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Elderly Care and Multiple Monies

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Abstract

This paper presents an overlapping generations model in which old generations require specific services from young generations due to idiosyncratic shocks. An example of such services is elderly care. The model shows that a two-money system in which fiat money for such services is separated from that for the other types of goods and services can replicate the resource allocation in a one-money system with a fair insurance. For this result, it is necessary to prohibit old generations from exchanging different types of fiat monies in the two-money system. The model implies that the introduction of fiat money for elderly care reduces the real value of government bonds outstanding in the country.

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1 Introduction

This paper experiments a two-money system in an overlapping generations model in which old agents demand specific services from young agents due to idiosyncratic shocks. An example of such services is elderly care. The model shows that the two-money system can achieve optimal risk sharing if fiat money for the specific services is separated from that for the other goods and services, and if old agents are prohibited from swapping the two types of fiat monies among themselves.

The illiquidity of fiat monies is necessary to replicate the effect of an insurance, as it allows only old people hit by idiosyncratic shocks to spend fiat money for the specific services. Furthermore, a competitive market ensures the fair pricing of fiat monies, and hence optimal risk sharing among old agents.

This result is related to Kocherlakota's (2003) model of a monetary economy. He shows that if there are two types of agents with different intertemporal rates of substitution, then the social optimum requires that impatient agents hold money while patient agents hold long-term bonds, and also that long-term bonds are not transferrable until maturity. This paper adds to his analysis by focusing on heterogeneity due to idiosyncratic needs for some specific services, rather than heterogeneity due to different intertemporal rates of substitution. This paper shows that restricting the liquidity of fiat monies for specific holders, i.e., old agents, is necessary to achieve optimal risk sharing in this case.

Also, Aoki, Nakajima, and Nikolov (2014) shows that fiat money can circulate in an equilibrium as an instrument for insuring idiosyncratic productivity shocks among ex-ante homogeneous agents. This paper's result adds to their analysis by showing that optimal risk sharing cannot be achieved by uniform fiat money if an insurance is necessary for idiosyncratic shocks to preferences over the composition of goods and services to consume.

In addition, fiat money in a one-money system in the model can be interpreted as the liabilities of the consolidated government, because it is held as a store of wealth and also because government bonds in reality are redeemable only in currency. Given this interpretation of fiat money and the interpretation of specific services provided by young agents as elderly care, the model implies that if the government has been running an elderly care insurance, and introduces new fiat money for elderly care to pay to elderly care providers, then the government can use traditional money pooled at the elderly care insurance to finance other government expenditures or to repay government bonds outstanding. In either case, the real value of government bonds declines through inflation or a reduction in the nominal face value of government bonds.

2 Baseline model with divisible labor and one money

Time is discrete and has an infinite horizon. For $t = 0, 1, 2, ...$, a $[0, N_t]$ continuum of agents are born in period t, and live for two periods. The set of agents born in period t is referred to as "cohort t", and agents in their first period and those in the second period are called "young" and "old", respectively. There exists a unit continuum of the initial old endowed with a fixed amount of fiat money, M, for each. The measure of agents in each cohort is defined by the Lebesgue measure over $[0, N_t]$.

Each old agent can gain utility from consuming goods. In addition, there is an i.i.d. health shock to each old agent that makes the agent needs elderly care with probability θ ($\in (0, 1)$). Each young agent is endowed with $\bar{\ell}$ (> 0) units of time, which can be allocated for the production of goods and the supply of elderly care. There exist competitive markets for goods and elderly care in each period. Each young agent can also subscribe to a competitive elderly care insurance in which insurers earn a zero profit.

