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Does high-frequency trading improve market quality?

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Abstract

We study a model of a limit-order market with high-frequency (HF) and non-HF traders. Our HF traders have a speed advantage over non-HF trader, which allows them to pick off existing limit orders in the order book or revise the quotes of their limit orders upon arrival of fundamental news. Our traders estimate non-execution and picking-off probabilities at different limit prices from the actual transaction and order placement history, and optimally select a limit price that maximizes their expected profit. We evaluate benefits and costs of HF trading by investigating its impacts on price discovery, volume, spread, volatility, and welfare. We demonstrate a larger spread and wider volatility, and a social welfare loss in a market with HF traders than without HF traders. However, a transaction tax imposed on HF traders improves social welfare; however, a cancellation fee on HF traders widens income inequality. Thus, we suggest that the transaction tax is a possible policy choice for regulating HF trading in the market dominated by HF traders.

JEL classification: G02; G12; G14

Keywords: High-frequency trading; limit order market; market quality; agent-based modeling; transaction tax; cancellation fee

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1. Introduction

Upon the development of computer technology, major stock exchanges replaced human intermediation with an automated order-matching platform for stock trading. Recently, financial markets have further transformed the computer trading system, and thus have increased the speed for order acceptance notices as well as for information distribution of transaction prices and quotes to stock traders. For example, on January 4, 2010, the Tokyo Stock Exchange introduced the Arrowhead, which can confirm an order acceptance in 1 millisecond and distribute the updated order book information to the public in 2.5 milliseconds. In response to the introduction of this sophisticated trading platform, market participants attempt to obtain milliseconds speed advantages by investing in computer technologies, and take extraordinarily quick actions on arbitrage chances. As a result, high-frequency (hereafter HF) trading has been pervasive in recent stock trading. HF traders conduct stock trading with its programmed computer system, which quickly implements trading decisions, sending orders to the exchanges and managing them. Brogaard (2011), for example, shows that 77% of all trades were conducted by HF traders in his Nasdaq data for 120 stocks. While improving the trading system, we observed the Flash Crash in the e-mini S&P 500 futures market on May 6, 2010, which was accompanied by a considerable price drop of almost 10% within 15 minutes before rebounding. Kirilenko, Kyle, Samadi, and Tuzun (2011) claim that HF traders liquidated their positions and exacerbated the downturn, and thus they are a new source of market fragility.

The flash crash during a period of growing and substantial use of HF trading has let researchers, policy makers, and practitioners to hotly debate the impact of HF trading on market quality. Why does it account for a large fraction of trading volume? Furthermore, does HF trading make stock markets more fragile or robust? Does it foster price discovery and liquidity of the market? How about the impact on welfare in the economy? If HF trading

has negative effects on the economy, what regulations should policy makers impose on it? This study provides possible answers to these questions, not only by proposing an agent-based limit order market with HF and non-HF traders, but also by demonstrating under which conditions HF trading benefits and costs the market. In the case where HF trading negatively contributes to the market, our model suggests that a transaction tax imposed on it is a possible policy choice for mitigating the negative impact, but a cancellation fee worsens the market. We choose the two regulations for investigating the impact on the market due to the facts on not only the recent growing volume and market share by HF trading, but also on their cancellation frequency.¹

In our model, traders place market or limit orders to the limit order book. HF traders have fast trading technologies, and thus can revise the quotes of their limit orders at a faster rate than slower traders, denoted as non-HF traders in our study, after the arrival of fundamental news. The difference in information processing speed generates an information asymmetry between HF and non-HF traders. Thus, HF traders have a reduced opportunity of being picked off. Instead, they can pick off the limit order placed by non-HF traders and have a chance to make a profit, and thus non-HF traders increase the picking-off risk. Our traders make their trading decisions, such as placing a market or limit order, or revise their previously submitted limit order as follows. First, they compute execution likelihoods and picking-off risks at multiple limit prices from the actual history of transactions and order placements. Then, they select a price, at which they are willing to trade, by maximizing their expected profits among a variety of limit prices, given the computed execution and picking-off probabilities. Thus, our modeling allows us to examine not only complicated interactions among order book dynamics, execution and picking-off risks, traders' order

¹ Hasbrouck and Saar (2009) show that 36.69% of all limit orders are cancelled within two seconds of submission on INET. In addition, spoofing and smoking are two of the very popular trading strategies by HF traders that involve cancellations. See, for example, Biais (2011).

choice, and its impact on market quality but also how a transaction tax and cancellation fee impact the market through the interactions, which is unique in the literature on limit order markets with HF traders. This study evaluates the benefits and costs of HF trading by investigating its impacts on volume, spread, volatility, price efficiency, and overall welfare.

We demonstrate that, after the arrival of fundamental news, HF traders tend to place more aggressive orders, such as market orders, to capture the profitable chance quickly. They execute limit orders at the best prices and remove them from the order book, pushing the best quotes toward the fundamental values. As a result, we observe a widening of the spread, increasing volatility, and more efficient prices. When HF traders are dominant in the market, profitable chances tend to disappear soon, as a large number of them execute existing limit orders, moving the best prices toward the fundamental values quickly. The higher tendency of market order placements increases an execution likelihood of limit orders, causing both HF and non-HF traders to place limit orders. However, the action enhances the picking-off risk for limit order traders, which increases the possibility of incurring a large loss for both types of traders. Therefore, we conclude that HF trading hurts the market when HF traders are dominant.

In addition, we demonstrate that, in a market with HF traders, a transaction tax improves the market, while a cancellation fee further hurts it. On one hand, we show that a transaction tax contributes to obtaining higher profits for both types of traders. The transaction tax is a direct cost for HF traders that discourages market order placements. However, we demonstrate that, in the end, it reduces the execution probabilities of limit orders, improving expected profits from market order submissions relative to those from limit order strategies for HF and non-HF traders. Therefore, we observe a higher frequency of market order submissions by both types of traders. Less frequent limit order provisions by non-HF traders reduce the chance for them to be picked off and, consequently, increase their

profits. On the other hand, the cancellation fee discourages HF traders from placing limit orders, and thus encourages their market order placements. It increases the execution opportunity of a limit order for non-HF traders, and thus contributes to their placements of limit orders with higher frequency. However, it also increases the chances for non-HF traders to be picked off by HF traders. Therefore, the cancellation fee widens the income difference between HF and non-HF traders by increasing the chance for HF traders to make greater profits, and for non-HF traders to incur a loss. The results indicate that the impacts vary according to the regulations, and thus careful considerations are required on the choice of regulations on HF traders in a market dominated by them as in actual stock markets. We suggest that the transaction tax is a possible policy choice for regulating HF trading in the market dominated by HF traders.

Despite increasing research attention, there is not yet sufficient empirical and theoretical research on HF trading, and the results regarding its costs and benefits are mixed. Thus, there is no general conclusion on the overall impact of HF trading on market quality. Furthermore, the regulations on HF trading and its impact on the market are not theoretically examined. The costs and benefits can be assessed by quantifying its impact on market quality. In previous empirical and theoretical studies, market quality has been measured by volume, spread, volatility, price efficiency, and overall welfare in the market. HF trading benefits the market if it increases the volume and tightens the bid-ask spreads due to lower transaction costs. It is also beneficial when it reduces volatility, and thus makes the market stable. Price efficiency can be measured by the speed at which HF traders detect and exploit discrepancies between stock prices, pushing them back to the fundamental value. HF trading improves the market when the recovery speed of the price efficiency as well as the overall welfare in the economy increases. HF trading costs the market in all opposite cases.

