigma} N_{D,t}^{\psi(\omega-1)-1} \left(\frac{\rho_{D,t}(z)}{q(z)} \right)^{1-\omega} C_t,$$

$$d_{X,t}(z) = \frac{Q_t}{\sigma} N_{X,t}^{\psi(\omega-1)-1} \left(\frac{\rho_{X,t}(z)}{q(z)} \right)^{1-\omega} C_t^* - \frac{w_t f_X}{Z_t}, \text{ if firm } z \text{ exports,}$$

2.3 Firm Averages

Given a distribution $G(z)$, a mass of $N_{D,t}$ of domestically producing firms has a distribution of productivity levels over $[z_{\min}, \infty)$. Among these firms, there is a mass of $N_{X,t} = [1 - G(z_{X,t})]\, N_{D,t}$ exporters in Home. Following Melitz (2003), we define two average productivity levels, $\widetilde{z}_D$ for domestically producing firms and $\widetilde{z}_{X,t}$ for exporters, as follows:

$$\widetilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}, \quad z \equiv \left[\frac{1}{1 - G(z_{X,t})} \int_{z_{X,t}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}.$$

These average productivity levels summarize all the information about the distribution of productivities. Provided these averages, I define average real domestic and export prices as $\widetilde{\rho}_{D,t} \equiv \rho_{D,t}(\widetilde{z}_D)$ and $\widetilde{\rho}_{X,t} \equiv \rho_{X,t}(\widetilde{z}_{X,t})$, respectively. Similarly, average domestic and export quality are provided by $\widetilde{q}_D \equiv q_D(\widetilde{z}_D)$ and $\widetilde{q}_{X,t} \equiv q_{X,t}(\widetilde{z}_{X,t})$. Additionally, I define average real profits from domestic sales and export sales as $\widetilde{d}_{D,t} \equiv d_{D,t}(\widetilde{z}_D)$ and $\widetilde{d}_{X,t} \equiv d_{X,t}(\widetilde{z}_{X,t})$. Finally, average real profits among all Home firms are given by $\widetilde{d}_t^s = \widetilde{d}_{D,t} + (N_{X,t}/N_{D,t})\, \widetilde{d}_{X,t}$.

2.4 Firm Entry and Exit

A mass of $N_{E,t}$ entrants at time t is assumed to start producing only at time $t+1$. Their expected post-entry value is

$$\widetilde{v}_t^s = E_t \sum_{i=t+1}^{\infty} \beta^{i-t} \left(\frac{C_i}{C_t} \right)^{-\gamma} (1-\delta)^{s-t} \widetilde{d}_i^s$$

The above is the sum of discounted expected profits. Entry occurs until the above post-entry value is equalized with entry costs as

$$\widetilde{v}_t^s = \frac{w_t}{Z_t} f_E.$$

The timing of entry and production implies that the number of domestically producing firms evolves according to the following motion: $N_{D,t} = (1-\delta)(N_{D,t-1} + N_{E,t-1})$.

2.5 Parametrization of Productivity Draws

The following Pareto distribution for $G(z)$ is assumed:

$$G(z) = 1 - \left(\frac{z_{\min}}{z} \right)^{\kappa},$$

where κ $(> \sigma - 1)$ is the shape parameter. With the above distribution, we have

$$\widetilde{z}_D = z_{\min} \left[\frac{\kappa}{\kappa - (\sigma - 1)} \right]^{\frac{1}{\sigma-1}}, \quad \widetilde{z}_{X,t} = z_{X,t} \left[\frac{\kappa}{\kappa - (\sigma - 1)} \right]^{\frac{1}{\sigma-1}}$$

Additionally, the share of exporters in the total number of domestic firms is given by

$$\frac{N_{X,t}}{N_{D,t}} = z_{\min}^{\kappa} \left(\widetilde{z}_{X,t} \right)^{-\kappa} \left[\frac{\kappa}{\kappa - (\sigma - 1)} \right]^{\frac{\kappa}{\sigma-1}}.$$

In the end, there exists the firm with cutoff level productivity $z_{X,t}$ that earns zero profits from exporting with $d_{X,t}(z_{X,t}) = 0$. Combined the above Pareto distribution and the expression of total average profits, a zero cutoff profit (ZCP) condition implies that

$$\widetilde{d}_{X,t} = \frac{w_t f_X}{Z_t} \frac{\sigma - 1}{\kappa - (\sigma - 1)}.$$

2.6 Household Budget Constraint and Intertemporal Choices

There are two types of financial assets – equities and bonds. Here, I present the case of financial autarky. In a later section, I relax this assumption by allowing international borrowing and lending and present the model with state noncontingent bonds.

Gross returns of Home equities and bonds between t and $t+1$ (in units of Home consumption) are defined respectively as

$$R_{h,t+1}^{s} \equiv (1-\delta)\,\frac{\widetilde{v}_{t+1}^{s} + \widetilde{d}_{t+1}^{s}}{\widetilde{v}_{t}^{s}}, \quad R_{h,t+1}^{b} \equiv 1 + r_{t+1}.$$

Equity returns are adjusted by $1-\delta = N_{D,t+1}/\left(N_{D,t} + N_{E,t}\right)$, the surviving rate of producing firms and entrants between two consecutive time periods. Bonds returns are defined in terms of the Home consumption basket.

[3] The period budget constraint of the representative household in Home (defined in units of Home consumption) is given by[4]

$$C_t + \widetilde{v}_t^{s}\left(N_{D,t} + N_{E,t}\right) s_{h,t+1} + b_{h,t+1} = w_t L_t + R_{h,t}^{s}\widetilde{v}_{t-1}^{s}\left(N_{D,t-1} + N_{E,t-1}\right) s_{h,t} + R_{h,t}^{b} b_{h,t}. \quad (2)$$

The Home representative household finances the entry cost of new entrants and all producing firms in Home at time t by purchasing a share of Home equities $s_{h,t+1}$. The representative household maximizes the expected intertemporal utility with respect to $s_{h,t+1}$, $b_{h,t+1}$, L_t and C_t subject to (2) for all time periods. The first-order condition with

[3] The return of Foreign equity and bonds is $R_{f,t+1}^{s} \equiv (1-\delta)\frac{\widetilde{v}_{t+1}^{s*} + \widetilde{d}_{t+1}^{s*}}{\widetilde{v}_t^{s*}}\frac{Q_{t+1}}{Q_t}$, $R_{f,t+1}^{b} \equiv \left(1 + r_{t+1}^{*}\right)\frac{Q_{t+1}}{Q_t}$. Both returns are denominated in the Home consumption basket.

[4] The corresponding budget constraint for Foreign households is

$$C_t^* + \widetilde{v}_t^{s*}\left(N_{D,t}^* + N_{E,t}^*\right) s_{f,t+1}^* + b_{f,t+1}^* = w_t^* L_t^* + \frac{Q_t}{Q_{t+1}} R_{f,t}^{s}\widetilde{v}_{t-1}^{s*}\left(N_{D,t-1}^* + N_{E,t-1}^*\right) s_{f,t}^* + \frac{Q_t}{Q_{t+1}} R_{f,t}^{b} b_{f,t}^*$$

respect to equity holdings is

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R^s_{h,t+1} \right].$$

Additionally, the first-order condition with respect to bond holdings is

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R^b_{h,t+1} \right].$$

Finally, the optimal labor supply is given by

$$\chi \left(L_t \right)^{\frac{1}{\psi}} = w_t C_t^{-\gamma}.$$

2.7 General Equilibrium and Balanced Trade

Supplied labor units L_t are required for fixed costs of exporting and firm creation and for production of domestic and tradable goods. Accordingly, labor market clearings (LMC) in Home imply that

$$L_t = \frac{N_{E,t}\widetilde{v}_t^s}{w_t} + \frac{(\sigma - 1) N_{D,t}\widetilde{d}_t}{w_t} + \frac{\sigma N_{X,t} f_X}{Z_t}.$$

The model is completed by considering the balanced trade condition such that

$$\int_0^{N^*_{X,t}} p^*_{X,t}(\vartheta^*) c_{X,t}(\vartheta^*) d\vartheta^* = \int_0^{N_{X,t}} p_{X,t}(\vartheta) c^*_{X,t}(\vartheta) d\vartheta.$$

Using the demand system found previously, the above expression is equivalent to

$$N_{X,t}^{\psi(\sigma-1)} \left(\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}} \right)^{1-\sigma} \rho_{H,t}^{*\sigma-\omega} Q_t C_t^* = N_{X,t}^{*\psi(\sigma-1)} \left(\frac{\widetilde{\rho}^*_{X,t}}{\widetilde{q}^*_{X,t}} \right)^{1-\sigma} \rho_{F,t}^{\sigma-\omega} C_t. \quad (3)$$

The whole system of equations is summarized in Table 2.

