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# A Characterization of Optimum Fee Schemes for Delegated Portfolio Management

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Abstract

We examine a moral hazard problem on delegated portfolio management. Focusing on amounts of assets under management (AUM), we investigate the existence of optimal fee schemes for asset owners. First, we present the solutions of the first and the second best problem. Next, under some additional restrictions, we characterize the solution of the second best problem. As a result, a kind of incentive fee (an incentive fee with upper bound) is shown to be the only optimum fee scheme.

*Keywords:* Delegated portfolio management; Incentive fee scheme; Principal-agent model; Alignment; Asset under management

*JEL classification:* G19; G23

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# 1 Introduction

On active management, while excess return merely represents the relative value of active management against passive management, net excess return corresponds to final gains of asset owners who make contracts of active management. This implies that net excess return is more important for asset owners than excess return.

It has been pointed out for a long time that active management products realizing positive net excess returns are difficult to be found for asset owners (e.g., Jensen (1968), Carhart (1997)). Berk and Green (2004) attribute this phenomenon to cash inflow into products with proven track records. They show that, assuming the delegate investment market is perfectly competitive and the marginal cost of active fund management is positive and increasing, net excess return of every successful product comes, inevitably and immediately, to zero.

Typically, however, contracts between a management company and asset owners do not be made simultaneously. For asset owners ahead of the others in contracts, there can be room for preventing cash inflow by devising fees they propose to the management company.

Relating to reality, GPIF, the largest pension fund in Japan, has introduced a performance based fee structure (a kind of incentive fee) in active manager selection in 2018. According to Jimba (2018), the new structure emphasizes *alignment* so that the level of management fee is determined corresponding to levels of realized excess returns against the benchmark, guaranteeing the level of fee payed for the passive product as minimum.

While several works deal with incentive fees on active management (e.g., Ross(1973), Chevalier and Elison (1997), Ou-Yang (2003)), their desirability and relation to AUM have not been examined yet.

We present a model taking into account the time structure of bargaining. We introduce an extensive form game to consider AUM under asymmetric information between an asset owner and a manager, and examine the existence of optimum fee schemes for the asset owner. After presenting the first best solution, we show that second best solution does not exist under a mild condition. Then, under additional conditions, we show that a specific incentive fee with upper bound is optimal. This fee scheme not only prevents AUM from being enlarged, but also has some reasonable properties such as *alignment* and independence of risk characteristics.

The model and the first best solution are presented in Section 2. In Section 3, we examine the second best problem with a mild condition. In Section 4, we introduce a few axioms and characterize optimum incentive fees. Simulation results are presented in Section 5, and concluding remarks are in Section 6.

# 2 Model and the First Best Problem

## 2.1 Model

There are two (relatively massive) asset owners, 1 and 2, and an asset management company, $M$. Each asset owner $i$ ($i = 1, 2$) has its asset as $A_i$.

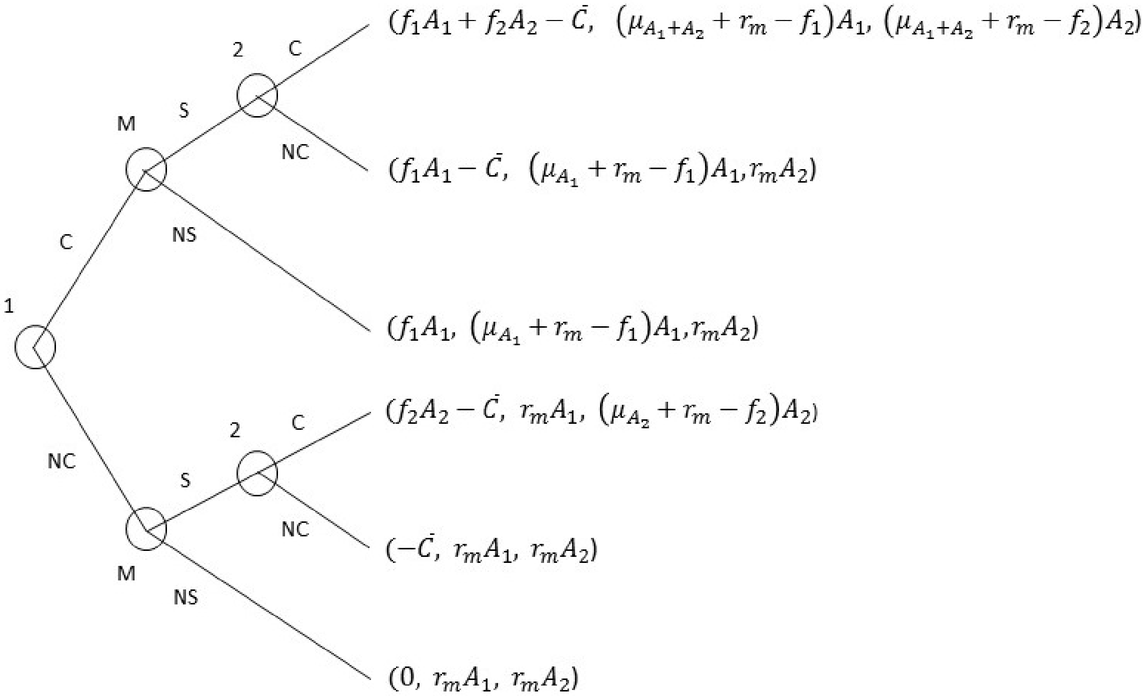


**Figure1: The Game Tree of $\Gamma$**

We put payoffs in the order as (M's payoff, 1's payoff, 2's payoff).
$S$ denotes *Sales*, $NS$ denotes *Not Sales*, $C$ denotes *Contract*, and $NC$ denotes *Not Contract*.

The extensive form game $\Gamma$ is illustrated in **Figure1**. 1 plays first and decides whether 1 contracts with $M$ for an active fund. On the one hand, if 1 contracts with $M$ for the active fund, then $M$ decides whether $M$ tries to enlarge the fund size with cost $\bar{C}$. While 2 decides whether 2 contracts with $M$ for the active fund when $M$ tries to contract with 2, 2 invests in the passive fund when $M$ closes the active fund.

On the other hand, if 1 invests in the passive fund at the beginning, then $M$ decides whether $M$ sells the active fund for 2 with cost $\bar{C}$. While 2 decides whether 2 contracts with $M$ for the active fund when $M$ tries to contract with 2, 2 invests in the passive fund when $M$ closes the active fund.

Assume that passive management is costless and available at no charge. By making a contract of active management, each asset owner expectedly gains (expected total returns minus fees) $\times$ (its asset). Notations are as follows:

**Notations.**

- $r_m$ : The expected market return (expected return of the passive fund).
- $C(\cdot)$ : The cost function of assets for active management.
- $\mu_A \equiv \alpha + (\beta - 1)r_m - \frac{C(A)}{A}$ : Expected excess returns per $A$ against $r_m$.
- $\alpha + \beta r_m - \frac{C(A)}{A}$ $(= r_m + \mu_A)$ : Expected total returns per $A$.
- $\bar{f}$ : The minimum fee.
- $f_i$ : Fixed fees proposed by asset owner $i$ $(i = 1, 2)$.

Note that all players know the whole structure of the game. All players are risk neutral. $M$ reports a realized total return at the end of this one-shot game.

Assumptions are as follows:

**Assumptions.**

- $\forall A \geq 0,\ C(A) \geq 0,\ C'(A) > 0,$ and $C''(A) > 0$.
- $C(0) = 0,\ \lim_{A \to \infty} C'(A) = \infty$.
- $\forall i,\ \mu_{A_i} > \mu_{A_1 + A_2} \geq f_i \geq \bar{f} > 0$.
- $0 < f_2 A_2 - \bar{C} < (\frac{C(A_1+A_2)}{A_1+A_2} - \frac{C(A_1)}{A_1})A_1$.

In short, we assume that the marginal cost is positive and increasing, expected net excess returns are non-negative, and 1's loss caused by $M$'s sales is larger than $M$'s gain through $M$'s sales.

We focus on fee schemes which 1 offers to $M$. At the beginning, as illustrated in **Figure1**, 1 proposes $f_1$ alone (2 proposes $f_2$ alone throughout this paper). From **Figure1**, the sub-game perfect equilibrium of $\Gamma$ is (Sales $\cdot$ Sales, Contract, Contract $\cdot$ Contract), and the equilibrium path is ($M$: Sales, 1: Contract, 2: Contract). This consequence corresponds to the result obtained by Berk and Green (2004): because assets flow as much as possible in the active fund of $M$, gains of asset owners decrease. In short, asset owner 1 dose not prevent cash inflow when 1 merely proposes a fixed fee $f_1$. Note that this result holds regardless of the level of $f_1$ [1].

[1] Throughout this paper, we assume that players choose strategies which are analytically more important if several strategies of the players produce the same payoff to them.

