WINPEC Working Paper Series No.E1906
May 2019

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March 19, 2019

Abstract

Political parties and candidates usually prefer making ambiguous promises. This study identifies the conditions under which candidates choose ambiguous promises in equilibrium, given convex utility functions of voters. The results show that in a deterministic model, no equilibrium exists when voters have convex utility functions. However, in a probabilistic voting model, candidates make ambiguous promises in equilibrium when, (i) voters have convex utility functions, and (ii) the distribution of voters' most preferred policies is polarized.

Keywords: elections, political ambiguity, public promise, campaign platform, probabilistic voting, polarization

JEL Classification Numbers: D71, D72

*The author has benefitted from the comments of Amihai Glazer, Airo Hino, Hideki Konishi, Michael Munger, Tetsuro Okazaki, and Orestis Troumpounis. Financial support from JSPS KAKENHI (Grant Number 26780178) is gratefully acknowledged.

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1 Introduction

Politicians prefer using vague words and announce several policies in their electoral promises, a practice referred to as "political ambiguity." A standard and classical interpretation of political ambiguity is a lottery, that is, a probability distribution on policies. This can be explained in the following manner: candidates announce a lottery, and voters choose the candidate who announces the better lottery (Zeckhauser, 1969; Shepsle, 1972; Aragones and Postlewaite, 2002; Callander and Wilson, 2008). One possible reason why candidates make such vague promises is because voters have convex utility functions. Zeckhauser (1969) was the first to interpret political ambiguity as a lottery, and showed that the median policy, which is most preferred by the median voter, can be defeated by a risky lottery when the voter's utility function is convex. Shepsle (1972) generalizes the findings of Zeckhauser (1969) and shows that a Condorcet winner does not exist when voters have convex utility functions. However, they do not establish the existence of equilibria in which candidates announce ambiguous promises. Aragones and Postlewaite (2002) show political ambiguity as an equilibrium phenomenon using voters' convex utility functions. However, they assume that candidates need to provide a positive probability for their most preferred policy. Thus, a campaign promise is always ambiguous when candidates commit to implementing a policy other than their own most preferred policy. To the best of my knowledge, no existing studies show that a candidate chooses to make an ambiguous promise in equilibrium because of the convex utility functions of voters, without any restriction on the candidate's choices.

This study identifies the conditions under which candidates choose ambiguous promises in equilibrium when voters have convex utility functions, and there is no restriction on the candidate's choices. It extends the standard Downsian model with fully office-motivated candidates to allow a candidate chooses a lottery. Voters vote sincerely, and two candidates announce a binding promise before an election: a candidate will implement a policy according to the probability distribution of the announced promise after s/he wins the election.[1]

[1] The assumption of a binding promise is employed by electoral competition models in the Downsian tradition, but the Downs model does not consider commitment to a lottery. It is possible to suppose that in this study, the model used implicitly assumes a repeated game between candidates and voters. In order to induce voters to believe a probability distribution announced by a candidate in the future, candidates have

The findings are as follows. First, in a deterministic model without any uncertainty, the unique Condorcet winner is the median policy when voters have concave or linear utility functions. However, no Condorcet winner exists when voters have convex utility functions. Therefore, two candidates choose the median policy in equilibrium when voters have concave or linear utility functions, but no equilibrium exists in the case of convex utility functions. On the other hand, in a probabilistic voting model, where candidates are uncertain about voters' preferences, they choose ambiguous promises in equilibrium when (i) voters have convex utility functions, and (ii) the distribution of voters' most preferred policies is polarized. Therefore, for political ambiguity to be considered as an equilibrium phenomenon with convex utility functions, voters must be polarized, and voting must be probabilistic.

Most prior studies assume that voters are risk-averse. However, there is no robust and clear evidence that voters have concave utility functions for all political issues. Osborne (1995) states that, "I am uncomfortable with the implication of concavity that extremists are highly sensitive to differences between moderate candidates (p. 275)," and "it is not clear that evidence that people are risk-averse in economic decision-making has any relevance here (p. 276)." Furthermore, Kamada and Kojima (2014) state that, "(e)conomic policy is arguably a concave issue, given the evidence that individuals are risk-averse in financial decisions. By contrast, voters may have convex utility functions on moral or religious issues (p.204)." Their findings imply that an ambiguous promise tends to be used for non-economic issues, which may be a convex issue. Shepsle (1972) states the following:

> In the 1968 presidential campaign, both Nixon's "I have a plan" statements on the Vietnam issue and Humphrey's "law and order with justice" slogan on "the social issue" suggest that equivocal pronouncements during the course of campaign are a common and recurrent theme in American electoral politics. (p. 555)

These are examples of ambiguity regarding non-economic issues, and public opinion on the Vietnam war was almost equally divided and polarized between pro-escalation and anti-escalation (Verba et al., 1967). Therefore, this model shows one possible explanation for

an incentive to keep their promises.

why Nixon chose an ambiguous promise.

Through this study, I do not intend to say that convexity of utility functions is the only reason why political ambiguity emerges; many reasonable mechanisms have been suggested, as discussed in the literature review. However, although prior studies recognize the convexity of a voter's utility function as one reason for the emergence of ambiguity, none show it as an equilibrium phenomenon without any restrictions on a candidate's strategy. Thus, one of the main contributions of this study is to show additional conditions (i.e., probabilistic voting and polarization) in which candidates choose a vague promise in equilibrium, given voters' convex utility functions.