2.8 Calibration

I calibrate the theoretical models with parameter values, as in Table 3. The calibration is conducted on a quarterly basis. The value of constant risk aversion γ, the steady-state discount factor β, the Frisch elasticity of the labor supply φ and the elasticity of

Table 2: The Model

Price indices	$\rho_{H,t}^{1-\omega}+\rho_{F,t}^{1-\omega}=1,\ \rho_{H,t}=N_{D,t}^{-\psi}\frac{\widetilde{\rho}_{D,t}}{\widetilde{q}_D},\ \rho_{F,t}=N_{X,t}^{*-\psi}\frac{\widetilde{\rho}^*_{X,t}}{\widetilde{q}^*_{X,t}}$
	$\rho_{F,t}^{*1-\omega}+\rho_{H,t}^{*1-\omega}=1,\ \rho^*_{F,t}=N_{D,t}^{*-\psi}\frac{\widetilde{\rho}^*_{D,t}}{\widetilde{q}^*_D},\ \rho^*_{H,t}=N_{X,t}^{-\psi}\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}$
Pricing	$\widetilde{\rho}_{D,t}=\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z_t\widetilde{z}_D},\quad \widetilde{\rho}_{X,t}=\tau_t\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z_t\widetilde{z}_{X,t}}Q_t^{-1},$
	$\widetilde{\rho}^*_{D,t}=\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z^*_t\widetilde{z}^*_D},\quad \widetilde{\rho}^*_{X,t}=\tau_t\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w^*_t}{Z^*_t\widetilde{z}^*_{X,t}}Q_t$
Profits	$\widetilde{d}_t=\widetilde{d}_{D,t}+\frac{N_{X,t}}{N_{D,t}}\widetilde{d}_{X,t},\quad \widetilde{d}_{D,t}=\frac{1}{\sigma}N_{D,t}^{\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{D,t}}{\widetilde{q}_D}\right)^{1-\omega}C_t$
	$\widetilde{d}_{X,t}=\frac{Q_t}{\sigma}N_{X,t}^{\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}\right)^{1-\omega}C^*_t-\frac{w_tf_X}{Z_t}$
	$\widetilde{d}^*_t=\widetilde{d}^*_{D,t}+\frac{N^*_{X,t}}{N^*_{D,t}}\widetilde{d}^*_{X,t},\quad \widetilde{d}^*_{D,t}=\frac{1}{\sigma}N_{D,t}^{*\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}^*_{D,t}}{\widetilde{q}^*_D}\right)^{1-\omega}C^*_t$
	$\widetilde{d}^*_{X,t}=\frac{Q_t^{-1}}{\sigma}N_{X,t}^{*\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}^*_{X,t}}{\widetilde{q}^*_{X,t}}\right)^{1-\omega}C_t-\frac{w^*_tf^*_X}{Z^*_t}$
Free entry	$\widetilde{v}^s_t=\frac{w_t}{Z_t}f_E,\qquad \widetilde{v}^{s*}_t=\frac{w^*_t}{Z^*_t}f^*_E$
LMC	$w_tL_t=N_{E,t}\widetilde{v}^s_t+(\sigma-1)\,N_{D,t}\widetilde{d}_t++\sigma N_{X,t}\frac{w_tf_X}{Z_t}$
	$w^*_tL^*_t=N^*_{E,t}\widetilde{v}^{s*}_t+(\sigma-1)\,N^*_{D,t}\widetilde{d}^*_t+\sigma N^*_{X,t}\frac{w^*_tf^*_X}{Z^*_t}$
Export share	$\frac{N_{X,t}}{N_{D.t}}=z_{\min}^{\kappa}\left(\widetilde{z}_{X,t}\right)^{-\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}},\quad \frac{N^*_{X,t}}{N^*_{D,t}}=z_{\min}^{\kappa}\left(\widetilde{z}^*_{X,t}\right)^{-\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}$
ZCP	$\widetilde{d}_{X,t}=\frac{w_tf_X}{Z_t}\frac{\sigma-1}{\kappa-(\sigma-1)},\quad \widetilde{d}^*_{X,t}=\frac{w^*_tf^*_X}{Z^*_t}\frac{\sigma-1}{\kappa-(\sigma-1)}$
Export quality	$\widetilde{q}_{X,t}=\left(\frac{\phi}{1-\phi}\widetilde{z}_{X,t}\right)^{\phi},\ \widetilde{q}^*_{X,t}=\left(\frac{\phi}{1-\phi}\widetilde{z}^*_{X,t}\right)^{\phi}$
Number of firms	$N_{D,t+1}=(1-\delta)\,(N_{D,t}+N_{E,t}),\quad N^*_{D,t+1}=(1-\delta)\left(N^*_{D,t}+N^*_{E,t}\right)$
Euler shares	$1=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^s_{h,t+1}\right]$
	$1=\beta E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}R^s_{f,t+1}\frac{Q_t}{Q_{t+1}}\right]$
Euler bonds	$1=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^b_{h,t+1}\right]$
	$1=\beta E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}R^b_{f,t+1}\frac{Q_t}{Q_{t+1}}\right]$
Balanced trade	$N_{X,t}^{\psi(\sigma-1)}\left(\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}\right)^{1-\sigma}\rho_{H,t}^{*\sigma-\omega}Q_tC^*_t=N_{X,t}^{*\psi(\sigma-1)}\left(\frac{\widetilde{\rho}^*_{X,t}}{\widetilde{q}^*_{X,t}}\right)^{1-\sigma}\rho_{F,t}^{\sigma-\omega}C_t$

substitution between local goods and imported goods ω are in line with the literature on open macroeconomics. The value of the death shock δ, the elasticity of substitution among product varieties σ, the preference for variety ψ, fixed export costs f_X and the shape of the Pareto distribution κ are set following Ghironi and Melitz (2005). These values are based on the empirical findings of Bernard et al. (2003), which also document

Table 3: Baseline Parameter Values

γ	constant risk aversion	2
β	discount factor	0.99
φ	Frisch elasticity of labor supply	2
σ	elasticity of substitution among varieties	3.8
ω	between Home and Foreign goods	2
τ	steady-state trade cost	1.3
δ	death shock	0.025
κ	Pareto distribution	3.34
ψ	Preference for variety	Dixit-Stiglitz
ϕ	quality ladder	0.61

that the share of exporters for the United States. The value of fixed export costs f_X is taken such that in the steady state, the share is 21% accordingly. The parameter value that determines quality ladder ϕ comes from Feenstra and Romalis (2014), who estimate the elasticity of firm-specific quality with respect to firm-specific productivity using world trade data.

The productivity process is selected from Backus et al. (1992) such that Z_{t+1}=$\Omega\mathsf{Z}_t$+ξ_t, where $\mathsf{Z}_t = \begin{bmatrix} Z_t, & Z_t^* \end{bmatrix}'$, $\xi_t = \begin{bmatrix} \xi_t, & \xi_t^* \end{bmatrix}'$ and

$$\Omega = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix} \text{ and } V(\xi) = \begin{bmatrix} 0.73 & 0.19 \\ 0.19 & 0.73 \end{bmatrix},$$

where ξ_t is assumed to be zero mean i.i.d..

2.9 The Model with State Noncontingent Bonds

Although the main intuition of the model and its consequence arising from firm heterogeneity is perfectly described by the above benchmark model under financial autarky, I discuss here an alternative financial market structure to argue the robustness of the results. Specifically, the assumption of financial autarky is relaxed and internationally exchanged state noncontingent bonds are introduced instead. With this specification, net foreign assets fluctuate. Since the model is almost identical as the benchmark model, only modified points are discussed below.

2.10 Households

With internationally held bonds, the budget constraint of the Home representative households is

$$C_t + \widetilde{v}_t^s \left(N_{D,t} + N_{E,t}\right) s_{h,t+1} + b_{h,t+1} + Q_t b_{f,t+1} + \frac{\vartheta}{2} b_{h,t+1}^2 + \frac{\vartheta}{2} Q_t b_{f,t+1}^2$$
$$= w_t L_t + R_{h,t}^s \widetilde{v}_{t-1}^s \left(N_{D,t-1} + N_{E,t-1}\right) s_{h,t} + R_{h,t}^b b_{h,t} + R_{f,t}^b Q_{t-1} b_{f,t} + T_t^f. \quad (4)$$

To precisely determine the equilibrium international bond holding positions and nonstationarity of dynamics, quadratic adjusting costs of bond holdings, ϑ, are introduced. T_t^f is a free rebate of adjusting costs. The representative household maximizes the expected intertemporal utility with respect to $s_{h,t+1}$, $b_{h,t+1}$, $b_{f,t+1}$, L_t and C_t, subject to (4) for all periods. Euler equations for bond holdings are given by

$$1 + \vartheta b_{h,t+1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{h,t+1}^b \right], \quad 1 + \vartheta b_{f,t+1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{f,t+1}^b \right].$$

Other first-order conditions are identical to the benchmark model.

Similar conditions hold for Foreign.

2.11 General Equilibrium and Net Foreign Asset Dynamics

We have the same labor market clearing condition as in the benchmark model. The balanced trade condition, however, is replaced by the following net foreign asset dynamics. Net foreign assets (denominated in Home consumption unit) at the end of period t are defined as

$$NFA_{t+1} \equiv b_{f,t+1} Q_t - b_{h,t+1}^*.$$

Since there are no cross-border equity holdings by assumption, only cross-border bond holdings appear in the definition. With the above definition of the net foreign assets, the budget constraint (4) can be rewritten, and the following net foreign asset dynamics are derived:

$$NFA_{t+1} = NX_t + NFA_t R^b_{h,t} + \xi_{h,t},$$

where NX_t denotes net exports and $\xi_{h,t}$ stands for the "excess returns" between $t-1$ and t relative to returns on Home bonds $R^b_{h,t}$. Precisely, NX_t and $\xi_{h,t}$ are given by

$$NX_t = \frac{1}{2}\left[w_t L_t + N_{D,t}\widetilde{d}_t - Q_t\left(w^*_t L^*_t + N^*_{D,t}\widetilde{d}^*_t\right)\right] - \frac{1}{2}\left[(C_t - N_{E,t}\widetilde{v}^s_t) - Q_t\left(C^*_t - N^*_{E,t}\widetilde{v}^{s*}_t\right)\right],$$

and

$$\xi_{h,t} \equiv b_{f,t} Q_t \left(R^b_{f,t} - R^b_{h,t}\right).$$

Note that the excess returns are zero in the first-order dynamics because of zero bond holdings due to adjustment costs in the steady state. Finally, asset markets clear for all time periods as

$$b_{h,t+1} + b^*_{h,t+1} = b_{f,t+1} + b^*_{f,t+1} = 0.$$

Table 4 summarizes the set of equations replaced or added. The symmetric steady state remains the same as in the model with balanced trade.