Some empirical studies on the cost and benefits of HF trading show declining bid-ask spreads (Boehmer, Fong, and Wu, 2014, Brogaard, Hagströmer, Norden, and Riordan, 2013, Carrion, 2013, Hasbrouck and Saar, 2013, Riordan and Storckenmaier, 2012, Hendershott, Jones, and Menkveld, 2011, Jovanovic and Menkveld, 2011, Malinova, Park, and Riordan, 2013, Riordan and Storckenmaier, 2012) and more efficient prices (Boehmer, Fong, and Wu, 2014, Brogaard, 2011, Carrion, 2013, Chaboud, Chiquoine, Hjalmarsson, and Vega, 2014, Hendershott, Jones, and Menkveld, 2011, Hendershott, and Riordan, 2014). Some demonstrate smaller volatility (Brogaard, 2011, Chaboud, Chiquoine, Hjalmarsson, and Vega, 2014, Hagströmer and Norden, 2013, Hasbrouck and Saar, 2013), while others find higher volatility (Boehmer, Fong, and Wu, 2014). Kirilenko, Kyle, Samadi, and Tuzun (2011) demonstrate that, while HF traders did not cause the flash crash, they exacerbated the price drop, worsening the volatility. HF trading increases depth in the order book (Brogaard, Hagströmer, Norden, and Riordan, 2013, Hasbrouck and Saar, 2013, Jovanovic and Menkveld, 2011), while Brogaard (2011) and Easley, Lopez de Prado, and O'Hara (2011) demonstrate that HF traders withdraw their liquidity from the market in bad times. HF trading takes a speed advantage and makes profits (Brogaard, 2011, Menkveld, 2013). However, previous empirical works do not argue the overall impact of HF trading on welfare. Limited amounts of empirical research have been conducted so far, and they show mixed results. Therefore, existing empirical conclusions must be regarded as tentative; further theoretical works are necessary in order to fully understand how HF trading impacts the market.

One of the theoretical works related to this paper is the one by Hoffman (2014), who constructs a model of a dynamic limit order market, where fast traders can revise their limit price at a faster rate than slow traders after fundamental news arrives. Thus, the fast traders can face a reduced picking-off risk, while slow traders face an increased one. They also increase their market power and extract rents from slow traders, inducing slow traders to

place limit orders strategically far from the best quotes. The model predicts a social welfare loss.

In addition, the theory by Biais, Foucault, and Moinas (2014) shows social inefficiency when institutions overinvest in speed technology. Jovanovic and Menkveld (2011) construct a model generating adverse selection costs for slower traders and leading to lower trade and reduced welfare. Liquidity supply increases because the middleman can avoid the picking-off risk by cancelling and revising limit orders at the arrival of new information. Cartea and Penalva (2013) show that HF trading raises volatility and volume. Pagnotta and Philippon (2011) show that the speed competition generally increases welfare, but decreases it when speed becomes inefficiently high. Rosu (2014) theoretically shows that volume, volatility, and liquidity increase when the number of HF traders increases, but foster market efficiency because trades by HF traders incorporate new information quickly. The model by Foucault, Hombert, and Rosu (2013) demonstrates that the speed difference generates adverse selection costs, lowering liquidity (higher price impact of trades) in the short run, but increasing it in the long run. Therefore, on average, there is no overall effect of HF trading on volatility, regardless of whether the informed have a speed advantage or not.

This paper brings the following four contributions. First and most significantly, it demonstrates the conditions in which HF trading benefits and costs the market, as well as the impact of transaction and cancellation fees on the market. While sufficient amounts of theoretical research have not been conducted to reach a consistent conclusion on the impact of HF trading, previous studies mentioned above do not demonstrate that a financial market both benefits and suffers from HF trading simultaneously in one model. In addition, the previous theoretical analyses do not argue which regulations, in particular a transaction tax or cancellation fee, should be imposed for reducing the negative effects of HF trading, and thus

improving total welfare in the economy. The present paper distinguishes itself by addressing these issues.

Previous theoretical models add a speed advantage of the millisecond for HF traders into the models that generate an adverse selection cost, and attempt to investigate the market quality under equilibrium. However, to be analytically tractable, modeling a limit order market requires imposing several assumptions that restrict the behavior of the limit order book and of traders. Therefore, this paper makes unique contributions to the body of knowledge on modeling HF trading as follows.

Our second contribution is that our model allows traders to compute the observed probabilities of being picked off and of non-execution at multiple limit prices, and the observed probabilities influence trading decisions. In particular, the model in the present study stores limit orders in the order book for multiple periods, and thus our traders can transact based on this information and past transaction history. In this setup, we can explain in detail not only how the order book information influences an adverse selection raised by the difference in information processing speed, but also how trading decisions affect the dynamics of volume, spread, and volatility. In previous studies, like the ones by Foucault (1999) and Hoffman (2014), the limit order can stay in the order book for one period only. Thus, the models do not explain how the limit order remaining in the order book influences the risk of being picked off or the market spread and volatility. In addition, the setup is more consistent with real stock trading than previous theoretical studies, given the fact that major stock exchanges adopt the open order book. Therefore, our modeling is very important in order to understand the impact of HF trading on market quality.

Third, this paper endogenously decides an optimal limit price that maximizes expected profits. Several previous agent-based theoretical studies, such as Chiarella, Iori, and Perelló (2009) and Anufriev and Panchenko (2009), assume that limit prices are randomly

selected. However, placement decisions by our agents are determined by comparing expected profits at different limit prices computed by the non-execution and picking-off risks from the actual history of order placements and transactions. Thus, unlike previous studies, our limit price selection is consistent with the concept of endogenous profit maximization, and is a more general case for characterizing a limit order market.

Fourth, our HF traders are different in that some of them can take a speed advantage over others and pick them off. This is a more general model than previous theoretical studies in which HF traders have the same information processing speed, so they are always successful in avoiding the picking-off risk, and thus gain from trade, meaning that slow traders always lose by construction. This results in an increase in trades for fast traders, and a decrease for slow traders. However, HF traders also compete in speed with each other, and thus there should be certain types of HF traders who incur an adverse selection cost and make a loss by trading with other HF traders. Our extension allows us to examine how the adverse selection costs as well as the resulting trades and welfare become larger or smaller for HF traders in response to the change in the number of HF traders in the market. We show that certain HF traders actually lose when there is a large number of this type of traders in the market.

The rest of the paper is structured as follows: Section 2 introduces our agent-based model with HF and non-HF traders; Section 3 presents the results from computer simulations; Section 4 summarizes our findings and concludes.

2. The model

This section extends the model by Hoffman (2014) and designs our continuous double-auction or order-driven economy with two types of traders: HF and non-HF traders. Our traders place limit or market orders, and trade through order book once the orders are

matched. Transactions are executed according to the time and price priority rule. Our basic framework resembles the order-driven trading mechanism in major stock exchanges, which is used after the opening session and before the closing session.