1.1 Related Literature

1.1.1 Causes of Political Ambiguity

Prior studies use formal models to indicate various mechanisms that generate political ambiguity. There are two main types of models: voter-centered models and candidate-centered models. Voter-centered models suppose that voters prefer a higher degree of ambiguity, and that candidates choose an ambiguous promise to win an election. This category includes models with convex utility functions of voters. Callander and Wilson (2008) indicate that voters may develop a taste for ambiguity using the notion of context-dependent voting, as introduced in their earlier paper (2006).[2] Kartik, Van Weelden, and Wolton (2017) suppose that voters are uncertain about their own ideal policy, and only politicians receive policy-relevant information after an election. In this case, voters prefer a politician who announces a vague promise when the politician shares the voters' policy preferences, because they prefer to allow the politician the discretion to adopt policies. Moreover, if voters believe incorrectly that their favored candidate's position is closer to their most preferred policy than it actually is (Jensen, 2009), or that they are not expected-utility maximizers and have Knightian uncertainty (Berliant and Konishi, 2005), candidates who make ambiguous promises may win the election.

[2]Context-dependent voting means that voters are interested not only in the policies of the party in question, but also in the relative attractiveness of the oppositions' policies.

On the other hand, candidate-centered models suppose that voters dislike ambiguity, but that candidates prefer ambiguity. First, candidates may prefer political ambiguity because of its direct (non-electoral) benefits. For example, they may not know which policy is most expedient (Aragones and Neeman, 2000), and may want the flexibility to implement their own preferred policy (Alesina and Cukierman, 1990). Furthermore, a party may be able to recruit a greater number of elites by allowing for ideological diversity (Jensen and Lee, 2017). Second, candidates may be able to obtain indirect (electoral) benefits from political ambiguity, even though voters prefer a less ambiguous policy. For example, when candidates are uncertain about the position of the median policy, they may prefer to maintain ambiguity (Glazer, 1990). This is especially true in primary elections because candidates have less information about voters' preferences (Meirowitz, 2005). Moreover, if campaign platforms are decided sequentially, the follower, who makes policy decisions later than his/her opponent, has a significant advantage when a Condorcet winner does not exist. As a result, candidates prefer to retain political ambiguity (Kamada and Sugaya, 2018).

1.1.2 Definitions of Political Ambiguity

We interpret "political ambiguity" as a lottery that includes several policies; however, it has alternative definitions. Some studies interpret a set of policies as an ambiguous policy, and do not consider a candidate's decision-making on a probability distribution in this set (Glazer, 1990; Jensen, 2009; Aragones and Neeman, 2000; Kartik, Van Weelden, and Wolton, 2017; Kamada and Sugaya, 2018). Meirowitz (2005) and Berliant and Konishi (2005) suppose that ambiguity exists when candidates do not announce anything in their campaigns.[3] Most of these studies suppose that candidates do not have the discretion to decide the probability distribution on policies, which is given exogenously in the model. That is, candidates cannot change the degree of ambiguity totally. On the other hand, prior studies that suppose political ambiguity as a lottery assume that candidates can choose any probability distribution freely. The reality should be between these two definitions; candidates have some (but not perfect) discretion to decide a probability distribution. However, this paper supposes

[3] On the other hand, Alesina and Cukierman (1990), and Jensen and Lee (2017) define the level of ambiguity as the variance in the noise of the policy outcomes observed by voters.

political ambiguity as a lottery to clearly investigate the strategic choices of candidates on the degree of ambiguity.

It may be unrealistic that candidates announce a specific probability distribution on policies in his/her campaign. There are two justifications for this point. First, using words, candidates may induce voters to have specific expectations. For example, in Japan, Prime Minister Shinzo Abe made an ambiguous announcement stating that he will increase the consumption tax rate in 2019 if a severe financial crisis (or a similar event) does not occur. Thus, voters may think that although the probability for such an event is high since financial crises do not occur frequently, it is not 100% certain. Second, candidates may induce voters to have specific expectations by allocating weights (emphasis) to each policy (Page, 1976).[4] Voters believe that a policy with higher weight is more likely to be implemented. These interpretations suppose a probability distribution of an ambiguous promise as beliefs held by voters. In other words, a candidate can use weights or words to induce voters to form specific beliefs.

1.1.3 Political Ambiguity and Divergence

The probabilistic voting model adopted here is based on that of Kamada and Kojima (2014), who suppose that candidates can choose only a single policy (not a lottery). They show that with convex utility functions of voters and a polarized voter distribution, perfectly divergent candidates result in a unique equilibrium. Here, perfect divergence means that without exception, the left candidate chooses a left policy, while the right candidate chooses a right policy. On the other hand, we allow candidates to choose a lottery instead of a single policy, which increases the number of equilibria. Thus, ambiguity can arise in the form of equilibrium strategies in the context of convex voter utilities. In some equilibria, candidates choose the same ambiguous lottery, so policy divergence does not occur. On the other hand, perfectly divergent equilibrium shown by Kamada and Kojima (2014) also exists in this

[4] Page (1976) considers that political ambiguity arises when candidates allocate their limited resources (emphasis) among several policies. If candidates do not allocate sufficient resources to a policy, its promise to voters becomes vague. Chappell (1994), Dellas and Koubi (1994), and Chu and Niou (2005) follow a very similar interpretation.

model. Therefore, this model shows that a probability voting model with convex utilities is useful to show not only political polarization but also political ambiguity.