Table 4: The Mode with International Bonds

Euler bonds	$1 + \vartheta b_{h,t+1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R^b_{h,t+1}\right]$
	$1 + \vartheta b_{f,t+1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R^b_{f,t+1}\right]$
	$1 + \vartheta b^*_{f,t+1} = \beta E_t \left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma} R^b_{f,t+1}\frac{Q_t}{Q_{t+1}}\right]$
	$1 + \vartheta b^*_{h,t+1} = \beta E_t \left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma} R^b_{h,t+1}\frac{Q_t}{Q_{t+1}}\right]$
Bond market clearing	$b_{h,t+1} + b^*_{h,t+1} = 0, \quad b_{f,t+1} + b^*_{f,t+1} = 0.$
Net foreign asset	$NFA_{t+1} = NX_t + NFA_t\left(1 + r_{t+1}\right) + \xi_t$
Net export	$NX_t = \frac{1}{2}\left[w_t L_t + N_{D,t}\widetilde{d}_t - Q_t\left(w^*_t L^*_t + N^*_{D,t}\widetilde{d}^*_t\right)\right]$
	$-\frac{1}{2}\left[(C_t + N_{E,t}\widetilde{v}^s_t) - Q_t\left(C^*_t + N^*_{E,t}\widetilde{v}^{s*}_t\right)\right]$
Excess returns	$\xi_{h,t} = Q_t B_{*,t}\left(r^*_{t+1} - r_{t+1}\right)$

3 International Risk Sharing with Heterogeneous Firms

Even under financial autarky, it is known that consumption risk can be insured through appropriate fluctuations in the terms of trade (Cole and Obstfeld, 1991). Does this conclusion still hold in the world where firms are heterogeneous, and hence, does the reallocation of them result in fluctuations in the number of product varieties and their qualities? It is shown that this situation is indeed the case. Furthermore, with wealth transfer due to the reallocation, the Kollmann-Backus-Smith correlation is found to be structurally conditional on changes in the number of product varieties and product quality driving a wedge between relative consumption growth rate correlations and the real exchange rate fluctuations.

3.1 Complete Financial Markets and the Puzzle

In the presence of nontraded goods or trade costs, the price level across countries can differ. In such a general case, under complete financial markets, the marginal utility stemming from one additional unit of nominal wealth should be equalized across countries:

$$U_{C,t} = U^*_{C^*,t} Q_t^{-1}$$

where $U_{C,t}$ and $U^*_{C^*,t}$ represent the marginal utility of consumption in Home and Foreign, respectively. With CRRA utility function and separability between leisure and consumption, the above condition is specified as

$$\mathsf{C} - \mathsf{C}^* = \frac{1}{\gamma}\mathsf{Q} \tag{5}$$

In the above expression, the parameter γ determines the extent of relative risk aversion. Sans Serif font denotes the first-order deviations, and time indices are dropped when there is no room for confusion henceforth. Since typically $\gamma \geq 1$, the correlation between the relative consumption across countries and the real exchange is *positive*: consumption growth in Home relative to that in Foreign must be associated with a real depreciation for Home. However, this is not the case in the data. Correlations between relative consumption and real exchange rate are *close to zero or even negative* for the large number of countries, which is known as the Kollmann-Backus-Smith puzzle (Kollmann, 1995 and Backus and Smith, 1993).

3.2 Financial Autarky

The absence of positive KBS correlations that we expect to see under complete markets (5) would be attributed to the absence of complete asset markets itself (Obstfeld and Rogoff, 2000). I follow Corsetti et al. (2008) and discuss the implication of firm heterogeneity under financial autarky. To see the point, I begin by expressing fluctuations in the real exchange rate. With firm heterogeneity, fluctuations in the real exchange rate are expressed as

$$\mathsf{Q} = (2S_{ED} - 1)\,\mathsf{TOL} + \psi S_{ED}\mathsf{N}_D^R - (1 - S_{ED})\left[\psi \mathsf{N}_X^R + \widetilde{\mathsf{q}}_X^R + \widetilde{\mathsf{z}}_X^R\right], \tag{6}$$

where $\mathsf{TOL} \equiv -\left(\mathsf{w}^R - \mathsf{Z}^R\right)$ represents fluctuations in *the terms of labor* (Ghironi and Melitz, 2005) in which $\mathsf{w}^R \equiv \mathsf{w} - \mathsf{w}^*$ and $\mathsf{Z}^R \equiv \mathsf{Z} - \mathsf{Z}^*$ represent fluctuations in real wage and productivities across countries, respectively Similarly, $\mathsf{N}_D^R \equiv \mathsf{N}_D - \mathsf{N}_D^*$, $\mathsf{N}_X^R \equiv \mathsf{N}_X - \mathsf{N}_X^*$, $\widetilde{\mathsf{z}}_X^R \equiv \widetilde{\mathsf{z}}_X - \widetilde{\mathsf{z}}_X^*$ and $\widetilde{\mathsf{q}}_X^R \equiv \widetilde{\mathsf{q}}_X - \widetilde{\mathsf{q}}_X^*$ are relative changes in the number of domestically available varieties, the number of exported varieties, the quality of export and the cutoff level productivities of exporters across countries, respectively. S_{ED} ($> 1/2$) and ψ represent the steady-state expenditure share on domestically produced goods and the marginal utility stemming from one additional product variety, respectively. Furthermore, Q is referred to as "welfare-based" fluctuations since it fully captures changes in the number of product varieties and qualities. Importantly, Q now includes not only the fluctuations in the terms of labor as in the standard model but also those in the number of product varieties and their qualities.

Having the above fluctuations in Q in hand, the relationship between relative consumption growth and real exchange rate growth with firm heterogeneity under financial autarky is expressed as[5]

[5]In our model, using the demand systems found previously, the balanced trade condition (3) can be

$$\mathsf{C}-\mathsf{C}^{*}=\frac{2S_{ED}\omega-1}{2S_{ED}-1}\mathsf{Q}+\frac{(\omega-1)\,S_{ED}}{2S_{ED}-1}\left[\psi(\mathsf{N}_{D}^{R}-\mathsf{N}_{X}^{R})+\widetilde{\mathsf{q}}_{X}^{R}+\widetilde{\mathsf{z}}_{X}^{R}\right]. \tag{7}$$

The first term on the right-hand side of the equation is basically the same one argued in Corsetti et al. (2008) in the absence of changes in the number of product varieties and qualities. Importantly, this first term is *conditional* on changes in product quality and the number of product varieties with firm heterogeneity. As a result, the *unconditional* coefficient can be different from the structural coefficient, $\frac{2S_{ED}\omega-1}{2S_{ED}-1}$. Shutting down firm heterogeneity, however, the discrepancy between the conditional and unconditional relationship disappears. In such a case, we find the same expression as found in Hamano (2013) with *homogeneous* firms.[6] The above discussion is summarized by the following proposition.

Proposition 1. *With firm heterogeneity, there exists a wedge between unconditional KBS correlations and conditional KBS correlations.*

For instance, with expenditure share on domestic goods, which is higher than one-half as $S_{ED} > 1/2$, and the elasticity of substitution, which is higher than unity as $\omega > 1$, the conditional correlation is unambiguously positive as $\frac{2S_{ED}\omega-1}{2S_{ED}-1} > 0$. However, since the welfare-based KBS correlation is conditional on the fluctuations in the number of product varieties and their qualities, the unconditional KBS correlation can be negative or close to zero.

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expressed as

$$\omega\mathsf{Q}-(\mathsf{C}-\mathsf{C}^{*})+\psi\,(\omega-1)\,\mathsf{N}_{X}^{R}-(\omega-1)\left[\mathsf{w}^{R}-\mathsf{Z}^{R}-\widetilde{\mathsf{q}}_{X}^{R}-\widetilde{\mathsf{z}}_{X}^{R}\right]=0.$$

Plugging the decomposition of the real exchange rate (6) into the above balanced trade condition, we obtain (7).

[6]By comparing the expression found in Hamano (2013), which has only the first term on the right-hand side of the equation (7), the second term in the square brackets arises due to the presence of fixed exporting costs. By setting $f_X = f_X^{*} = 0$, all firms export independent of their specific productivities. As a result, we do not see any changes in cutoff and quality as $\widetilde{\mathsf{z}}_{X}^{R} = \widetilde{\mathsf{q}}_{X}^{R} = 0$ and the number of exporters and domestic producers coincide as $\mathsf{N}_{D}^{R} = \mathsf{N}_{X}^{R}$. With homogeneous firm setting as in Hamano (2013), a wedge between unconditional and conditional KBS correlations appears only when consumption and the real exchange rate are measured in empirical basis.

