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Does time inconsistency differ between gain and loss?

An intra-personal comparison using a non-parametric  
elicitation method (A revised version)

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# Does time inconsistency differ between gain and loss? An intra-personal comparison using a non-parametric elicitation method (A revised version)

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## Abstract

Several studies in the time preference literature have found time inconsistency (TI) of both gain and loss preferences. However, the relationship between the two within the same person remains unclear; that is, does an individual who demonstrates TI for gain outcomes do so for loss as well? This paper reports the individual's TI for both gain and loss in a laboratory setting. To compare individuals' TI for gain and loss precisely, we allowed the experiment to test for so-called *future bias*, which has been a focus area in recent TI literature. Further, we measured the level of TI rather than only identifying whether TI was present. We used a non-parametric elicitation method to avoid any specification error in the analysis. Based on this setting, we could examine the intra-personal relationship of TI for gain and loss—whether the same person shows similar TI of gain preference and TI of loss preference. Our findings are as follows: First, we found future bias in preference for not only gain but also loss, and confirmed that this tendency was consistent with previous findings on preference for gain. Second, such TI tended to have a positive relationship at the individual level for both gain and loss. Participants who exhibited TI when they chose gain tended to exhibit similar TI when they chose loss. These results suggest that people's perception of time is important in time preference; how far they perceive the future in their mind may play a crucial role in TI.

Keywords: time inconsistency, sign effect, non-parametric elicitation, future bias, intra-personal comparison

## 1 Introduction

It is common knowledge that people occasionally make time-inconsistent decisions; that is, they change their previous decisions without situational changes for both gain and loss outcomes. For example, we may incur additional costs to accelerate the delivery of a new television (gain) or knowingly delay the payment of a debt (loss) simply because time passes. Such decisions are the most severe violations in an exponential discounting model, a standard model that is used to explore individuals' intertemporal choices but requires the assumption of a constant discount rate, according to which people never change their previous decisions when time passes (for a review, see Frederick et al., 2002).

Usually, TI is explained as follows in the literature (it is called *present bias*). Suppose an individual prefers to receive (and therefore decides to choose) 105 dollars after 13 months rather than 100 dollars after 12 months. However, 12 months later, the person can claim 105 dollars after a month versus 100 dollars today. Now he or she prefers to

receive 100 dollars today and changes the previous decision. Such a preference (i.e., one can wait for an additional five dollars when the outcomes are in the far future, but not when they are close to the present) may be quite intuitive, but cannot be explained by exponential discounting. Thus, many researchers have empirically and theoretically examined time inconsistency (TI) to improve the understanding of people's time preferences for gain and loss.

Moreover, present bias provides an incentive for people to accelerate future utility (gain) or procrastinate future disutility (loss) from events that they themselves have planned before. From this property, present bias relates to an individual's tendency to various "irrational" behaviors such as over-consumption of goods and procrastination of bads, inefficient medical treatment, and addiction.

When people have no time-consistent preference for both gain and loss, as reported in many studies (see Frederick et al., 2002), the question naturally arises whether the same person's TI for both gain and loss are related to each other. Does an individual who exhibits TI for gain manifest similar TI for loss, or are such tendencies independent of each other? We conducted a laboratory experiment to examine the intra-personal relationship of TI for monetary gain and loss. We offered our participants various intertemporal binary choices and measured the level of TI when the outcomes were closer

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to the present. What we are interested in is how participants change their decision over time when they choose future gain, and how they will behave when they choose future loss.

To compare TI of both gain and loss preferences of the same person, we considered two things. First, we measured the degree of TI exhibited by each individual, which was comparable between gain- and loss-related choices. If we only consider whether or not participants exhibit TI, how the level of TI is related between gain and loss preferences would not be clear. Therefore, we measured how strongly participants exhibited TI with gain and loss outcomes and tried to compare the TI levels. More concretely, we offered our participants intertemporal binary choices for both gain and loss, with various temporal conditions (more than 3 or 6 months' delay with 1 or 5 weeks' waiting for later alternatives), and for each of them we elicited the TI level when delay decreases gradually (to 29 days, 8 days, and 1 day from the present).<sup>1</sup> This TI comparison, which is based on Prelec (2004), requires only standard conditions of rational preference, so that no functional specifications are required (see the appendix).

Second, we considered not only present bias but also *future bias*. While most studies of TI have traditionally focused only on present bias, its inverse, which is called future bias, has received research attention only in this decade. For example, Sayman and Öncüler (2009) reported that 19 of 38 participants chose 7 euro in a day rather than 10 euro in three days, but changed their decision to 10 after a day. This is clearly the opposite phenomenon of what we described before, since they became more patient as time passed. Similar to present bias, future bias has been found to be robust by recent empirical works (e.g., Attema et al., 2010; Takeuchi, 2011). Importantly, the two directions of TI (present and future biases) are not incompatible so that the same person exhibits both of them. What makes people show present or future bias is the temporal condition of their facing alternatives; future bias is observed only when the delay to and between the alternatives is sufficiently small. This preference is characterized by *inverse S-shaped discounting* (hereafter, ISD; we explain this later), and its parametric function fitted the data better than alternative functions that are exponential or hyperbolic in some empirical works (e.g., Abdellaoui et al., 2013a). In our measurement, future bias is described as negative present bias (zero present bias implies time consistency). Therefore, a positive correlation between TI levels for gain and loss means that individuals who exhibit strong present bias for gain are likely to exhibit strong present bias

<sup>1</sup>Theoretically, decreasing the delay in a choice and passing time are not the same phenomenon. Nevertheless, we do not consider the difference between them and assume that both are equivalent to allow comparison with important previous studies. In other words, we assume time invariance (i.e., the preference does not change over time). It is noteworthy that our results remain valid even without this simplification when “TI” and “present bias” are replaced with “non-stationary” and “common-difference effect,” respectively. See Halevy (2015) for a detailed discussion.

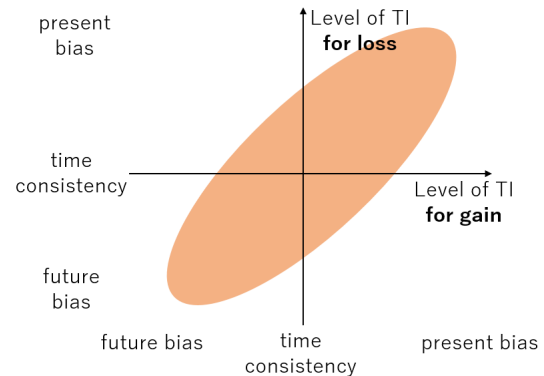


FIGURE 1: Image of the intra-personal relationship of TI for gain and loss.

Note: The  $x$  and  $y$  axes represent exhibiting TI levels when the choice concerns gain and loss, respectively. A positive TI implies present bias, whereas a negative one indicates future bias. For example, the extreme north-eastern point represents strong present bias for both gain and loss. Similarly, the south-east area represents present bias for gain and future bias for loss. A uniform distribution of observations over the entire area implies that the TI for gain and loss in the same person is independent of each other, whereas a distribution extending from north-west to south-east indicates a negative TI relationship between gain and loss. We expect a positive relationship, which is shown by a circle in the figure.

for loss and that those who exhibit strong future bias for gain also tend to have future bias for loss (see figure 1).

Our contributions are as follows. First, we show positive evidence of ISD explanations even for loss preference. Most future bias studies focus only on gain preference, although loss (procrastination) is associated with many interesting TI phenomena. While no study has empirically found future bias for loss, we identified future bias for loss outcomes and found that its frequency was consistent with previous findings (i.e., it was characterized by ISD) *on average*.

Second, we found a strong relationship between TI levels for gain and loss in the same person. The correlation between TI levels for gain and loss was relatively high and positive in every condition; in addition, they did not significantly differ from each other in most cases. While differences in impatience levels (the discount rates) for gain and loss have been reported in the literature, our results clearly indicated that TI levels for gain and loss are strongly related: an individual who exhibited strong present bias for loss did so for gain too.

Our findings suggest that people’s time perception is important to understanding TI. The shift in impatience may be caused by a distortion in the perception of how far the future is. People change their previous decisions because they do not perceive one week from two months later and one week from tomorrow in the same light. This explanation about TI (e.g., Kim and Zauberman, 2009) is consistent with our findings, while the argument that future risk causes TI (e.g., Halevy, 2008) somehow does not fit our analysis.

The rest of this paper is constructed as follows. Section 2

reviews the TI literature. Section 3 describes our experiment and explains how we measure TI and compare TI levels. TI is formally defined and the comparison technique is described in the appendix. Section 4 provides the results. Section 5 discusses our findings and the experimental limitations. Section 6 concludes the paper.

## 2 Time Inconsistency

### 2.1 future bias and ISD

Empirical research suggests two TI directions. One is present bias, in which people are less patient for nearer future choices. That is, for an intertemporal binary choice (between SS and LL, the shorter-sooner and larger-later alternatives, respectively), SS tends to be relatively less discounted (than LL) as time passes. When present bias occurs, people accelerate the receiving of desired goods, and procrastinate the timing of undesired events that they themselves have planned before. This tendency has been reported in many empirical studies (e.g., Thaler, 1981) and has been explained by *hyperbolic discounting* (this tendency itself is sometimes called hyperbolic discounting).

The other direction, called future bias, is the inverse of present bias. Here, people are more patient for nearer choices. Some recent studies have reported that people exhibit not only present bias but future bias (Attema et al., 2010; Sayman & Öncüler, 2009; Takeuchi, 2011). For example, Sayman and Öncüler (2009) conducted laboratory experiments and asked participants about their preference for an intertemporal binary choice at two different timings; once participants chose either SS or LL, they were given a chance to change their decisions later (at a point in time when SS can be received immediately). According to the study, many participants changed their decision from SS to LL, which is the inverse change of present bias, because they became more patient for a near-future decision.

In sum, there are two opposite effects on impatience with decreasing delay (i.e., passing time). However, this does not mean that the two effects are incompatible. According to Sayman & Öncüler (2009), people exhibit one of the effects depending on the temporal condition of the choice. In their study, participants exhibited future bias more frequently when the (original) alternatives were sufficiently close to the present and the delay between them was sufficiently short (less than a few weeks), but present bias was mainly observed for other choices.