## 2.2 The First Best Problem

In the previous subsection, asset owner 1 cannot prevent another asset owner 2 from contracting with management company $M$ after 1 contracts with $M$, even 1 knows that the contract between $M$ and 2 makes 1's payoff lower. In this subsection, however, under symmetric information between 1 and $M$, 1 may compensate some amount of $X$ ($\geq 0$) for $M$ if $M$ does business only with the existing customer 1. That is, 1 offers to $M$ a vector ($f_1^{cp}$(fixed fee), $X$(compensation for $NS$)). A new game $\hat{\Gamma}$ is illustrated in **Figure2**. 1's strategy $(f_1, 0)$ in **Figure2** coincides with the 1's strategy $C$ in **Figure1**.

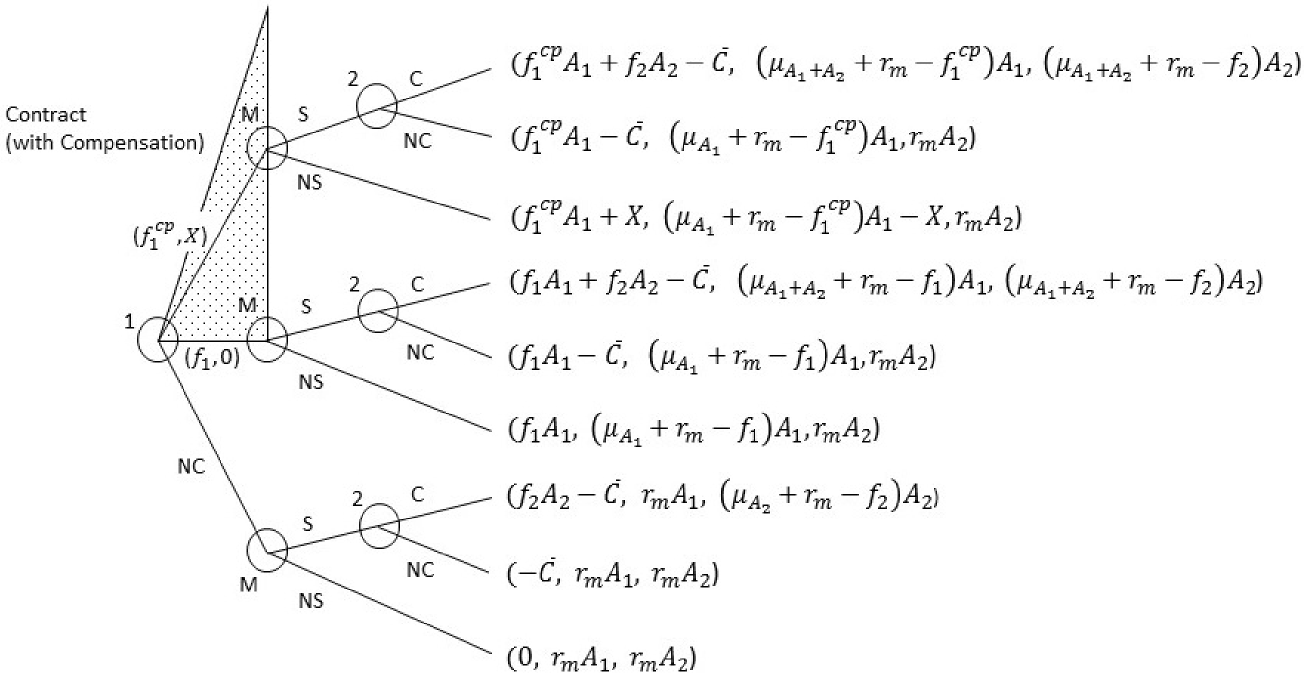


**Figure2: The Game Tree of $\hat{\Gamma}$**

We put payoffs in the order as (M's payoff, 1's payoff, 2's payoff).
$S$ denotes *Sales*, $NS$ denotes *Not Sales*, $C$ denotes *Contract*, and $NC$ denotes *Not Contract*.

The payoff maximization problem for 1 on this game is the first best problem stated below:

[[The First Best Problem]]

$$\max_{f_1^{cp},\ X(\geq 0)} \ (\mu_{A_1} + r_m - f_1^{cp})A_1 - X$$
$$s.t.\ \ f_1^{cp}A_1 + X \ \geq\ f_1^{cp}A_1 + f_2A_2 - \bar{C}.$$

The solution of this problem is stated as follows:

[Proposition 1]

The solution of the first best problem is as follows:

$$(f_1^{cp}, X) = (\bar{f}, f_2A_2 - \bar{C}).$$

(Proof)

It is immediately derived from **Figure2**. ■

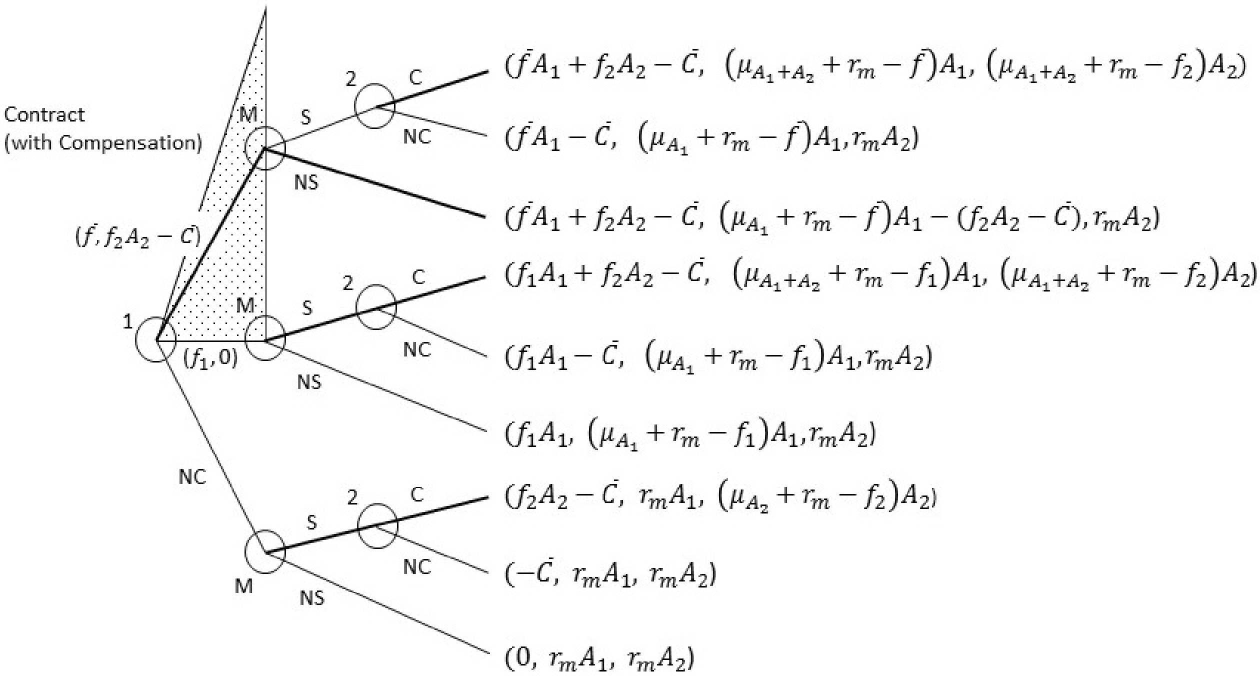


**Figure3: The Subgame Perfect Equilibrium of $\hat{\Gamma}$**

We put payoffs in the order as (M's payoff, 1's payoff, 2's payoff).
$S$ denotes *Sales*, $NS$ denotes *Not Sales*, $C$ denotes *Contract*, and $NC$ denotes *Not Contract*.

The subgame perfect equilibrium of $\hat{\Gamma}$ is

$$((((\{S\}(\text{if } X < f_2A_2 - \bar{C}), \{NS\}(\text{if } X \geq f_2A_2 - \bar{C})) \cdot S), (\bar{f}, f_2A_2 - \bar{C}), (\{C\})),$$

and it is illustrated in **Figure3**.

In this equilibrium, 1 gains the following amount more than its payoff in the equilibrium of $\Gamma$:

$$\begin{aligned}&(f_1 - \bar{f})A_1 + (\mu_{A_1} - \mu_{A_1+A_2})A_1 - (f_2A_2 - \bar{C})\\&\qquad = (f_1 - \bar{f})A_1 + (\frac{C(A_1 + A_2)}{A_1 + A_2} - \frac{C(A_1)}{A_1})A_1 - (f_2A_2 - \bar{C}).\end{aligned}$$

# 3 The Second Best Problem and Lower Bound Constraint of Fee

Assume, in this section, that there exists asymmetric information between 1 and $M$. In other words, 1 has no way of detecting whether $M$ contracts with 2 after 1 and $M$ make a contract [2].

