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## Abstract

Negative correlations between inflation and demographic aging were observed across developed nations recently. To understand the phenomenon from a politico-economic perspective, we embed the fiscal theory of the price level into an overlapping-generations model. In the model, successive short-lived governments choose income tax rates and bond issues considering the political influence of existing generations and the policy response of future governments. The model sheds new light on the traditional debate about the burden of national debt. Because of price adjustments, the accumulation of government debt does not become a burden on future generations. Our analysis reveals that the effects of aging depend on its causes. Aging is deflationary when caused by an increase in longevity but inflationary when caused by a decline in birth rate. Numerical simulation shows that aging over the past 40 years in Japan generated deflation of about 0.6 percentage points annually.

Keywords: Deflation, Fiscal theory of the price level; Politico-economic equilibrium

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# 1 Introduction

Recently, negative correlations between inflation and demographic aging were observed in developed nations. Figure 1 shows a negative relationship between the GDP deflator and growth of the working-age population in the Organisation for Economic Co-operation and Development (OECD) countries, implying that countries facing population aging were more likely to experience disinflation in the previous decade (see Shirakawa [2012]). In particular, as shown in Figure 2, Japan suffered from concurrent aging and deflation during the past two decades. While disinflation is presently being experienced only by developed countries, demographic aging is manifesting in some emerging economies such as China as well. This implies that the interconnection between aging and deflation will be an important issue for the emerging countries in the near future.

In this study, we analyze the impact of price changes on income distribution across generations and investigate the reason for the concurrent population aging and deflation in the framework of the fiscal theory of the price level (FTPL). The FTPL seems to be an appropriate unified framework to understand the recent disinflation wherein the central banks of the developed countries, namely, the Bank of Japan, European Central Bank, and the Federal Reserve, adopted a *passive* policy, to use the terminology of Leeper (1991), by maintaining nominal interest rates around zero, partly due to the zero lower bound of nominal interest rates, while fiscal debt accumulated. However, when attempting to understand the concurrent deflation and aging by applying the FTPL framework, one might encounter a puzzle. According to the conventional FTPL, the current price level adjusts to equate the real value of the government's outstanding debt with the discounted sum of current and future fiscal surpluses in real terms. In this context, population aging is a factor expected to reduce future fiscal surpluses owing to the increasing social security expenditure and declining income tax revenue. Thus, one would expect aging to generate an *inflationary* pressure (that is, an upward pressure on current prices) rather than the very low rate of inflation observed recently in Japan and other developed countries.

In this study, we mainly undertake the following two extensions to the standard FTPL framework. First, we embed the FTPL into an overlapping-generations (OLG) model instead of a model with an infinitely lived representative consumer, the latter of which is employed in previous studies using a standard FTPL framework. By this extension, we highlight how aging affects intergenerational income distribution by inducing price changes through its impacts on government budget. Second, and more importantly, we consider the political economy of fiscal

policy and incorporate the effect of aging on the choice of income tax rates and bond issues. This contrasts with the standard FTPL framework that considers the policy variables as given. Specifically, we set up a model in which policies are chosen by a succession of short-lived governments. Those governments make policy decisions under the political influence of existing generations, taking into account the general equilibrium effects on the price level and the real interest rate as well as the expected strategic responses of future governments.<sup>1</sup>

Let us now introduce the three recent features of Japan's political economy that are behind the motivation of this research.

1. Japan's nominal interest rates have been set close to zero for at least a decade now. Owing to *de facto* fixed nominal interest rates, fluctuations in the real interest rates have been correlated negatively with inflation.
2. To some extent, the demographic aging in Japan was unexpected. While Figure 3 shows that the official forecasts of Japan's birth rate (formally, the total fertility rate) were repeatedly revised downward, Figure 4 shows that the official forecasts of Japan's life expectancy were continually revised upward.<sup>2</sup>
3. In Japan, the political influence of the older generation recently overtook that of the younger generation owing to changes in political participation. Figure 5 not only shows that the voter turnout rate (the fraction of eligible voters who cast a ballot) in elections to the Japanese lower house increased with age, but also that the differences in turnout rates between the age groups widened.