2.1. Fundamental news

Our model consists of two types of traders, HF and non-HF traders, and the total number of traders is N . The fraction of HF traders is exogenously given by α , and thus there are $N \times \alpha$ HF traders and $N \times (1 - \alpha)$ non-HF traders in our market. The traders transact a single risky asset, but differ in the timing of receiving the fundamental news information, the order book conditions, or arbitrage chance, as well as in the information processing speed. Our HF traders are able to acquire profitable information, make decisions for order placements and revisions, and send orders to the order book at faster rates than non-HF traders. Thus, they have a speed advantage. Our non-HF traders are considered to be human traders in real financial markets whose reactions to updates of the news and order book are much slower than those of HF traders.

Since traders in a continuous double-auction market trade through the bid and ask sides of the order book, profitable chances arise from information on the changes in bid and ask prices. We assume that profitable information arises from changes in the fundamental values of the bid and ask prices, or in the bid and ask prices from a different trading platform. The fundamental values of the bid and ask prices, or the bid and ask prices from a different trading platform are assumed to grow with the following random walk process:

$$V_{a,t} = V_{a,t-1} + \varepsilon_{a,t-1} \tag{1}$$

$$V_{b,t} = V_{b,t-1} + \varepsilon_{b,t-1} \tag{2}$$

where subscripts a and b represent ask and bid prices respectively, and random variables $\varepsilon_{a,t-1}$ and $\varepsilon_{b,t-1}$ are identically and independently distributed. $\varepsilon_{a,t-1}$ and $\varepsilon_{b,t-1}$ take integers in the range of $+\sigma$ and $-\sigma$ with equal likelihood. In our model, HF traders can exactly observe $v_{a,t}$ and $v_{b,t}$, meaning that they know the realized values of $\varepsilon_{a,t-1}$ and $\varepsilon_{b,t-1}$. Therefore, the information advantage for HF traders can be interpreted in one of the following two ways: first, as in previous theoretical studies on higher frequency trading, our HF traders are quickly able to acquire information on the fundamental price of the asset; second, as in the real financial market, the markets are assumed to be fragmented so that HF traders can quickly find arbitrage chances in different trading platforms, but non-HF traders cannot. In particular, defining the best bid and ask prices at t as B_t^{Best} and S_t^{Best} respectively, the profitable conditions are $B_t^{Best} > v_{a,t}$ and $S_t^{Best} < v_{b,t}$ so that traders can earn positive returns by buying at a lower price and selling at a higher price.

We assume that non-HF traders can observe the past values $v_{a,t-1}$ and $v_{b,t-1}$, but not the current values $v_{a,t}$ and $v_{b,t}$. Thus, they attempt to predict the values of $v_{a,t}$ and $v_{b,t}$ based on the past values. Consistent with the results of the survey studies by Lui and Mole (1998) and Menkoff and Taylor (2007), and of previous agent-based models (Hommes (2006) for the survey), our non-HF traders utilize past price information for making their expectations as:

$$E_{t-1}^i(v_{k,t}) = v_{k,t-1} + \beta_k^i \times I(v_{k,t-1} - v_{k,t-2}) \quad (3)$$

where k denotes either a (best ask price) or b (best bid price). $I(\bullet)$ is an indicator function that takes 1 if $v_{k,t-1} - v_{k,t-2} > 0$ or $v_{k,t-1} - v_{k,t-2} < 0$, and zero otherwise. Non-HF traders are trend followers when β_k^i takes a positive value. They are contrarians attempting to find the turning points of the current market trend if the value of β_k^i is negative. For all experiments,

the values of β_k^i are uniformly distributed within the range of integers between $[-\sigma_\beta, \sigma_\beta]$, and randomly and independently determined across non-HF traders.

2.2. Time sequence of events

In our model, time subscript t refers to the time when the new profitable information arrives, and agents trade during the time between the old and new information, meaning that transactions occur between t and $t+1$. We assume that, at the beginning of t , a trader becomes a buyer or seller with equal probability, as in the model by Hoffman (2014), in which HF and non-HF traders sequentially enter the market and the order is determined randomly, implying that non-HF traders can take trading actions as quickly as HF traders do, even though they do not know the fundamental value. However, in our market, HF traders can observe the profitable information and place orders at a faster rate than non-HF traders. The speed advantage leads us to assume that all of our HF traders take actions before non-HF traders, as follows: when the profitable news arrives in the market, all HF traders receive it at once; then, they start trading sequentially by placing market or limit orders, or cancelling and resubmitting their previous limit orders if doing so increases their profits.

In addition, HF traders also have the ability to quickly observe the order book updates. Therefore, they update the order book information with every action by investors. All actions by HF traders are taken immediately after the news but before non-HF traders make any trading decisions. We denote the time period when all HF traders sequentially take actions by $t'(\geq t)$. We further assume that they take countervailing trades at the fundamental or best prices in a different trading platform at $t''(\geq t')$ if it is profitable.

Non-HF traders are slow to realize the fundamental news, and HF traders tend to remove profitable chances before they take actions. Our non-HF traders are slow traders, and thus predict a relatively larger future price and attempt to raise profits from the price

difference between the current and future prices. Based on predictions, they sequentially place market or limit orders at $t'''(\geq t'')$. As in the model by Hoffman (2014), since non-HF traders do not frequently check the order book updates, they are not able to revise and cancel their previously placed limit orders. We assume that the time sequence of our traders' actions, which is summarized in Figure 1, is repeated T times.

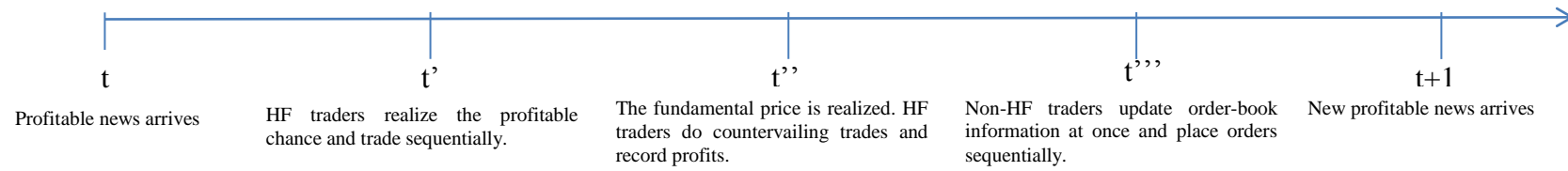


Figure 1: Time sequence of events

2.3. Non-execution risk, picking-off risk, and expected profits at multiple limit prices

Limit prices at which our HF or non-HF trader i is willing to trade at t are denoted by $B_t^{i,j}$ for a buy limit price and $S_t^{i,j}$ for a sell limit price. j represents the tick(s) above and below the best prices at the last period, namely B_{t-1}^{Best} and S_{t-1}^{Best} , where $j = -J, \dots, -1, Best, +1, \dots, +J$, and the positive (negative) numbers show how much higher (lower) the price is compared to the best prices. Traders submit a market buy order when the limit buy price exceeds the best ask, and they place a market order to sell when the limit sell price is below the best bid. They compare expected profits at all possible limit prices and choose the one that produces the maximum expected profits. As in Hoffman (2014), we assume the order size to be one.