2 Analysis

2.1 Deterministic Voting

First, this subsection presents the implications of the deterministic model as a benchmark. Let us denote X as the set of policies, and define $g(x, y)$ as the majority margin for $x, y \in X$; the number of voters who prefer x to y minus the number of voters who prefer y to x, where x and y are single policies. A policy x is the Condorcet winner when $g(x, y) \geq 0$ for all $y \in X$ (Black, 1948). Let us denote ΔX as the set of probability distributions over X. Define $g(p, q)$ as the majority margin for lotteries $p, q \in \Delta X$. We call a Condorcet winner on ΔX a Condorcet winning lottery, which is defined as follows:[5]

Definition 1 *A Condorcet winning lottery is a lottery p, such that $g(p, q) \geq 0$ for all $q \in \Delta X$.*

Suppose three policies, $X = \{L, M, R\}$, where an element of ΔX is $(p_L, p_M, p_R) \in \Delta X$, and $p_x \geq 0$ is the probability that $x \in X$ occurs, where $p_L + p_M + p_R = 1$. In addition, suppose there is a population of voters of mass one, divided into three discrete groups, l, m, and r, and the proportion of voters in each group is less than 1/2; that is, no group constitutes a majority. Denote the set of groups as $G = \{l, m, r\}$, and its element as $g \in G$. Suppose that members of each group have the following preference relations:

$$\begin{aligned} l &: L \succ M \succ R \\ m &: M \succ L \succsim R \\ r &: R \succ M \succ L. \end{aligned} \tag{1}$$

[5] This differs from the maximal lottery (probabilistic/randomized Condorcet winner) proposed by Fishburn (1984). A maximal lottery supposes that voters make a decision after a policy is revealed from each lottery, whereas a Condorcet winning lottery supposes that voters choose before the outcomes of the lotteries are revealed. More precisely, p is a maximal lottery if $\sum_{x,y \in X} p(x)q(y)g(x, y) \geq 0$ for all $q \in \Delta X$.

These preference relations satisfy single-peakedness. Furthermore, the median point is M, which is the Condorcet winner in Black (1948).[6] These preference relations of voters in group g can be represented by the Von Neumann–Morgenstern utility function $u_g : X \to \{0, v, 1\}$, with $v \in (0,1)$. The function assigns the value one to the most preferred alternative, v to the second-best alternative, and zero to the worst alternative. A voter has a concave utility function if $v > 1/2$, a linear utility function if $v = 1/2$, and a convex utility function if $v < 1/2$. Note that if a member of group m has $L \sim R$, the utilities of both L and R are $v = u_m(L) = u_m(R)$. We refer to the lottery $(p_L, p_M, p_R) = (0, 1, 0)$ simply as M. Suppose that voters choose to abstain when they are indifferent. Note that the result does not change even if voters choose a lottery with an equal probability when they are indifferent. Then, we have the following proposition.[7]

Proposition 1 *A Condorcet winning lottery is M when $v \geq 1/2$, and does not exist when $v < 1/2$.*

Proof See Appendix A.1. □

The rationale is as follows. Policy M cannot be the Condorcet winning lottery if voters have convex utility functions (i.e., $v < 1/2$). If M is chosen, the utilities of voters in groups l, m, and r are v, 1, and v, respectively. On the other hand, if lottery q_1 with $(p_L, p_M, p_R) = (1/2, 0, 1/2)$ is chosen, the utilities of voters in l, m, and r are $1/2$, $v/2$, and $1/2$, respectively. Thus, if $v < 1/2$, the voters in l and r prefer q_1 to M, and M is defeated by q_1 in a pairwise election. Moreover, q_1 cannot be a Condorcet winning lottery. If lottery q_2 with $(p_L, p_M, p_R) = (2/3, 1/3, 0)$ is chosen, the utilities of voters in l, m, and r are $(2 + v)/3$, $(1 + 2v)/3$, and $v/3$, respectively. Thus, the voters in l and m prefer q_2 to q_1. However, q_2 is also defeated by q_3 with $(p_L, p_M, p_R) = (0, 2/3, 1/3)$. As in these cases, for any lottery, there is another that will receive the majority's support. The sum of the probabilities of choosing each policy is one. Thus, at least one group has a positive probability of its best policy being chosen. This probability can be divided between the remaining two groups' most preferred

[6] If a Condorcet winner does not exist, a Condorcet winning lottery does not exist either (Fishburn, 1972).

[7] In the following proposition, Shepsle (1972) demonstrates the case with risk-loving voters, and Aragones and Postlewaite (2002) demonstrate the case with risk-averse voters. I merge these two findings by adding an analysis with risk-neutral voters.

policies. This new lottery can then defeat the original lottery. On the other hand, when $v \geq 1/2$, M is not defeated by q_1 (or any other lotteries).

Such a preference cycle usually occurs when a policy space has multiple dimensions. Indeed, when we suppose that candidates can choose a lottery instead of a single policy, the space of lotteries has two dimensions, since p_L and p_M should be identified (and p_R is determined by $p_R = 1 - p_L - p_M$), even though the dimension of a single policy is one. However, if they have concave utility functions, all voters prefer the least risk, that is, to make a certain choice. If they have linear utility functions, voters in l and r are indifferent, but voters in m still prefer M to q_1 because their utility is maximized when M is chosen for sure. Consequently, if voters have concave or linear utility functions, they all (weakly) prefer a less ambiguous choice, in which case the dimension of the space can be considered to be one ($p_L = 1$, $p_M = 1$, or $p_R = 1$). Therefore, a Condorcet winning lottery exists if $v \geq 1/2$.

On the other hand, if voters have convex utility functions, conflicts of interest will arise among them: voters in group m prefer M to q_1 because their utility is maximized when M is chosen for sure. However, others prefer q_1 to M because q_1 is riskier. Thus, both the position of a lottery and its degree of ambiguity matter, and this multi-dimensional space induces the non-existence of a Condorcet winning lottery.