Those researchers suggested that ISD could explain such behavior (e.g., Takeuchi, 2011). While hyperbolic discounting expects only present bias for any choice, ISD also allows future bias for specific choices. The difference between the two discounting types is that impatience increases in the near future period in ISD (the concave domain in the figure 2). Figure 2 describes TI in ISD. Because impatience increases

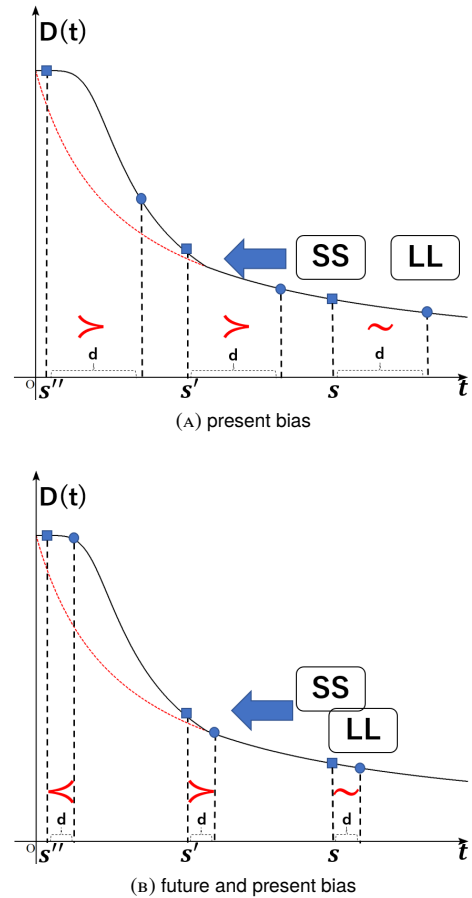


FIGURE 2: ISD and exhibited TI.

Note: The dotted line describes the usual hyperbolic discounting. Present bias is exhibited under ISD, as with hyperbolic discounting, if  $d$  is sufficiently large (panel A). That is, the individual becomes more impatient if the alternatives occur sooner (from  $s$  to  $s'$ ). Therefore, he or she is indifferent between SS and LL if SS is delayed to  $s$  but strictly prefers SS if it is delayed to  $s'$ . On the other hand, if  $d$  is small (panel B), the individual becomes impatient when SS happens sooner (from  $s$  to  $s'$ ), but then becomes more patient if it happens much sooner (from  $s'$  to  $s''$ ). Therefore, he or she exhibits present bias when SS moves to  $s'$  from  $s$ , but shows future bias when SS comes closer to the present (from  $s$  to  $s''$ ). These features of TI behavior are consistent with the findings in the literature.

from now to the near future, and decreases otherwise, future bias occurs only for future alternatives with a short waiting time, and only when the delay is sufficiently short ( $s''$  in panel B). Otherwise, present bias may occur ( $s'$  and  $s''$  in panel A and  $s'$  in panel B). More details are provided in the note below the figure.

Positive evidences of ISD have been reported in this decade. Whereas Sayman and Öncüler's (2009) study was based on participants' behavior for given alternatives, other recent studies have elicited indifference among alternatives to obtain TI including future bias.<sup>2</sup> Takeuchi (2011) found

<sup>2</sup>Obviously, change in response for given alternatives implies TI, but not vice versa.

that a large proportion of behaviors indicated future bias. Indeed, based on a functional specification of ISD, they reported that the ISD function explained the behaviors of more than half of the participants well. Attema et al. (2010) also found future bias under a different setting. Median and classification analyses in their study suggest that the frequency of future bias remained low even as time passed so long as the alternatives were still in the far future (the front end delay became 5 months from larger delay, i.e.,  $5 \text{ months} < s' < s$ ). Abdellaoui et al. (2013a), assuming various functional forms of the discount function, reported that ISD fitted the data better than exponential or hyperbolic function (we explain this result in more detail in the next subsection).

An important point with ISD is that while people may exhibit both present and future biases, they do so not at random but exhibit future bias only when facing a choice that i) results in a sufficiently short delay from SS to LL and ii) accelerates the outcome sufficiently close to the present. Hence, the TI direction (and level) depends on the temporal condition of the choice faced, which is tested in our experiment more directly than in previous research.

## 2.2 TI for loss

Previous empirical research has found TI for losses too. Ben-zion et al. (1989) found a decreasing discount rate under the linear utility assumption, which corresponds to present bias. Using the same assumption, Thaler (1981) found that the discount rate is not constant, but does not decrease. This implies that people’s impatience for loss does not uniformly decrease. Abdellaoui et al. (2013a) relaxed this linear assumption and found decreasing impatience preference for loss outcomes. In addition, they compared the explanatory power for various discount functions and showed that functions that allowed future bias fitted the data better than those that did not allow future bias. While Abdellaoui et al. emphasized the importance of future biases for loss, no study has yet directly examined future biases in the loss domain.

Abdellaoui et al. (2013a) also mentioned an intra-personal TI relationship with both gain and loss. Focusing on the difference between time preferences for gain and loss in terms of utility and discount functions, they estimated the parameters of both functions, which were assumed to be sign-dependent; that is, the parameters for loss could differ from those for gain. They measured the level of TI by the parameter  $\alpha$  of the generalized hyperbolic discounting (GHD) function (Loewenstein and Prelec, 1992). GHD is represented by the function  $D(t) = (1 + \alpha t)^{-\frac{\beta}{\alpha}}$  with  $\alpha > 0$ . In this function, for any given  $\beta$  (which represents the level of impatience),  $\alpha \rightarrow 0$  implies time consistency, and  $\alpha > 0$  implies present bias for any intertemporal choice. Thus, the parameter  $\alpha$  can be thought of as an index of the TI level. Using this function, they concluded that TI was stronger for loss than for gain at the individual level and that its strength had no

correlation. However, the GHD specification allowed for the existence of only present biases, not future biases, so their measurement of TI should be biased. Drawing on Abdellaoui et al.’s (2013a) research interest, our study focuses on the TI difference for the individual, and not on the utility function. We measure TI allowing future bias, and avoid specification errors on both functions.

It is worth noting that previous research suggests that people discount delayed gain more heavily than delayed loss. This means that individuals’ time discounting for gain and loss differs. However, even so, we may expect a positive relationship between TI for gain and loss because the level of TI (and its direction) is independent of the level of impatience. Even if people evaluate waiting differently when they choose gain or loss, they may deviate from their previous decision in the same way. In addition, some researchers explain TI by the distortion in time perception (see section 5.2). Accordingly, we may expect a positive relationship of TI because the time perception of waiting for the outcome may be the same for both gain and loss. Our study controlled for individuals’ impatience and examined the difference between TI for gain and loss regardless of the level of impatience.

## 3 Experiment

### 3.1 Design

In this section, we describe our experiment and explain how we compare TI levels in the analysis. To begin with, we should note two points. First, our experimental design follows Rohde (2010). Although he did not consider the time preference relationship for gain and loss, and was primarily interested in hyperbolic discounting and its functional forms, the basic idea of our experiment is similar to that of Rohde (2010). Second, we only describe our comparison of the TI level in this section, and place the formal definition of TI and the comparison method in the appendix. We follow the most widely used comparison method, based on the TI criteria of Prelec (2004).

Let  $X = \mathbb{R}$  be a set of outcomes and  $T = \mathbb{R}^+$  a set of time periods. We assume that individuals have a time preference  $\succeq$  over  $X \times T$ , which is a weak order. Each element  $(x, t)$  of  $X \times T$  denotes a delayed outcome, “receiving outcome  $x$  at time  $t$ .” We assume this preference is continuous, monotonic, and impatient. The preferences for gain  $\succeq^+$  and loss  $\succeq^-$  are subsets of  $\succeq$  on  $X^+ \times T$  and  $X^- \times T$ , where  $X^+$  and  $X^-$  are sets of gain and loss. For each individual, we assume a reference point of 0, that is,  $X^+ = \mathbb{R}^+$ . Finally, we define the indifference  $\sim$  and the strict relation  $>$  in a manner commonly done in the literature.

See figure 3. Suppose we obtain four indifferent pairs of the same SS and LL as shown in this figure. The black arrows represent the indifference of delayed alternatives and the differences between those pairs are the temporal conditions,

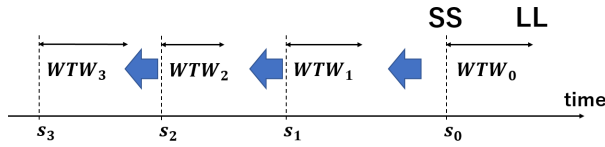


FIGURE 3: WTW Sequence

Note: The black arrows represent the indifference between the outcomes SS and LL over time. The length of the arrows represents WTW until LL from SS at each  $s$ . Since the outcomes are common in every indifference,  $WTW_i$  suggests how the individual dislikes future waiting, that is, the level of impatience for the future choice.

namely, the timing of SS and the length of waiting until LL from SS. Here, since the arrows are indifferences, the later one represents how long an individual can wait for monetary improvement  $LL - SS$  from SS at its timing, which is called willingness to wait (WTW). Therefore, this figure shows the sequence of WTW until LL, instead of SS, when SS is in  $s_0$ ,  $s_1$ ,  $s_2$  and  $s_3$  respectively. It is clear that each degree of WTW represents the level of impatience for the future outcome, so that sequentially decreasing WTW (i.e.,  $WTW_t < WTW_{t+1}$ ) implies increasing impatience (i.e., present bias). More precisely, if an individual exhibits  $WTW_t < WTW_{t+1}$ , since he or she cannot wait until LL,  $WTW_t$  days, when SS occurs in  $s_{t+1}$ , although the person could do so when SS was in  $s_t$ , present bias will be exhibited for the choice between  $(SS, s_t)$  and  $(LL, s_t + WTW_t)$  when SS occurs in  $s_{t+1}$ . Likewise, increasing WTW corresponds to future bias.

Such changes in WTW over time indicates the dynamic changes in participants' impatience. However, in general, such change is not a comparable measure across participants and between gain and loss preference in the same person; that is, a large change in WTW does not imply strong TI. This is because when  $WTW_t - WTW_{t+1}$  is 2 for individual  $i$  and 3 for  $j$ , but 8 for  $i$   $WTW_t$  and 100 for  $j$   $WTW_t$ , the intensity of change in impatience can be greater for  $i$  than  $j$ , even though size of the change is smaller. The point here is that since the patience differs between  $i$  and  $j$ , it is difficult to consider the level of change in impatience as a measure of TI.