The expected profit for trader i at a limit price that is j tick from the best price is denoted by $Vb_t^{i,j}$ for a buy limit order, and by $Vs_t^{i,j}$ for a sell limit order. They are computed based on the traders' valuation of the future asset price, a limit price $B_t^{i,j}$ or $S_t^{i,j}$, and an execution likelihood at $B_t^{i,j}$ or $S_t^{i,j}$, computed from the past transaction history. We first define the execution probability before formulating the expected profits.

The execution likelihood or non-execution risk by an agent i is computed independently on the buy and sell orders, and is denoted by $qb_t^{i,n}$ for a buy limit order, and by $qs_t^{i,n}$ for a sell limit order. At each point in time, the agents compute the execution likelihood from the observation of the order book condition and the history of all order placements. Agents observe the number of limit orders that exist at each limit price and at the prices that are better than a certain limit order. We denote the amount of limit orders waiting in front of a certain limit order as n . They also know the time until the limit order is executed, where the time is the number of agent actions (placing market or limit orders). We assume that agent i is assigned a parameter γ_i , which is a unit in ticks and measures the patience of

the agent towards the time until execution, where γ_i is randomly and independently drawn from $\gamma_i \in [\gamma_{\min}, \gamma_{\max}]$ across agents. When the time until a limit order is actually executed is longer than γ_i , we assume that agent i considers that the order is not executed because he/she is impatient. Therefore, the execution probabilities are computed as follows. For a limit buy order placed at t by agent i , where n limit orders are waiting in front of his/her limit order,

$$qb_t^{i,n} = \frac{\# \text{ of the limit buy order executed within } \gamma_i \text{ periods}}{\# \text{ of the limit buy orders placed}} \quad (4)$$

Similarly, for a sell limit order where n limit orders exist in front of his/her limit order,

$$qs_t^{i,n} = \frac{\# \text{ of the limit sell order executed within } \gamma_i \text{ periods}}{\# \text{ of the limit sell orders placed}} \quad (5)$$

Due to the time and price priority rule in an order book market, the execution probability of a limit order increases the fewer limit orders the book contains. Since we assume that HF traders quickly observe order book changes, they update the execution likelihood at each limit price with every action by investors. On the other hand, non-HF traders are slower to update the order book information in our model, and thus we assume that they simultaneously update the order book information before they take actions. Since each agent has a different value of γ_i , our setup indicates that agents are heterogeneous towards the non-execution risk.

Given the best prices B_t^{best} and S_t^{best} , the expected profits for HF traders are defined as follows:

$$Vb_t^{i,j} = qb_t^{i,n} (v_{b,t} - B_t^{i,j}) \text{ for a buy limit order at } B_t^{i,j}, \text{ where } j=-J, \dots, +J$$

$$Vb_t^{i,j} = v_{b,t} - S_t^{best} \text{ for a buy market order}$$

$$Vs_t^{i,j} = qs_t^{i,n} (S_t^{i,j} - v_{a,t}) \text{ for a sell limit order at } B_t^{i,j}, \text{ where } j=-J, \dots, +J$$

$V_{S_t^{i,j}} = B_t^{best} - v_{a,t}$ for a sell market order

The expected profits for non-HF traders are determined as follows:

$V_{B_t^{i,j}} = qb_t^{i,n} \times (E(v_{b,t}) - B_t^{i,j})$ for a buy limit order at $B_t^{i,j}$, where $j=-J, \dots, +J$

$V_{S_t^{i,j}} = E(v_{b,t}) - S_t^{best}$ for a buy market order.

Our traders will be picked off the existing limit orders when the fundamental value or prices in a different trading platform crosses the limit prices. This risk is higher for non-HF traders than HF traders, because non-HF traders have a speed disadvantage on acquiring the fundamental information and they cannot cancel and revise their limit orders. In particular, non-HF traders will be picked off and thus make a loss when the limit price is expected to be lower than the future best bid, because they have to sell at a lower price in the future than their buying price, that is to say $v_{b,t} - B_t^{i,j} < 0$. In such a case, the expected loss from being picked off at a limit price $B_t^{i,j}$ is $qb_t^{i,n} \times (E(v_{b,t}) - B_t^{i,j})$ where $j=-J, \dots, +J$. In addition, when the fundamental ask price or ask price in a different trading platform becomes higher than the limit ask price that the agent is going to place, his/her limit sell order will be picked off and he/she will buy it back at a higher price in the future. Therefore, the agent incurs a lose due to the risk of being picked off by $qs_t^{i,n} (S_t^{i,j} - E(v_{a,t}))$, when $S_t^{i,j} - v_{a,t} < 0$.

Given the execution probability, the expected profit increases the farther the limit price is from the best price, and thus traders buy at a lower price and sell at a higher price. In addition, profits increase with a higher execution probability and a lower chance to be picked off.

2.4. Expected profit maximization and order cancellation

Our HF buyer's maximization problem is given by:

$$\max_{B_t^{i,j}} [qb_t^{i,n}(v_{b,t} - B_t^{i,j}), v_{b,t} - S_t^{best} \mid j = -J, \dots, +J]$$

For a HF seller,

$$\max_{S_t^{i,j}} [qs_t^{i,n}(S_t^{i,j} - v_{a,t}), B_t^{best} - v_{a,t} \mid j = -J, \dots, +J]$$

A non-HF buyer chooses $B_t^{*i,j}$ by solving the following maximization problem:

$$\max_{B_t^{i,j}} [qb_t^{i,n}(E(v_{b,t}) - B_t^{i,j}), E(v_{b,t}) - S_t^{best} \mid j = -J, \dots, +J]$$

A non-HF seller selects $S_t^{*i,j}$ by:

$$\max_{S_t^{i,j}} [qs_t^{i,n}(S_t^{i,j} - E(v_{a,t})), B_t^{best} - E(v_{a,t}) \mid j = -J, \dots, +J]$$

If more than one limit price shows the same maximum expected profit, we randomly choose one among the candidates. In addition, we assume that our HF traders have an option to cancel their previously submitted limit orders and resubmit them at the newly selected limit price when higher profits are expected to be obtained. Denoting the previously submitted limit buy order by $B_{t-k}^{i,j}$, a HF buyer cancels it and resubmits it at a selected limit price $B_t^{*i,j}$ if $qb_t^{i,n}(v_{b,t} - B_t^{*i,j}) > qb_{t-k}^{i,n_k}(v_{b,t} - B_{t-k}^{i,j})$ for any k (>0), where k indicates the past period ($k = 1, \dots, K$) and qb_t^{i,n_k} is the execution probability at t for the previously placed limit order $B_{t-k}^{i,j}$. A HF seller replaces $S_{t-k}^{i,j}$ with $S_t^{*i,j}$ when a limit sell order at $S_{t-k}^{i,j}$ is not executed yet and $qs_t^{i,n}(S_t^{*i,j} - v_{a,t}) > qs_{t-k}^{i,n_k}(S_{t-k}^{i,j} - v_{a,t})$.

3. Simulations and results

This section conducts computer simulations on our model and presents the results. We change the value of the fraction of HF traders α to be 0.1, 0.5, or 0.9, and investigate how it impacts order placement patterns as well as market quality, which is measured by volume, spread, volatility, price efficiency, and profits. The results on order patterns are

presented by types of traders, which are HF and non-HF traders. For our simulation experiments, we set T , which denotes the last period of simulation, equal to 7,000 rounds for each simulation, and analyze the data from the last 5,000 periods where one round consists of actions by all agents.² We run simulations 20 times for the given fraction of HF traders α . Our benchmark parameter values are assigned as in Table 1.