2.2 Probabilistic Voting

2.2.1 Settings

A Condorcet winning lottery does not exist when voters have convex utility functions, as in the case of multiple policy dimensions. One method of finding an equilibrium when there are multiple policy dimensions is to introduce probabilistic voting, in which candidates are uncertain about voters' preferences.

The voters' preference relations on policies and utilities are the same as those in (1). However, suppose that the members of group m have $L \sim R$, such that the utilities from L and R are both v. A continuum of voters is distributed to each group according to a probability mass function $f : G \to [0, 1/2)$, with $f(m) = \lambda$ and $f(l) = f(r) = (1 - \lambda)/2$, where $\lambda \in [0, 1/2)$; that is, we consider a symmetric distribution. The parameter λ represents the degree of centralization of the voter distribution. Two candidates 1 and 2 simultaneously determine the weight to allocate to each policy, $\Sigma_i = (\sigma_i^L, \sigma_i^M, \sigma_i^R) \in \Delta X$ before the election, where $i = 1$ or 2. The value of $\sigma_i^x \in [0,1]$ is the weight assigned to policy $x \in X$, where $\sigma_i^L + \sigma_i^M + \sigma_i^R = 1$. Note that Σ_i is not a mixed strategy on X, because the policy is chosen after the election, while in a mixed strategy, a policy is chosen before an election.[8] We also suppose that voters believe that the probability that policy x will be implemented after an election is the same as the weight on x; therefore, candidates can affect voters' beliefs by allocating weights, as Callander and Wilson (2008) supposed.

Candidate i obtains the share of voters given by

$$\Pi(\Sigma_i, \Sigma_{-i}) = \sum_{g \in G} f(g) \pi \left(\sum_{x \in X} \sigma_i^x u_g(x) - \sum_{x \in X} \sigma_{-i}^x u_g(x) \right),$$

where Σ_{-i} is chosen by i's opponent, and σ_{-i}^x is the weight on policy x. The function $\pi : \Re \to [0,1]$ is strictly increasing ($\pi'(t) > 0$), satisfying $\pi(t) + \pi(-t) = 1$ (thus, $\pi(0) = 1/2$), and is strictly concave ($\pi''(t) < 0$) for all $t \in [0, \infty)$. Since $\pi(t) + \pi(-t) = 1$, $\pi'(t) = \pi'(-t)$ for all $t \in [0, \infty)$. Here, $\sum_{x \in X} \sigma_i^x u_g(x)$ is the expected utility of a voter in group g when candidate i wins the election. In addition, $\sum_{x \in X} \sigma_i^x u_g(x) - \sum_{x \in X} \sigma_{-i}^x u_g(x)$ is the difference in the expected utility of a voter in group g between the promise of candidate i and that of his/her opponent. If this is positive (negative), candidate i's lottery gives a higher (lower) expected utility than that of his/her opponent. In the deterministic model, $\pi(t) = 1$, when $t > 0$, and $\pi(t) = 0$ when $t < 0$. However, in the case of probabilistic voting, even if $t > 0$, $\pi(t) \in (1/2, 1]$. One interpretation of this is that voters make decisions based not only on candidates' policies, but also on other factors, and therefore, their voting behavior is probabilistic. We suppose that an office-motivated candidate i maximizes $\Pi(\Sigma_i, \Sigma_{-i})$.

2.2.2 Equilibrium with Convergence

There exist multiple equilibria of this game. In order to clarify a situation where both candidates choose an ambiguous promise, we use the following corollary to show equilibria where

[8] The model used here implicitly supposes sequential decision-makings where candidates announce a promise before an election, and decide a policy after an election. On the other hand, if we suppose simultaneous decision-makings, Σ_i and a mixed strategy should be identical.

both candidates choose the same lottery ($\Sigma_i = \Sigma_{-i}$), i.e. both candidates are converged.

Corollary 1 *(i) If $v < 1/2$ and*

$$\lambda \leq \frac{1-2v}{3-4v},$$

an equilibrium with $\Sigma_i = \Sigma_{-i}$ must satisfy $\sigma_i^M = \sigma_{-i}^M = 0$. (ii) Otherwise, an equilibrium with $\Sigma_i = \Sigma_{-i}$ must satisfy $\sigma_i^M = \sigma_{-i}^M = 1$.

Proof See Appendix A.2. □

When voters have convex utility functions ($v < 1/2$), and the proportion of median voters (λ) is sufficiently small, an ambiguous lottery such as $\Sigma_i = \Sigma_{-i} = (1/2, 0, 1/2)$ can be an equilibrium. Otherwise, both candidates converge to the median policy M.

When voters have concave or linear utility functions ($v \geq 1/2$), there are no conflicts of interest with regard to the degree of ambiguity because all voters (weakly) prefer the lower degree of ambiguity. Therefore, the candidates should converge to M with certainty in equilibrium. This situation is the same as that of the Downsian model, and the median voter becomes critical in deciding the winner.

On the other hand, when voters have convex utility functions, conflicts of interest among the voters on the degree of ambiguity do exist, because the voters in groups l and r (extreme voters) prefer a higher degree of ambiguity, whereas those in group m (median voters) still prefer a less ambiguous policy. If the proportion of median voters λ is sufficiently high, candidates need to consider the median voters' interests, and thus, they converge to the median policy. However, if λ is low, candidates care more about the extreme voters than they do about median voters, and thus, choose an ambiguous policy. In many extensions of the Downsian model of electoral competitions, the candidate who wins the support of the median voter is the winner. However, when (i) voters have convex utility functions, (ii) the proportion of median voters is small, and (iii) candidates are allowed to announce a lottery as a policy platform, a candidate cannot win even if s/he gets the support of the median voter. Rather, candidates must ignore the interests of the median voter to win the election.