In addition to these features relating to Japan, the inflation rate of the Euro area was a mere 0.3% in September 2014, which is well lower than the definition of price stability, that is, 2%, in the midst of fiscal retrenchment and population aging. These recent stylized facts observed in Japan and the Euro area motivate us to develop a politico-economic model with FTPL to investigate the reasons for the concurrent aging and disinflation.

Our main findings are summarized as follows.

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<sup>1</sup>This extension contributes to the following three (of four) important research topics proposed by Leeper and Walker (2011) regarding the FTPL: integrating heterogeneity and policy uncertainty; identifying policy behavior; and quantifying fiscal limits.

<sup>2</sup>These figures are from a speech made by Nishimura (2012), the former Deputy Governor of the Bank of Japan.

First, a current larger fiscal deficit produces a temporary price rise that only affects the well-being of the existing generations, but does not impact that of the future generations. In contrast to the previous literature such as Bowen *et al.* (1960) and Barro (1974), we present a novel argument that the burden of national debt does not shift from the current to future generations even in the absence of altruistic bequest motives between the generations because the price level is adjusted. As in the standard FTPL framework, the government debt outstanding at the beginning of each period is fixed in nominal terms in our model and the current price level is adjusted to equate its real value with the discounted sum of the current and future fiscal surpluses. Because of such price adjustments, the intertemporal fiscal linkage between the governments in different periods disappears and each government can choose its fiscal policy without being constrained by public debt inherited from its predecessors.

Second, and related to the above, we show that short-lived governments have no incentive to strategically accumulate debt because the adjustments in price level make the past accumulation of nominal government debt irrelevant to current governments' policy choice. The argument of strategic debt accumulation was pioneered by Tabellini and Alesina (1990) and Persson and Svensson (1989); recently Rohrs (2010), Song, Storesletten, and Zilibotti (2012), and Ono (2013) incorporated national debt into a political economy framework like ours. All these previous studies, however, assume that government debts are specified in real terms and therefore adjustments in the price level do not change the intertemporal budget constraint of the public sector. In the FTPL framework, irrespective of the amount of nominal debt accumulated, each government can pursue a fiscal policy in its own interest, thanks to the adjustments in the price level.<sup>3</sup> Consequently, debt accumulation has no commitment effect on the budgetary decisions of a succeeding government. A government cannot "tie the hands" of a successor it disapproves of through strategic debt creation.

Third, population aging affects the price level differently depending on its causes. The price level rises when the birth rate unexpectedly declines and decreases when the life expectancy unexpectedly becomes longer. Political factors can explain these different consequences. If the birth rate declines, the resultant contraction in the tax base reduces the fiscal surplus. The

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<sup>3</sup>Aghion and Bolton (1990) analyze strategic debt creation in the presence of future government defaults. Reinhart and Rogoff (2011) point out that governments have used inflation to wipe out part of domestic local currency bonds. Hall and Sargent (2011) report that the US federal government's debt-to-GDP ratio declined by 80.3% from 1945 to 1974, 50.2% of which was attributed to inflation.

government is then inclined to maintain its solvency partly by generating inflation at the cost of the older generation's well-being and partly by making the younger generation pay more taxes. In contrast, if the life expectancy increases and older persons survive longer than expected, they might face a shortage of savings for their retirement period. The government then, led by the strengthened political influence of the older generation, attempts to suppress inflation and increase the real value of the government bonds held by the older generation. Our numerical simulation shows that aging over the past 40 years in Japan brought about an annual deflation of about 0.6 percentage points.