This section is divided into two subsections. The first one presents the results without any taxes or fees on HF traders, and we conclude with our simulation results that HF trading hurts the market, suggesting that certain regulations may be necessary to improve market quality. The next subsection imposes a transaction tax and cancellation fee on HF trading and discusses how market quality improves in these circumstances.

Table 1: Parameters

Number of agents: N	1,000
Variation of the fundamental value: σ	5
Variation of the expectation component by non-HF traders: σ_β	2
Min and max ticks of a limit price from the best prices: J	10
A range of agents' patience parameter: $\gamma_i \in [\gamma_{\min}, \gamma_{\max}]$	50 and 2,500

3.1. Simulations without a tax or fee

Table 2 presents the results of order placements, which are categorized by the percentages of market and limit orders at the best prices, inside and outside of the spread. In the table, the numbers without brackets represent the averages over 20 simulations, and those with brackets are standard deviations across 20 simulations.

² We eliminate the first 2,000 simulated data periods in order to remove the effect from initializations for simulating our market, such as the initialization of the order book.

Table 2: Order placements by HF and non-HF traders in economies with 0, 10, 50, and 90% of HF traders. The numbers without brackets are the averages over 20 simulations and those with brackets are standard deviations over 20 simulations.

	% market order	% limit order Inside spread	At the best prices	Outside spread	% cancellation
<hr/>					
HF(0%)-nonHF(100%)					
Non-HF	0.443 [0.136]	0.014 [0.009]	0.273 [0.145]	0.270 [0.245]	
<hr/>					
HF(10%)-nonHF(90%)					
HF	0.795 [0.043]	0.089 [0.044]	0.026 [0.017]	0.089 [0.038]	0.093 [0.039]
Non-HF	0.420 [0.061]	0.034 [0.018]	0.276 [0.071]	0.269 [0.108]	
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HF(50%)-nonHF(50%)					
HF	0.697 [0.077]	0.101 [0.070]	0.055 [0.048]	0.148 [0.097]	0.171 [0.138]
Non-HF	0.301 [0.115]	0.033 [0.024]	0.321 [0.087]	0.345 [0.173]	
<hr/>					
HF(90%)-nonHF(10%)					
HF	0.605 [0.119]	0.078 [0.054]	0.108 [0.035]	0.208 [0.146]	0.223 [0.195]
Non-HF	0.213 [0.130]	0.026 [0.025]	0.298 [0.086]	0.462 [0.226]	
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On the first row, we present order placement patterns when there are only non-HF traders in the market. This is done in order to highlight the impact of HF trading. 44.3% of the order submissions are market orders, while the sum of limit order placements at and outside the best prices is 54.3%. However, the distribution of order placements dramatically changes when HF traders are active in the market. In economies with HF traders, both types of traders tend to place less aggressive orders when a larger fraction of HF traders stays in the market. For example, when the fraction of HF traders is 10%, the percentages of market orders amount to 79.5% by HF traders and 42.0% by non-HF traders, while limit order placements at or outside the best prices consist of 11.5% by HF traders and 54.5% by non-HF traders. However, the frequency of aggressive order placement decreases as HF traders become the majority in the market. When the fraction increases to 90%, 60.5% of HF traders' submission and 21.3% of non-HF traders' submission are market orders, and 31.6% of HF orders and 76.0% of non-HF orders are placed through limit orders at or outside the best quotes. Since HF traders are more likely to place limit orders, as their fraction increases from 10% to 90%, we observe an increased tendency for the cancellation of limit orders from 9.3% to 22.3%. The observation is explained as follows.

Our HF traders can take actions immediately after fundamental information arrives in the market. In our market, profitable chances arise when the current best bid is greater than the fundamental ask price $B_t^{Best} > v_{a,t}$, or when the current best ask is lower than the fundamental bid $S_t^{Best} < v_{b,t}$. In these cases, HF traders can immediately place market orders in order to make profits. On the other hand, they can also acquire profits by $Vb_t^{i,j} = qb_t^{i,n}(v_{b,t} - B_t^{i,j})$ through the placement of a limit buy order at $B_t^{i,j}$, or by $Vs_t^{i,j} = qs_t^{i,n}(S_t^{i,j} - v_{a,t})$ through a limit sell order at $S_t^{i,j}$. In our model, the choice of market or limit orders depends on expected profits and, more fundamentally, on the execution

probability of limit orders. Our results suggest that HF traders attempt to acquire sure profits by sending market orders when their fraction is small, like 10%. However, as they place market orders, limit orders at the best quotes are executed and removed from the order book. Then, the best prices tend to move toward fundamental prices, for example when $B_t^{Best} > v_{a,t}$ and HF traders place market sell orders and make profits. When HF traders remove all of the limit orders, the new best bid becomes lower than the previous one, moving closer to the fundamental ask price $v_{a,t}$. When $B_t^{Best} = v_{a,t}$, market sell orders do not generate profits. However, when HF traders hit the limit orders at the best prices, this action increases the execution probability of limit orders and, consequently, the expected profits from limit order strategies. An increased execution probability is likely to be observed as more HF traders exist in the market and place market orders. Thus, HF traders have a higher chance of placing limit orders when their fraction is high. In our example, in an economy with a higher fraction of HF traders, they frequently place limit buy orders at a certain limit price $B_t^{i,j}$, due to an increased profitability by $qb_t^{i,n}(v_{b,t} - B_t^{i,j})$. Due to this increased execution probability of limit orders caused by HF traders, our non-HF traders place less aggressive orders as HF traders dominate the market.

Our results, particularly in an economy where HF traders dominate the market, provide a possible explanation on the order placement behavior from previous research on HF trading as well as from actual HF trading firms. For example, theoretical studies by Hoffman (2014) and Jovanovic and Menkveld (2011) show that fast (HF) traders are more likely to trade through limit orders than slow (non-HF) traders. The result comes from the speed advantage of HF traders. They can act fast in light of new information, and thus they can eliminate the picking-off risk by cancelling and revising their limit orders quickly. This result is consistent with ours, particularly when the number of HF traders in the market is

high. In addition, from previous empirical evidence by Chaboud, Chiquoine, Hjalmarsson, and Vega (2014), Hagströmer and Norden (2013), Malinova, Park, and Riordan (2013), and Menkveld (2013), HF traders tend to be makers rather than takers. Furthermore, the market-making behavior is observed by actual HF trading firms, such as Citadel, Getco, and so on. However, HF traders place more market orders than limit orders as empirically shown in Brogaard, Hendershott, and Riordan (2014). Our results suggest that HF traders can be takers and makers, depending on the fraction of HF traders in the market and the execution probability of limit orders. They are more likely to act as takers in an economy with a smaller fraction of HF traders. However, a greater number of HF traders leads to the elimination of profitable chances from the quick placement of market orders, but contributes to a higher chance for limit orders to be executed and profited. As a result, they become makers. In Table 4, we confirm this interpretation by comparing order placements to overall profits.