Therefore, we need to control for participants' level of impatience. Therefore, our experiment was conducted in two stages in accordance with Rohde (2010)). Figure 4 presents an image of our experiment. In stage 1, we asked participants to indicate their willingness to accept (WTA) for a delay of  $d_0$  days of SS, that is, the stake size of LL, which is needed to examine an individual's SS and LL by keeping  $WTW_0$  for LL common for all participants. In stage 2, we asked participants, using their WTA data, to indicate a delay, at which they would be indifferent between SS at timing  $s$  and WTA in the future.

Once  $WTW_0$  was common across participants,  $\delta = WTW_0 - WTW$  (the red arrow in the figure 4) can be a compa-

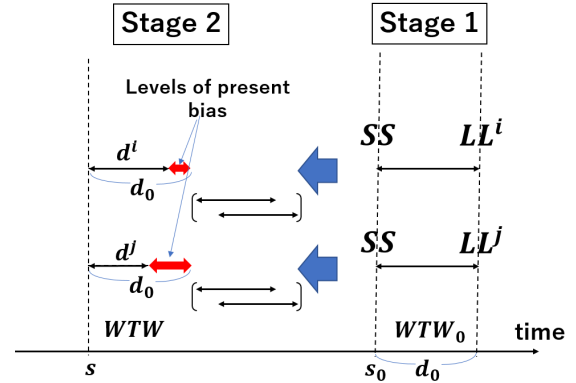


FIGURE 4: Comparison of TI when the choice comes closer to the present.

Note: In the first stage, we obtain the WTA ( $LL^i$ ) for delay  $d_0 (= WTW_0)$ . In stage 2, we then identify WTW at  $s < s_0$  according to the WTA obtained 2. Since the choice in the first stage becomes a choice in the second stage with delay  $d_0$  after  $s_0 - s$  days, a positive difference in WTW ( $\delta = d_0 - d > 0$ ) indicates present bias, represented by the red arrow in the figure. Indeed, a larger difference for individual  $j$  than  $i$  implies that the delay of LL compensates for the present bias of individual  $i$  with  $\delta^i$ , for but it is not enough for  $j$ . In this sense,  $\delta^j > \delta^i$  suggests that  $j$  exhibits stronger present bias than  $i$ .

rable measure of TI when the choice shifts from  $s_0$  to  $s$  (where  $s$  takes  $s_1, s_2$ , and  $s_3$ ). That is, when  $\delta^j = WTW_0^j - WTW^j$  is larger than  $\delta^i = WTW_0^i - WTW^i$ ,  $j$  has stronger present bias than  $i$  in the following sense:  $j$ 's present bias is compensated by accelerating LL by  $\delta^j$ , whereas  $j$ 's present bias still remains by doing so. In figure 4,  $s_0$  is the timing of the original SS,  $d_0$  is the WTW for LL, and  $s$  and  $d^i$  are the revised timing and WTW, respectively, when the choice comes closer. Then,  $d^i < d_0$  indicates that the individual strictly prefer SS rather than LL when SS and  $LL^i$  with waiting  $d_0$  comes closer to  $s$ . This present bias is compensated by advancing  $LL^i$  by  $d_0 - d^i$ , so that SS and  $LL^i$  become indifferent. However, this acceleration is not enough for  $j$ ; that is,  $j$  still prefers SS and, therefore, exhibits stronger present bias than  $i$ . Here, future bias is described as a negative present bias ( $\delta < 0$ ).

This comparison between  $\delta^i$  and  $\delta^j$  applies to the relationship between the gain and loss preference of the same person. If  $\delta^-$  compensates for present bias for loss, but not for gain, the latter is stronger than the former. That is, if  $\delta^+$ , which compensates for present bias for the gain outcome, is larger than  $\delta^-$ , then present bias is stronger for gain than for loss. This comparison is discussed in more detail in the appendix.

Therefore, once we identify SS and LL, with WTW until LL  $d_0$ , we obtain  $WTW_1, WTW_2$ , and  $WTW_3$  by changing the delay in the second stage. Here, we provide an example. Suppose, we identify the WTA in the first stage and obtain  $\{WTW_t^+\}_{t=0}^3 = \{7, 4, 5, 8\}$  from one participant in the second

stage. Then, our TI measure is calculated by  $\delta_1^+ = 7 - 4 = 3$ , which represents the TI level when the SS at  $s_0$  comes closer to the present, until  $s_1$ . Likewise,  $\delta_2^+ = 7 - 5 = 2$  and  $\delta_3^+ = 7 - 8 = -1$  are the TI levels when outcomes are accelerated from  $s_0$  to  $s_2$  and  $s_3$  respectively. In this case, when SS and LL for gain get closer to the present, the individual exhibits present bias until SS is attached at  $s_2$ , but displays future bias SS at  $s_3$  ( $\delta_1^+$  and  $\delta_2^+$  are positive whereas  $\delta_3^+$  is negative). If the same person has  $\delta_1^- = 1$ , his or her present bias for gain is stronger than that for loss when the choice moves closer from  $s_0$  to  $s_1$ .

### 3.2 Procedures

We set SS at 10,000 yen. Thus, SS results in a gain or loss of 10000 yen. In the first stage, subjects were asked to indicate the WTA that would offset a delay  $d_0$  in SS. The following is an example of the question:

Please input a number (X) that would make you feel that B is as good as A.

-A: Receiving **10,000** yen in **92** days.

-B: Receiving X yen in **99** days.

For example, if one participant answers 11,000 to the question above, he or she is indifferent between (10000, 92) and (11000, 99). We provided such questions with various values of  $s_0$  (92 and 183) and  $d_0$  (7 and 35). We replaced the word “good” and “receiving” with “not good” and “paying” in the loss questions.

In the second stage,  $WTW_k$  is obtained in the same way as in the first stage; for example, a participant who answered 11000 in the first stage with  $(s_0, d_0) = (92, 7)$  will be asked,

Please input a number ( $T'$ ) that would make you feel that B is as good as A.

-A: Receiving **10,000** yen in **1** day.

-B: Receiving **11,000** yen in  $T'$  days.

We set  $s$  from 1, 8, and 29, which are about a day, a week, and a month respectively. Here, if a participant answers  $T' = 6$  for  $(s_0, d_0, s) = (92, 7, 1)$ , then he or she can wait only five days for LL rather than SS in a day, although the person could wait seven days for LL when SS was available in 92 days. This is due to present bias.

The subjects were not informed that the parameters in this stage ( $LL^+$  and  $LL^-$ ) are from their previous answers.<sup>3</sup> They were not allowed to go to the previous pages on the computer, so they could not change these parameters.

We asked a total of 32 questions, of which 8 were about WTA (4 each for gain and loss) and 24 about WTW (12 each for gain and loss). To control for the order effect, we divided

<sup>3</sup>To avoid a consistency effect in the second stage, we randomly ordered the questions under each  $s$  condition.

the subjects into two groups, and asked each group a series of questions, progressing from loss to gain related (for one group) and from gain to loss related (for the other group) during the two stages ( $n = 57$  and 52). Prior to answering the questions, the subjects were asked two training questions.

### 3.3 Elicitation Method

We used matching-based elicitation in every question. There is extensive literature on the advantages of matching and choice tasks to elicit decision-making behavior (e.g., Bostic et al., 1990) and both methods coexist especially in the time preference studies. For example, Abdellaoui et al. (2013a) and Sayman and Öncüler (2009) used a choice task, which is sometimes recommended in other literature strands such as risk preference. We used matching-based elicitation because the choice task could only provide too coarse a grid point, which was not appropriate for an experiment with a non-parametric elicitation. In fact, because of this, most non-parametric studies use matching-based elicitation. Moreover, although a matching task may bias the participant’s responses, we do not think that the results were conditional on our elicitation method; that is, such biases did not harm our findings seriously. We will discuss this point in section 5.3. We note here that this elicitation method also helps maintain subjects’ concentration due to the huge number of questions. To aid their decision-making and ease the process, we allowed each subject access to a calculator and calendar printed with the date and number of days from the days of the experiment.

## 4 Result

### 4.1 Participants

We conducted a one-day experiment with 109 students from various departments in Waseda University on 23/01/2017. The subjects were recruited using the university’s online portal and asked to visit our experimental laboratory any time during the day to participate in the study. Of the participants, 69 were male and 40 female. Most of them (78 participants) were 20 to 24 years old, 28 were under 19, and 3 were 25 to 29. Fifteen participants studied natural science, and 94 did social science or general arts, of whom 7 students studied economics. The largest number of subjects (28 participants) studied literature. The questions included choices of losses and, thus, were all hypothetical. We offered the subjects 800 yen (more than 7 US dollars) for participation, which was reasonable since most subjects took about 15–45 minutes to complete the experiment. The instructions and questions were given on the computer screen.



TABLE 1: Descriptive statistics of participants' answers.