The utility maximization problem for a young agent in cohort t is specified as

$$\begin{aligned}
\max_{\{c_{i,O,t+1}, h_{i,O,t+1}, c'_{i,O,t+1}, b_{i,O,t+1}, b'_{i,O,t+1}, \ell_{i,Y,t}, \hat{\ell}_{i,Y,t}\}} \quad & \theta(u(c_{i,O,t+1}) - g(\bar{h} - h_{i,O,t+1})) + (1-\theta)u(c'_{i,O,t+1}) \\
\text{s.t.} \quad & c_{i,O,t+1} + q_{t+1}h_{i,O,t+1} = v_{t+1}b_{i,O,t+1} \\
& c'_{i,O,t+1} = v_{t+1}b'_{i,O,t+1} \\
& \theta b_{i,O,t+1} + (1-\theta)b'_{i,O,t+1} = \frac{\alpha \ell_{i,Y,t}}{v_t} + \frac{\beta \hat{\ell}_{i,Y,t}}{q_t} \\
& \ell_{i,Y,t} + \hat{\ell}_{i,Y,t} = \bar{\ell} \\
& c_{i,O,t+1},\ h_{i,O,t+1},\ c'_{i,O,t+1},\ \ell_{i,Y,t},\ \hat{\ell}_{i,Y,t} \geq 0
\end{aligned} \tag{1}$$

where $c_{i,O,t+1}$ and $c'_{i,O,t+1}$ denote the amounts of goods consumed when hit by an health shock and when not, respectively; $h_{i,O,t+1}$ is the amount of elderly care received when hit by an health shock; q_t is the real price of elderly care in terms of goods in period t; v_t is the real value of a unit of fiat money in period t; $b_{i,O,t+1}$ and $b'_{i,O,t+1}$ are the amounts of fiat money paid by the elderly care insurance when hit by a health shock and when not, respectively; $\ell_{i,Y,t}$ and $\hat{\ell}_{i,Y,t}$ denote the units of time used for the production of goods and the supply of elderly care, respectively; and α and β are the labor productivity in the production of goods and that in the supply of elderly care, respectively. The function $u : \mathbf{R}_+ \to \mathbf{R}_+$ is twice differentiable, satisfying $u' > 0$, $u'' < 0$, and $\lim_{x\to 0} u'(x) = \infty$. The function $g : [0, \bar{h}] \to \mathbf{R}_+$ is twice differentiable, satisfying $g(0) = 0$, $g' > 0$, $g'' > 0$, $\lim_{x\to 0} g'(x) = 0$, and $\lim_{x\to \bar{h}} g'(\bar{h}) = \infty$, where $\bar{h}$ is a positive constant.

In (1), the objective function is expected utility from goods consumption and elderly care. The first constraint is the flow of funds constraint for an old agent hit by a health shock, whereas the second constraint is that for an old agent without a health shock. The third constraint is the flow of funds constraint for a young agent. The right-hand side of this constraint is the total amount of fiat money earned by each young agent; thus, the left-hand side implies that young agents buy elderly care insurance benefits by fiat money in a competitive market. A zero profit condition for an insurer requires the discounted price of an insurance benefit for each contingency to be the probability of the contingency, given health shocks to old agents being i.i.d. The fourth constraint is the feasibility constraint on labor supply. The last constraint lists non-negativity constraints on choice variables.

The market clearing conditions for goods, elderly care, and fiat money are

$$\theta \int_0^{N_{t-1}} c_{i,O,t}\, di + (1-\theta) \int_0^{N_{t-1}} c'_{i,O,t}\, di = \int_0^{N_t} \alpha \ell_{i,Y,t}\, di \tag{2}$$

$$\theta \int_0^{N_{t-1}} h_{i,O,t}\, di = \int_0^{N_t} \beta \hat{\ell}_{i,Y,t}\, di \tag{3}$$

$$\int_0^{N_t} \theta b_{i,Y,t} + (1-\theta) b'_{i,Y,t}\, di = M \tag{4}$$

respectively for $t = 0, 1, 2, ...$ In (2) and (3), the left-hand side and the right-hand side are demand and supply, respectively. Note that each young agent chooses the values of $c_{i,O,t}$, $h_{i,O,t}$, and $c'_{i,O,t}$.