Tables 3 and 4 summarize the results on market quality, which is measured by volume, spread, volatility, price efficiency, and profits. Volume is the total number of units traded in a simulation. We record spread at the end of each round, volatility as a standard deviation of mid-quotes during a trading round, and price efficiency as a percent deviation of a mid-quote from a mid-fundamental price. Profits are calculated in terms of the amount of dollars that traders pay and receive by transactions over each simulation. All of the recorded measures are cumulated and averaged over 20 simulations, and presented by the numbers without parentheses in Tables 3 and 4. The numbers with parentheses are the standard deviations over 20 runs. Table 3 reports the results on volume, spread, volatility, and price efficiency. Table 4 summarizes the results on profits, including the distribution of profits, which are percentages that traders receive less than -100,000, -10,000, and -100 units, and more than 100 and 1,000 units.

Table 3: Market quality. The impacts of HF trading on volume, spread, volatility, and price efficiency.

	Volume	Spread	Volatility	Price efficiency
HF(0%)-nonHF(100%)	572816 (655119)	1.130 (0.393)	0.069 (0.196)	0.075 (0.060)
HF(10%)-nonHF(90%)	1275099 (697382)	1.513 (0.899)	0.431 (0.656)	0.073 (0.093)
HF(50%)-nonHF(50%)	1903397 (1590283)	1.402 (0.654)	0.560 (0.759)	0.050 (0.041)
HF(90%)-nonHF(10%)	2828824 (2107313)	1.470 (1.342)	0.595 (1.224)	0.041 (0.040)

Table 4: Market quality. The impacts of HF trading on profits.

	Mean	% less than -100,000	-10,000	-100	% greater than 100	1,000
<hr/>						
HF(0%)-nonHF(100%)						
Non-HF	0 (27321)	0.004	0.211	0.424	0.419	0.367
<hr/>						
HF(10%)-nonHF(90%)						
HF	360591 (434223)	0	0	0	0.9995	0.998
Non-HF	915 (69398)	0.073	0.351	0.432	0.564	0.558
HF+non-HF	361506					
<hr/>						
HF(50%)-nonHF(50%)						
HF	258584 (308870)	0	0	0.028	0.939	0.848
Non-HF	8100 (114188)	0.134	0.356	0.410	0.587	0.582
HF+non-HF	266684					
<hr/>						
HF(90%)-nonHF(10%)						
HF	183561 (190096)	0	0	0.076	0.853	0.818
Non-HF	54403 (266242)	0.228	0.451	0.475	0.525	0.520
HF+non-HF	237964					

In Table 3, the volume is 572816 in an economy with non-HF traders only, but amounts to 1275099 when the fraction of HF traders is 10%. This number significantly increases as the fraction of HF traders increases further. For example, the volume becomes 1903397 and 2828824 in the case of 50% and 90% HF traders, respectively. The spread is 1.130 when there are no HF traders in the market, but it increases when they are active. For example, it is 1.513 in the economy with 10% HF traders, and it amounts to 1.402 and 1.470 when HF traders consist of 50% and 90% of the market, respectively. The mid-quote volatility is 0.069 when there are no HF traders in the market, but increases to 0.431, 0.560, and 0.565 when the fraction (and thus the number) increases from 10% to 50% and 90%. The price efficiency improves as more HF traders are in the market. It is 0.075 in an economy without HF traders, but it decreases to 0.041 in an economy consisting of 90% HF traders.

We relate the results to the interpretations of those in Table 2. Our HF traders attempt to obtain profits by placing market orders when profitable chances are observed. The percentage of market orders placed in an economy with HF traders is likely to be higher than in one without HF traders. The volume is greater in markets with HF traders than in ones without them, as more HF traders take this action. As they place market orders, limit orders at the best prices are executed and removed from the order book, causing best prices to move toward fundamental prices. Therefore, we observe a larger spread and higher price efficiency in markets with HF traders. The average volatility is greater in economies with HF traders as well. Their profit-taking behavior increases the frequency of market order placements, removing limit orders at the best prices. This action causes the best prices to change often, generating higher volatility.

Our results show a worsening of market quality in terms of spread and volatility, but improvement in terms of volume and price efficiency, as more HF traders exist in the market. However, Table 4 shows that the market quality measured by profits, and thus social welfare,

worsens in an economy with more HF traders. Therefore, we conclude that HF trading hurts the market, indicating that we need to impose certain regulations on it in order to improve social welfare.

In Table 4, the sum of the average profits (welfare) changes from 361506 to 237964 as the fraction of HF traders in the market increases from 10% to 90%. In addition, as more HF traders exist, the percentages of non-HF traders who incur a big loss increase. For example, the probability that non-HF traders lose less than 100,000 units is only 0.4% in an economy with only non-HF traders, but it increases to 22.8% in an economy consisting of 90% HF traders. This is due to the increased likelihood for non-HF traders to be picked off by HF traders. The picking-off behavior leads to high percentages of HF traders making profits of more than 100 and 1000 units: 99.95% and 99.8% of them respectively when there are 10% HF traders in the market. However, the percentages that both types of traders earn positive profits are significantly reduced in economies with more HF traders. For example, 99.8% of HF traders and 55.8% of non-HF traders make profits of more than 1000 units in an economy with 10% HF traders. However, the percentages decrease to 81.8% and 52.0% when HF traders consist of 90% of the market. The profit taking behavior of HF traders causes best prices to move towards fundamental prices. Consequently, the profit chances disappear but the higher frequency of market order placement leads to the increased likelihood of limit order executions. Then, both HF and non-HF traders are more likely to place limit orders in order to obtain the benefits from the increased execution probability. This increases the competition to make profits from limit order provisions, and thus reduces the profits for both types of traders. Therefore, we conclude that HF traders worsen the market quality when they dominate the market. In such a case, certain regulations on HF trading activities may be necessary for the improvement of the overall welfare in the economy, which is the issue that we investigate in the next subsection.

The result suggests that market orders are a better strategy for HF traders than limit orders, particularly when there is a small number of them in the market. However, limit orders can become a better choice as more HF traders enter the market. This result is attributed to the profit-maximizing behavior of our HF traders that are influenced by execution probabilities computed by the actual transaction and order placement history. Chaboud, Chiquoine, Hjalmarsson, and Vega (2014), Hagströmer and Norden (2013), Malinova, Park, and Riordan (2013), and Menkveld (2013), and actual HF trading firms like Citadel and Getco, show a higher likelihood of HF traders using limit order strategies in the market, while Brogaard, Hendershott, and Riordan (2014) show a higher frequency of market order placements by HF traders. Therefore, our results provide an economic justification of why empirical studies show mixed results on trading strategies by HF traders.

Our results are also comparable to previous empirical and theoretical works on HF trading, which show mixed results about its impact. For example, HF trading improves depth, spread, and volatility in empirical studies by Brogaard (2010), Chaboud et al. (2014), Hasbrouck and Saar (2010), and Hendershott et al. (2011). On the other hand, certain empirical studies, like the ones by Boehmer, Fong, and Wu (2014), Kirilenko, Kyle, Samadi, and Tuzun (2011), Brogaard (2011), and Easley, Lopez de Prado, and O'Hara (2011), show a worsened market quality, as measured by depth, spread, and volatility. On the theoretical side of the research, Hoffman (2014), Biais, Foucault, and Moinas (2014), and Jovanovic and Menkveld (2011) show welfare loss. Jovanovic and Menkveld (2011) and Rosu (2014) show enhanced depths, while volatility increases in the model by Cartea and Penalva (2013) and Rosu (2014). Our results show that the impact depends on the fraction of HF traders in the market, and thus depends on how fast profitable chances disappear and how the execution likelihood of limit orders varies. In our economy, the market worsens when it is dominated by HF traders.