## 3.1 The Second Best Problem

We proceed to solve the second best problem and find the optimum fee schemes. Fee scheme consists of the two parts: one is the lowest fixed fee $\bar{f}$ which is guaranteed regardless of the levels of the realized excess returns, and the other is an additional fee scheme which is a function of excess return and denoted as $x(r)$.

Since excess returns of the active fund depend on AUM, we introduce the following two normal distributions $g_{A_1}$ and $g_{A_1+A_2}$ with the same variance $\sigma^2$:

$$\forall r,\ g_{A_1}(r) = \frac{1}{(2\pi)^{1/2}\sigma}\ exp\{-\frac{(r - \mu_{A_1})^2}{2\sigma^2}\}.$$

$$\forall r,\ g_{A_1+A_2}(r) = \frac{1}{(2\pi)^{1/2}\sigma}\ exp\{-\frac{(r - \mu_{A_1+A_2})^2}{2\sigma^2}\}.$$

Note that $g_{A_1}$ ($g_{A_1+A_2}$) is a probability distribution function of excess returns when $M$ chooses $NS$ ($S$) after $M$ makes a contract with 1. The difference between the two is only in their mean values. These are illustrated in **Figure4**:

[2] This means, in practice, that $M$ may sell for 2 a remarkably similar but different active fund comparing to the fund offered for 1 so that $M$ needs not inform 1 of the contract with 2.

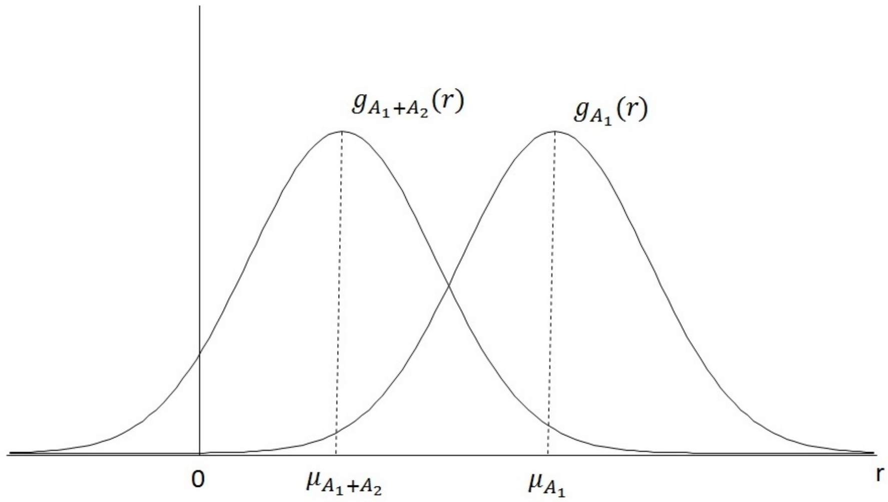


**Figure4: Two Normal Distributions ($g_{A_1}$ and $g_{A_1+A_2}$)**

In order to find optimum additional fee schemes for 1, we formulate the constraint minimization problem as below:

[[The Second Best Problem]]

$$\min_{x(r)} \; A_1 \int_{-\infty}^{\infty} g_{A_1}(r)x(r)dr$$

$$\begin{aligned} s.t. \;\; & A_1 \int_{-\infty}^{\infty} (g_{A_1}(r) - g_{A_1+A_2}(r))x(r)dr \geq f_2 A_2 - \bar{C}, \\ & A_1 \int_{-\infty}^{\infty} g_{A_1+A_2}(r)x(r)dr \geq 0, \\ & \{\mu_{A_1} - (\bar{f} + \int_{-\infty}^{\infty} g_{A_1}(r)x(r)dr)\}A_1 \geq 0. \end{aligned}$$

The objective function is the total additional payment under $x(r)$ when $M$ chooses $NS$. The first constraint requires that $M$ has no incentive to choose $S$ under $x(r)$. The second constraint requires that the total additional payment cannot be negative under $x(r)$. The third constraint requires that choosing $NC$ does not produce more gain for 1 under $x(r)$.

The solution of this problem is stated as follows:

[Proposition 2]
The solution of the second best problem produces the same payoff for 1 obtained by the first best solution only if $x(r) < 0$ for some $r$.

(Proof)
From the second constraint, we have

$$A_1 \int_{-\infty}^{\infty} g_{A_1+A_2}(r)x(r)dr = 0.$$

From this and the first constraint, we have

$$A_1 \int_{-\infty}^{\infty} g_{A_1}(r)x(r)dr = f_2 A_2 - \bar{C}.$$

Note that these two equations hold if and only if 1 obtains the same payoff as that of the first best solution. Obviously, these two equations cannot simultaneously hold if $x(r) \geq 0$ for all $r$. ■

### 3.2 Lower Bound Constraint of Fee

By Proposition 2, we find that the moral hazard problem is resolved if asset owners are allowed to introduce fees which take a value less than $\bar{f}$ on some excess return. However, these fees are prohibited in the market of delegated portfolio management. Hence, we have to impose the condition stated below:

**(a1)**

$$\forall r,\ x(r) \geq 0.$$

This axiom requires that, whatever excess returns $M$ realizes, $M$ surely receives the minimum payment $\bar{f}A_1$.

Unfortunately, under **(a1)**, we have no solutions:

[Proposition 3]
Under **(a1)**, there exist no solutions of the second best problem.

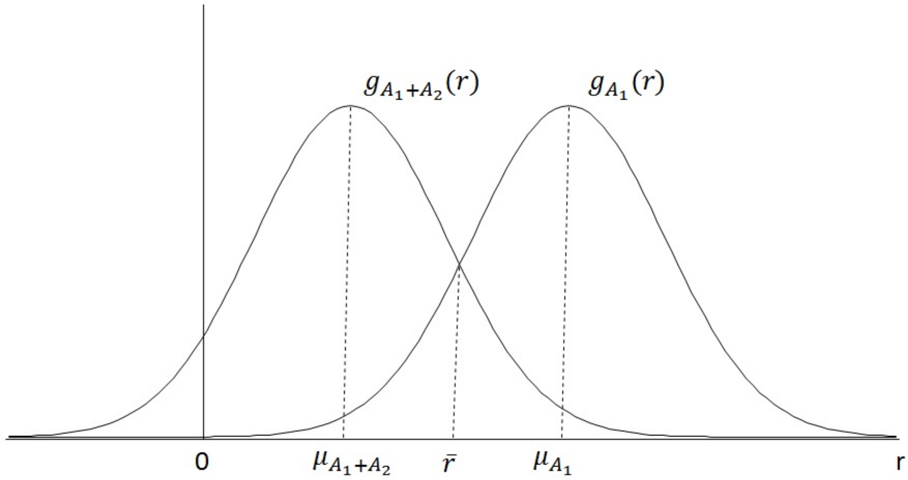


**Figure5: Proof of the Proposition 3**

(Proof) See **Figure5**. $\bar{r}$ denotes the excess return which satisfies $g_{A_1}(\bar{r}) = g_{A_1+A_2}(\bar{r})$.

Suppose that there exists a solution of the second best problem under **(a1)**, and let it be $x(r)$.

Suppose that there exists $r_1 \leq \bar{r}$ such that $x(r_1) > 0$. It turns out that $x(r)$ is not a solution of the problem because there exists $\epsilon > 0$ such that $x(r_1) - \epsilon \geq 0$ and another additional fee scheme

$$x^*(r) = \begin{cases} x(r) & \text{if } r \neq r_1 \\ x(r) - \epsilon & \text{if } r = r_1 \end{cases}$$

produces less cost than $x(r)$, while satisfying three constraints. Therefore, we have that $\forall r \leq \bar{r},\ x(r) = 0$.

Next, suppose that there exists $r_1 > \bar{r}$ such that $x(r_1) > 0$. Consider the following additional fee scheme

$$x^*(r) = \begin{cases} x(r) & \text{if } r \neq r_1, r_2 \\ x(r) - \epsilon & \text{if } r = r_1 \\ x(r) + \frac{g_{A_1}(r_1) - g_{A_1+A_2}(r_1)}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}\epsilon & \text{if } r = r_2 \end{cases}$$

where $r_2 > r_1$, $\epsilon > 0$ and $x(r_1) - \epsilon \geq 0$.