Following Sargent and Wallace (1981), the foundations of the FTPL literature were built by Leeper (1991), Sims (1994), Woodford (1995), and Cochrane (2005), among others.<sup>4</sup> While several studies incorporate the FTPL framework into an OLG model (e.g., Cushing [1999] and Braun and Nakajima [2012]), Sims (2013) is the one most closely related to our study in terms of model structure. He uses a two-period OLG model with nominal government debt, as in our study, and discusses how the price level is determined within the FTPL mechanism. Although not dealing with the FTPL, Bullard *et al.* (2012) is also similar to our analysis in that it focuses on income redistribution to the older generation through deflation.<sup>5</sup> While these studies develop models and provide policy implications similar to ours, they do not endogenize political decisions on fiscal policy variables. To the best of our knowledge, our study is the first to analyze the public choice of fiscal policy variables such as income tax rates and bond issues in the FTPL framework.<sup>6</sup>

The remainder of this paper is organized as follows. Section 2 constructs a simple endowment economy model with two overlapping generations and analyzes the Markov perfect equilibrium wherein a succession of short-lived governments choose tax rates and bond issues. Section 3 generalizes the model in the previous section and conducts numerical simulation. Section 4 provides our concluding remarks.

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<sup>4</sup>See Bassetto (2002) and Buiter (2002) for a critical view. Empirical studies were conducted by Canzoneri, Cumby, and Diba (2001) and Bianchi and Ilut (2014).

<sup>5</sup>Other studies on aging and the nominal side of the macroeconomy include Fujiwara and Teranishi (2008) on optimal monetary policy and Heer, Maussner, and McNelis (2011) on money holdings.

<sup>6</sup>Christiano and Fitzgerald (2000) consider optimal distorting taxation to a certain extent, but not a politico-economic aspect.

## 2 An Endowment Economy with Overlapping Generations

We consider an endowment economy with overlapping generations in which the government imposes taxes on the younger generation and issues new public bonds to repay those held by the older generation. The government behaves in a *non-Ricardian* manner, to use the terminology of Woodford (1995) and Cochrane (2005), and the price level is based on a valuation equation for government debt. In this section, we first describe the economic equilibrium assuming that taxes are exogenous. We then endogenize them by embedding political decision-making into the model and characterize a politico-economic equilibrium.

### 2.1 Economic Equilibrium with Exogenous Fiscal Policy

Assume two overlapping generations, the young and the old, in each period. In period  $t$ , the population consists of  $N_t$  young households and  $\theta_t N_{t-1}$  old ones, where  $\theta_t$  is an independent survival probability of the young born in period  $t-1$ . That is, a young household born in period  $t$  will survive and become old in period  $t+1$  with probability  $\theta_{t+1} \in (0, 1]$  or will die at the end of period  $t$  with probability  $1 - \theta_{t+1}$ . We interpret the survival probability  $\theta_t$  as a measure of longevity for the older generation in period  $t$ . We also define  $n_t := N_t/N_{t-1}$  and call it the birth rate. Demographic aging occurs in period  $t$  either by an increase in  $\theta_t$  or by a decline in  $n_t$ . The streams of  $\theta_t$  and  $n_t$  are deterministic and so the economy exhibits no aggregate uncertainty.

This is an endowment economy with no storable technology. Each young household born at the beginning of period  $t$  receives one unit of endowment income and consumes  $c_t^y$  units in period  $t$  and  $c_{t+1}^o$  units in period  $t+1$  if it remains alive. It can save and insure against the risk of longevity by buying *nominal* annuities in competitive insurance markets. Each annuity unit contracted in period  $t$  promises to provide benefits defined in nominal terms as  $R_t^A$ , which will be paid out in period  $t+1$ . Owing to market incompleteness, annuity contracts that define benefit in real terms are not available. Let  $A_t$  be the amount of nominal annuity each young household buys in period  $t$ . Its budget constraint is then expressed as

$$c_t^y + \frac{A_t}{P_t} = 1 - \tau_t \tag{1}$$

and

$$c_{t+1}^o = \left( \frac{P_t R_t^A}{P_{t+1}} \right) \frac{A_t}{P_t}, \tag{2}$$

where  $P_t$  is the price level in period  $t$  and  $\tau_t$  is the income tax rate in period  $t$ .