| condition<br>waiting | condition<br>timing of SS | sign      | WTA (stage 1) |          |         | condition<br>new timing | WTW (stage 2) |       |       |    |
|----------------------|---------------------------|-----------|---------------|----------|---------|-------------------------|---------------|-------|-------|----|
|                      |                           |           | Mean          | S.D.     | Obs.    |                         | Mean          | S.D.  | Obs.  |    |
| $d_0=7$              | $s_0=92$                  | gain      | 10722.36      | 877.23   | 90      | s=1                     | 5.35          | 6.02  | 85    |    |
|                      |                           | Loss      | s=8           | 5.47     | 6.43    | 87                      |               |       |       |    |
|                      |                           |           | s=29          | 5.37     | 4.70    | 86                      |               |       |       |    |
|                      | $s_0=183$                 | gain      | 10877.26      | 1673.07  | 73      | s=1                     | 8.30          | 11.30 | 66    |    |
|                      |                           | loss      | s=8           | 7.06     | 9.36    | 66                      |               |       |       |    |
|                      |                           |           | s=29          | 8.46     | 12.12   | 72                      |               |       |       |    |
| $d_0=35$             | $s_0=92$                  | gain      | 10881.04      | 1455.66  | 90      | s=1                     | 5.57          | 7.12  | 84    |    |
|                      |                           | loss      | s=8           | 5.25     | 6.31    | 84                      |               |       |       |    |
|                      |                           |           | s=29          | 6.09     | 7.33    | 88                      |               |       |       |    |
|                      |                           |           | s=1           | 8.94     | 15.22   | 67                      |               |       |       |    |
|                      |                           | $s_0=183$ | gain          | 10674.03 | 1386.66 | 71                      | s=8           | 9.02  | 16.11 | 65 |
|                      |                           |           | loss          | s=8      | 9.75    | 15.71                   | 71            |       |       |    |
|                      | s=1                       |           |               | 17.39    | 18.12   | 97                      |               |       |       |    |
|                      | s=8                       |           |               | 15.45    | 16.37   | 99                      |               |       |       |    |
|                      | $s_0=92$                  |           | gain          | 12792.49 | 3031.14 | 98                      | s=29          | 16.37 | 15.10 | 99 |
|                      |                           |           | loss          | s=1      | 18.77   | 22.00                   | 80            |       |       |    |
|                      |                           | s=8       |               | 18.35    | 22.04   | 81                      |               |       |       |    |
|                      |                           | s=29      |               | 16.32    | 14.68   | 81                      |               |       |       |    |
| $s_0=183$            |                           | gain      | 12311.34      | 2466.88  | 85      | s=1                     | 21.78         | 31.53 | 83    |    |
|                      |                           | loss      | s=8           | 20.20    | 32.24   | 85                      |               |       |       |    |
|                      | s=29                      |           | 19.27         | 24.16    | 84      |                         |               |       |       |    |
|                      | s=1                       |           | 12840.25      | 2998.44  | 99      |                         |               |       |       |    |
|                      | $s_0=92$                  | gain      | 12056.44      | 2458.16  | 87      | s=8                     | 20.20         | 32.24 | 85    |    |
|                      |                           | loss      | s=8           | 20.20    | 32.24   | 85                      |               |       |       |    |
| s=29                 |                           |           | 19.27         | 24.16    | 84      |                         |               |       |       |    |

Note: The total number of valid subjects is 108. The ‘‘Obs.’’ column indicates the number of observations satisfying our assumption on preference.

## 4.2 Analysis

### 4.2.1 Dynamics of impatience over time

Table 1 summarizes the responses. We eliminated one participant (subject ID 42) owing to unreliable answers.<sup>4</sup> As a result, the total number of observations for  $\delta$  is  $24 \times 108$ . We included all data for the two ordered groups and corrected certain answers for the following analysis.<sup>5</sup> For the analysis, we adopted a set of general assumptions (i.e., weak order, monotonicity, impatience, and continuity) and used only those observations that satisfied these assumptions. This resulted in 2,002 observations of the  $24 \times 108$ .<sup>6</sup>

<sup>4</sup>In the first stage, the subject responded with ‘‘10,000’’ for all the questions and, in the second, provided confusing numbers.

<sup>5</sup>Although we found significant differences between the two groups in terms of their responses for the loss segment in the second stage, the results do not considerably differ from the main results for each ordered group. In addition, some answers appeared to be incorrectly inputted; thus, we either corrected the answers or excluded them from the analysis. However, the results remain essentially unchanged.

<sup>6</sup>In most cases of violation of these assumptions, participants answered  $\leq 10,000$  yen, which directly violated impatience (151 answers in the first stage or 453 observations for  $\delta$ .) and monotonicity (154 observations in rest data). In other cases, they answered WTW in the second stage, which exceeded the LL date of the first stage (34 observations in addition to those mentioned immediately above). For example, one subject answered  $(-10000, 92) \sim (-12000, 127)$  in the first stage and  $(-10000, 1) \sim (-12000, 180)$  in the second. These two relations violated impatience under transitivity because impatience requires  $(-10000, 92) > (-10000, 1)$  and  $(-12000, 180) > (-12000, 127)$ , which are inconsistent with the two indifferent relations. We discuss the deviation

From this table, we see that our participants' WTA for seven days delay of receiving 10000 in 92 days was 10722 on average, i.e., participants were indifferent between receiving 10000 in 92 days and receiving 10722 in 99 days on average. The annual discount rate under linear utility was 3600%, which was quite high, but this is usual for the choice between alternatives with small  $d$  (e.g., Kirby, 1997; Kirby et al., 1999). In the second stage, the average WTW in the first line was 5.35, which could roughly mean that our participants were indifferent between  $(10000, 1)$  and  $(10722, 1 + 5.35)$ , as long as we remember that the LL value differs across participants so that 10722 was not the exact choice parameter in this stage but its average. The point here is that, *on average*, the participants exhibited present bias. This is because their WTW was less than seven days, and so it indicates  $(10000, 92) \sim (10722, 99)$  but  $(10000, 1) > (10722, 8)$ . An individual with this preference should have an incentive to change his or her decision from LL to SS after 91 days.

The dynamic change in impatience is almost consistent with ISD on average. Let us see the change in  $WTW_t$ . Figure 5 describes the mean of WTW for each pair of SS and LL. From panel A,  $WTW^+$  until LL that is WTA for 35 days delay of SS at 92 days, decreases as SS moves closer from 92 to 29 days and from 29 to 8 days, but then it increases when SS shifts from 8 to 1. In most cases, except for loss

of the non-negligible number of observations from the general assumptions in section 5.3.



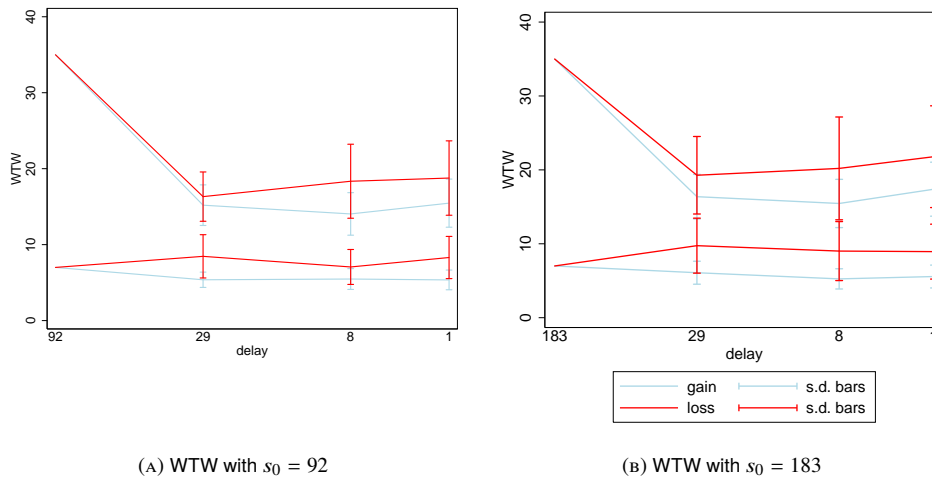


FIGURE 5: Average WTW with respect to a delay shift from the far future to the present.

Note: The upper lines are of  $d_0 = 35$  whereas the lower ones are of  $d_0 = 7$ . The lines represent average  $\{WTW_k\}_{k=0}^3$  with respect to each condition.

TABLE 2: Ratios of participants who exhibited future bias in each condition.

|          | $s_0 = 92, d_0 = 7$ |         | $s_0 = 183, d_0 = 7$ |         | $s_0 = 92, d_0 = 35$ |         | $s_0 = 183, d_0 = 35$ |         |
|----------|---------------------|---------|----------------------|---------|----------------------|---------|-----------------------|---------|
|          | gain                | loss    | gain                 | loss    | gain                 | loss    | gain                  | loss    |
| $s = 1$  | 24%(85)             | 27%(66) | 23%(84)              | 30%(67) | 7%(96)               | 13%(80) | 10%(98)               | 13%(93) |
| $s = 8$  | 17%(87)             | 21%(66) | 17%(84)              | 20%(65) | 5%(98)               | 12%(81) | 8%(99)                | 11%(85) |
| $s = 29$ | 19%(86)             | 24%(72) | 23%(88)              | 28%(71) | 5%(98)               | 10%(81) | 8%(99)                | 12%(84) |

Note: The number of observations that satisfied our axioms is presented in parentheses.

with  $d_0 = 7$ , WTW decreases first and then increases.<sup>7</sup> On average, as the future choice comes closer to the present, our participants become impatient until the choice is sufficiently close and then, they become patient, which is a character of ISD.

On the other hand, other alternative discountings don't fit our observation much. We rejected exponential discounting because the hypothesis that  $WTW_k$  is constant is rejected statistically (Kruskal-Wallis test of  $WTW_k \equiv WTW_0$ ; the  $p$ -values are  $< 0.01$  under every condition of  $(s_0, d_0)$  for both gain and loss).<sup>8</sup> Indeed, hyperbolic discounting requires that  $WTW$  monotonically decreases over time, but the figure does not show this tendency (this point is further discussed in 5.1).

When we compare  $WTW_k$  with  $WTW$  at  $s_0$  (i.e.,  $WTW_0$ ), all  $WTW_k$  is smaller than  $WTW_0$  for all  $i$  except for the case

<sup>7</sup>Even for those exceptional cases, the median of WTW is always lower than  $d$ . For example, the medians of  $WTW^-$  under  $(92, 7)$  at  $s = 92, 29, 8$ , and 1 are 7, 3.5, 2.5, and 5 days, respectively, and those under  $(183, 7)$  are 7, 4, 3, and 4, respectively. This implies that these exceptional cases occur because only a few subjects chose relatively large  $d$ . The median of  $d$  is always lower than  $d_0$ .

<sup>8</sup> $H_0$  is the constant median of  $WTW_k$  over  $k = 0$  to 3. Note that the median of  $WTW_0$  is always  $d_0$ .

of loss with  $d_0 = 7$ . It seems that the participants exhibited present bias for most choices that were obtained in the first stage on average, regardless of how close they came to the present. Since present bias was observed almost robustly in TI studies, this tendency is not surprising. Rather, what we are interested in is its inverse effect.