This fee scheme produces less cost than $x(r)$ while satisfying three constraints. This is shown as follows:

Increment of the left hand side of the first constraint

$$= A_1[\{(g_{A_1}(r_1) - g_{A_1+A_2}(r_1))\}(-\epsilon) + \{(g_{A_1}(r_2) - g_{A_1+A_2}(r_2))\frac{g_{A_1}(r_1) - g_{A_1+A_2}(r_1)}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}\epsilon\}]$$
$$= A_1\epsilon\{-(g_{A_1}(r_1) - g_{A_1+A_2}(r_1)) + (g_{A_1}(r_1) - g_{A_1+A_2}(r_1))\}$$
$$= 0.$$

Increment of the left hand side of the second constraint

$$= A_1\{g_{A_1+A_2}(r_1)(-\epsilon) + g_{A_1+A_2}(r_2)\frac{g_{A_1}(r_1) - g_{A_1+A_2}(r_1)}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}\epsilon\}$$
$$= A_1\epsilon\{\frac{-g_{A_1+A_2}(r_1)(g_{A_1}(r_2) - g_{A_1+A_2}(r_2)) + g_{A_1+A_2}(r_2)(g_{A_1}(r_1) - g_{A_1+A_2}(r_1))}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}\}$$
$$= A_1\epsilon\{\frac{-g_{A_1+A_2}(r_1)g_{A_1}(r_2) + g_{A_1}(r_1)g_{A_1+A_2}(r_2)}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}\}$$
$$\geq A_1\epsilon\{\frac{-g_{A_1+A_2}(r_1)g_{A_1}(r_2) + g_{A_1+A_2}(r_1)g_{A_1+A_2}(r_2)}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}\}$$
$$= -A_1 g_{A_1+A_2}(r_1)\epsilon.$$

Increment of the cost

$$= -A_1 g_{A_1}(r_1)\epsilon + A_1 g_{A_1}(r_2)\epsilon\frac{g_{A_1}(r_1) - g_{A_1+A_2}(r_1)}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}$$
$$= A_1\epsilon\frac{-g_{A_1}(r_1)(g_{A_1}(r_2) - g_{A_1+A_2}(r_2)) + g_{A_1}(r_2)(g_{A_1}(r_1) - g_{A_1+A_2}(r_1))}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}$$
$$= A_1\epsilon\frac{g_{A_1}(r_1)g_{A_1+A_2}(r_2) - g_{A_1+A_2}(r_1)g_{A_1}(r_2)}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}$$
$$= A_1\epsilon\frac{g_{A_1}(r_2)g_{A_1+A_2}(r_2)}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}\ (\frac{g_{A_1(r_1)}}{g_{A_1}(r_2)} - \frac{g_{A_1+A_2}(r_1)}{g_{A_1+A_2}(r_2)})$$
$$= A_1\epsilon\frac{g_{A_1}(r_2)g_{A_1+A_2}(r_2)}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}\ [exp\{-\frac{(r_1-\mu_{A_1})^2}{2\sigma^2} + \frac{(r_2-\mu_{A_1})^2}{2\sigma^2}\} - exp\{-\frac{(r_1-\mu_{A_1+A_2})^2}{2\sigma^2} + \frac{(r_2-\mu_{A_1+A_2})^2}{2\sigma^2}\}]$$
$$= A_1\epsilon\frac{g_{A_1}(r_2)g_{A_1+A_2}(r_2)}{g_{A_1}(r_2) - g_{A_1+A_2}(r_2)}\ [exp\{\frac{(r_1+r_2-2\mu_{A_1})(r_2-r_1)}{2\sigma^2}\} - exp\{\frac{(r_1+r_2-2\mu_{A_1+A_2})(r_2-r_1)}{2\sigma^2}\}]$$
$$< 0$$

Note that, since $x(r_1)-\epsilon \geq 0$, the left hand side of the second constraint is more than or equal to $A_1 g_{A_1+A_2}(r_1)\epsilon$ under $x^*(r)$. Also note that, (Increment of the cost $< 0$) implies that increment of the left hand side of the third constraint is positive.

As is shown above, it turns out that $x(r)$ is not a solution of the problem. Therefore, we have that $\forall r > \bar{r},\ x(r) = 0$.

Hence, we obtain $x(r) = 0$ for all $r$. However, this does not satisfy the first constraint. Therefore, under **(a1)**, there exist no solutions of the second best problem. ■

# 4 Modified Second Best Problems

In the previous section, we find that there exist no optimum solutions of the second best problem under non-negative constraint of $x(r)$. In this section, by properly restricting the set of $x(r)$ further, we examine modified second best problems. We find solutions and characterize the optimum additional fee scheme $x(r)^*$ by some desirable properties of the fee schemes.

## 4.1 Additional Axioms

The second axiom requires that, 1 should not borrow money for payment:

**(a2)**

$$\forall r \geq \bar{f},\ x(r) \leq r - \bar{f}.$$

The third axiom is related to the performance of the active fund relative to the passive management:

**(a3)**

$$\forall r \leq \bar{f},\ x(r) = 0.$$

This axiom insists that, if $M$ underperforms the passive management, then additional payment for $M$ should be equal to 0. That is, if total return (- the minimum fee $\bar{f}$) under-performs the market return, then the level of additional fee is inevitably equal to 0.

The fourth axiom requires that, additional fee schemes should be designed to prevent another moral hazard action possibly taken by $M$. $M$ may rearrange its portfolio so as to increase its expected reward. $M$ possibly takes more risks without 1's agreement, and this tendency is reinforced if relatively higher management fees will be payed for relatively higher excess returns. In short, this axiom requires that additional fee schemes should be independent of risk characteristics. There are two versions:

**(a4-1)**

$$A_1 \int_{-\infty}^{\infty} g_{A_1}(r)x(r)dr = A_1 \int_{-\infty}^{\infty} h_{A_1}(r)x(r)dr$$

for all $h_{A_1}(r)$, which is a normal distribution function whose mean value is $\mu_{A_1}$.

**(a4-2)**

$$A_1 \int_{-\infty}^{\infty} g_{A_1}(r)x(r)dr \geq A_1 \int_{-\infty}^{\infty} h_{A_1}(r)x(r)dr$$

for a specific $h_{A_1}(r)$, where a probability distribution function $h_{A_1}(r)$ is made from $g_{A_1}(r)$ by increasing its variance.

**(a4-1)** represents a situation where 1 has no ability of monitoring $M$'s actions or $M$ confronts no technical constraints for taking more risks.

Relating to **(a4-1)**, we propose the following lemma which strongly influences the final result:

**Lemma.**
Suppose that $x(r)$ is continuous. Then, a necessary and sufficient condition of **(a4-1)** is that $x(r)$ is an odd function around $(\mu_{A_1}, x(\mu_{A_1}))$.

**Proof.**
Suppose that $x(r)$ is continuous.

$(\Leftarrow)$

Suppose that $x(r)$ is an odd function around $(\mu_{A_1}, x(\mu_{A_1}))$. Then, for all $h_{A_1}(r)$ which is made from $g_{A_1}(r)$ by increasing its variance,

$$\begin{aligned} A_1 \int_{-\infty}^{\infty} (g_{A_1}(r) - h_{A_1}(r))x(r)dr &= A_1 \int_{-\infty}^{\infty} g_{A_1}(r)x(r)dr - A_1 \int_{-\infty}^{\infty} h_{A_1}(r)x(r)dr \\ &= A_1 x(\mu_{A_1}) - A_1 x(\mu_{A_1}) \\ &= 0. \end{aligned}$$

$(\Rightarrow)$

Suppose that **(a4-1)** holds and that $x(r)$ is not an odd function around $(\mu_{A_1}, x(\mu_{A_1}))$. Then, we can represent $x(r)$ as the sum of an even function around $r = \mu_{A_1}$ and an odd function around $(\mu_{A_1}, 0)$ as follows:

$$x(r) = x_{even}^{\mu_{A_1}}(r) + x_{odd}^{\mu_{A_1}}(r),$$

where

$$x_{even}^{\mu_{A_1}}(r) \equiv \frac{x(r) + x(2\mu_{A_1} - r)}{2}$$

and

$$x_{odd}^{\mu_{A_1}}(r) \equiv \frac{x(r) - x(2\mu_{A_1} - r)}{2}.$$

Note that $x_{even}^{\mu_{A_1}}(r) \neq x(\mu_{A_1})$ because $x(r)$ is not an odd function around $(\mu_{A_1}, x(\mu_{A_1}))$, and that since $x_{even}^{\mu_{A_1}}(r)$ is continuous, any jumps at $\pm\infty$ are excluded.

Also, we represent $h_{A_1}(r)$ as

$$h_{A_1}(r) = \frac{1}{(2\pi)^{1/2}k\sigma}\ exp\{-\frac{(r-\mu_{A_1})^2}{2(k\sigma)^2}\}$$

for all $r$, where $k > 1$.