The last condition, (4), implies that old agents pay an amount M of fiat money in total for goods and elderly care supplied by young agents in each period. Young agents pay the fiat money for contingent claims to insurance benefits, whereas Insurers pool the received fiat money and repay the claims when the agents become old. An equilibrium is characterized by the solution to (1), given $\{q_t, v_t, q_{t+1}, v_{t+1}\}$ for $t = 0, 1, 2, ...$; and $\{q_t, v_t\}$ that satisfy (3) and (4) for all $t = 0, 1, 2, ...$, given (2) being satisfied by the Walrus' law.

2.1 Monetary equilibrium

In any monetary equilibrium, the real price of elderly care in terms of goods, q_t, equals the ratio of labor productivity between the production of goods and the supply of elder care:

$$q_t = \frac{\alpha}{\beta} \tag{5}$$

Therefore, young agents become indifferent to the allocation of their labor into the production of goods and the supply of elder care. As a result, the real labor income for each young agent can be evaluated by the labor productivity for the production of goods, α. Because young agents receive the fixed-supplied fiat money every period as implied by the market clearing condition for fiat money, (4), the real value of a unit of fiat money in terms of goods, v_t, equals between the real labor income and the fixed money supply:

$$v_t = \frac{\alpha N_t \bar{\ell}}{M} \tag{6}$$

An elderly care insurance allows each agent to smooth consumption across contingent states when old. Also, old agents hit by a health shock buy goods and elderly care in such a way that the ratio between marginal utilities from consumption of goods and elderly care equals the relative price—i.e., the real price of elderly care in terms of goods, q_t:

$$c_{i,O,t} = c'_{i,O,t} = \frac{\alpha N_t \bar{\ell}}{N_{t-1}} - \frac{\theta \alpha h_{i,O,t}}{\beta} \tag{7}$$

$$\frac{\alpha}{\beta} u' \left(\frac{\alpha N_t \bar{\ell}}{N_{t-1}} - \frac{\theta \alpha h_{i,O,t}}{\beta} \right) = g'(\bar{h} - h_{i,O,t}) \tag{8}$$

for all $i \in [0, N_t]$ and $t = 0, 1, 2, ...$ There exists a unique monetary equilibrium, as (8) has a unique root for $h_{i,O,t}$, given the characteristics of the functions u and g assumed above. Each young agent's labor supplies for the production of goods and elderly care, i.e., $\ell_{i,Y,t}$ and $\ell'_{i,Y,t}$, respectively, are determined in such a way that (8) and the market clearing conditions for goods and elderly care, (2) and (3), respectively, are satisfied simultaneously.

3 Introducing two monies

3.1 Environment

Now assume that the initial old are endowed with an amount M of fiat money for goods, and an amount $\hat{M}_0$ of fiat money for elderly care separately. The two types of fiat monies are distinguishable. Also assume that the government can ban the exchange between the two types of fiat monies.

In this case, the utility maximization problem for a young agent in cohort t is specified as

$$
\begin{aligned}
\max_{\{c_{i,O,t+1},h_{i,O,t+1},c'_{i,O,t+1},m_{i,Y,t},\hat{m}_{i,Y,t},\ell_{i,Y,t},\hat{\ell}_{i,Y,t}\}} \quad & \theta(u(c_{i,O,t+1})-g(\bar{h}-h_{i,O,t+1}))+(1-\theta)u(c'_{i,O,t+1}) \\
\text{s.t.} \quad & c_{i,O,t+1}=v_{t+1}m_{i,Y,t} \\
& h_{i,O,t+1}=\hat{q}_{t+1}\hat{m}_{i,Y,t} \\
& c'_{i,O,t+1}=v_{t+1}m_{i,Y,t} \\
& v_t m_{i,Y,t}=\alpha\ell_{i,Y,t} \\
& \hat{q}_t\hat{m}_{i,Y,t}=\beta\hat{\ell}_{i,Y,t} \\
& \ell_{i,Y,t}+\hat{\ell}_{i,Y,t}=\bar{\ell} \\
& c_{i,O,t+1},\ h_{i,O,t+1},\ c'_{i,O,t+1},\ m_{i,Y,t},\ \hat{m}_{i,Y,t},\ \ell_{i,Y,t},\ \hat{\ell}_{i,Y,t}\geq 0
\end{aligned}
\tag{9}
$$