How far then is the allocation under financial autarky from that implied under complete asset markets? It is well known that even under financial autarky, the mechanism of international risk sharing is present due to desirable fluctuations in relative prices (Cole and Obstfeld, 1991, Acemoglu and Ventura, 2002). As seen from (7), the model under financial autarky potentially can generate too little or too much international risk sharing depending on the parameters' values and specifically the wealth transfer due to the reallocation (the second term on the right-hand side of (7)). Importantly, the following proposition is derived:

Proposition 2. *In the model with firm heterogeneity with which product qualities and the number of product varieties fluctuate endogenously, the equilibrium allocation under financial autarky (7) perfectly mimics that obtained with complete asset markets (5) when* $\omega = \gamma = 1$.

The above proposition is a generalization of the result discussed in Cole and Obstfeld (1991). Importantly, with firm heterogeneity, the international transmission through which the complete market allocation is reproduced depends not only on fluctuations in the terms of labor but also on the reallocation in the relative number of varieties and their qualities based on firm heterogeneity.

3.3 From Welfare to Empirically Based Fluctuations

In investigating the KBS puzzle with actual data, one should notice that, as is the case for economic growth (Aghion et al., 2017), fluctuations in the number of varieties and product qualities are only imperfectly measured (Broda and Weinstein, 2004, 2006). I characterize such an unavoidable feature by defining the empirically relevant fluctuations in the following way:[7]

[7]Ghironi and Melitz (2005) and Hamano (2015) provide a similar decomposition but without quality. In particular, Ghironi and Melitz (2005) argue the Harrod-Balassa-Samuelson effect that based on entry and exit of firms between *endogenously determined* traded and nontraded sector.

$$\begin{aligned}\widehat{\mathsf{Q}} \equiv \mathsf{Q} - \psi\lambda_1 \mathsf{N}_D^R + \psi\lambda_2 \mathsf{N}_X^R + \lambda_3 \widetilde{\mathsf{q}}_X^R \\ = (2S_{ED} - 1)\,\mathsf{TOL} - (1 - S_{ED})\,\widetilde{\mathsf{z}}_X^R + \psi(S_{ED} - \lambda_1)\mathsf{N}_D^R \\ - \psi\,(1 - S_{ED} - \lambda_2)\,\mathsf{N}_X^R - (1 - S_{ED} - \lambda_3)\,\widetilde{\mathsf{q}}_X^R, \quad (8)\end{aligned}$$

where $\widehat{\mathsf{Q}}$ is referred as the "empirically based" measure of fluctuations of the real exchange rate. Accordingly, the parameters λ_1, λ_2 and λ_3 capture the extent of (in)efficiency of statistical agencies in measuring fluctuations in the number of domestic varieties N_D^R, export (import) varieties N_X^R and product qualities of export (import) $\widetilde{\mathsf{q}}_X^R$.[8] Depending on the value of these parameters, the definition of $\widehat{\mathsf{Q}}$ can be different. On the one hand, when $\lambda_1 = \lambda_2 = \lambda_3 = 0$, there is no discrepancy between the welfare-based measures and the empirically relevant measures as $\widehat{\mathsf{Q}} = \mathsf{Q}$. On the other hand, when $\lambda_1 = S_{ED}$ and $\lambda_2 = \lambda_3 = 1 - S_{ED}$, the statistical agency completely ignores the fluctuations in the number of varieties and their qualities.[9] In general, when $\lambda_1 > 0$ ($\lambda_1 < 0$), the statistical agents underestimate (overestimate) the impact of domestic varieties in the consumption basket. In a similar way, when $\lambda_2 > 0$ ($\lambda_2 < 0$) and $\lambda_3 > 0$ ($\lambda_3 < 0$), they under (over) estimate the impact of import varieties and product quality. In a similar way, empirical fluctuations in relative consumption are defined as

$$\widehat{\mathsf{C}} - \widehat{\mathsf{C}}^* \equiv \mathsf{C} - \mathsf{C}^* - \psi\lambda_1 \mathsf{N}_D^R + \psi\lambda_2 \mathsf{N}_X^R + \lambda_3 \widetilde{\mathsf{q}}_X^R. \quad (9)$$

Finally, using the above mentioned empirically based fluctuations (8) and (9), we can rewrite the welfare-based relation (7) as the empirically based one as

$$\begin{aligned}\widehat{\mathsf{C}} - \widehat{\mathsf{C}}^* = \frac{2S_{ED}\omega - 1}{2S_{ED} - 1}\widehat{\mathsf{Q}} + \frac{(\omega - 1)\,S_{ED}}{2S_{ED} - 1}\widetilde{\mathsf{z}}_X^R + \frac{\psi\,(2\lambda_1 - 1)\,(\omega - 1)\,S_{ED}}{2S_{ED} - 1}\mathsf{N}_D^R \\ - \frac{\psi\,(2\lambda_2 - 1)\,(\omega - 1)\,S_{ED}}{2S_{ED} - 1}\mathsf{N}_X^R - \frac{(2\lambda_3 - 1)\,(\omega - 1)\,S_{ED}}{2S_{ED} - 1}\widetilde{\mathsf{q}}_X^R \quad (10)\end{aligned}$$

[8] Of course, the possibility of time-variant measurement parameters λ_1, λ_2 and λ_3 over the business cycles that might influence explanatory power of variety and quality cannot be excluded. Aghion et al. (2017) explore how systematically these coefficients are endogenously determined, while I leave the issue for the sake of simplicity.

[9] This is indeed the case in Corsetti et al. (2007) when they discuss the empirically relevant measures.

Note that the first term of the right-hand side of the equation has exactly the same coefficient as in (7), which determines the welfare-based relation. Here again, as put forward in proposition 1, there exists a wedge between unconditional KBS relation and conditional KBS relation. The signs on the number of domestic varieties N_D^R, export (import) varieties N_X^R and the product quality $\widetilde{\mathsf{q}}_X^R$ also depend on the values of λ_1, λ_2 and λ_3. Importantly, what drives the wedge between relative consumption growth and real exchange rate growth is not the above procedure passing from welfare to empirical basis. Different from Hamano (2013), the wedge exists fundamentally because of the reallocation of product varieties and qualities based on firm heterogeneity.[10]

Provided the above definition in the empirically relevant fluctuations, it is straightforward to derive empirically based fluctuations under complete asset markets. By plugging (8) and (9) into the first-order deviation version of the complete asset market condition (5), we obtain

$$\widehat{\mathsf{C}}-\widehat{\mathsf{C}}^{*}=\frac{1}{\gamma}\widehat{\mathsf{Q}}-\left(1-\frac{1}{\gamma}\right)\left\{\psi\left[S_{ED}\lambda_1\mathsf{N}_D^R-(1-S_{ED})\lambda_2\mathsf{N}_X^R)\right]-(1-S_{ED})\lambda_3\widetilde{\mathsf{q}}_X^R\right\}.$$

As can be seen, the tight positive link between relative consumption growth and real exchange rate growth in the original welfare-based relation is broken.[11] Note that this wedge is only possible in empirical basis: in welfare basis, the KBS correlation is always positive as (5) under complete financial markets. Under complete asset markets, firm heterogeneity alone cannot create a wedge between relative consumption growth and real exchange rate growth.

To summarize, the presence of firm heterogeneity and implied reallocation, as well as extent of market completeness and measurement errors, altogether may contribute to a biased conclusion about the state of international risk sharing. In the following section,

[10] Again, when $\lambda_1=\lambda_2=\lambda_3=0$, the expression (10) coincides to (7). In particular, by removing the fixed cost for exporting and setting $\lambda_1=S_{ED}$ and $\lambda_2=1-S_{ED}$, the expression (10) becomes identical to the one found in Hamano (2013) with changes only in the number of product varieties.

[11] Indeed, product quality and variety work here as a preference shock that breaks the tight relationship between the relative consumption and the real exchange rate implied by complete asset markets; see Stockman and Tesar (1995), Raffo (2010) and Mandelman et al. (2011).

I test the implication of the theoretical model and explore the existence of bias and its direction with actual data. Before moving on the empirical analysis, however, I discuss how the observable close to zero or even negative KBS correlation can be obtained in the theoretical model.

4 KBS Correlation in the Theoretical Model with Firm Heterogeneity

Does the theoretical model with firm heterogeneity reproduce a plausible KBS correlation as we see in the data? As argued in the preceding literature (Corsetti et al., 2008), the key driver of such a realistic KBS correlation is the wealth effect. I emphasize in particular the role played by product quality among complementary mechanisms.

4.1 Unconditional KBS Correlations

Table 5 reports empirically relevant unconditional KBS correlations implied by the theoretical model, namely, $Corr(\widehat{\mathsf{Q}}, \widehat{\mathsf{C}} - \widehat{\mathsf{C}}^*)$, under different degrees of financial market imperfections, together with those obtained with actual data. In particular, in specifying the (in)efficiency of empirically based measures, I use similar parameter values as in Feenstra (1994) and Ghironi and Melitz (2005) such that $\lambda_1 = \lambda_2 = \lambda_3 = 1$. The world average KBS correlation and that of the OECD average and the OECD average with respect to the United States are -0.025, -0.017 and 0.06, respectively. With these values, the KBS correlation is 0.16 under financial autarky. The bond economy provides a very similar correlation that is close to zero, 0.18. With an alternative empirically based measure such that $\lambda_1 = S_{ED}$, $\lambda_2 = \lambda_3 = 1 - S_{ED}$, the KBS correlations tend to become more negative: It is -0.3 under financial autarkey and -0.14 under bond economy. However, with complete financial markets, the empirically based KBS correlations are close to unity for either degree of misspecification indicating the difficulty in breaking up the tight positive link that we see in the welfare-based relation (5). In summary, the theoretical model can successfully reproduce the observed KBS correlation which is close to zero or even

Table 5: The KBS Correlation in the Models

	$Corr(\widehat{\mathsf{Q}}, \widehat{\mathsf{C}} - \widehat{\mathsf{C}}^*)$
World Average (Median)	-0.025 (-0.028)
OECD Average (Median)	-0.017 (-0.039)
OECD Average with US (Median)	0.06 (0.09)
$\lambda_1 = \lambda_2 = \lambda_3 = 1.$	
Financial Autarkey	0.16
Bond Economy	0.18
Complete Markets	1.00
$\lambda_1 = S_{ED}$, $\lambda_2 = \lambda_3 = 1 - S_{ED}$	
Financial Autarkey	-0.3
Bond Economy	-0.14
Complete Markets	1.00

Note: Data on per capital consumptions and real effective exchange rates are taken from Penn World Table, version 9.0 for the period of 1984 to 2011.

negative. I explore in the following how these plausible correlations are achieved in the theoretical model.