Annuities are backed by government bonds, and insurance companies invest their sales of annuities in these bonds. Let  $R_t$  be the one-period gross nominal interest that a one-dollar nominal government bond sold in period  $t$  promises to pay in period  $t+1$ . Because no aggregate uncertainty exists, competitive markets ensure that insurance companies break even in equilibrium; that is,

$$R_t^A = \frac{R_t}{\theta_{t+1}}. \quad (3)$$

A young household in period  $t$  chooses  $c_t^y$  and  $c_{t+1}^o$  to maximize its expected life-time utility subject to the budget constraints (1) and (2), respectively. For simplicity, we suppose that households' utility functions are logarithmic; thus, their expected lifetime utility can be expressed as

$$\log c_t^y + \beta \theta_{t+1} \log c_{t+1}^o \quad (4)$$

where  $\beta$  is a discount factor. The utility is normalized to zero when the household is dead in period  $t+1$ . By combining (1), (2), and (3), we reduce the budget constraints into

$$c_t^y + \frac{\theta_{t+1}}{r_{t+1}} c_{t+1}^o = 1 - \tau_t, \quad (5)$$

where  $r_{t+1} \equiv R_t P_t / P_{t+1}$  is the real interest rate a young household faces in period  $t$ . It depends on the realization of the price level in the next period.<sup>7</sup> A young household's utility maximization yields its demand for goods and nominal annuities as follows:

$$c_t^y = \frac{1 - \tau_t}{1 + \beta \theta_{t+1}}, \quad (6)$$

$$c_{t+1}^o = \frac{\beta r_{t+1} (1 - \tau_t)}{1 + \beta \theta_{t+1}} \quad (7)$$

and

$$A_t = \frac{P_t \beta \theta_{t+1} (1 - \tau_t)}{1 + \beta \theta_{t+1}}. \quad (8)$$

Following our specification, a higher real interest rate increases the consumption of the older generation without affecting the savings of the younger generation.

The government in period  $t$  imposes a tax,  $\tau_t$ , on each young household in real terms and issues nominal bonds,  $B_t$ , to repay its outstanding debt,  $R_{t-1} B_{t-1}$ , inherited from its predecessor

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<sup>7</sup>No physical capital exists in the economy. If it exists, the real return on government bonds may be constrained by the real return from the physical capital due to the arbitrage condition. Sims (2013), however, argues that investments in physical capital cannot be an equilibrium when its rate of return is less than unity.



and held by the older generation in period  $t$ . The budget balance is expressed as

$$R_{t-1}B_{t-1} = B_t + P_t N_t \tau_t. \quad (9)$$

We assume a situation of *fiscal dominance*, or, to use the terminology of Woodford (1995) and Cochrane (2005), governments follow a *non-Ricardian* or *active fiscal policy* with *passive monetary policy*. This means that the price level,  $P_t$ , is adjusted to hold (9), taking fiscal policies as given.

Now, let us consider the general equilibrium of the economy. The stream of real interest rates is determined to clear the markets of government bonds. The condition for period  $t$  is

$$N_t A_t = B_t, \quad (10)$$

which is equivalent to the goods-market clearing condition,  $N_t c_t^y + \theta_t N_{t-1} c_t^o = N_t$ , whenever (9) is met, owing to Walrus' law. By plugging (8) and (10) into (9), we determine the equilibrium real interest realized in period  $t + 1$  such that

$$\frac{r_{t+1}}{n_{t+1}} = \frac{1 + \beta\theta_{t+1}}{\beta\theta_{t+1}(1 - \tau_t)} \frac{\beta\theta_{t+2} + \tau_{t+1}}{1 + \beta\theta_{t+2}}. \quad (11)$$

This shows that the equilibrium real interest rate depends on not only the current and future tax rates but also on the future birth rate and survival probabilities. More specifically, (11) demonstrates that increases in the current as well as future *expected* tax rates raise it. Because these increases in tax rates lead each one of the younger generation to consume less, a rise in the real interest rate can stimulate the consumption of each one of the older generation. An *expected* decline in the birth rate, on the other hand, decreases the real interest rate since the older generation's consumption has to decrease in proportion to the reduction in savings of the younger generation. Further, if we assume that  $\theta_{t+1} = \theta_{t+2}$ , from (11) an *expected* future increase in survival probability lowers the real interest rate for the following reason. In response to a higher survival probability, nominal annuity benefits fall, decreasing the consumption of each one of the older generation as well as that of the younger generation. However, the increase in the population of the older generation functions to increase aggregate consumption. To satisfy the goods-market clearing condition, the consumption of each one of the older generation should decrease further, which leads to a fall in the real interest rate.