Note that when $v < 1/2$ and $\lambda \leq (1-2v)/(3-4v)$, there exist many equilibria with $\Sigma_i = \Sigma_{-i}$ and $\sigma_i^M = \sigma_{-i}^M = 0$ such that $\Sigma_i = \Sigma_{-i} = (1/3, 0, 2/3)$. There also exist other equilibria with $\Sigma_i \neq \Sigma_{-i}$. Proposition 2 in the next subsection shows such equilibria with divergence.

2.2.3 Equilibrium with Divergence

Denote

$$\overline{\sigma} \equiv \sigma_i^L - \sigma_{-i}^L, \text{ and}$$
$$\overline{\lambda} \equiv \frac{(1-2v)\pi'(\overline{\sigma})}{2(1-v)\pi'(0) + (1-2v)\pi'(\overline{\sigma})}.$$

We then have the following proposition.[9]

Proposition 2 *Suppose $v < 1/2$. A strategy profile with $\sigma_1^M = \sigma_2^M = 0$ and $\overline{\sigma} \equiv \sigma_i^L - \sigma_{-i}^L$ (hence $\sigma_i^R - \sigma_{-i}^R = -\overline{\sigma}$) is a Nash equilibrium when $\lambda \leq \overline{\lambda}$.*

Proof See Appendix A.2. □

As in Corollary 1, when the degree of political centralization λ is sufficiently small, political ambiguity can emerge. Note that when voters have concave or linear utility functions, both candidates choosing M for sure, is a unique equilibrium.

Corollary 2 *If $v \geq 1/2$, $\sigma_1^M = \sigma_2^M = 1$ is a unique equilibrium.*

Proof See Appendix A.2. □

Kamada and Kojima (2014) consider probabilistic voting where candidates can choose only a single policy (not a lottery) and show that a strategy profile with $\Sigma_i = (1, 0, 0)$ and $\Sigma_{-i} = (0, 0, 1)$ is an equilibrium in the case of convex utility functions and a polarized voter distribution. On the other hand, we allow candidates to choose a lottery rather than a single policy. As a result, they may choose partially divergent policies: They combine policy divergence and political ambiguity (i.e., $\sigma_i^L > 0$ and $\sigma_i^R > 0$ with $\Sigma_i \neq \Sigma_{-i}$).

Note that the equilibrium with a perfectly divergent and certain policy shown by Kamada and Kojima (2014) (i.e., $\sigma_i^L = 1$ and $\sigma_{-i}^R = 1$) is one of equilibria in Proposition 2. Also, equilibrium with a perfectly convergent and ambiguous policy such that $\Sigma_i = \Sigma_{-i} = (1/2, 0, 1/2)$ shown by Corollary 1 is also one of equilibria in Proposition 2.

In addition, the following corollary is obtained.

[9] It does not consider an indifferent case which rarely occurs. See the appendix for more details.

Corollary 3 *As $\pi'(\overline{\sigma})$ increases, $\overline{\lambda}$ increases.*

This corollary has two implications. First, it is less likely to lead to an equilibrium with more divergence, i.e., higher $\overline{\sigma}$. Because $\pi''(t) < 0$ for all $t \in [0, \infty)$, a policy with more divergence has lower $\pi'(\overline{\sigma})$, which decreases $\overline{\lambda}$. Thus, the condition $\lambda \leq \overline{\lambda}$ becomes more difficult to satisfy. Second, if voters are more sensitive to differences between candidates, candidates tend to be converged. Suppose two functions π and $\overline{\pi}$ such that $\pi(t) < \overline{\pi}(t)$ for all $t \in [0, \infty)$, that is voters are more sensitive to policy divergence with $\overline{\pi}$ than π. Since $\pi(t) < \overline{\pi}(t)$ for all $t \in [0, \infty)$, $\pi'(t) < \overline{\pi}'(t)$ with low t while $\pi'(t) > \overline{\pi}'(t)$ with high t where $t \in [0, \infty)$. This means that the condition $\lambda \leq \overline{\lambda}$ is more likely to be satisfied with low $\overline{\sigma}$, but it becomes difficult to be satisfied with high $\overline{\sigma}$.

2.3 Discussion

When (i) voters have a convex utility function, and (ii) the distribution of their most preferred policies is polarized, candidates choose policy divergence, political ambiguity, or any combination of the two. As discussed in the introduction, voters may have convex utility functions on non-economic issues. Although policy divergence is observed for some non-economic issues with polarized voters, such as the debate around same-sex marriage in the United States (Kamada and Kojima, 2014), candidates also prefer choosing an ambiguous position. Another example of political ambiguity is the constitutional reform in Japan.

The Constitution of Japan was enacted in 1947 as the new constitution for post-war Japan. In 1947, Japan was occupied by the Allies, mainly the United States. Thus, the Constitution was written by non-Japanese, although the opinion of many Japanese were taken into account. Therefore, constitutional reform has been a topic of frequent discussion since Japan gained independence. Article 9 is the most controversial, as it prohibits Japan from holding any military power. Nevertheless, Japan has had a defense force that has held military power since 1954. Public opinion on constitutional reform is divided. According to a 2017 poll conducted by NHK (the public broadcaster in Japan), 43% of the responses were in favor of the reform, while 34% of the responses were against the reform. These values have remained much the same over time[10]. This issue is not related to the economy; therefore, it may be a convex issue and the distribution of voters' opinions are polarized. Thus, the conditions for political ambiguity are satisfied.