4.2 Wealth Effects

The imperfect international risk sharing or a close to zero or even negative KBS correlation in the model can be provided through a strong wealth effect that reverses the wealth redistributive movement of the terms of trade (the terms of labor). Corsetti et al. (2008) note a lower elasticity of substitution between domestically produced goods and imported goods and/or a high shock persistence as a driver of such a strong wealth effect. Hamano (2013) argues that a higher number of product varieties than what exists abroad brings the terms of labor into appreciation.

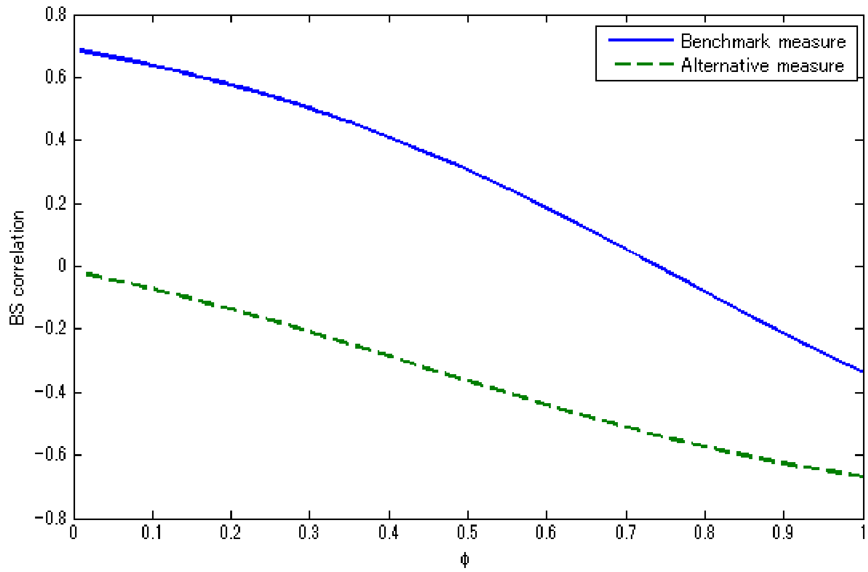
The theoretical model considered here embeds all the abovementioned wealth effects discussed in the literature. With firm heterogeneity, the transmission of the wealth effects materializes through a Harrod-Balassa-Samuelson (HBS) mechanism with endogenous entry and exit of exporters. Complementary to the possible devices, I focus on the role played by product quality as a driver of a strong wealth effect. In the model, as indicated in the equation (1), when the quality ladder in the economy increases (a higher value of ϕ), marginal costs of production increase for the country that produces higher quality goods and the terms of labor appreciate for that country. As seen in equation (8), such an appreciation in the terms of labor results in an appreciation in empirically based real exchange rate $\widehat{\mathsf{Q}}$, together with a higher consumption in the country compared to what exists abroad. In Figure 1, sensitivity analysis against the value of quality ladder, ϕ is reported. It is observed that as ϕ increases from zero, the KBS correlation changes from positive to negative in the benchmark calibration. The similar pattern is observed for the alternative calibration. Thus, a high value of quality ladder amplifies the HBS mechanism through a strong wealth effect and a resulting appreciation in the terms of labor.[12]

5 The Kollmann-Backus-Smith Correlation with Data and Its Systematic Bias

As argued in the previous section, the structural relationship between relative consumption and the observed real exchange rates across countries is conditional on changes in product qualities and the number of product varieties. This consideration may imply a bias in assessing the extent of international risk sharing. In this section, a regression analysis is performed. The KBS coefficients are found to be more positive once they are controlled with fluctuations in the number of traded varieties and their qualities.

[12]Figure 3 in Appendix B provides the result of the sensitivity analysis of the KBS correlation with respect to quality ladder ϕ under a bond economy. The similar pattern is observed as is the case under balanced trade. Additionally, Figure 4 gives the result of a sensitivity analysis with respect to the shock persistence of common productivity. Corsetti et al. (2008) notes the role played by shock persistence in generating a realistic KBS correlation. Higher income anticipated in the future due to higher persistence increases wealth today, providing a sharper appreciation in the terms of labor in transitory dynamics. In the model, I confirm the wealth effect due to a high shock persistence as a complementary mechanism.

Figure 1: The Kollmann-Backus-Smith Correlation and Quality Ladder (Balanced Trade)



Note: The figure reports the sensitivity result of the unconditional KBS correlation in the theoretical model against the quality ladder, ϕ, with the benchmark measurement error ($\lambda_1 = \lambda_2 = \lambda_3 = 1$) and the alternative measurement error ($\lambda_1 = S_{ED}$, $\lambda_2 = \lambda_3 = 1 - S_{ED}$) obtained under balanced trade.

The result indicates underestimation about the extent of international consumption risk sharing.

5.1 Data

For the analysis, a panel data set of 178 countries from 1984 to 2011 is used. Feenstra and Romalis (2014) provide a data set of their estimates of quality of exports and imports for each good (defined in four-digit SITC codes) for each country in the world for the period from 1984 to 2011.[13] Their estimates of product qualities are defined with respect to the world average, which is normalized to unity. Based on their estimates, I compute the *aggregate* quality of exports and imports for each country in each year in the sample. Specifically, based on the estimated quality of a particular good k of export ($s = X$) or import ($s = M$) of a country i for a year t, q^i_{kst}, the aggregated quality of that country's exports or imports for year t is defined as

[13] http://cid.econ.ucdavis.edu/Html/Quality_Data_Page.html

$$q^i_{st} = \sum^{N^i_{st}} ts^i_{kst} q^i_{kst}$$

where N^i_{st} is the number of exported or imported varieties (or precisely, the number of categories of goods defined in terms of four-digit SITC codes) with the ROW, and ts^i_{kst} is the share of exports or imports of that particular good k in total value of exports or imports of country i.

In Appendix C, I present descriptive statistics and the evolution of the number of varieties of exports and imports, as well as their quality for a number of selective countries.[14] The average number of export varieties (categories of goods) amounts to 299.6, while that of import varieties is 487.6 for each year. The number of import varieties tends to be much higher than the number of export varieties for the emerging and less developed countries such as Egypt, Mexico, Malaysia, Thailand and Zimbabwe, specifically at the beginning of the sample years. We see a large drop for the number of both export and import varieties beginning in the year 2009, the time of the "great trade collapse" following the financial crisis. Contrary to the number of traded varieties, the aggregate measure of quality of trade record much lower standard deviations as reported in the table. The quality of export tends to be higher than the quality of import for advanced economies (Canada, Germany, France, United Kingdom and United States), while we observe some catch up and an upward trend over time for emerging countries such as India, Malaysia and Thailand. China exports a large number of varieties while its quality stays at lower level over time. Overall, the aggregate measures indicate similar patterns about the trade of quality and product varieties in the world as observed in the literature and consistent with those in Feenstra and Romalis (2014).

The data on real per capital consumption, price level of consumption goods and per capital income are taken from the Penn World Table (pwt90).

[14]These countries are ARG (Argentina), CAN (Canada), CHE (Chile), CHN (China), DEU (Germany), EGY (Egypt), FRA (France), GBR (United Kingdom), IND (India), ITA (Italy), JPN (Japan), MEX (Mexico), MYS (Malaysia), THA (Thailand), USA (United States) and ZWE (Zimbabwe). The full data set of all countries is available upon request.

5.2 Empirical Analysis

Provided the abovementioned data, bilateral consumption growth, real exchange rate fluctuation, bilateral growth in the number traded varieties and their qualities are computed for each country pair in the world. The consumption growth rate of country i, ΔC_t^i, is defined as $\Delta C_t^i = \ln C_t^i - \ln C_{t-1}^i$.[15] The growth rate of the price level is defined as $\Delta P_t^i = -(\ln P_t^i - \ln P_{t-1}^i)$. The growth rates of the number of export or import varieties and that of quality are defined as $\Delta N_t^i = \ln N_{X,t}^i - \ln N_{M,t}^i - (\ln N_{X,t-1}^i - \ln N_{M,t-1}^i)$ and $\Delta q_t^i = \ln q_{X,t}^i - -\ln q_{M,t}^i - (\ln q_{X,t-1}^i - \ln q_{M,t-1}^i)$.

To roughly determine the implication in conditioning the KBS correlations with changes in the number of product varieties and their qualities as described in the previous section, I first present unconditional and conditional KBS correlations for each OECD country (country i) with respect to the United States during the entire sample period. In Figure 2, unconditional KBS correlations, $Corr(\Delta C_t^i - \Delta C_t^{USA},\ \Delta P_t^{USA} - \Delta P_t^i)$ and conditional correlations, $Corr(\Delta C_t^i - \Delta C_t^{USA},\ \Delta P_t^{USA} - \Delta P_t^i \mid \Delta N_t^i - \Delta N_t^{USA},\ \Delta q_t^i - \Delta q_t^{USA})$, are plotted, together with a 45-degree line. Unconditional correlations are close to zero or take even negative values for some countries. However, once they are conditioned with changes in the number of product varieties and their qualities, the correlations improve for a large number of countries: conditional correlations are situated above the 45-degree line.