#### 4.2.2 future bias

Let us compare the  $\delta$  values of gain and loss preferences of the same person. Recall that the original pair of alternatives is set to be indifferent in the first stage, and for each pair,  $\delta$  represents the level of TI when SS shifted from  $s_0$  to at 29 days, 8 days, and 1 day. Table 2 describes the ratio of participants who exhibited future bias (i.e.,  $\delta < 0$ ) in each temporal condition of  $s_0, d_0$ , and  $s$ . Although most of the subjects demonstrated present bias or time consistency for both gain and loss, an unignorable number of observations showed future bias, with the ratios being slightly higher for loss. The table clearly shows certain common tendencies; future bias was more likely to be observed with a short ( $d_0 = 7$ ) rather than long ( $d_0 = 35$ ) waiting for LL and with small ( $s = 1$ ) rather than large ( $s = 8, 29$ ) delay. These

TABLE 3: Comparison of test statistics of TIs between gain and loss.

| $d_0$                              | 7    |      |      |      |      |       | 35   |      |      |      |      |      |       |
|------------------------------------|------|------|------|------|------|-------|------|------|------|------|------|------|-------|
| $s_0$                              | 92   |      | 183  |      | 92   |       | 183  |      | 92   |      | 183  |      |       |
| $s$                                | 1    | 8    | 29   | 1    | 8    | 29    | 1    | 8    | 29   | 1    | 8    | 29   | total |
| <b>measure <math>\delta</math></b> |      |      |      |      |      |       |      |      |      |      |      |      |       |
| Spearman's $\rho$                  | 0.54 | 0.61 | 0.45 | 0.43 | 0.52 | 0.55  | 0.74 | 0.61 | 0.44 | 0.63 | 0.54 | 0.43 |       |
| p-value                            | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00  | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |       |
| Sign-rank                          | **   | n.s. | n.s. | n.s. | n.s. | n.s.* | n.s. | n.s. | n.s. | n.s. | n.s. | n.s. |       |
| Obs.                               | 64   | 63   | 68   | 63   | 62   | 68    | 77   | 79   | 78   | 81   | 84   | 82   | 869   |
| <b>measure <math>\alpha</math></b> |      |      |      |      |      |       |      |      |      |      |      |      |       |
| Spearman's $\rho$                  |      |      |      |      |      |       |      |      |      |      |      |      | 0.64  |
| p-value                            |      |      |      |      |      |       |      |      |      |      |      |      | 0.00  |
| Sign-rank                          |      |      |      |      |      |       |      |      |      |      |      |      | n.s.  |
| Obs.                               |      |      |      |      |      |       |      |      |      |      |      |      | 75    |

Note: In the sign-rank rows, \*\*\* and \*\* denote that the difference is significant at the 1% and 5% levels, respectively. In addition, "n.s." indicates "not significant," whereas "n.s.\*" implies that the difference is significant at the 10% level. The number of observations differ because we use only observations that satisfy our axioms.

features are consistent with ISD and the findings of Sayman and Öncüler (2009), who set  $s = 0$  and changed  $s_0$ . Here, we found that the future bias frequency increases when  $s$ , but not  $s_0$ , changes. Note that Sayman and Öncüler reported more future bias observations when  $s_0$  was just a few weeks away. In our setting,  $s_0$  was much larger, and there was no salient difference in frequency with changes in such  $s_0$ .

One remarkable behavior in this table is that the ratio was also higher when  $s$  was sufficiently large ( $s = 29$ ) compared to  $s = 8$ , especially with the  $d_0 = 7$  condition. This implies that some participants exhibited future bias when the SS delay shifted to 29 days from  $s_0$  ( $= 92$  or  $183$ ), but showed present bias when SS shifted to more closer (decreased to 8 days) from  $s_0$  days. Since, ISD claims that future and present biases occur for near and far future choices respectively, those participants did not follow ISD in the experiment.

#### 4.2.3 Intra-personal relationship with the measure $\delta$

We found a positive correlation between the degree of TI for gain and loss. Table 3 presents the correlations of the calculated  $\delta$  (and the number of observations) for each condition. Since we focus on the relationship between TI for gain and loss, we included in the table only those observations that satisfied the general assumption for both gain and loss. Figure 6 depicts the correlations of our measure at the individual level and the lack of clear differences under most conditions. These are similar in each condition. That is, participants exhibited similar TI for given SS and LL for gain and loss. If participants exhibit strong present bias for gain when the alternative is 29 days away, for SS at 92 days, they would also exhibit similar present bias for loss when the alternative is 29 days away. This relationship also holds when the alternative is 8 (instead of 29) days into the future.

In fact, Spearman's correlation  $\rho$  was relatively high in every condition (see the first two rows in Table 3). Moreover,

$\delta^-$  did not significantly differ from  $\delta^+$  in most conditions (see the results of the sign rank test in Table 3)<sup>9</sup>, although  $y^+$  and  $y^-$  in the first stage significantly differed each other (sign rank test;  $p$ -values are 0.02, 0.00, 0.00, and 0.00 in condition  $(s_0, d_0) = (92, 7)$ ,  $(183, 7)$ ,  $(92, 35)$ , and  $(183, 35)$  respectively). Thus, we conclude that people's TI for gain and loss is possibly related.<sup>10</sup>

#### 4.2.4 Intra-personal relationship with the $\alpha$ of GHD

Whereas we found negative evidences for most common discounting functions, including GHD (see footnote 10), numerous studies, including Abdellaoui et al. (2013a), have measured TI assuming a GHD function. Thus, we also adopted  $\alpha$  of GHD here as a measure and estimated the parameters  $\alpha^+$  and  $\alpha^-$  of GHD using a non-linear least-square method. As we stated, we have two indifferences,  $(x, s_0) \sim (y, s_0 + d_0)$  and  $(x, s) \sim (y, s + d)$ , for each condition  $(s_0, d_0, s)$ , where  $s_0 > s$ . Under the usual discounted

<sup>9</sup>The sign rank test is less likely to identify a significant difference than the  $t$ -test. While we rejected the normality of  $\delta$  using the Shapiro-Wilk  $W$ -test under all conditions ( $p$ -values were 0 for every condition), we performed a  $t$ -test and obtained almost identical results. We found significant differences in two conditions at the 5% but not 1% level (and in two more conditions even at the 10% level).

<sup>10</sup>Rohde (2010) provided an adjusted measure of  $\delta$ , called *hyperbolic factor*. Using this measure, we confirmed that the results did not change much. What is more interesting is that our analysis using hyperbolic factors yielded negative evidence of most widely used discount functions, including GHD. Rohde (2010) showed that this measure was constant across temporal conditions under most standard functional specifications (e.g., exponential, quasi-hyperbolic discounting, and GHD) and coincided with parameter  $\alpha$  under the GHD specification (Rohde, 2010, Theorems 8–10). However, hyperbolic factors varied across conditions in our experiment (Kruskal-Wallis test; the  $p$ -value is 0.00 for both gain and loss) so that the usual functions cannot explain our observations. While Abdellaoui et al. (2013a) assumed such functions in most parts of their analysis, our measure  $\delta$  remains valid owing to its non-parametric setting.

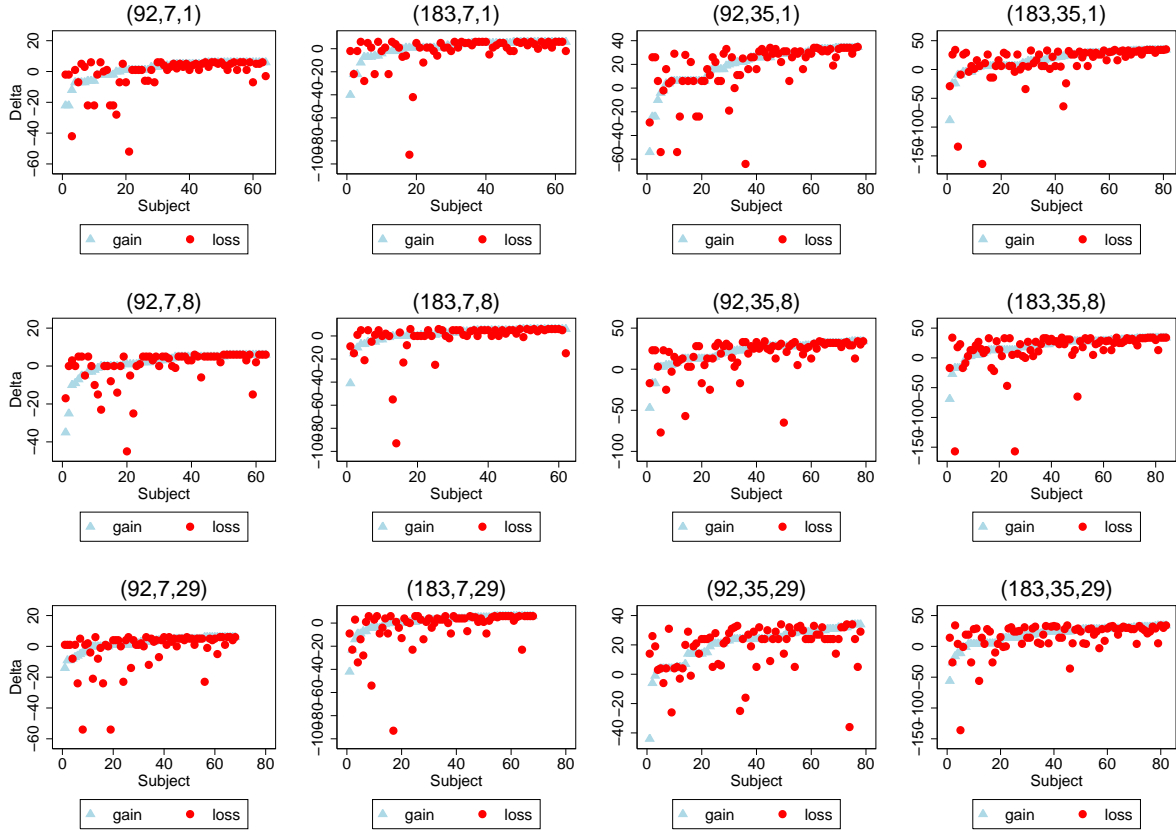


FIGURE 6: Correlations of  $\delta$  in each condition.

Note: The subjects are ordered by degree of  $\delta^+$ . Each sub-figure corresponds to individual experimental conditions characterized by  $(s_0, d_0, s)$  at the top of each sub-figure.

utility model of intertemporal choice, we have

$$D(s_0)u(x) = D(s_0 + d_0)u(y) \quad \text{and} \quad D(s)u(x) = D(s + d)u(y),$$

so that

$$d = D^{-1}\left(\frac{D(s)}{D(s_0)}D(s_0 + d_0)\right) - s. \quad (1)$$

Assume  $D$  is GHD function  $D(t) = (1 + \alpha t)^{-\frac{\beta}{\alpha}}$ . Then, under those specifications, WTW in stage 2 ( $d$ ) must satisfy

$$d = \frac{1}{\alpha} \left[ \frac{1 + \alpha s}{1 + \alpha s_0} \left( 1 + \alpha(s_0 + d_0) \right) - 1 \right] - s. \quad (2)$$

We should note here that even if we assumed a parametric specification for discounting, the present experiment would be free from specifications on the utility function  $u$  because the two intertemporal choices contain the same outcomes,  $x$  and  $y$ , and thus  $u(\cdot)$  is canceled out in equation (1). This structure is identical to those in Attema et al.'s (2010) and Takeuchi's (2011) models, so that such experiments are also called "utility-free" methods.