Then,

$$\begin{aligned} A_1 \int_{-\infty}^{\infty} h_{A_1}(r)x(r)dr &= A_1 \int_{-\infty}^{\infty} h_{A_1}(r)(x_{even}^{\mu_{A_1}}(r) + x_{odd}^{\mu_{A_1}}(r))dr \\ &= A_1 \int_{-\infty}^{\infty} h_{A_1}(r)x_{even}^{\mu_{A_1}}(r)dr \\ &= \frac{A_1}{(2\pi)^{1/2}\sigma}\frac{1}{k}\int_{-\infty}^{\infty} exp\{-\frac{(r-\mu_{A_1})^2}{2(k\sigma)^2}\}x_{even}^{\mu_{A_1}}(r)dr. \end{aligned}$$

Since **(a4-1)** holds, differentiating the right hand side with $k(> 1)$, we should have 0:

$$\begin{aligned} \frac{A_1}{(2\pi)^{1/2}\sigma}(-\frac{1}{k^2}\ \phi(k) + \frac{1}{k}\ \phi'(k)) = 0 \\ \Leftrightarrow \phi'(k) - \frac{1}{k}\ \phi(k) = 0, \end{aligned}$$

where

$$\phi(k) \equiv \int_{-\infty}^{\infty} exp\{-\frac{(r-\mu_{A_1})^2}{2(k\sigma)^2}\}x_{even}^{\mu_{A_1}}(r)dr.$$

Solving this linear differential equation, we have

$$\phi(k) = C \times exp(lnk),$$

where $C$ is a constant of integration. Then, we have

$$C = exp(-lnk)\int_{-\infty}^{\infty} exp\{-\frac{(r-\mu_{A_1})^2}{2(k\sigma)^2}\}x_{even}^{\mu_{A_1}}(r)dr.$$

The right hand side is obviously a function of $k$, not a constant. This is a contradiction. Hence, **(a4-1)** implies that $x(r)$ is an odd function around $(\mu_{A_1}, x(\mu_{A_1}))$. ■

Finally, we introduce a principle which indicates that, if $M$ performs better (worse), then both $M$ and 1 should have more (less) money. We define a notion of *alignment-domination* as follows:

**Definition.**
Let $x(r)$ and $y(r)$ be additional fee schemes. Then, $x(r)$ *alignment-dominates* $y(r)$ if either (i) or (ii) stated below holds:

(i) $x(r)$ is continuous and increasing while $y(r)$ is discontinuous or non-increasing.

(ii) Both $x(r)$ and $y(r)$ are continuous and increasing. Besides, there is a proportional part in $x(r)$ which is longer than any proportional parts in $y(r)$.

Although this definition is primitive, it is enough to obtain the final results stated in the next subsection.

## 4.2 Results

**Theorem 1.**
Suppose **(a1)**, **(a3)** and **(a4-1)** are satisfied. Then, the following sentences are equivalent:

(i) The solution $x(r)$ of the second best problem is *alignment-maximum*[3].

(ii) The solution $x(r)$ of the second best problem is equal to $x(r)^*$ below:

$$x(r)^* = \begin{cases} 0 & (r \le \bar{f}) \\ S(r - \bar{f}) & (r \in [\bar{f}, 2\mu_{A_1} - \bar{f}]) \\ 2S(\mu_{A_1} - \bar{f}) & (r \ge 2\mu_{A_1} - \bar{f}), \end{cases}$$

where

$$S = \frac{f_2 A_2 - \bar{C}}{A_1}[\int_{\bar{f}}^{2\mu_{A_1}-\bar{f}} (g_{A_1}(r) - g_{A_1+A_2}(r))(r - \bar{f})dr + \int_{2\mu_{A_1}-\bar{f}}^{\infty} (g_{A_1}(r) - g_{A_1+A_2}(r))\{2(\mu_{A_1} - \bar{f})\}dr]^{-1}.$$

Furthermore, (ii) implies that $x(r)^*$ satisfies **(a2)**.

**Proof.**
Suppose that **(a1)**, **(a3)** and **(a4-1)** hold.

((ii) $\Rightarrow$ (i).) Obvious.

((ii) implies that $x(r)^*$ satisfies **(a2)**.)
Since $x(r)^*$ satisfies the third constraint of the second best problem, we have

[3] This means that $x(r)$ is *alignment-maximum* on **(a1)**, **(a3)** and **(a4-1)**

$$\{\mu_{A_1} - (\bar{f} + \int_{-\infty}^{\infty} g_{A_1}(r)x(r)^* dr)\}A_1 \geq 0 \;\Leftrightarrow\; \mu_{A_1} - (\bar{f} + x(\mu_{A_1})^*) \geq 0$$
$$\Leftrightarrow\; \mu_{A_1} - \bar{f} \geq x(\mu_{A_1})^*$$
$$\Leftrightarrow\; \mu_{A_1} - \bar{f} \geq S(\mu_{A_1} - \bar{f})$$
$$\Leftrightarrow\; 1 \geq S.$$

((i) $\Rightarrow$ (ii).)

Suppose that $x(r)^*$ is not a solution of the second best problem. Note that $x(r)^*$ satisfies the first constraint of the second best problem:

$$A_1 \int_{-\infty}^{\infty} (g_{A_1}(r) - g_{A_1+A_2}(r))x(r)^* dr$$
$$= A_1[\int_{\bar{f}}^{2\mu_{A_1}-\bar{f}} (g_{A_1}(r) - g_{A_1+A_2}(r))S(r-\bar{f})dr + \int_{2\mu_{A_1}-\bar{f}}^{\infty} (g_{A_1}(r) - g_{A_1+A_2}(r))\{2S(\mu_{A_1} - \bar{f})\}dr]$$
$$= A_1 S\ [\int_{\bar{f}}^{2\mu_{A_1}-\bar{f}} (g_{A_1}(r) - g_{A_1+A_2}(r))(r-\bar{f})dr + \int_{2\mu_{A_1}-\bar{f}}^{\infty} (g_{A_1}(r) - g_{A_1+A_2}(r))\{2(\mu_{A_1} - \bar{f})\}dr]$$
$$= f_2 A_2 - \bar{C}.$$

Also note that $x(r)^*$ satisfies the second constraint of the second best problem. It is easily found that $x(r)^*$ satisfies **(a1)** and **(a3)**. Moreover, from **Lemma**, we find that $x(r)^*$ satisfies **(a4-1)**. Hence, if $x(r)^*$ is not a solution of the second best problem under **(a1)**, **(a3)** and **(a4-1)**, there are two cases:

**Case1.** *$x(r)^*$ does not satisfy the third constraint of the second best problem.*

As calculated above, this implies $S > 1$. In this case, it is impossible to find any fee schemes which satisfy **(a1)**, **(a3)**, **(a4-1)** and the constraints of the problem. Hence, there is no solutions of the problem.

**Case2.** *There exists another fee scheme which produces less cost than $x(r)$.*

Then, such a fee scheme can not be alignment-maximum on **(a1)**, **(a3)** and **(a4-1)**.

From the discussions above, we obtain (i) $\Rightarrow$ (ii). ■

The characterized solution is illustrated in **Figure6**:

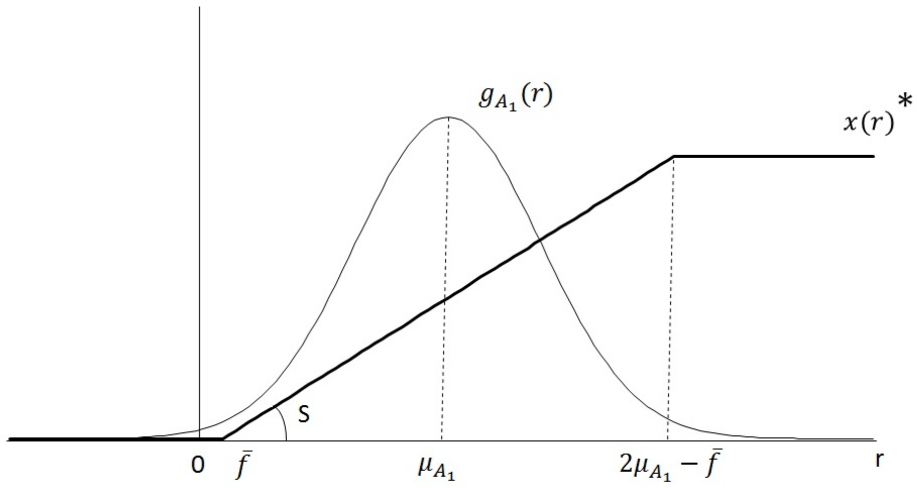


**Figure6: The Optimum Additional Fee Scheme** $x(r)^*$

Are there any solutions which *alignment-dominates* the solution in **Theorem 1** under different environment ? An answer is in the following theorem:

**Theorem 2.**
Suppose **(a1)**, **(a3)** and **(a4-2)** are satisfied. If

$$x(r)^* = \begin{cases} 0 & (r \leq \bar{f}) \\ S(r - \bar{f}) & (r \in [\bar{f}, \mu_{A_1}]) \\ S'(r - \mu_{A_1}) + S(\mu_{A_1} - \bar{f}) & (r \geq \mu_{A_1}), \end{cases}$$

where $S, S' \in (0, 1]$ and $S > S'$, is the solution of the second best problem, then

$$\frac{S'}{S} \in [\frac{\frac{f_2 A_2 - \bar{C}}{S A_1} - \int_{\bar{f}}^{\mu_{A_1}} (G_{A_1+A_2}(r) - G_{A_1}(r))dr}{\int_{\mu_{A_1}}^{\infty} (G_{A_1+A_2}(r) - G_{A_1}(r))dr},$$

$$\min\{\frac{\frac{\mu_{A_1} - \bar{f}}{S} - \{(\mu_{A_1} - \bar{f}) - \int_{\bar{f}}^{\mu_{A_1}} G_{A_1}(r)dr}{\int_{\mu_{A_1}}^{\infty} g_{A_1}(r)(r - \mu_{A_1})dr}, \frac{\int_{\bar{f}}^{\mu_{A_1}} (H_{A_1}(r) - G_{A_1}(r))dr}{\int_{\mu_{A_1}}^{\infty} (G_{A_1}(r) - H_{A_1}(r))dr}\}].$$

The possible solution is illustrated in **Figure7**:

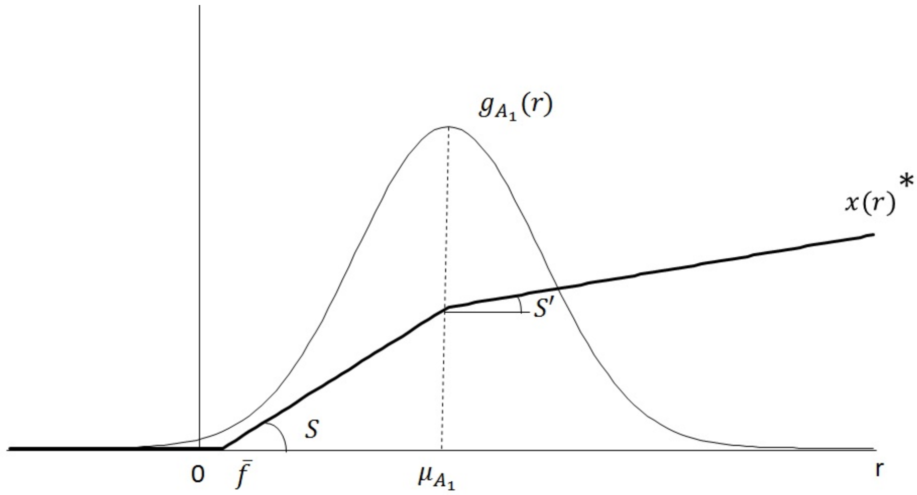


**Figure7: The Optimum Additional Fee Scheme $x(r)^*$**

**Proof.**

From the first constraint of the second best problem, we have

$$A_1 \int_{-\infty}^{\infty} (g_{A_1}(r) - g_{A_1+A_2}(r))x(r)^* dr \geq f_2 A_2 - \bar{C}$$

$$\Leftrightarrow \int_{\bar{f}}^{\mu_{A_1}} (g_{A_1}(r) - g_{A_1+A_2}(r))S(r - \bar{f})dr$$

$$+ \int_{\mu_{A_1}}^{\infty} (g_{A_1}(r) - g_{A_1+A_2}(r))\{S'(r - \mu_{A_1}) + S(\mu_{A_1} - \bar{f})\}dr \geq \frac{f_2 A_2 - \bar{C}}{A_1}$$

$$\Leftrightarrow S \int_{\bar{f}}^{\mu_{A_1}} (G_{A_1}(r) - G_{A_1+A_2}(r))'(r - \bar{f})dr$$

$$+S' \int_{\mu_{A_1}}^{\infty} (G_{A_1}(r) - G_{A_1+A_2}(r))'(r - \mu_{A_1})dr$$

$$+S(\mu_{A_1} - \bar{f}) \int_{\mu_{A_1}}^{\infty} (G_{A_1}(r) - G_{A_1+A_2}(r))' dr \geq \frac{f_2 A_2 - \bar{C}}{A_1}$$

$$\Leftrightarrow S[(G_{A_1}(r) - G_{A_1+A_2}(r))(r - \bar{f})]_{\bar{f}}^{\mu_{A_1}} - S \int_{\bar{f}}^{\mu_{A_1}} (G_{A_1}(r) - G_{A_1+A_2}(r))dr$$

$$+S'[(G_{A_1}(r) - G_{A_1+A_2}(r))(r - \mu_{A_1})]_{\mu_{A_1}}^{\infty} - S' \int_{\mu_{A_1}}^{\infty} (G_{A_1}(r) - G_{A_1+A_2}(r))dr$$

$$+S(\mu_{A_1}-\bar{f})[(G_{A_1}(r)-G_{A_1+A_2}(r))]_{\mu_{A_1}}^{\infty} \geq \frac{f_2A_2-\bar{C}}{A_1}$$

$$\Leftrightarrow S(G_{A_1}(\mu_{A_1})-G_{A_1+A_2}(\mu_{A_1}))(\mu_{A_1}-\bar{f}) - S\int_{\bar{f}}^{\mu_{A_1}}(G_{A_1}(r)-G_{A_1+A_2}(r))dr$$

$$+S'\lim_{r\to\infty}(G_{A_1}(r)-G_{A_1+A_2}(r))(r-\mu_{A_1}) - S'\int_{\mu_{A_1}}^{\infty}(G_{A_1}(r)-G_{A_1+A_2}(r))dr$$

$$-S(\mu_{A_1}-\bar{f})(G_{A_1}(\mu_{A_1})-G_{A_1+A_2}(\mu_{A_1})) \geq \frac{f_2A_2-\bar{C}}{A_1}$$

$$\Leftrightarrow S\int_{\bar{f}}^{\mu_{A_1}}(G_{A_1+A_2}(r)-G_{A_1}(r))dr + S'\int_{\mu_{A_1}}^{\infty}(G_{A_1+A_2}(r)-G_{A_1}(r))dr \geq \frac{f_2A_2-\bar{C}}{A_1}$$

$$\Leftrightarrow \frac{S'}{S} \geq \frac{\frac{f_2A_2-\bar{C}}{SA_1} - \int_{\bar{f}}^{\mu_{A_1}}(G_{A_1+A_2}(r)-G_{A_1}(r))dr}{\int_{\mu_{A_1}}^{\infty}(G_{A_1+A_2}(r)-G_{A_1}(r))dr}.$$

From the third constraint of the second best problem, we have

$$\{\mu_{A_1}-(\bar{f}+\int_{-\infty}^{\infty}g_{A_1}(r)x(r)^*dr)\}A_1 \geq 0$$

$$\Leftrightarrow \mu_{A_1}-\bar{f} \geq \int_{\bar{f}}^{\mu_{A_1}} g_{A_1}(r)S(r-\bar{f})dr + \int_{\mu_{A_1}}^{\infty} g_{A_1}(r)\{S'(r-\mu_{A_1})+S(\mu_{A_1}-\bar{f})\}dr$$

$$\Leftrightarrow \mu_{A_1}-\bar{f} \geq S\int_{\bar{f}}^{\mu_{A_1}} G'_{A_1}(r)(r-\bar{f})dr + S'\int_{\mu_{A_1}}^{\infty} g_{A_1}(r)(r-\mu_{A_1})dr + S(\mu_{A_1}-\bar{f})\int_{\mu_{A_1}}^{\infty} G'_{A_1}(r)dr$$

$$\Leftrightarrow \mu_{A_1}-\bar{f} \geq S[G_{A_1}(r)(r-\bar{f})]_{\bar{f}}^{\mu_{A_1}} - S\int_{\bar{f}}^{\mu_{A_1}} G_{A_1}(r)dr + S'\int_{\mu_{A_1}}^{\infty} g_{A_1}(r)(r-\mu_{A_1})dr + S(\mu_{A_1}-\bar{f})[G_{A_1}(r)]_{\mu_{A_1}}^{\infty}$$

$$\Leftrightarrow \mu_{A_1}-\bar{f} \geq SG_{A_1}(\mu_{A_1})(\mu_{A_1}-\bar{f}) - S\int_{\bar{f}}^{\mu_{A_1}} G_{A_1}(r)dr + S'\int_{\mu_{A_1}}^{\infty} g_{A_1}(r)(r-\mu_{A_1})dr + S(\mu_{A_1}-\bar{f})(1-G_{A_1}(\mu_{A_1}))$$

$$\Leftrightarrow \mu_{A_1}-\bar{f} \geq S\{(\mu_{A_1}-\bar{f}) - \int_{\bar{f}}^{\mu_{A_1}} G_{A_1}(r)dr\} + S'\int_{\mu_{A_1}}^{\infty} g_{A_1}(r)(r-\mu_{A_1})dr$$

$$\Leftrightarrow \frac{S'}{S} \leq \frac{\frac{\mu_{A_1}-\bar{f}}{S} - \{(\mu_{A_1}-\bar{f}) - \int_{\bar{f}}^{\mu_{A_1}} G_{A_1}(r)dr}{\int_{\mu_{A_1}}^{\infty} g_{A_1}(r)(r-\mu_{A_1})dr}.$$