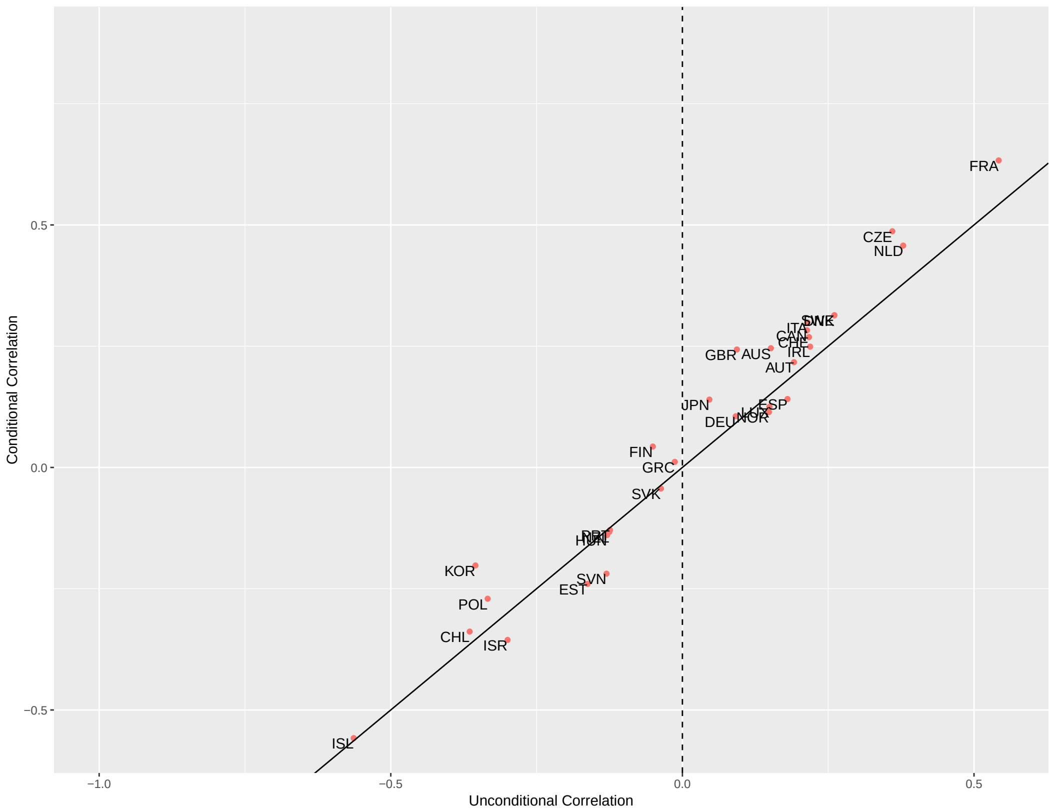
To investigate further the above systematic bias, I now use the entire sample and perform a panel regression. Based on the structural relation (10), whether the stability of KBS coefficients without or with controls of changes in variety of trade and their qualities is tested. The benchmark specifications are thus as follows:

$$\Delta C_t^i - \Delta C_t^j = \beta_0 + \beta_1(\Delta P_t^j - \Delta P_t^i) + \mu_j^i + \nu_t + \xi_t, \tag{11}$$

$$\Delta C_t^i - \Delta C_t^j = \beta_0 + \beta_1(\Delta P_t^j - \Delta P_t^i) + \beta_2\left(\Delta N_t^i - \Delta N_t^j\right) + \beta_3\left(\Delta q_t^i - \Delta q_t^j\right) + \mu_j^i + \nu_t + \xi_t, \tag{12}$$

[15] Income growth rate of country i and the world average are defined in a similar way.

Figure 2: Unconditional vs. Conditional KBS Correlation



Note: Unconditional and conditional Kollmann-Backus-Smith correlations of each OECD countries against the United States are plotted for the period from 1984 to 2011. The solid straight line shows 45-degree line.

where μ_j^i and ν_t represent country-pair-specific fixed effects and time fixed effects, respectively. ξ_t denotes i.i.d. shock.

Table 6 shows the results of estimation. As shown, by conditioning the KBS relation with changes in the number of varieties and quality, the KBS coefficients (β_1 of equation (12)) become more positive and significant compared to unconditioned KBS coefficients (β_1 of equation (11)) which are less positive and often insignificant. For panel regression with country-pair and time fixed effects, the coefficient changes from 0.037 to 0.051. For a further robustness check, I also include the relative income growth rate $\Delta Y_t^i - \Delta Y_t^j$ as a control variable, as in Kose et al. (2009), Hess and Shin (2010) and Baxter (2012). This can be considered as a proxy of relative changes in the domestic number of varieties N_D^R across countries in the theoretical relation (10). Interestingly, the KBS coefficient becomes insignificant and negative (−0.002) with GDP growth in the standard KBS regression without any changes in the number of traded varieties and qualities. This disappearing explanatory power of real exchange rate growth on relative consumption growth is also reported in Hess and Shin (2010). However, as can be seen, the KBS coefficients become more positive and significant (0.015) once they are controlled with changes in the number of product varieties and their qualities. I also present the result among OECD countries in Table 7, which shows a similar pattern. The unconditional KBS coefficients are negative; however, they become less negative by controlling for variety and quality growth.

To summarize, there exists a systematic bias of the KBS coefficients that arises by controlling for cross-country differences in product variety and quality of trade. By conditioning, the KBS coefficients increase, indicating a partial resolution of the puzzle and a better international risk sharing across countries.[16]

6 Conclusion

The paper explores the implication of the reallocation effect arising from firm heterogeneity on international consumption risk sharing. In the theoretical model, firms that are heterogeneous in their specific productivities choose their product qualities endogenously. Assuming that the creation of high-quality product variety requires higher marginal costs, a wealth effect that brings the real exchange rate into appreciation is generated, together with other complementary mechanisms that drive a realistic correlation between relative consumption and real exchange rates.

Specifically, the Kollmann-Backus-Smith correlation is shown to be conditional on turnover in the number of product varieties and product qualities based on heterogeneous

[16]Fitzgerald (2012) estimates the extent of international risk sharing based on the gravity equation in the trade literature. She shows that measured price indices tend to provide puzzling coefficients as the original KBS puzzle, indicating less risk sharing across countries. Instead of using such observable real exchange rates, however, by relying on fixed effect estimates of price indices in the gravity equation, she finds more favorable evidence for the presence of international risk sharing, especially among OECD countries. Price indices estimated using country fixed effect are welfare-consistent and thus include fluctuations in product quality and variety by definition. Her results note the same type of bias that could be provided by unobservable fluctuations in product quality and variety.

Table 6: KBS Regression: Full Sample

Dep Var:	$\Delta C_t^i - \Delta C_t^j$			
	(1)	(2)	(3)	(4)
$\Delta P_t^j - \Delta P_t^i$	0.037^{***}	0.051^{***}	-0.002	0.015^{***}
	(0.003)	(0.003)	(0.003)	(0.003)
$\Delta N_t^i - \Delta N_t^j$		-0.026^{***}		-0.026^{***}
		(0.002)		(0.002)
$\Delta q_t^i - \Delta q_t^j$		-0.054^{***}		-0.052^{***}
		(0.002)		(0.002)
$\Delta Y_t^i - \Delta Y_t^j$			0.218^{***}	0.195^{***}
			(0.003)	(0.004)
Country Pair FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	405,248	368,227	405,248	368,227
R^2	0.0005	0.005	0.011	0.012
Adjusted R^2	0.0004	0.005	0.010	0.012

Note: $\Delta C_t^i - \Delta C_t^j$, $\Delta P_t^j - \Delta P_t^i$, $\Delta N_t^i - \Delta N_t^j$, $\Delta q_t^i - \Delta q_t^j$ and $\Delta Y_t^i - \Delta Y_t^j$ represent the growth rate of consumption, the real exchange rate, the number of traded varieties, the quality of traded products and the income for country i with respect to country j. In parentheses, standard errors are reported. ***, ** and * indicate significance at the 10 % 5% and 1% levels, respectively.

Table 7: KBS Regression: OECD Countries

Dep Var:	$\Delta C_t^i - \Delta C_t^j$			
	(1)	(2)	(3)	(4)
$\Delta P_t^j - \Delta P_t^i$	-0.143^{***}	-0.103^{***}	-0.113^{***}	-0.086^{***}
	(0.007)	(0.007)	(0.006)	(0.006)
$\Delta N_t^i - \Delta N_t^j$		0.069^{***}		0.058^{***}
		(0.017)		(0.017)
$\Delta q_t^i - \Delta q_t^j$		-0.032^{***}		-0.036^{***}
		(0.007)		(0.006)
$\Delta Y_t^i - \Delta Y_t^j$			0.516^{***}	0.401^{***}
			(0.011)	(0.013)
Country Pair FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	12,684	11,372	12,684	11,372
R^2	0.037	0.023	0.174	0.102
Adjusted R^2	0.034	0.021	0.173	0.100

Note: $\Delta C_t^i - \Delta C_t^j$, $\Delta P_t^j - \Delta P_t^i$, $\Delta N_t^i - \Delta N_t^j$, $\Delta q_t^i - \Delta q_t^j$ and $\Delta Y_t^i - \Delta Y_t^j$ represent the growth rate of consumption, the real exchange rate, the number of traded varieties, the quality of traded products and the income for country i with respect to country j. In parentheses, standard errors are reported. ***, ** and * indicate significance at the 10 % 5% and 1% levels, respectively.

firms, and the conventional unconditional correlation can be biased. I test the implied conditional relation using world trade data and find that the KBS correlations become more positive, indicating underestimation of the extent of international risk sharing. Therefore, there is an unexplored gain of international trade with respect to international risk sharing. For future research, a detailed analysis about the conditional KBS correlation for different country groups and over time would be interesting.