According to equation (2), parameter  $\beta$  of GHD, and thus the value of function  $D(t)$ , cannot be estimated in the present study. Takeuchi (2011) proposed estimating them by deriving the ratio of utility  $\frac{u(x)}{u(y)}$  from risk preferences in an expected utility (EU) model. However, the EU specification for risk preference is not reasonable (e.g., Kahneman & Tversky, 1979). In addition, the implicit assumption that the utility functions for risk and time are identical may lead to bias (Abdellaoui et al., 2013b; Andreoni & Sprenger, 2012a). Since this study focuses on TI, and not the degree of time discounting, we do not follow this method.

For the entire data, the best-fit estimates of  $\alpha^+$  and  $\alpha^-$  are 4.30 and 2.57, respectively, implying that time preference deviates from exponential discounting more for gain than for loss. At the individual level, the medians of  $\alpha^+$  and  $\alpha^-$  are 8.07 (87 subjects) and 4.62 (78 subjects), respectively, considering that certain subjects could not be analyzed because their observations that satisfied our assumptions were too few or the estimator was rather large ( $\geq 10^{35}$ ). These parameters demonstrate a positive correlation (Spearman's

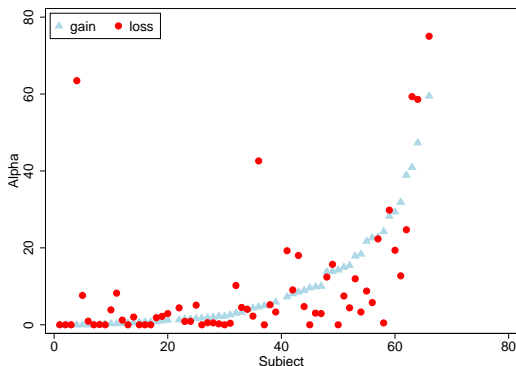


FIGURE 7: Correlation of  $\alpha$  for observations whose parameters are less than 100.

TABLE 4: Direction of impatience shift in the second stage. The case of gain with  $d_0 = 7$  and  $s_0 = 92$ .

| $s = 29 \rightarrow 8$ | $s = 8 \rightarrow 1$ |    |             | Total |
|------------------------|-----------------------|----|-------------|-------|
|                        | present bias          | TC | future bias |       |
| present bias           | 12                    | 8  | 12          | 32    |
| TC                     | 8                     | 15 | 2           | 25    |
| future bias            | 15                    | 2  | 7           | 24    |
| Total                  | 35                    | 25 | 21          | 81    |

Note: This table shows the number of participants who became impatient (patient) when delay decreased from 29 to 8, and from 8 to 1. TC indicates that the person exhibited same patience levels, i.e.,  $WTW_t = WTW_{t+1}$ .

$\rho = 0.64$ ,  $p = 0.00$ ) and no significant difference (sign rank test;  $p = 0.31$ ) for subjects whose parameters could be fully estimated (75 subjects). A graph of the individual relationships is presented in figure 7. The mean values for these subjects are considerably large because some of the subjects reported a high  $\alpha$ .

## 5 Discussion

### 5.1 Classes of Discounting

Most studies assume exponential discounting or hyperbolic discounting, which cannot explain future bias. However, in our experiment, an unignorable number of participants exhibited future bias in each condition, whose general tendency was consistent with the ISD prediction. Here, we again examine the dynamic change in impatience (only in the second stage) and try to classify our participants into discount classes.

To simplify the classification, we start with table 4, which uses limited data from stage 2. This table shows the numbers of observations that exhibited  $WTW_1 > (= <) WTW_2$  and

$WTW_2 > (= <) WTW_3$  when  $LL^+$  represents a WTA for 7 days waiting from SS at 92 days. Thus, this table shows how many participants became patient (or impatient) when SS shifted from 29 to 8 and 8 to 1 with  $LL^+$  of  $(s_0, d_0) = (92, 7)$ . In this table, only 15 out of 81 participants exhibited constant impatience in the second stage, which is a necessary condition of the exponential discounting model. Twelve participants always exhibited present bias and 8 exhibited constant impatience for the shift from 29 to 8 but present bias from 8 to 1, where those two tendencies are hyperbolic discounting properties where people become impatient when the choice comes closer to the present. In sum, hyperbolic discounting can explain only 35 out of 81 participants' behavior. Regarding the other participants in the upper right triangle, ISD explains their behavior. Twelve of them exhibited present bias when the delay decreased from 29 to 8 days and future bias when it decreased again to 1. On the other hand, seven participants always exhibited future bias, which implies that their discounting function has a long domain of a concave shape. The number of participants who were consistent with ISD is 64 out of 81.

Thus, we classified the participants into discounting classes using all observations. Recall that Table 3 describes the WTW structure or sequence of our observations. If participants follow exponential discounting, they would exhibit constant impatience, i.e., constant  $WTW_i$  for  $i = 0, 1, 2, 3$ . Likewise, hyperbolic discounting allows decreasing impatience, implying non-increasing  $WTW_i$  in  $i$  without the relation of  $WTW_{i+2} = WTW_{i+1} < WTW_i$ . All classification rules are stated in table 5. We obtain a nested structure since ISD and hyperbolic discounting are generalizations of hyperbolic and exponential discounting, respectively.

Table 6 presents this classification. As shown in the table, almost no observation follows exponential discounting, and only 20% at most are classified under hyperbolic discounting. However, ISD explains 50-77% of observations in every condition. This suggests that we need to apply ISD to validate the explanation.

### 5.2 The Source of TI

TI has sometimes been explained in the literature as a distortion of individuals' time perception. Kim and Zauberman (2009), for instance, showed that a preference could be time inconsistent even when a person is constantly impatient but his or her time perception follows Stevens' power law, which is not linear. In their model, people differently perceive a delay of one day from a day later and from a year later. While Kim and Zauberman (2009) considered only present bias as TI, Takeuchi (2011) suggested that future bias could be also explained when people perceive the very close future as almost the "present" (which he called *extended present*). With such a perception, an individual will stop discounting two

TABLE 5: Classification Table

| Classification | Change of Imp. over time  |
|----------------|---|
| EXP            | $WTW_3 = WTW_2 = WTW_1 = WTW_0$ .   |
| HD             | $WTW_3 \leq WTW_2 \leq WTW_1 \leq WTW_0$ but “=” can place only from a left-hand side.  |
| ISD            | $WTW_3 > WTW_2 \leq WTW_1 \leq WTW_0$ , or<br>$WTW_3 > WTW_2 > WTW_1 \leq WTW_0$ , or<br>$WTW_3 > WTW_2 > WTW_1 > WTW_0$ , or<br>$WTW_3 \leq WTW_2 \leq WTW_1 \leq WTW_0$ but not $WTW_3 < WTW_2 = WTW_1 < WTW_0$ . |

Note: EXP, HD and ISD are exponential, hyperbolic and inverse S-shaped discountings.

TABLE 6: Classification of discounting classes.

|          | EXP  | HD    | ISD   | Other | Total |
|----------|------|-------|-------|-------|-------|
| gain     |      |       |       |       |       |
| (92,7)   | 3.7% | 13.6% | 58.0% | 42.0% | 81    |
| (183,7)  | 2.4% | 9.8%  | 57.3% | 42.7% | 82    |
| (92,35)  | 1.0% | 20.8% | 77.1% | 22.9% | 96    |
| (183,35) | 1.0% | 18.6% | 75.3% | 24.7% | 97    |
| loss     |      |       |       |       |       |
| (92,7)   | 0.0% | 4.9%  | 59.0% | 41.0% | 61    |
| (183,7)  | 0.0% | 9.2%  | 50.8% | 49.2% | 65    |
| (92,35)  | 0.0% | 20.3% | 66.2% | 33.8% | 74    |
| (183,35) | 0.0% | 10.3% | 59.0% | 41.0% | 78    |

Note: The nested structures are EXP, HD, and ISD.

alternatives once both come into “present” as time passes; thus, he or she exhibits future bias.

Our result on the intra-personal TI relationship is consistent with this explanation of TI. We found no persuasive evidence of differences between TI with gain and loss. This implies that people’s TI tendencies may be common for both delayed gain and delayed loss. This relationship would be natural if we consider time perception as a source of TI. Since waiting for the outcome is the same regardless of whether the outcome is gain or loss, perceived time length itself should be independent of the sign of outcomes, though its evaluation (how much they dislike waiting so long) may not be. If this is the case, the sign of the outcome may affect the evaluation of the outcome (e.g., loss aversion and the effect of the sign on the utility curve) and of the delay (effect of the sign on the discount rate), but not TI itself. This intuition is shared by Loewenstein and Prelec (1992), who reconstructed the sign difference of the imputed discount rate by assuming the utility of the prospect theory.

Another explanation of the source of TI is the potential risk in the future. The future is not always predictable, and future events (receiving or paying) are not certain (e.g., one may

die before receiving). This risk changes over time for both SS and LL, so people may change their previous decisions at a later time period.<sup>11</sup> According to this argument, the potential risks should have differed between gain and loss in our experiment. Therefore, our finding of no significant difference in TI and its sign might be confusing. Some risks like the possibility of death in the future might be common for both gain and loss, but these were possibly too small and ignorable (the time horizon was one-half of a year). However, other risks such as financial problems or breaking of promises by the experimenter should be much larger, and are basically only relevant to gains, not losses. That is, the probability of not realizing a gain must be much larger than that of not incurring a loss, and the degree of TI varies accordingly. Our results did not exhibit this tendency. One may argue that potential risk truly affects TI, and the finding of no difference in the signs was only due to the hypothetical setting of our experiment. If all events were hypothetical, the risk of realization would not be a concern, and one would not sufficiently consider potential risk. If this is the case, however, one cannot explain why TI (present bias and future bias) was observed so frequently even in the hypothetical setting, because a no-risk situation is needed to drive time-consistent behaviors.