3.2. Transaction tax and cancellation fee on HF traders

This subsection investigates the impacts on market quality of a transaction tax and cancellation fee imposed on HF traders, and demonstrates that the transaction tax improves market quality while the cancellation fee worsens it. In particular, on one hand, the transaction tax increases the possibility for both HF and non-HF traders to receive positive profits, but reduces the probability of incurring a big loss. On the other hand, the cancellation fee significantly contributes to higher profits for HF traders but lowers profits for non-HF traders. Thus, the cancellation fee worsens the profit distribution among HF and non-HF traders.

The transaction tax that we analyze in this subsection is a direct tax incurred by HF traders for each transaction, while the cancellation fee is imposed on each cancellation by them. The tax or fee is $P_{sell}(1-c)$ for a sell order and $P_{buy}(1+c)$ for a buy order, where P_{sell} is the transaction price at which an investor sells the share, P_{buy} is the transaction price to buy, and c is the one-way transaction tax or cancellation fee expressed as a fraction of the price. The specific rates of the transaction tax and cancellation fee that we should analyze are not known. Thus, we proceed with our subsequent analyses by imposing a one-way transaction tax with 10 and 100 basis points and cancellation fee with 1 and 50 basis point(s), and present the results on order patterns and market quality. We select the rates for which we clearly observe the influence of the tax and fee. We have observed that welfare worsens as the fraction of HF traders becomes as high as 0.9 when the tax and fee are imposed on HF trading. Therefore, we present the results from the value of α equal to 0.9 in order to investigate whether market quality improves through the addition of a transaction tax or cancellation fee.

3.2.1. Transaction tax on HF traders

Table 5 reports results on order placement patterns with the transaction tax on HF traders. Our results show that both types of traders become slightly more aggressive on order choice with transaction tax than without it. Without a transaction tax, market order placements comprise of 60.5% for HF traders and 21.3% for non-HF traders, while the limit orders outside of the spread encompass 20.8% for HF traders and 46.2% for non-HF traders. When we impose a transaction tax of 10bps and 100bps, the percentages of market order placements by HF traders are 60.7% and 61.3% and those by non-HF traders are 24.9% and 28.8%, respectively. Limit order provisions outside of the spread consist of 22.3% and 22.0% by HF traders and 43.1% and 41.6% by non-HF traders, respectively. Our transaction tax is imposed on HF traders directly, which certainly reduces profits earned through both market and limit order executions. However, since the tax is paid with a multiple of the execution probability from limit order strategies, where the probability is usually less than one, the expected profit loss is less than that from market order placements, discouraging HF traders from submitting market orders. In addition, when market orders are less frequently submitted, the execution probabilities of limit orders also decrease, further declining expected profits from limit order submissions. The smaller execution probabilities directly affect the profits from limit order executions. In our model, the execution probabilities are endogenously determined from the past transaction and order placement history, and traders' order submission strategies are optimally chosen by maximizing the expected profits at different limit prices. Therefore, we observe a higher frequency of market order placements by both types of traders, particularly by non-HF traders, as we impose a transaction tax on HF traders.

Table 5: Order placements by HF and non-HF traders in economies with 90% of HF traders, when transaction taxes are 10 and 100 basis points and cancellation fee is zero. The numbers without brackets are the averages over 20 simulations and those with brackets are standard deviations over 20 simulations.

	% market order	% limit order Inside spread	At the best prices	Outside spread	% cancellation
No transaction tax					
HF	0.605 [0.119]	0.078 [0.054]	0.108 [0.035]	0.208 [0.146]	0.223 [0.195]
Non-HF	0.213 [0.130]	0.026 [0.025]	0.298 [0.086]	0.462 [0.226]	
Transaction tax = 10 bps					
HF	0.607 [0.113]	0.088 [0.061]	0.082 [0.034]	0.223 [0.165]	0.210 [0.174]
Non-HF	0.249 [0.126]	0.033 [0.027]	0.288 [0.098]	0.431 [0.240]	
Transaction tax = 100 bps					
HF	0.613 [0.084]	0.087 [0.058]	0.081 [0.035]	0.220 [0.124]	0.209 [0.137]
Non-HF	0.288 [0.097]	0.038 [0.028]	0.258 [0.094]	0.416 [0.204]	

Tables 6 and 7 summarize the effects of the transaction tax on market quality. In Table 6, although we do not see significant changes on volume and price efficiency, the spread and volatility become worse with the transaction tax. For example, the spread, volatility, and price efficiency are 1.470, 0.595, and 0.041 without transaction tax, respectively. However, they are 1.516, 0.777, and 0.047 as the transaction tax becomes 100 basis points. The results are related to the finding from Table 5 that the transaction tax on HF traders reduces the execution probabilities of limit orders by both HF and non-HF traders, encouraging market order placements by both types of traders. The higher frequency of market order submissions consumes limit orders at the best quotes, which changes best prices, increasing spread and volatility.

While the spread and volatility worsen, the transaction tax improves overall welfare. On one hand, the percentages of receiving benefits more than 100 units are 85.3% for HF traders and 52.5% for non-HF traders without transaction tax, but they improve to 93.9% and 61.2% respectively with 100 basis points of transaction tax. On the other hand, 45.1% of non-HF traders lose more than 10,000 units without the transaction tax, but 37.4% of them lose the same degree when we impose 100 basis points of transaction tax. In our model, only HF traders know the fundamental price or profitable opportunity, and thus non-HF traders are more likely to be picked off by HF traders than HF traders themselves. However, due to the transaction tax, non-HF traders have a higher incentive to place market orders, reducing the possibility of loss from the picking-off risk. The results indicate that the fractions of both HF and non-HF traders receive higher profits when we impose the transaction tax on HF traders. Therefore, we conclude that the transaction tax contributes to improving the overall market quality.

Table 6: Market quality. The impacts of transaction tax on volume, spread, volatility, and price efficiency

	Volume	Spread	Volatility	Price efficiency
No transaction tax	2828824 (2107313)	1.470 (1.342)	0.595 (1.224)	0.041 (0.040)
Transaction tax = 10 bps	2901318 (2396223)	1.558 (1.299)	0.711 (1.380)	0.045 (0.039)
Transaction tax = 100 bps	2558075 (2131856)	1.516 (1.305)	0.777 (1.526)	0.047 (0.045)

Table 7: Market quality. The impacts of transaction tax on profits.