Since 1955, the Liberal Democratic Party of Japan (LDP) has run the government, except during the periods, 1993–1994 and 2008–2012. In the early period of the LDP administration (e.g., the Hatoyama administration, 1954–1956), many claimed that the Constitution should be written by the Japanese people. However, since the 1960s, LDP administrations have avoided discussing (and almost given up on) this issue because public opinion was so divided and an intra-LDP faction hesitated to implement reforms (Machidori, 2016, p.4). Consequently, the Japanese Constitution has not yet been revised. Recently, Prime Minister Shinzo Abe explicitly promised to reform the Constitution in the 2017 general election. The 2017 LDP manifesto (a booklet containing campaign promises) devoted about two pages (out of a total of 38 pages) to this promise. In contrast, in the 2012 and 2014 elections, in which Abe was also the party leader, the LDP manifestos devoted only one-sixth to half a page (out of 26 pages) to this issue. Moreover, even in the 2017 manifesto, details on the reform remained vague. Voters believed that the LDP was more likely to revise the Constitution than its opponents ($\sigma_i^R > \sigma_i^L$); however, it was ambiguous ($\sigma_i^R \neq 1$).

In an election, parties and candidates usually announce and promote their economic policies. Indeed, most LDP manifestos in 2012, 2014, and 2017 laid significant emphasis (large weights) on explaining economic policies (popularly known as *Abenomics* in Japan). On the other hand, candidates prefer maintaining a degree of ambiguity on social and national-security issues. Possibly, they prefer specifying an economic policy because voters have concave utility functions for such economic policies. However, they prefer ambiguity for non-economic and polarized issues, where voters may have convex utility functions.

3 Conclusion

Prior studies usually interpret political ambiguity as a lottery. This study supposes that voters can choose between lotteries, rather than a single policy. Further, it identifies the

[10]From the website of NHK (https://www3.nhk.or.jp/news/special/kenpou70/yoron2017.html)

conditions under which political ambiguity occurs in equilibrium, given the convex utility functions of voters. In the deterministic model, if voters have concave or linear utility functions, the median policy is still the Condorcet winner. However, if voters have convex utility functions, the existence of the Condorcet winning lottery is not ensured because the space of campaign promises has multiple dimensions. On the other hand, in the probabilistic voting model, candidates choose an ambiguous promise in equilibrium when (i) voters have convex utility functions and (ii) the distribution of voters' most preferred policies is polarized. Therefore, to have political ambiguity as an equilibrium phenomenon with convex utility functions of voters, voters need to be polarized, and candidates must be uncertain about voters' preferences.

There are several directions for future research. First, it is important to identify the policy issues on which voters have convex utility functions. Second, many extensions of the Downsian model suppose that candidates can choose a single policy only. However, in reality, this is rare. When voters have concave or linear utility functions, the main finding, where both candidates converge to the median policy, does not change, which may be a reasonable simplification. However, if voters have convex utility functions, the implications can change as a result of allowing candidates to choose a lottery instead of a single policy, as we have shown here. Thus, any extension of the Downsian model should generate additional (or different) implications by supposing a lottery instead of a single policy. The third issue is to suppose a multidimensional policy space. This paper assumes that all policy issues are separable, but they should be intertwined with each other in reality. Finally, generalization to continuous policy space would be important extensions. Kamada and Kojima (2014) indicate that a "three-point model is advantageous because it is tractable and allows us to unambiguously define the degree of convexity of any voter function" (p. 219). However, most past studies based on Downsian settings suppose a continuous policy space with a continuum of voters. In order to understand political ambiguity, this extension should be useful. Moreover, if we suppose a continuous policy space, it is also possible to analyze a more realistic case where utilities are neither convex nor concave globally.

A Proof

A.1 Proposition 1

First, suppose a lottery q_L with $(p_L > 0, p_M \geq 0, p_R \geq 0)$, and a second lottery q'_L with $(p'_L = 0, p'_M \geq 0, p'_R \geq 0)$. When a member of group m has $L \succ R$, the voters in m prefer q'_L to q_L if $p_L v + (1 - p_L - p_R) < (1 - p'_R)$, which is $p_R + p_L(1 - v) > p'_R$. When a member of group m has $L \sim R$, they prefer q'_L to q_L if $p_L v + (1 - p_L - p_R) + p_R v < (1 - p'_R) + p'_R v$, which is $p_R + p_L > p'_R$. On the other hand, the voters in r prefer q'_L to q_L if $(1 - p_L - p_R)v + p_R < (1 - p'_R)v + p'_R$, which is $p'_R > p_R - (vp_L)/(1 - v)$. Because $v \in (0, 1)$, $p_R + p_L > p_R + p_L(1 - v) > p_R - (vp_L)/(1 - v)$, which means there exists q'_L, which defeats q_L. Thus, no lottery with $p_L > 0$ can be a Condorcet winning lottery.

Second, suppose a lottery q_R with $(p_L \geq 0, p_M \geq 0, p_R > 0)$ and a second lottery q'_R with $(p'_L \geq 0, p'_M \geq 0, p'_R = 0)$. When a member of group m has $L \succ R$, the voters in m prefer q'_R to q_R if $p_L v + (1 - p_L - p_R) < p'_L v + (1 - p'_L)$, which is $p_L + p_R/(1 - v) > p'_L$. When a member of m has $L \sim R$, they prefer q'_R to q_R if $p_L v + (1 - p_L - p_R) + p_R v < p'_L v + (1 - p'_L)$, which is $p_L + p_R > p'_L$. On the other hand, the voters in l prefer q'_R to q_R if $p_L + (1 - p_L - p_R)v < p'_L + (1 - p'_R)v$, which is $p'_L > p_L - (vp_R)/(1 - v)$. Because $v \in (0, 1)$, $p_L + p_R > p_L + p_R/(1 - v) > p_L - (vp_R)/(1 - v)$, there exists q'_R, which defeats q_R. Thus, no lottery with $p_R > 0$ can be a Condorcet winning lottery.