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A Steady State

At the symmetric steady state, I assume without loss of generality that $Z = Z^* = f_E = f_E^* = z_{\min} = z_{\min}^* = 1$. In this symmetric steady state, I drop the asterisks, which denote Foreign variables and time indices. Note that $NFA = NX = 0$ and $Q = 1$ in the symmetric steady state. I choose the parameter χ so that the steady-state labor supply reaches unity as $L = 1$.

First, I solve the value of f_X so that it matches the empirical findings on the share of exporters. The free-entry condition gives $\widetilde{v}^s = w$. Thus, using the steady-state Euler equation for shareholdings, we have

$$\widetilde{d} = \frac{1 - \beta\,(1-\delta)}{\beta\,(1-\delta)} w. \tag{13}$$

Therefore, by the definition of $\widetilde{d}$, we obtain

$$\widetilde{d}_D + \frac{N_X}{N_D}\widetilde{d}_X = \frac{1 - \beta\,(1-\delta)}{\beta\,(1-\delta)} w. \tag{14}$$

Now, we rewrite $\widetilde{d}_D$ and $\widetilde{d}_X$ in the above expression. From the zero-profit export cutoff condition, we have

$$\widetilde{d}_X = w f_X \frac{\sigma - 1}{k - (\sigma - 1)}. \tag{15}$$

With the above expression and using the steady-state average domestic and export profits $\widetilde{d}_D$ and $\widetilde{d}_X$, $\widetilde{d}_D$ can be rewritten as

$$\widetilde{d}_D = \frac{1}{\tau^{1-\omega}} \left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)} \left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} \left[\frac{\sigma - 1}{k - (\sigma - 1)} + 1\right] w f_X, \tag{16}$$

where we use the fact that $\widetilde{\rho}_D/\widetilde{q}_D = \frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w}{\widetilde{q}_D\widetilde{z}_D}$, $\widetilde{\rho}_X/\widetilde{q}_X = \frac{\sigma}{\sigma-1}\tau\frac{1}{1-\phi}\frac{w}{\widetilde{q}_X\widetilde{z}_X}$ and $\widetilde{q}_D = \left(\frac{\phi}{1-\phi}\widetilde{z}_D\right)^{\phi}$, $\widetilde{q}_X = \left(\frac{\phi}{1-\phi}\widetilde{z}_X\right)^{\phi}$.

Plugging (16) and (15) into (14), we obtain

$$\left[\frac{1}{\tau^{1-\omega}}\left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)}\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)}\frac{k}{k-(\sigma-1)}+\frac{N_X}{N_D}\frac{\sigma-1}{k-(\sigma-1)}\right]f_X = \frac{1-\beta(1-\delta)}{\beta(1-\delta)}. \quad (17)$$

In the above expression, $\widetilde{z}_D$ is given by Pareto distribution. $\frac{N_X}{N_D}$ is set to 0.21. Given this value, which is also from the Pareto distribution, $\widetilde{z}_X = 2.9425$ is required with the values of parameters in the benchmark calibration. By plugging these values into the above equation, f_X can be solved.

Provided this subsidy, the steady-state labor supply is set to unity by controlling χ. Thus, the labor market clearing condition in the steady state gives

$$w = \left[N_E\widetilde{v}^s + (\sigma-1)N_D\widetilde{d} + +\sigma N_X w f_X\right].$$

The equation about the motion of firms gives $N_E = \frac{\delta}{1-\delta}N_D$. Using (13) and replacing $\widetilde{v}^s$ as previously, the above expression can be rewritten as

$$N_D = \frac{1}{\frac{\delta}{1-\delta}+(\sigma-1)\frac{1-\beta(1-\delta)}{\beta(1-\delta)}+\sigma\frac{N_X}{N_D}f_X}. \quad (18)$$

This is the solution for N_D.

Finally, the second equation can be obtained using the steady-state price index as

$$\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} + \tau^{1-\omega}\left(\frac{N_X}{N_D}\right)^{-\psi(1-\omega)} = \left(\frac{N_D^{\psi}}{\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w}{\widetilde{q}_X\widetilde{z}_X}}\right)^{1-\omega} \quad (19)$$

By rearranging this equation, we have the solution for w:

$$w = \left\{\left(N_D^{\psi}\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{1}{\widetilde{q}_X\widetilde{z}_X}\right)^{1-\omega}\left[\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)}+\tau^{1-\omega}\left(\frac{N_X}{N_D}\right)^{-\psi(1-\omega)}\right]\right\}^{\frac{1}{-(1-\omega)}}.$$

Once w is found, N_D can be found from (18). The steady-state values of the other variables are relatively easy to find. In particular, the value of parameter χ is set by

$\chi = wC^{-\gamma}$ so that $L = 1$. It gives 0.1829 with the parameter values of the benchmark calibration.

Finally, we define steady-state shares that appear in calibrating the first-order set of equations. The share of domestic and imported goods in total expenditures is

$$S_{ED} \equiv \rho_H^{1-\omega} \text{ and } 1 - S_{ED} \equiv \rho_F^{1-\omega}.$$

The steady-state share of fixed export costs, dividends on domestic, export and total sales relative to C are respectively defined as

$$S_{FX} \equiv \frac{N_X w f_X}{C}, \;\; S_{DD} \equiv \frac{N_D \widetilde{d}_D}{C}, \;\; S_X \equiv \frac{N_X \widetilde{d}_X}{C}, \;\; S_D \equiv \frac{N_D \widetilde{d}}{C}.$$

The steady-state share of investments, wage and consumption relative to C are respectively defined as

$$S_I \equiv \frac{N_E v^s}{C}, \quad S \equiv \frac{w}{C}, \quad S_M \equiv \frac{M}{C}.$$

B Sensitivity Analysis

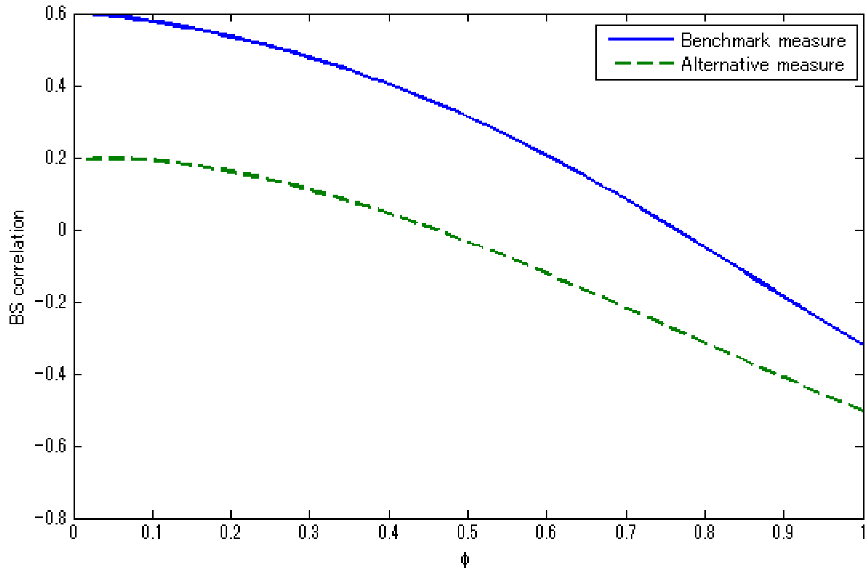
C Data

Table 8: Descriptive Statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Nb of exported varieties	5,012	299.6	255.5	0	71	550	816
Quality of exported goods	4,673	1.0	0.3	0.2	0.8	1.1	8.0
Nb of imported varieties	5,012	487.6	220.8	0	340	670	845
Quality of imported goods	4,698	1.0	0.1	0.3	0.9	1.0	1.6

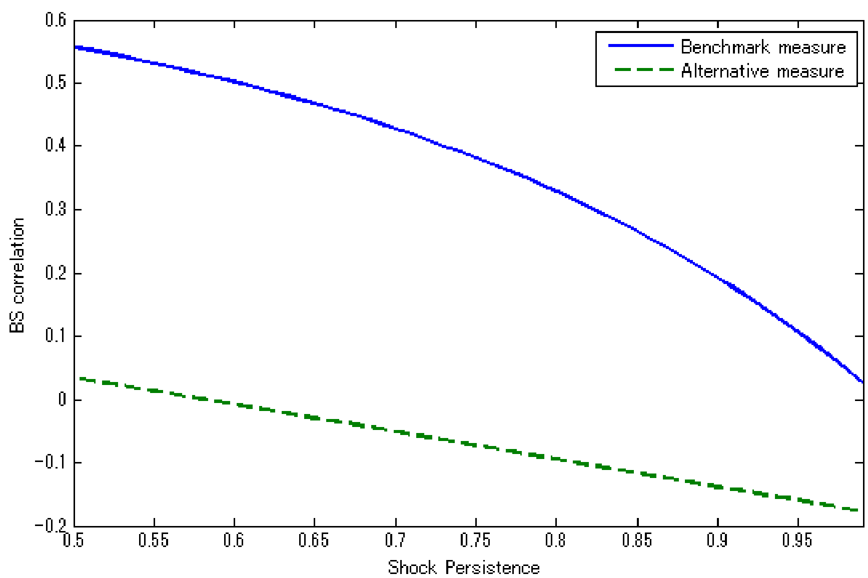
Source: Feenstra and Romalis (2014) and the author's calculation.