where $m_{i,O,t}$ and $\hat{m}_{i,O,t}$ denotes the amount of fiat money for goods and that for elderly care, respectively, acquired by the agent when young; and $\hat{q}_t$ denotes the real value of a unit of fiat money for elderly care in terms of elderly care. The first three constraints are the flow of funds constraints for goods and elderly care when the agent is old. They imply that the agent cannot buy goods with fiat money for elderly care, or elderly care with fiat money for goods. The fourth and the fifth constraint imply that the agent must supply labor for the production of goods and elderly care to acquire fiat monies for goods and elderly care, respectively. The rest of the constraints are similar to those in (1).

The market clearing conditions for goods and elderly care remain the same as (2) and (3), respectively. Those for fiat monies are

$$\int_0^{N_t} m_{i,Y,t}\, di = M \tag{10}$$

$$\int_0^{N_t} \hat{m}_{i,Y,t}\, di = \begin{cases} \theta\hat{M}_0 & \text{if } t=0 \\ \theta\int_0^{N_{t-1}} \hat{m}_{i,Y,t-1}\, di & \text{if } t=1,2,3,.... \end{cases} \tag{11}$$

where (10) is for fiat money for goods, and (11) is for fiat money for elderly care. In each equation, the left-hand side is the aggregate amount of a type of fiat money received by young agents, whereas the right-hand side is that paid by old agents in the same period. An equilibrium is characterized by the solution to (1), given $\{v_t, \hat{q}_t, v_{t+1}, \hat{q}_{t+1}\}$ for $t = 0, 1, 2, ...$; and $\{v_t, \hat{q}_t\}$ that satisfy (10) and (11) for all $t = 0, 1, 2, ...$ Note that (2) is satisfied by the Walrus' law, whereas (11) and the second and the fifth constraint in (9) are sufficient for (3).

3.2 Monetary equilibrium

The first and the third constraint in (9) imply perfect consumption smoothing across all states:

$$c_{i,O,t+1} = c'_{i,O,t+1} \tag{12}$$

This result holds because the amount of fiat money for goods held by each old agent is determined before the realization of health shocks. Also, substituting the fourth and the fifth constraint in the sixth constraint yields

$$\frac{v_t m_{i,Y,t}}{\alpha} + \frac{\hat{q}_t \hat{m}_{i,Y,t}}{\beta} = \bar{\ell} \tag{13}$$

Then, the first-order conditions for fiat monies for goods and elderly care, $m_{i,Y,t}$ and $\hat{m}_{i,Y,t}$, respectively, imply that

$$\frac{\alpha v_{t+1}}{v_t}[\theta u'(c_{i,O,t+1}) + (1-\theta)u'(c'_{i,O,t+1})] = \frac{\beta \hat{q}_{t+1}}{\hat{q}_t}\theta g(\bar{h} - h_{i,O,t+1})) \tag{14}$$

in any monetary equilibrium, where the left-hand side and the right-hand side are the marginal gains from allocating labor to acquire fiat monies for goods and elderly care, respectively.