Therefore, only M $((p_L, p_M, p_R) = (0, 1, 0))$ can be a Condorcet winning lottery. From M, the utilities of the voters in l, m, and r are v, 1, and v, respectively. Because members of m earn the highest utility from M, they do not have an incentive to deviate. Suppose we have another lottery q_M with (p'_L, p'_M, p'_R). Then, the utilities of the voters in l and r are $p'_L + v(1 - p'_L - p'_R)$ and $p'_R + v(1 - p'_L - p'_R)$, respectively, from this lottery. If $p'_L \leq v$ or $p'_R \leq v$, q_M would never be able to defeat M. On the other hand, if $p'_L > v$ and $p'_R > v$, q_M can defeat M because the members of l and r prefer q_M to M. The conditions $p'_L > v$ and $p'_R > v$ can be satisfied at the same time only if $v < 1/2$. Therefore, a Condorcet winning lottery is M if and only if $v \geq 1/2$, and does not exist otherwise. □

A.2 Proposition 2, and Corollaries 1 and 2

First, we obtain the following lemma. Denote $\alpha \in [0,1]$, the weight on policy L such that $\sigma_i^L = \alpha(1-\sigma_i^M)$ and $\sigma_i^R = (1-\alpha)(1-\sigma_i^M)$.

Lemma 1 *Suppose $\sigma_i^M = \sigma_{-i}^M$. Then, given the opponent's strategy, and σ_i^M, a candidate is indifferent among any $\Sigma_i = (\sigma_i^L, \sigma_i^M, \sigma_i^R) = (\alpha(1-\sigma_i^M), \sigma_i^M, (1-\alpha)(1-\sigma_i^M))$ with $\alpha \in [0,1]$.*

Proof Suppose that the opponent of candidate i chooses Σ_{-i}, and from this policy, the voters in each group get u_l, u_m, and u_r, respectively. Then, candidate i chooses $\Sigma_i = (\sigma_i^L, \sigma_i^M, \sigma_i^R)$, such that it maximizes

$$\begin{aligned}\Pi(\Sigma_i, \Sigma_{-i}) &= \frac{1-\lambda}{2}\pi\left(\alpha(1-\sigma_i^M) + \sigma_i^M v - u_l\right) \\ &\quad +\lambda\pi\left(\sigma_i^M + (1-\sigma_i^M)v - u_m\right) \\ &\quad +\frac{1-\lambda}{2}\pi\left((1-\alpha)(1-\sigma_i^M) + \sigma_i^M v - u_r\right). \end{aligned} \tag{2}$$

where $\alpha \in [0,1]$. Suppose $\sigma_i^M = \sigma_{-i}^M$, and denote $\widehat{\sigma} \equiv \alpha(1-\sigma_i^M) + \sigma_i^M v - u_l$. Then, $(1-\alpha)(1-\sigma_i^M) + \sigma_i^M v - u_r = -\widehat{\sigma}$ when $(1-\sigma_i^M) + 2\sigma_i^M v = u_l + u_r$ is satisfied. Because $u_l = \sigma_{-i}^L + \sigma_{-i}^M v$ and $u_r = \sigma_{-i}^R + \sigma_{-i}^M v$, $u_l + u_r = (1-\sigma_{-i}^M) + 2\sigma_{-i}^M v$. Therefore, when $\sigma_i^M = \sigma_{-i}^M$, $(1-\sigma_i^M) + 2\sigma_i^M v = u_l + u_r$ and $\sigma_i^M + (1-\sigma_i^M)v - u_m = 0$ are satisfied. Then, (2) can be written to

$$\Pi(\Sigma_i, \Sigma_{-i}) = \frac{1-\lambda}{2}\pi\left(\widehat{\sigma}\right) + \lambda\pi\left(0\right) + \frac{1-\lambda}{2}\pi\left(-\widehat{\sigma}\right).$$

Because $\pi(t) + \pi(-t) = 1$ for any $t \in [0,\infty)$, it becomes

$$\Pi(\Sigma_i, \Sigma_{-i}) = \frac{1-\lambda}{2} + \lambda\pi\left(0\right).$$

which does not depend on α. It means that the probability of winning does not depend on a value of α when $\sigma_i^M = \sigma_{-i}^M$. □

Differentiate (2) with respect to σ_i^M. Then, the first derivative is

$$\begin{aligned}\frac{\partial\Pi(\Sigma_i, \Sigma_{-i})}{\partial\sigma_i^M} &= -\frac{(1-\lambda)}{2}\left(\alpha - v\right)\pi'\left(\alpha(1-\sigma_i^M) + \sigma_i^M v - u_l\right) \\ &\quad +\lambda(1-v)\pi'\left(\sigma_i^M(1-v) + v - u_m\right) \\ &\quad -\frac{(1-\lambda)}{2}[(1-\alpha) - v]\pi'\left((1-\alpha)(1-\sigma_i^M) + \sigma_i^M v - u_r\right). \end{aligned} \tag{3}$$