Figure 3: The Kollmann-Backus-Smith Correlation and Quality Ladder (Bond Economy)



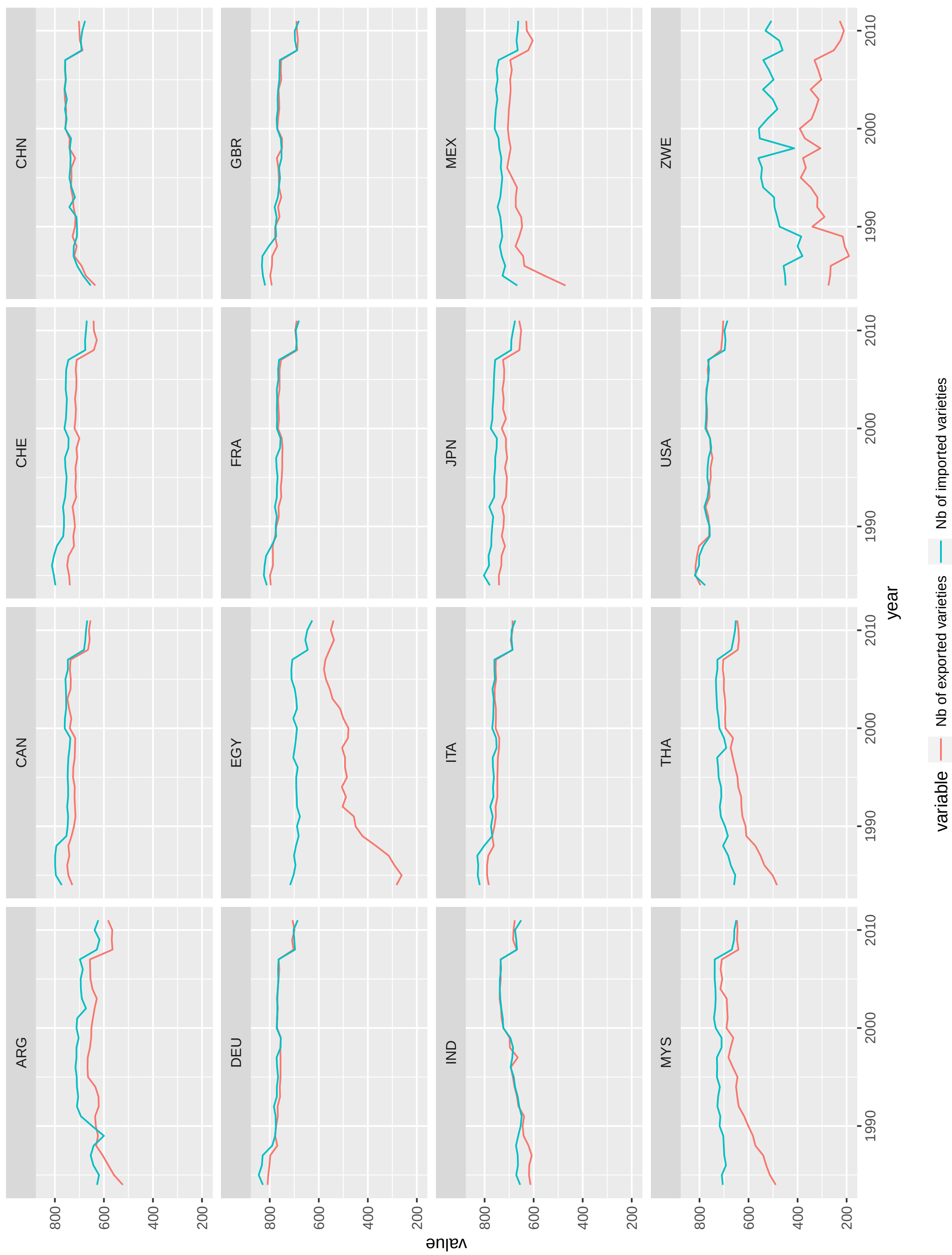
Note: The figure reports the sensitivity result of the unconditional KBS correlation in the theoretical model against the quality ladder, ϕ, with the benchmark measurement error ($\lambda_1 = \lambda_2 = \lambda_3 = 1.$) and the alternative measurement error ($\lambda_1 = S_{ED}$, $\lambda_2 = \lambda_3 = 1 - S_{ED}$) obtained under the bond economy.

Figure 4: The Kollmann-Backus-Smith Correlation and Shock Persistence (Bond Economy)



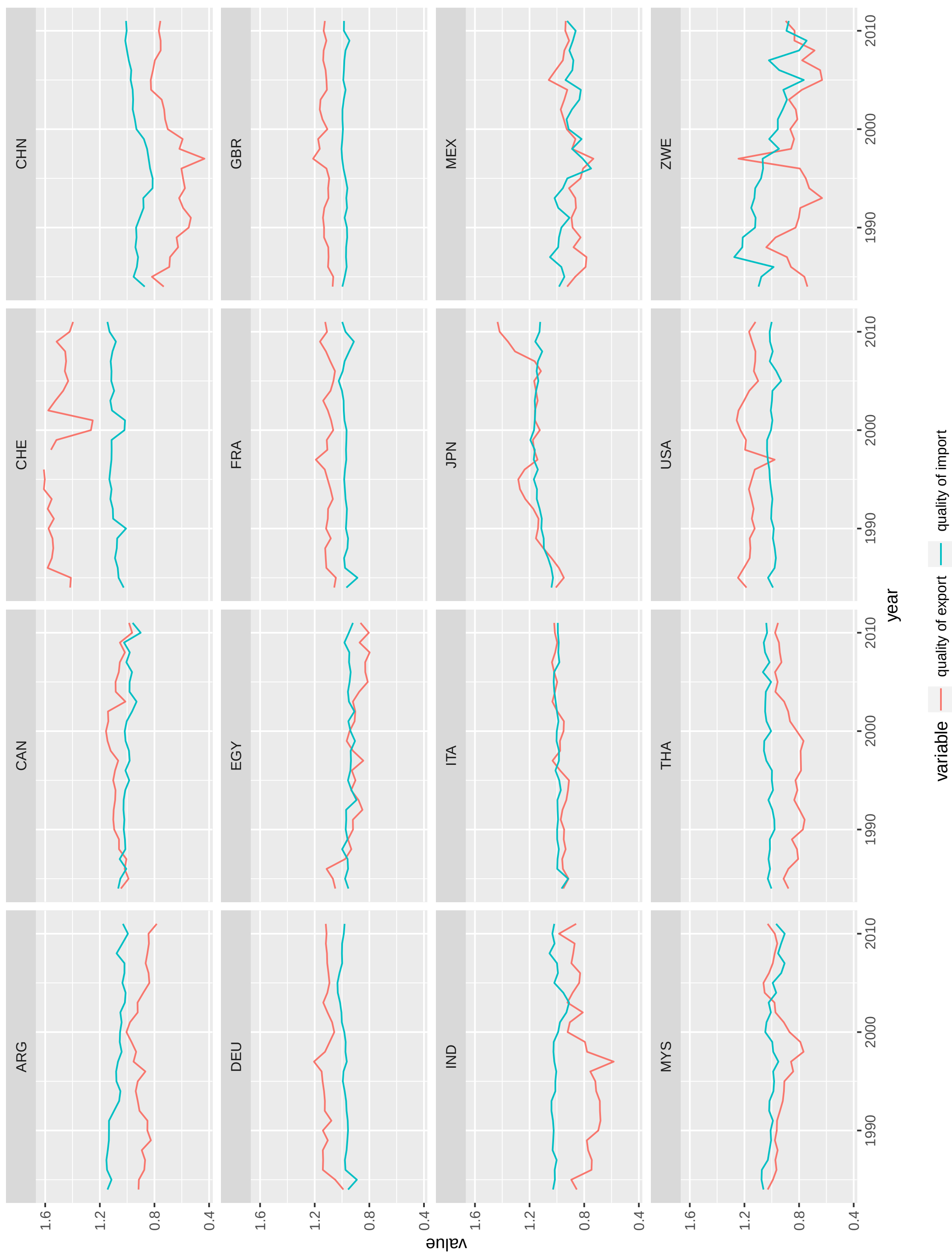
Note: The figure reports the sensitivity result of the unconditional KBS correlation in the theoretical model against the shock persistence of the productivity process, Z_t, with the benchmark measurement error ($\lambda_1 = \lambda_2 = \lambda_3 = 1.$) and the alternative measurement error ($\lambda_1 = S_{ED}$, $\lambda_2 = \lambda_3 = 1 - S_{ED}$) obtained under the bond economy.

Figure 5: Evolution of the Number of Exported and Imported Varieties



Note: Evolution of the number of exported and imported varieties of the selected countries from 1984 to 2011. Source: Feenstra and Romalis (2014) and the author's calculation.

Figure 6: Evolution of Exported and Imported Quality



Note: Evolution of the number of exported and imported varieties of the selected countries from 1984 to 2011. Source: Feenstra and Romalis (2014) and the author's calculation.