Our result of positive correlation with TI was contrary to the finding of Abdellaoui et al. (2013a). Initially, we believed that their result was distorted by an error in their measurement  $\alpha$  of the GHD function, because it did not allow for future biases. However, our experiment reported a positive correlation even between  $\alpha^+$  and  $\alpha^-$ , so this contrary

<sup>11</sup>This explanation may be too simplistic. Specifically, Halevy (2008) and Epper et al. (2011) explained TI by such risks through non-expected utility models. With different risks, however, TI behaviors must also differ unless the evaluation of the non-expected utility risk counteracts the difference. Indeed, the literature provides other risk specifications and explanations of TI. For instance, Dasgupta and Maskin (2005) specified future risk based on the timing of an event, and explained that TI occurs through updating the risk over time. Pennesi (2017) focused on the uncertainty of individuals’ impatience, and explained TI by differences in sensitivity to delay between states. Regardless of which specification of risk is considered, the potential risks for gain and loss should differ from each other, resulting in the difference in TIs with signs.

result was apparently due to a different reason, possibly the variation in choice in the experiments or the specification error in the utility function.<sup>12</sup> Indeed, the contrary result could also be attributed to differences in the length of time.

While delays of alternatives varied from three months to five years in Abdellaoui et al.'s experiment, those in the present study ranged from a day to about 50 days, with the latest alternative being 218 days. Considering this difference, we may expect that TI for gain and loss may be correlated only when both delay and waiting time are sufficiently short, and not otherwise (similarly to the directions of TI in ISD). Since some researchers have stated that time preference depends on the time horizon (Ebert & Prelec, 2007; Read, 2001), this hypothesis may not be an uncommon one. If this is the case, the source of TI may differ when choice involves a long or short time horizon. It is surely important to know how people perceive waiting time in order to understand how they evaluate delay and why time-inconsistent behavior occurs. We leave to further research an exploration of the switches in TI from short- to long-term horizons.

### 5.3 Experimental Strengths and Limitations

As we noted in the introduction, our result is free from any specification errors. We do not specify any curvature of the utility function, loss aversion, and sign-dependency of evaluation. What we suppose regarding preferences are just standard assumptions in the literature: weak order, continuity, (strict) monotonicity, and (strict) impatience. Moreover, our study avoids the potential effect of the existence of immediate alternatives in the choice set. Most studies of future bias set  $s = 0$ ; however, immediacy may practically have a special impact, i.e., receiving a good “today” may be much different from receiving a good on “the day” when the experiment is carried out (e.g., difference in transaction costs; see Collier & Williams, 1999 for evidences). Existence of an immediate alternative may cause some discontinuity on intertemporal choices, which is however not our main research target. Therefore, we set positive front-end delay (more than a day) in our experiment to control for this effect. Thus, we confirmed the existence of future bias and the validity of ISD without the impact of immediacy.

However, in spite of the generality and validity of our results, this study and its results are not free from limitations. First, and most important, two types of questionnaires were used in the study. We asked our participants about both WTA and WTW and combined the answers. It is however well known in the risk preference literature that results depend on the target of the question; i.e., the certainty equivalent (asking outcome to be equivalent) and probability equivalent (asking probability to be indifferent) are not consistent with

<sup>12</sup>They also used preference for payoff streams, and assumed additivity.

each other (Hershey & Schoemaker, 1985).<sup>13</sup> This problem is still not clearly investigated in the time preference literature as far as we know. Our analysis of the intra-personal relationship of TI requires combining WTA and WTW, so it might well be the case that we connected responses that used different decision processes. On the other hand, the validity of ISD still holds even when we use only the indifferent pairs obtained in the second stage because the same classification in 5.1 suggests that 10.3%, 37.4%, and 83.8% of participants are classified as EXP, HD and ISD, respectively.

Second, some participants did not conform to (strict) monotonicity, impatience, and transitivity. Since these axioms are fundamental in utility theory, such violations should be considered. Although violation of impatience is not commonly observed in the literature, it is by no means rare. Negative discounting (i.e., preferring a delayed outcome) is mostly found for health, not monetary, outcomes (e.g., Ganiats et al., 2000), but Warner and Pleeter (2001) reported that many older officers might exhibit zero or negative discounting for money. Prelec and Loewenstein (1998) also found debt aversion, that is, a tendency to pay even before consumption.<sup>14</sup> Some studies have also reported violations of transitivity (e.g., Roelofsma & Read, 2000).

On the other hand, violations of monotonicity must be more serious. Since we use money as the outcome, strict monotonicity should hold. However, our participants might have thought that the set  $T$  is  $\mathbb{N}$  and not  $\mathbb{R}^+$ . If this is the case, answering  $d = 0$  does not imply a direct violation of monotonicity, but it only means the participants felt  $d = 1$  was too much. Actually, only one participant answered an unnatural number in the second stage (e.g., (10000, 8)  $\sim$  (13000, 10.1)). However, basically, WTW is difficult to interpret with an unnatural number (e.g., what did an individual mean by 8/7 days of WTW). This may be one of the practical problems of matching-based experiments, especially if the length of time is asked. We end this discussion noting that our results did not change appreciably when we included observations that satisfied monotonicity, but not strict monotonicity.<sup>15</sup>

Lastly, the use of matching-based elicitation might bias participants' responses. In the decision making literature, it is known that the response through matching-task elicitations is or may be biased (e.g., anchoring effect and inconsistency of the choice). We tried to minimize such biases in the experiment, but probably not to a sufficient level. In addition,

<sup>13</sup>We thank Professor Kontek and Professor Nishimura for pointing out this effect.

<sup>14</sup>This study does not focus on debt aversion, which is a choice between alternatives including both gain and loss, namely, consuming (gain) with payment (loss). However, according to standard theory or, more specifically, additivity of evaluation functions and impatience, a decision maker would prefer to pay after consumption because post-payment is discounted more than pre-payment.

<sup>15</sup>Of course, “inconsistency” is difficult to define, from the observed choices, if the strictness condition is dropped.

matching tasks sometimes leads to responses that are not based on serious thinking. In fact, several participants in our experiment might be guilty of this, even they had satisfied the basic preference conditions.<sup>16</sup> Of course, such potential biases can be avoided through the use of choice tasks, in exchange for giving up fine grid points of preference relations and fine bridges between the responses in the two stages.

However, we do not think that our elicitation method harmed all our results, because our observations are essentially not much different from those of previous studies that used choice-based elicitation. The finding of future bias and its tendency across temporal conditions are almost consistent with Sayman and Öncüler (2009). Even the high imputed discount rate was consistent with studies on time preference with small temporal differences, as mentioned in section 4.2.1. In addition, one well-controlled result was provided by Breichrodt et al. (2016). They replicated Attema et al.’s (2010) experiment in which respondents were asked their WTW through a choice-based matching task and found the same behavior. This indicates that WTW may not be affected by the elicitation method. Of course, although none of these facts is a direct proof that the matching task did not harm the results, they indirectly validate our observations and results. Thus, we believe that the results would not change if we use a choice task in the experiment.

## 6 Conclusion Remarks

We focused on the intra-personal relationship of TI with gain and loss, and obtain degrees of TI in a laboratory setting under various temporal gain and loss conditions, using minimal preference assumptions. Our first finding is that participants exhibited not only present bias but also future bias even for loss—a result first observed in our study. The frequency of future bias over the conditions was not random, but consistent with previous findings, which were characterized by ISD.

The second finding is that the degrees of TI for gain and loss were positively correlated to each other, and not significantly different. That is, even if people’s impatience for gain and loss outcomes are different, their TI tendency for gain and loss are positively related (and may be identical). If anything, an individual’s TI strength for gain and loss may

<sup>16</sup>For instance, we found answers in stage 1 that followed a rule like “WTA for delay of 35 instead of 10000 yen in 92 must be almost 13805, because  $10385/10000 = (92 + 35)/92$ .” Though this rule has no reasonable background, 9 out of 108 participants used this rule in entire tasks (we distributed calculators, which might enforce such behavior). Indeed, we found another type of rule, for example, “(10000, 92) must be indifferent to (10700, 99), because temporal difference was 7 days.” Only one participant followed this rule among entire tasks. There can be other types of “intuitive” rules, but fortunately, at least our main result does not change essentially when we drop these observations. Therefore, we suppose that “intuitive” responses would not seriously affect our findings. We thank Professor Baron for pointing out this possibility.

be the same even when the degree of impatience (discount rate) differs between gain and loss.

The results of our study suggest that people may have a common (or similar) TI system for gain and loss. This is possibly consistent with the existing explanation that TI is caused by a distorted perception of the time dimension. Indeed, the source of TI may also differ between time horizons in the near and far future. Our results offer a key to discover the source of TI; how far do people perceive the future play a crucial role in TI.

## Appendix: Comparison of the level of TI

For  $s_0$ ,  $s$ , and  $d_0$  such that  $s_0 > s > 0$  and  $d_0 > 0$ , we say that  $\succeq$  exhibits present bias for gain from  $s_0$  to  $s$  with waiting time  $d_0$  when  $(x, s_0) \sim^+ (y, s_0 + d_0)$  but  $(x, s) \succ^+ (y, s + d_0)$  hold. That is, although an individual is indifferent between SS in  $s_0$  and LL with  $d_0$ , he or she strictly prefers SS when both alternatives near the present (in  $s$ ). *present bias for loss* can also be defined by  $(x, s_0) \sim^- (y, s_0 + d_0)$  but  $(y, s + d_0) \succ^- (x, s)$ . Then, future bias is defined by reversing the relations of these equations.

The most standard measurement of TI uses the  $\alpha$  parameter of the GHD function  $D(t) = (1 + \alpha t)^{-\frac{\beta}{\alpha}}$  with  $\alpha > 0$ . In this function, for any given  $\beta$  (which represents the level of impatience),  $\alpha \rightarrow 0$  implies time consistency and  $\alpha > 0$  implies present bias for any intertemporal choice. Therefore,  $\alpha$  can be interpreted as a level of TI<sup>17</sup>, so that  $\alpha_i > \alpha_j$  implies that “i is more time inconsistent than j,” where i and j are the indexes of individuals. As we mentioned before, this function does not allow future bias.

Our comparison of TI is not based on functional specification, but on a pair of choices defined by Prelec (2004).