Now consider a monetary equilibrium in which the real value of each type of fiat money held by a young agent, i.e., $v_t m_{i,Y,t}$ and $\hat{q}_t \hat{m}_{i,Y,t}$, stays constant in each period. Hereafter, $\hat{M}_t$ denotes the total amount of fiat money for elderly care held by old agents at the beginning of period t:

$$\hat{M}_t \equiv \int_0^{N_{t-1}} \hat{m}_{i,Y,t-1}\, di \tag{15}$$

Given the homogeneity of young agents and the market clearing conditions for the two types of fiat monies, (10) and (11), aggregating the fourth and the fifth constraint in (9) implies that the real values of fiat monies take the following values in each period:

$$v_t = \frac{\alpha l_{i,Y,t} N_t}{M} \tag{16}$$

$$\hat{q}_t = \frac{\beta \hat{l}_{i,Y,t} N_t}{\hat{M}_t} \tag{17}$$

Thus, $v_t m_{i,Y,t}$ stays constant if

$$\frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} \tag{18}$$

$$\tag{19}$$

as implied by (10). Also, (11) implies

$$\hat{M}_t = \theta \hat{M}_{t-1} \tag{20}$$

for $t = 1, 2, ...$, because only a fraction θ of old agents are hit by health shocks and spend fiat money for elderly care in each period. Hence, $\hat{q}_t \hat{m}_{i,Y,t}$ stays constant if

$$\frac{\hat{q}_{t+1}}{\hat{q}_t} = \frac{N_{t+1}}{\theta N_t} \tag{21}$$

By substituting (12), (18), and (21) into (14), it can be shown that (7) and (8) hold in the monetary equilibrium.[1] Therefore, the allocation of labor into the production of goods and the supply of elderly care, i.e., $\ell_{i,Y,t}$ and $\hat{\ell}_{i,Y,t}$, respectively, remains the same as in the baseline model, given the market clearing conditions for goods and elderly care, i.e., (2) and (3), respectively, being the same.

3.3 Implications of the model

Fiat money for goods in the model can be interpreted as the nominal liabilities of the consolidated government in general, as it is held as a store of wealth in the model. Thus, the

[1] Combining the first and the fourth constraint in (9) yields $c_{i,O,t+1} = \alpha \hat{\ell}_{i,O,t} N_{t+1}/N_t$, given (18). Similarly, the second and the fifth constraint in (9) imply $\theta h_{i,O,t+1} = \beta \hat{\ell}_{i,O,t} N_{t+1}/N_t$, given (21). Substituting these two equations into the sixth constraint in (9) yields (7).

model does not distinguish base money issued by the central bank and government bonds. Note that governments bonds in reality are redeemable only in currency. Also, they are eligible collateral for the central bank, against which financial institutions can borrow base money from the central bank elastically. Hence, base money and nominal government bonds are close substitutes. For simplicity, the model abstracts from nominal interest payments on government bonds and the difference between base money and government bonds in terms of liquidity.

Given this interpretation, the model demonstrates that introducing a second type of fiat money in addition to conventional currency and government bonds can replace an insurance in a one-money system, if there exists idiosyncratic risk that makes old generations demand some specific services supplied by younger generations, such as elderly care. For this result, the two fiat monies must not be exchangeable, so that old generations that do not have the realization of the risk cannot use the second type of fiat money for any purpose. This feature of the second type of fiat money is essential to replicate the function of an insurance.

This result implies that measuring wealth by multiple dimensions is necessary for optimal risk sharing in the society. The current one-money system, in which everything can be convertible into currency and thus measured by currency units, cannot achieve this goal unless there is a fair insurance. Introducing the same numbers of monies as the number of idiosyncratic risks would achieve the same goal without insurances, including social security.

In addition, comparison between (6) and (16) implies that the nominal price of goods, $1/v_t$, increases if the government distributes fiat money for elderly care to old agents in a period unexpectedly, so that the economy shifts from a one-money system to a multiple-money system afterward. This result holds because traditional money can buy a smaller range of goods and services in a multiple-money system, given the total supply of traditional money, M, being unchanged.

In reality, this case can be applied to a country with a public elderly care insurance, such as Japan. Given the presence of such an insurance, suppose that the government creates new fiat money for elderly care, and pays the new money to elderly care providers as insurance payments until young generations become old and start spending the fiat money for elderly care they acquire. In this case, the government can use traditional money pooled at the elderly care insurance for purchasing other goods and services without changing the total liabilities of the consolidated government, which corresponds to M. The model indicates inflation would occur in such a case.