From **(a4-2)**, we have

$$\Leftrightarrow A_1 \int_{-\infty}^{\infty} (g_{A_1}(r) - h_{A_1}(r))x(r)^* dr \geq 0$$

$$\Leftrightarrow \int_{\bar{f}}^{\mu_{A_1}} (g_{A_1}(r) - h_{A_1}(r))S(r-\bar{f})dr + \int_{\mu_{A_1}}^{\infty} (g_{A_1}(r) - h_{A_1}(r))\{S'(r-\mu_{A_1}) + S(\mu_{A_1} - \bar{f})\}dr \geq 0$$

$$\Leftrightarrow S \int_{\bar{f}}^{\mu_{A_1}} (G_{A_1}(r) - H_{A_1}(r))'(r-\bar{f})dr$$
$$+S' \int_{\mu_{A_1}}^{\infty} (G_{A_1}(r) - H_{A_1}(r))'(r-\mu_{A_1})dr$$
$$+S(\mu_{A_1} - \bar{f}) \int_{\mu_{A_1}}^{\infty} (G_{A_1}(r) - H_{A_1}(r))' dr \geq 0$$

$$\Leftrightarrow S[(G_{A_1}(r) - H_{A_1}(r))(r-\bar{f})]_{\bar{f}}^{\mu_{A_1}} - S \int_{\bar{f}}^{\mu_{A_1}} (G_{A_1}(r) - H_{A_1}(r))dr$$
$$+S'[(G_{A_1}(r) - H_{A_1}(r))(r-\mu_{A_1})]_{\mu_{A_1}}^{\infty} - S' \int_{\mu_{A_1}}^{\infty} (G_{A_1}(r) - H_{A_1}(r))dr$$
$$+S(\mu_{A_1} - \bar{f})[(G_{A_1}(r) - H_{A_1}(r))]_{\mu_{A_1}}^{\infty} \geq 0$$

$$\Leftrightarrow S(G_{A_1}(\mu_{A_1}) - H_{A_1}(\mu_{A_1}))(\mu_{A_1} - \bar{f}) - S \int_{\bar{f}}^{\mu_{A_1}} (G_{A_1}(r) - H_{A_1}(r))dr$$
$$+S' \lim_{r\to\infty} (G_{A_1}(r) - H_{A_1}(r))(r-\mu_{A_1}) - S' \int_{\mu_{A_1}}^{\infty} (G_{A_1}(r) - H_{A_1}(r))dr$$
$$-S(\mu_{A_1} - \bar{f})(G_{A_1}(\mu_{A_1}) - H_{A_1}(\mu_{A_1})) \geq 0$$

$$\Leftrightarrow S \int_{\bar{f}}^{\mu_{A_1}} (H_{A_1}(r) - G_{A_1}(r))dr - S' \int_{\mu_{A_1}}^{\infty} (G_{A_1}(r) - H_{A_1}(r))dr \geq 0$$

$$\Leftrightarrow \frac{S'}{S} \leq \frac{\int_{\bar{f}}^{\mu_{A_1}} (H_{A_1}(r) - G_{A_1}(r))dr}{\int_{\mu_{A_1}}^{\infty} (G_{A_1}(r) - H_{A_1}(r))dr}. \quad \blacksquare$$ [4]

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[4] $\lim_{r\to\infty}(G_{A_1}(r) - G_{A_1+A_2}(r))(r-\mu_{A_1}) = 0$ and $\lim_{r\to\infty}(G_{A_1}(r) - H_{A_1}(r))(r-\mu_{A_1}) = 0$ are obtained by using Maclaurin expansion.

# 5 Parameterization

The optimal fee schemes derived by **Theorem 1** and **Theorem 2** is complicated, and it is difficult to figure out how much returns can be obtained intuitively. Therefore, based on these fee schemes, we confirm the expected net returns and the expected fees by simulation. We describe the investment amount of the asset owner 1 as a percentage of the maximum capacity, and calculate the required payments for each AUM percentage to prevent asset manager from aiming for the expansion of AUM.

To simplify, as shown in **Table 1**, we set the minimum fee and the cost of sales to zero. We employ quadratic form cost function following Berk & Green (2004). We set the parameters of cost function so that the expected excess return is approximately equal to 0 at maximum capacity. The target return may be higher than actual return because investment managers intend to show well for business purposes. However, considering that the alpha used in this paper corresponds to the excess return before influenced by the size of AUM and that data of realized returns is inevitably influenced by AUM, the target return is suitable as a proxy of alpha in terms of the maximum return earned by the manager.

| | | | |
|---|---|---|---|
| Minimum Fixed Fee | f_bar | 0.00% | For simplicity |
| Fee by Asset Owner 1 | f1 | 0.50% | Average active management fee is about 0.5% given 100 billion yen. |
| Fee by Asset Owner 2 | f2 | 0.50% | Average active management fee is about 0.5% given 100 billion yen. |
| Cost of Sales | C_bar | 0 | For simplicity |
| Maximum Capacity | A1+A2 | 100 | For simplicity |
| Average Excess Return in Maximum Capacity | $\mu$(A1+A2) | 0.00% | Berk and Green 2004 |
| Standard Deviation of Active Management | $\sigma$ | 5.00% | Average target tracking error of active fund is about 5% in global equity. |
| Cost Function | | - 0.000325 * AUM^2 | It is in line with the setting in Berk and Green 2004. |
| Intercept of Excess Return Function | | 3.25 | Average target tracking error of active fund is about 3% in global equity. To fit excess return to zero at maximum capacity. |

**Table 1: Parameter Values**

**Table 2** shows a result of simulation. We compare the expected management fee and the expected net return on the fee schemes derived by **Proposition 1**, **Theorem 1** and **Theorem 2**. Overall, investing only 40% or 50% of the maximum capacity tends to maximize the expected net returns. Also, if asset owners do not find another attractive product, investing 60% of maximum capacity is optimal for all fee schemes as the amount of gain comes to be the largest. If invest amount exceeds 60%, the payment of asset owner 1 actually becomes lower, but the excess return will also decrease due to 1's investment itself. As a result, such investment makes less return to asset owner 1. Although this value itself is dependent on the shape of the cost function, these results imply that asset owner 1 should invest as to maintain net return through leaving enough capacity, even if 1 has only to pay relatively higher management fee. This implication holds even when asset owner 1 has large amount of asset comparing with maximum capacity of investment product.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | AUM of Asset Owner 1 per Max Capacity | 10% | 20% | 30% | 40% | 50% | 60% | 70% | 80% | 90% |
| | Expected Excess Return | 3.22% | 3.12% | 2.96% | 2.73% | 2.44% | 2.08% | 1.66% | 1.17% | 0.62% |
| Proposition 1 | Expected Total Fee | - | 2.50% | 1.67% | 1.25% | 1.00% | 0.83% | 0.71% | 0.63% | 0.56% |
| | Expected Net Return | - | 0.62% | 1.29% | 1.48% | 1.44% | 1.25% | 0.94% | 0.55% | 0.06% |
| | Gain of Asset Owner 1 | - | 0.12 | 0.39 | 0.59 | 0.72 | 0.75 | 0.66 | 0.44 | 0.06 |
| Teorem 1 | Slope | - | - | 0.93 | 0.69 | 0.57 | 0.51 | 0.51 | 0.58 | 0.92 |
| | Upper Bound of Fee | - | - | 11.01% | 7.55% | 5.54% | 4.25% | 3.36% | 2.73% | 2.27% |
| | Expected Total Fee | - | - | 2.75% | 1.89% | 1.39% | 1.06% | 0.84% | 0.68% | 0.57% |
| | Expected Net Return | - | - | 0.21% | 0.84% | 1.06% | 1.02% | 0.82% | 0.49% | 0.06% |
| | Gain of Asset Owner 1 | - | - | 0.06 | 0.34 | 0.53 | 0.61 | 0.58 | 0.39 | 0.05 |
| Teorem 2 | Slope 1 | - | - | - | - | 0.93 | 0.88 | 0.89 | - | - |
| | Slope 2 | - | - | - | - | 0.07 | 0.04 | 0.03 | - | - |
| | Expected Total Fee | - | - | - | - | 1.49% | 1.15% | 0.89% | - | - |
| | Expected Net Return | - | - | - | - | 0.95% | 0.93% | 0.76% | - | - |
| | Gain of Asset Owner 1 | - | - | - | - | 0.47 | 0.56 | 0.53 | - | - |
| | Cost of Asymmetric Information | - | - | 1.09% | 0.64% | 0.39% | 0.23% | 0.13% | 0.06% | 0.01% |
| | Cost of Alignment | - | - | - | - | 0.10% | 0.08% | 0.05% | - | - |

**Table 2: Comparison of the Main Results**

The expected fee of **Theorem 1** is higher than that of compensation scheme in **Proposition 1** because we assume that asset owner 1 cannot observe investment manager's action on **Theorem 1**. Therefore, the difference of fee rate between the scheme on **Theorem 1** and compensation scheme corresponds to information cost. According to **Table 2**, the difference of fee rate is 0.39% when asset owner 1 invests 50% of the maximum capacity, and 0.23% when 60% of the maximum capacity. This difference tends to decrease with the increase in asset value. Moreover, the expected fee rate equals 1.06% when 60% of the maximum capacity. This is about twice the value of the fixed fee rate 0.5% paid by the asset owner 2. The asset owner 1 needs to pay a quite expensive fee compared to the generally accepted fee rate.