**Definition 1** (Inter-comparison of TI). *Preference  $\succeq_2$  exhibits a stronger present bias for gain than  $\succeq_1$  from  $s_0$  to  $s$  with  $d_0$  when*

$$(x, s_0) \sim_1^+ (y^1, s_0 + d_0), (x, s) \sim_1^+ (y^1, s + d), \text{ and } (x, s_0) \sim_2^+ (y^2, s_0 + d_0) \text{ hold, but } (x, s) \succ_2^+ (y^2, s + d).$$

*$\succeq_2$  exhibits a stronger present bias for loss than  $\succeq_1$  from  $s_0$  to  $s$  with  $d_0$  when*

$$(x, s_0) \sim_1^- (y^1, s_0 + d_0), (x, s) \sim_1^- (y^1, s + d), \text{ and } (x, s_0) \sim_2^- (y^2, s_0 + d_0) \text{ hold but } (y^2, s + d) \succ_2^- (x, s). \text{ }^{18}$$

This definition states that for a fixed time period  $s_0$ , waiting time  $d_0$ , and common difference  $s_0 - s$ , if accelerating (or

<sup>17</sup>More specifically, it represents the level of deviation from exponential discounting. See Loewenstein and Prelec (1992) for details.

<sup>18</sup>This definition is a modified version of the comparison in Prelec (2004). Prelec’s definition required the above mentioned relationships even when  $x$  of SS for  $\succeq_1$  and  $\succeq_2$  differ. However, in the present experiment,  $x$  of SS does not vary by subject; that is, we focus on the case in which  $x^1 = x^2$ . Therefore, we use  $x$  for simplicity of definition.



delaying) LL  $d_0 - d$  compensates for the *particular* present bias of individual 1, although it does not for individual 2, then the latter present bias should be stronger than the former.

Similarly, we define a comparison between preferences for gain and loss:

**Definition 2** (Intra-comparison of TI).  $\succeq$  exhibits stronger present bias for loss than for gain from  $s_0$  to  $s$  with  $d_0$  when  $(x, s_0) \sim^+ (y^+, s_0 + d_0)$ ,  $(x, s) \sim (y^+, s + d)$  and  $(-x, s_0) \sim^- (-y^-, s_0 + d_0)$  hold, but  $(-y^-, s + d) >^- (-x, s)$ .

That is, when  $SS^+$  and  $LL^+$  as well as  $SS^-$  and  $LL^-$  are indifferent for an individual, the present bias for loss is stronger if the present bias for gain is compensated by  $d_0 - d$  whereas the present bias for loss remains even with  $d_0 - d$ .

Then,  $\delta = d_0 - d$  with  $(x, s_0) \sim (y, s_0 + d_0)$ , and  $(x, s) \sim (y, s + d)$  is our measurement of the level of TI depending on conditions. Under our preference assumptions, the comparison of  $\delta$  clearly coincides with the above definition.

**Proposition 1.** For a given  $(s_0, d_0, s; x)$  with  $s_0 > s > 0$  and  $d_0 > 0$ , the following statements are equivalent:

- i)  $\succeq$  exhibits present bias (future bias) for  $(s_0, d_0, s; x)$
- ii)  $\delta(s_0, d_0, s; x) > 0 (< 0)$ .

In addition, the following are equivalent:

- i)  $\succeq_i$  exhibits a stronger present bias for  $(s_0, d_0, s; x)$  than  $\succeq_j$
- ii)  $\delta_i(s_0, d_0, s; x) > \delta_j(s_0, d_0, s; x)$ ,

where  $\succeq_i$  and  $\succeq_j$  are  $\succeq_1$  and  $\succeq_2$  (inter-personal) or  $\succeq^+$  and  $\succeq^-$  (intra-personal).

Note that  $\delta$  is well defined for any given  $(s_0, d_0, s; x)$ . This is because  $y$  is unique up to  $s_0, d_0$ , and  $x$ , and for such  $y$  with  $(s_0, d_0, s; x)$ ,  $d$  is also unique. It is also worth noting that under GHD specifications, the comparison of the parameter  $\alpha$  and of  $\succeq$  in our definition should be the same when the latter comparison holds, regardless of SS, LL, and  $s < s_0$ . That is, under the GHD specification,  $\alpha_i > \alpha_j$  holds if and only if  $\succeq_i$  exhibits a stronger present bias than  $\succeq_j$  for any SS and LL. This fact was almost proved in Rohde (2010), which analyzed the hyperbolic factor  $h = \frac{1}{s_0 d - s d_0} \delta$  under the setting  $s > s_0$ .

**Proposition 2.** Under GHD specifications,  $\alpha_i > \alpha_j$  holds if and only if  $\delta_i > \delta_j$  for any  $(s_0, d_0, s; x)$ .

*Proof.* Using the same logic as that in Rohde (2010, Theorem 9, p.133), we can show that the denominator of  $h$  is positive (it is called *regularity*) under GHD and  $\alpha$  must coincide with  $h(s_0, d_0, s; x)$  for any  $(s_0, d_0, s; x)$  in our setting. From Theorem 4 (Rohde, 2010, p.131), it also follows that regularity guarantees the equivalence of  $h_i(s_0, d_0, s; x) > h_j(s_0, d_0, s; x)$  for all  $(s_0, d_0, s; x)$  and the latter condition of the statement. Therefore, under the GHD specification,  $\alpha_i > \alpha_j$  holds if and only if  $h_i > h_j$  holds for

any condition, and the latter holds if and only if  $\delta_i > \delta_j$  for any condition, i.e.,  $\succeq_i$  exhibits a stronger present bias than  $\succeq_j$  for any SS and LL.  $\square$

## References

- Abdellaoui, M., Bleichrodt, H., & l'Haridon, O. (2013a). Sign-dependence in intertemporal choice. *Journal of Risk and Uncertainty*, 47(3), 225–253.
- Abdellaoui, M., Bleichrodt, H., l'Haridon, O., & Paraschiv, C. (2013b). Is there one unifying concept of utility? An experimental comparison of utility under risk and utility over time. *Management Science*, 59(9), 2153–2169.
- Andreoni, J., & Sprenger, C. (2012). Estimating time preferences from convex budgets. *American Economic Review*, 102(7), 3333–3356.
- Attema, A. E., Bleichrodt, H., Rohde, K. I. M., & Wakker, P. P. (2010). Time-tradeoff sequences for analyzing discounting and time inconsistency. *Management Science*, 56(11), 2015–2030.
- Baucells, M., & Heukamp, F. H. (2012). Probability and time trade-off. *Management Science*, 58(4), 831–842.
- Benzion, U., Rapoport, A., & Yagil, J. (1989). Discount rates inferred from decisions: an experimental study. *Management Science*, 35(3), 270–284.
- Bleichrodt, H., Gao, Y., & Rohde, K. I. M. (2016). A measurement of decreasing impatience for health and money. *Journal of Risk and Uncertainty*, 52(3), 213–231.
- Bostic, R., Herrnstein, R. J., & Luce, R. D. (1990). The effect on the preference reversal of using choice indifferences. *Journal of Economic Behavior & Organization*, 13(2), 193–212.
- Coller, M., & Williams, M. B. (1999). Eliciting individual discount rates. *Experimental Economics*, 2(2), 107–127.
- Dasgupta, P., & Maskin, E. (2005). Uncertainty and Hyperbolic Discounting. *American Economic Review*, 95(4), 1290–1299.
- Ebert, J. E. J., & Prelec, D. (2007). The fragility of time: time-insensitivity and valuation of the near and far future. *Management Science*, 53(9), 1423–1438.
- Epper, T., Fehr-Duda, H., & Bruhin, A. (2011). Viewing the future through a warped lens: Why uncertainty generates hyperbolic discounting. *Journal of Risk and Uncertainty*, 43(3), 169–203.
- Frederick, S., Loewenstein, G., & O'Donogue, T. (2002). Time discounting and time preference: A critical review. *Journal of Economic Literature*, 40(2), 351–401.
- Ganiats, T. G., Carson, R. T., Hamm, R. M., Cantor, S. B., Sumner, W., Spann, S. J., Hagen, M. D., & Miller, C. (2000). Population-based time preferences for future health outcomes. *Medical Decision Making*, 20(3), 263–270.

- Halevy, Y. (2008). Strotz meets Allais: diminishing impatience and the certainty effect. *The American Economic Review*, 98(3), 1145–1162.
- Halevy, Y. (2015). Time consistency: Stationarity and time invariance. *Econometrica*, 83(1), 335–352.
- Hershey, J. C., & Schoemaker, P. (1985). Probability versus certainty equivalence methods in utility measurement: Are they equivalent? *Management Science*, 31(10), 1213–1231.
- Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society*, 47(2), 263–292.
- Kim, B. K., & Zauberman, G. (2009). Perception of anticipatory time in temporal discounting. *Journal of Neuroscience, Psychology, and Economics*, 2(2), 91–101.
- Loewenstein, G., & Prelec, D. (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *Quarterly Journal of Economics*, 107(2), 573–597.
- Pennesi, D. (2017). Uncertain discount and hyperbolic preferences. *Theory and Decision*, 83(3), 315–336.
- Prelec, D. (2004). Decreasing impatience: a criterion for non-stationary time preference and ‘hyperbolic’ discounting. *The Scandinavian Journal of Economics*, 106(3), 511–532.
- Prelec, D., & Loewenstein, G. (1998). The red and the black: Mental accounting of savings and debt. *Marketing Science*, 17(1), 4–28.
- Read, D. (2001). Is time-discounting hyperbolic or subadditive? *Journal of Risk and Uncertainty*, 23(1), 5–32.
- Roelofsma, P., & Read, D. (2000). Intransitive intertemporal choice. *Journal of Behavioral Decision Making*, 13(2), 161–177.
- Rohde, K. I. M. (2010). The hyperbolic factor: A measure of time inconsistency. *Journal of Risk and Uncertainty*, 41(2), 125–140.
- Sayman, S., & Öncüler, A. (2009). An investigation of time inconsistency. *Management Science*, 55(3), 470–482.
- Takeuchi, K. (2011). Non-parametric test of time consistency: Present bias and future bias. *Games and Economic Behavior*, 71(2), 456–478.
- Thaler, R. H. (1981). Some empirical evidence on dynamic inconsistency. *Economics Letters*, 8(3), 201–207.
- Warner, J. T., & Pleeter, S. (2001). The personal discount rate: Evidence from military downsizing programs. *American Economic Review*, 91(1), 33–53.