	Mean	% less than -100,000	-10,000	-100	% greater than 100	1,000
<hr/>						
No transaction tax						
HF	183561 (190096)	0	0	0.076	0.853	0.818
Non-HF	54403 (266242)	0.228	0.451	0.475	0.525	0.520
HF+non-HF	237964					
<hr/>						
Transaction tax = 10 bps						
HF	280568 (347761)	0	0	0.028	0.896	0.878
Non-HF	54262 (276162)	0.233	0.321	0.379	0.621	0.617
HF+non-HF	334830					
<hr/>						
Transaction tax = 100 bps						
HF	269684 (343641)	0	0	0.040	0.939	0.915
Non-HF	92708 (266091)	0.199	0.374	0.388	0.612	0.611
HF+non-HF	362392					

3.2.2. Cancellation fee on HF traders

For the case of a cancellation fee, we observe significant impacts on the order patterns in Table 8. The frequencies of placing market orders are greater, and those of limit order provisions become smaller than those in an economy without a cancellation fee. This tendency intensifies with a larger fee for cancellation. In Table 8, orders placed by HF traders consist of 60.5% market orders and 39.5% limit orders, and those by non-HF traders are 21.3% and 78.7% respectively in an economy without a cancellation fee. However, the percentages of market order submissions increase for HF traders and decrease for non-HF traders in economies with higher cancellation fees. For example, with 50 basis points of cancellation fee, HF traders place 71.4% market orders and non-HF traders, 5.9%. Limit orders by HF traders are 28.6%, and by non-HF traders, 94.1%. This result indicates that, although the cancellation fee is a regulation imposed only on HF traders, it also influences order submissions by non-HF traders. It discourages HF traders from placing limit orders that may involve order cancellations, and thus increases their incentives to place market orders. This behavior results in improvements of execution probabilities of limit orders, significantly increasing the incentive for non-HF traders to place limit orders.

Table 8: Order placements by HF and non-HF traders in economies with 90% of HF traders, when cancellation fees are 1 and 50 basis point(s) and transaction tax is zero. The numbers without brackets are the averages over 20 simulations and those with brackets are standard deviations over 20 simulations.

	% market order	% limit order Inside spread	At the best prices	Outside spread	% cancellation
No cancellation fee					
HF	0.605 [0.119]	0.078 [0.054]	0.108 [0.035]	0.208 [0.146]	0.223 [0.195]
Non-HF	0.213 [0.130]	0.026 [0.025]	0.298 [0.086]	0.462 [0.226]	
Cancellation fee = 1 bp					
HF	0.623 [0.131]	0.077 [0.053]	0.107 [0.033]	0.192 [0.171]	0.195 [0.198]
Non-HF	0.221 [0.131]	0.026 [0.024]	0.312 [0.114]	0.441 [0.262]	
Cancellation fee = 50 bps					
HF	0.714 [0.062]	0.096 [0.051]	0.076 [0.027]	0.114 [0.106]	0.099 [0.123]
Non-HF	0.059 [0.074]	0.018 [0.017]	0.398 [0.156]	0.525 [0.217]	

The impacts of the cancellation fee on volume, spread, volatility, and price efficiency are summarized in Table 9. They appear clearly when the fee becomes sufficiently large, such as 50 basis points. Spread and volatility become 3.649 and 0.642 from 1.470 and 0.595 respectively in an economy without the cancellation fee. As demonstrated in Table 8, the cancellation fee increases the cost of limit order provisions for HF traders. Thus, the regulation encourages their market order submissions, increasing the execution likelihood of limit orders. Since the cancellation fee does not cost non-HF traders, the greater likelihood of limit order executions encourages them to place limit orders. As shown in Table 8, non-HF traders are likely to place limit orders outside the spread, leading to a wider spread and greater volatility. The volume does not change much in our experiments (2828824 without the cancellation fee and 2802832 without the fee), possibly because the impact from the higher frequency of market order placements by HF traders offsets that from the lower tendency of market order submissions by non-HF traders. The volume indicates that trading removes the limit orders at the best prices and moves the price forward to the fundamental price. Thus, the larger volume contributes to greater price efficiency. However, our results show that price efficiency does not vary much because of the similar volumes with and without the cancellation fee.

Table 10 shows that the changes in the structure of order patterns caused by the cancellation fee have a negative effect on overall profits. Without the cancellation fee, 85.3% of HF traders and 52.5% of non-HF traders receive more than 100 units as their profits, and 45.1% of non-HF traders lose more than 10,000 units. When the cancellation fee is 50 basis points, the percentage of HF traders receiving more than 100 units increases to 91%, and that of non-HF traders decreases to 47%. With the same cancellation fee, the proportion of non-HF traders who lose more than 10,000 units increases to 50.9%. When HF traders pay the cancellation fee, they are more likely to place market orders, increasing the execution

probability of limit orders. This encourages non-HF traders to place limit orders to acquire the higher expected profits. However, this action provides more chances for HF traders to pick them off and make higher profits, resulting in a greater likelihood for non-HF traders to incur a big loss. Our result suggests that the cancellation fee significantly impacts the distribution of profits, in particular widening the profit differences between HF and non-HF traders. Therefore, we conclude that the cancellation fee worsens market quality.

Table 9: Market quality. The impacts of cancellation fee on volume, spread, volatility, and price efficiency

	Volume	Spread	Volatility	Price efficiency
No cancellation fee	2828824 (2107313)	1.470 (1.342)	0.595 (1.224)	0.041 (0.040)
Cancellation fee = 1 bp	2807267 (2385146)	1.597 (1.906)	0.507 (1.021)	0.048 (0.058)
Cancellation fee = 50 bps	2802732 (2121035)	3.649 (2.579)	0.642 (1.254)	0.044 (0.045)

Table 10: Market quality. The impacts of cancellation fee on profits.

	Mean	% less than -100,000	-10,000	-100	% greater than 100	1,000
No cancellation fee						
HF	183561 (190096)	0	0	0.076	0.853	0.818
Non-HF	54403 (266242)	0.228	0.451	0.475	0.525	0.520
HF+non-HF	237964					
Cancellation fee = 1 bp						
HF	325266 (431482)	0	0	0.049	0.903	0.869
Non-HF	-13294 (309806)	0.303	0.461	0.479	0.521	0.519
HF+non-HF	311972					
Cancellation fee = 50 bps						
HF	213812 (305426)	0	0	0.034	0.910	0.864
Non-HF	-15119 (193050)	0.324	0.509	0.529	0.470	0.468
HF+non-HF	198693					

4. Conclusion

The present study models a limit order market with HF and non-HF traders. We demonstrate that, without transaction and cancellation fees, profits worsen for both HF and non-HF traders as the former dominate the market. We investigate the impact of a transaction tax and cancellation fee on HF traders, and we show that the transaction tax improves market quality. However, we find that the cancellation fee worsens it, because it widens the income gap between HF and non-HF traders. We demonstrate that our results are attributed to our modeling, where execution probabilities of limit prices, order book dynamics, and traders' expected profits (and thus their order choice) are interrelated. In our model, execution probabilities of multiple limit prices are endogenously determined from and adjusted to the past transaction and order placement history, and our traders optimally choose a limit price by maximizing expected profits from multiple limit prices. This modeling distinguishes itself from previous theoretical models on limit order markets with HF traders.

As presented in the Introduction, there have been few theoretical works on HF trading, and none of them investigates the impact on market quality of a transaction tax and cancellation fee for HF traders. Although Biais, Foucault, and Moinas (2014) do not argue this issue, they show that a tax on investment in fast-trading technology, which dampens fast traders' activities, improves social welfare. However, our results indicate that the impacts of regulations on HF traders are very different, and consequently, careful considerations are required upon selecting the appropriate regulation for improving overall social welfare in an economy dominated by HF traders as in actual stock markets.

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