Suppose $\sigma_i^M = \sigma_{-i}^M$. From the proof of Lemma 1, $\alpha(1-\sigma_i^M) + \sigma_i^M v - u_l = -[(1-\alpha)(1-\sigma_i^M) + \sigma_i^M v - u_r]$, so

$$\pi'\left(\alpha(1-\sigma_i^M) + \sigma_i^M v - u_l\right) = \pi'\left((1-\alpha)(1-\sigma_i^M) + \sigma_i^M v - u_r\right)$$

since $\pi'(t) = \pi'(-t)$ for any $t \in [0, \infty)$. Thus, the first derivative becomes

$$\begin{aligned}\frac{\partial \Pi(\Sigma_i, \Sigma_{-i})}{\partial \sigma_i^M} &= -\frac{(1-\lambda)}{2}(1-2v)\,\pi'\left(\alpha(1-\sigma_i^M) + \sigma_i^M v - u_l\right) \\ &\quad + \lambda(1-v)\pi'\left(\sigma_i^M(1-v) + v - u_m\right). \end{aligned} \tag{4}$$

Suppose $v < 1/2$. A candidate will choose $\sigma_i^M = \sigma_{-i}^M = 0$ (when (4) is strictly negative) or $\sigma_i^M = \sigma_{-i}^M = 1$ (when (4) is strictly positive) when they are not indifferent. Suppose $\sigma_i^M = \sigma_{-i}^M = 0$. Then, since $\overline{\sigma} \equiv \sigma_i^L - \sigma_{-i}^L$, (4) becomes

$$\frac{\partial \Pi(\Sigma_i, \Sigma_{-i})}{\partial \sigma_i^M} = -\frac{(1-\lambda)}{2}(1-2v)\,\pi'(\overline{\sigma}) + \lambda(1-v)\pi'(0).$$

Then, $\partial \Pi(\Sigma_i, \Sigma_{-i})/\partial \sigma_i^M \leq 0$ when

$$\lambda < \frac{(1-2v)\pi'(\overline{\sigma})}{2(1-v)\pi'(0) + (1-2v)\pi'(\overline{\sigma})} \equiv \overline{\lambda} \tag{5}$$

is satisfied. Because $\partial \Pi(\Sigma_i, \Sigma_{-i})/\partial \sigma_i^M \leq 0$, a lower σ_i^M gives a (weakly) higher vote share. Therefore, $\sigma_i^M = 0$ is the best response for candidate i. Thus, a strategy profile with $\sigma_i^M = \sigma_{-i}^M = 0$ and $\overline{\sigma} \equiv \sigma_i^L - \sigma_{-i}^L$ is an equilibrium when $v < 1/2$ and $\lambda \leq \overline{\lambda}$ (Proposition 2).

In an equilibrium with convergence ($\Sigma_i = \Sigma_{-i}$), $\overline{\sigma} = 0$ is satisfied. Then, (5) can be rewritten as

$$\lambda \leq \frac{(1-2v)\,\pi'(0)}{2(1-v)\pi'(0) + (1-2v)\,\pi'(0)} = \frac{1-2v}{3-4v}.$$

Thus, if $\lambda \leq (1-2v)/(3-4v)$, a lower σ_i^M gives a (weakly) higher vote share, so $\sigma_i^M = \sigma_{-i}^M = 0$ in equilibrium (Corollary 1 (i)).

If $v \geq 1/2$, (4) is strictly positive for any value of λ. Thus, regardless of the opponent's strategy, $\Sigma_i = (0, 1, 0)$ is the best response.

Note that candidates may choose $\sigma_i^M \neq \sigma_{-i}^M$. A candidate chooses $\sigma_i^M \in (0,1)$ when a candidate is indifferent, i.e. (3) is satisfied with equality, but we ignore such an indifferent case since it rarely occurs with a specific value of λ. The remaining case of $\sigma_i^M \neq \sigma_{-i}^M$ is $\sigma_i^M = 0$ and $\sigma_{-i}^M = 1$. The vote share (2) becomes

$$\Pi(\Sigma_i, \Sigma_{-i}) = \frac{1-\lambda}{2}\pi(\alpha - v) + \lambda\pi(v-1) + \frac{1-\lambda}{2}\pi(1-\alpha-v).$$

Differentiate (2) with respect to α. Then, the first order condition is

$$\frac{\partial \Pi(\Sigma_i, \Sigma_{-i})}{\partial \alpha} = \frac{1-\lambda}{2}\pi'(\alpha - v) - \frac{1-\lambda}{2}\pi'(1-\alpha-v) = 0, \tag{6}$$

and the second derivative is strictly negative since $\pi''(t) < 0$ for all $t \in [0, \infty)$. The opponent also has the same first order condition. Condition (6) is satisfied if and only if $\alpha = 1/2$. Therefore, given $\sigma_i^M = 0$ and $\sigma_{-i}^M = 1$, both candidates choose $\alpha = 1/2$. When $\alpha = 1/2$, (3) becomes

$$\frac{\partial \Pi(\Sigma_i, \Sigma_{-i})}{\partial \sigma_i^M} = -(1-\lambda)\left(\frac{1}{2} - v\right)\pi'\left(\frac{1}{2} - v\right) + \lambda(1-v)\pi'(v-1) \tag{7}$$

for both candidates. In order to satisfy $\sigma_i^M = 0$ and $\sigma_{-i}^M = 1$, (7) must be strictly positive for one candidate and strictly negative for another candidate, but it is impossible. Thus, $\sigma_i^M = 0$ and $\sigma_{-i}^M = 1$ is not equilibrium regardless of the value of v. (Corollary 1 (ii) and Corollary 2)

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