The main difference of assumption between on the fee scheme derived by **Theorem 1** and on the fee scheme with compensation in **Proposition 1** is asymmetric information, which means asset owner cannot detect the increase in AUM or cannot make the payment lower when AUM increases. It is the fact that manager informs AUM of the product by quarterly reports etc. and asset owner can calculate approximate asset inflow. However, it is very difficult for asset owners to know true AUM which affects cost function because some products share internal alpha resources with other products, such as investment ideas of analyst teams and top down macroeconomic view. Hence, assumption of asymmetric information makes sense.

In **Theorem 2**, solutions exist only when asset owner 1 invests about 50% to 70% of the maximum capacity. This range is narrower than other fee schemes. Comparing **Proposition 1** and **Theorem 1**, fee scheme derived by **Theorem 2** tends to be a little bit expensive. For example, the difference of fee rate between **Theorem 1** and **Theorem 2** is 0.10% when asset owner 1 invests 50% of the maximum capacity, and it is 0.08% when 60% of maximum capacity.

In **Theorem 2**, we relax the restriction of changing risk characteristics comparing with **Theorem 1**. As stated in the previous section, it is not necessary to have a point-symmetric fee scheme because asset owner can monitor the range of active risk which manager takes. Therefore, we introduced a non-upper bound incentive fee scheme that would improve *alignment*. The difference between the expected management fee of **Theorem 1** and that of **Theorem 2** is considered to be the cost of *alignment*. The increase in the management fee is largely due to the *alignment* because asset owner 1 pays the high additional fee when manager achieves incredible return (with a low probability). In practice, it seems difficult to justify such expensive fee schemes from the viewpoint of *alignment* since many active managers struggle to maintain net excess return. There is no motivation for asset owners to propose non-upper bound fee schemes even if asset owners can monitor the risk profile. In addition, it takes costs to monitor the risk profile accurately.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | AUM of Asset Owner 1 per Max Capacity | 10% | 20% | 30% | 40% | 50% | 60% | 70% | 80% | 90% |
| | Expected Excess Return | 3.22% | 3.12% | 2.96% | 2.73% | 2.44% | 2.08% | 1.66% | 1.17% | 0.62% |
| Base Setting | Slope | - | - | 0.93 | 0.69 | 0.57 | 0.51 | 0.51 | 0.58 | 0.92 |
| | Upper Bound of Fee | - | - | 11.01% | 7.55% | 5.54% | 4.25% | 3.36% | 2.73% | 2.27% |
| | Expected Total Fee | - | - | 2.75% | 1.89% | 1.39% | 1.06% | 0.84% | 0.68% | 0.57% |
| | Expected Net Return | - | - | 0.21% | 0.84% | 1.06% | 1.02% | 0.82% | 0.49% | 0.06% |
| | Gain of Asset Owner 1 | - | - | 0.06 | 0.34 | 0.53 | 0.61 | 0.58 | 0.39 | 0.05 |
| Minimum Fixed Fee +0.05% | Slope | - | - | 0.95 | 0.70 | 0.58 | 0.52 | 0.52 | 0.61 | 1.00 |
| | Upper Bound of Fee | - | - | 10.99% | 7.54% | 5.53% | 4.24% | 3.35% | 2.72% | 2.27% |
| | Expected Total Fee | - | - | 2.80% | 1.94% | 1.43% | 1.11% | 0.89% | 0.73% | 0.62% |
| | Expected Net Return | - | - | 0.16% | 0.80% | 1.01% | 0.97% | 0.77% | 0.44% | 0.01% |
| | Gain of Asset Owner 1 | - | - | 0.05 | 0.32 | 0.50 | 0.58 | 0.54 | 0.35 | 0.01 |
| Standard Deviation *1.5 | Slope | - | - | - | 0.99 | 0.82 | 0.74 | 0.75 | 0.87 | - |
| | Upper Bound of Fee | - | - | - | 10.78% | 7.98% | 6.18% | 4.94% | 4.05% | - |
| | Expected Total Fee | - | - | - | 2.70% | 2.00% | 1.55% | 1.24% | 1.01% | - |
| | Expected Net Return | - | - | - | 0.04% | 0.45% | 0.54% | 0.43% | 0.16% | - |
| | Gain of Asset Owner 1 | - | - | - | 0.02 | 0.22 | 0.32 | 0.30 | 0.13 | - |
| Cost for Sales +0.01 | Slope | - | - | 0.90 | 0.67 | 0.55 | 0.48 | 0.47 | 0.52 | 0.73 |
| | Upper Bound of Fee | - | - | 10.70% | 7.30% | 5.32% | 4.03% | 3.13% | 2.45% | 1.81% |
| | Expected Total Fee | - | - | 2.68% | 1.83% | 1.33% | 1.01% | 0.78% | 0.61% | 0.45% |
| | Expected Net Return | - | - | 0.29% | 0.91% | 1.11% | 1.08% | 0.88% | 0.56% | 0.17% |
| | Gain of Asset Owner 1 | - | - | 0.09 | 0.36 | 0.56 | 0.65 | 0.61 | 0.45 | 15.20% |
| Fixed Fee by Asset Owner 2 +0.2% | Slope | - | - | - | 0.97 | 0.80 | 0.71 | 0.71 | 0.82 | - |
| | Upper Bound of Fee | - | - | - | 10.58% | 7.76% | 5.94% | 4.70% | 3.82% | - |
| | Expected Total Fee | - | - | - | 2.65% | 1.94% | 1.49% | 1.18% | 0.95% | - |
| | Expected Net Return | - | - | - | 0.09% | 0.50% | 0.60% | 0.49% | 0.22% | - |
| | Gain of Asset Owner 1 | - | - | - | 0.04 | 0.25 | 0.36 | 0.34 | 0.18 | - |

**Table 3: Simulation on Theorem 1**

In **Table 3**, we move each parameter from the setting of the previous simulation in the fee scheme of **Theorem 1** and examined the change of result. When the minimum fee is at 0.05%, the expected fee increases at all ratios. The slope of the incentive fee becomes larger and the upper limit of the reward becomes lower. The expected fee, the upper bound and the slope of the incentive fee increase when we set the standard deviation of excess return to 1.5 times (7.5%). Assuming that the asset owner 2 plans to pay 0.7% as a fixed management fee, the expected fee, the upper limit and the slope of the incentive fee also increase. Just like in the base case, when asset owner 1 invests 60% of the maximum capacity, the expected fee becomes 1.49%, which is about twice as much as the fixed fee that asset owner 2 plans to pay. When the sales cost is set to 0.01, the expected fee, the upper bound and the slope of the incentive fee become smaller. Since the increase of the sales cost makes the cost of acquiring an additional AUM higher, the manager comes to accept fees even if they are relatively low.

From this result, in order to prevent an increase in AUM, it is desirable to introduce fee scheme with compensation in **Proposition 1** or fee scheme derived by **Theorem 1** and set an expected fee higher than the fee level which asset owner 2 plans to pay . Moreover, in order to reduce fee, asset owner should select, for example, a manager with high sales costs, such as startup manager who has no large sales network. Alternatively, asset owner should select a manager with a high return efficiency because lower tracking error produces lower fee.

# 6 Conclusion

We show that a specific incentive fee with upper bound is optimal. This fee scheme, combined with the minimum fee, not only prevents AUM from being enlarged, but also has some reasonable properties: having a lower bound, paying no additional fee for excess returns not more than the minimum fee, being independent of risk characteristics and promoting *alignment*. Moreover, the optimality of this fee scheme is supported by simulation.

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