Alternatively, the government can use saved money to retire government bonds. This case corresponds to the case in which the government retire saved money, which reduces the value of M. In this case, the price level remains unchanged. In either case, the real value of government bonds falls. This effect of the introduction of fiat money for elderly care would help to mitigate a fiscal crisis in a debt-laden country like Japan.

4 Indivisible labor and two monies

In the previous section, all young agents supply elderly care to old agents. In reality, each individual tends to be specialized in one profession. In this section, labor is assumed to be indivisible so that each young agent must choose whether to supply labor for the production of goods or for the supply of elderly care. It will be shown that the result described above is robust, if agents can swap the two monies only when young.

4.1 Model with indivisible labor and one money

First, suppose that young agents' labor is indivisible in the baseline model described in section 2. In this case, the utility maximization problem of a young agent, (1), includes

$$l_{i,Y,t} \in \{0,\ \bar{\ell}\} \tag{22}$$

$$\hat{l}_{i,Y,t} \in \{0,\ \bar{\ell}\} \tag{23}$$

as an additional constraint for $i \in [0, N_t]$ and $t = 0, 1, 2, ...$ Because (5) makes compensation for labor equal between the production of goods and the supply of elderly care, young agents are indifferent to the allocation of labor if (5) holds. Thus, there exists a monetary equilibrium such that (5)-(8) hold. In this equilibrium, part of young agents supply labor for the production of goods while the other young agents supply labor for elderly care, so that (7) and (8) are satisfied with the market clearing conditions for goods and elderly care, (2) and (3), respectively.

4.2 Model with indivisible labor and two monies

Next, consider the model with the two types of fiat monies. If young agents' labor is indivisible, it is impossible for a young agent to work for both the production of goods and the supply of elderly care to acquire both types of fiat money. Thus, the government must allow young agents to swap the two types of fiat monies. Suppose that there exists a competitive exchange market for fiat monies. In this case, the fourth and the fifth constraint in the utility maximization problem for a young agent, (9), are replaced by an integrated flow of funds constraint for a young agent such that

$$m_{i,Y,t} + e_t \hat{m}_{i,Y,t} = \frac{\alpha l_{i,Y,t}}{v_t} + \frac{e_t \beta \hat{\ell}_{i,Y,t}}{\hat{q}_t} \tag{24}$$

where e_t denotes the nominal exchange rate between fiat money for goods and that for elderly care. Note that the right-hand side is the total real value of fiat monies earned by a young agent in terms of goods. Also, (22) and (23) are added to the constraints in (9).

Because young agents must be indifferent to whether to supply labor for the production of goods or for the supply of elderly care, the nominal exchange rate must take the following value in the monetary equilibrium:

$$e_t = \frac{\alpha \hat{q}_t}{\beta v_t} \tag{25}$$

This value of e_t makes the relative value of the two types of fiat monies equal to that in 13. Thus, each young agent's allocation of income on the two types of fiat monies, and hence the consumption of goods and elderly care by old agents, remains the same as in the model described in section 3.

5 Conclusions

This paper considers a model environment in which old people demand specific services from young people due to idiosyncratic shocks, such as elderly care, and shows that a two-money system can implement optimal risk sharing without a formal insurance if fiat money for these services are separated from fiat money for the other types of goods and services. To implement this result, it is necessary to prohibit old people from exchanging different types of fiat monies.

The model shows the equivalence of a one-money system with a fair insurance and a two-money system without a fair insurance. A question remains as to which system is better. The answer to this question depends on frictions associated with a fair insurance, such as whether insurers can be credibly committed to contingent insurance payments. The two-money system described in this paper can work even if insurers have a limited commitment problem.

In reality, registering the holders of fiat monies would be necessary to restrict the liquidity of fiat monies for a holder satisfying a certain demographic condition, such as the age. This feature of fiat monies in the optimal two-money system collides with the demand for anonymous money holdings and payments in reality. The investigation of this issue is left for future research.

References

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