The assumption of fiscal dominance translates (9) into the fundamental FTPL equation, a valuation equation for government bonds, which relates the price level to the discounted sum of

the current and future fiscal surpluses. By expanding (9) forward from period  $t$ , we have the valuation equation

$$\left(\frac{R_{t-1}B_{t-1}}{N_{t-1}}\right)\frac{1}{P_t} = n_t \left[ \tau_t + \sum_{k=t+1}^{\infty} \tau_k \left( \prod_{h=t+1}^k \frac{r_h}{n_h} \right)^{-1} \right], \quad (12)$$

as long as the economy tends to its steady state.<sup>8</sup> This equation, when combined with (11), shows that the price level in each period is pinned down by the streams of tax rates, survival probabilities, and birth rates, given the government's nominal debt outstanding at the beginning of that period.<sup>9</sup>

Valuation equation (12) provides a fiscal implication on the equilibrium price level that the impact of a rise in future tax rates on the current price level is generally *ambiguous*. This ambiguity stems from the fact that a change in future tax rates affects the allocation of consumption between the younger and older generations and thus changes the equilibrium real interest rates, as we saw in (11). This implication sharply contrasts with the one commonly reiterated in the FTPL literature, which neglected the repercussion between fiscal surpluses and the equilibrium real interest rates owing to the assumption of an infinitely lived representative household. In fact, if we employ it instead of overlapping generations in our model, consumption will be constant over time so that the equilibrium real interest rates too should be constant at  $1/\beta$  irrespective of the stream of tax rates. Then, (12) provides a standard prediction that a rise in future tax rates *unambiguously* deflates the current price level. It also follows that a future reduction in birth rate inflates the price level from a reduction in future tax revenues. Of course, we cannot discuss the fiscal implication of longevity in the standard FTPL models because of the assumption of infinitely lived households.

Our model provides more contrasting fiscal implications on the price level. By combining (8), (9), and (10), we can demonstrate that the equilibrium price level in period  $t$  is determined to

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<sup>8</sup>If the tax rate and survival probability are constant, we have  $r_{t+1}/n_{t+1} > 1$ .

<sup>9</sup>In many existing models, such as those in Sargent and Wallace (1981), Rohrs (2010), Song, Storesletten, and Zilibotti (2012), Ono (2013), the equation as (12) does not serve as valuation equation for government debt, but it serves as a budget constraint on the stream of tax rates. This is because they assumed that the government issues *real* bonds, each one promising to pay a fixed amount of numeraire goods for every contingency in the next period. In such a situation, the left-hand side of (12) is predetermined in real terms at the beginning of period  $t$  and is equal to  $r_{t-1}B_{t-1}/(N_{t-1}P_{t-1})$ ; thus, there is no room for fiscal policy to affect the price level.

satisfy

$$\left(\frac{R_{t-1}B_{t-1}}{N_{t-1}}\right)\frac{1}{P_t} = \frac{n_t(\tau_t + \beta\theta_{t+1})}{1 + \beta\theta_{t+1}}, \quad (13)$$

owing to the specification of logarithmic utility functions. Since the term in parenthesis on the left-hand side is given at the beginning of period  $t$ , from (13) we can say that the current price level depends only on the current tax rate, current birth rate, and future survival probability. More specifically, a higher current tax rate and a higher *expected* future survival probability deflate the price level, but a lower *unexpected* current birth rate inflates it.<sup>10</sup> Other tax and demographic parameters, however, do not affect the current price level because of the changes in the real interest rates they induce in equilibrium.

Finally, we consider the indirect utilities of the young and old households in period  $t$  achieved in equilibrium. First, from (4), (6), and (7), the young household's indirect utility can be shown as

$$v_t^y = (1 + \beta\theta_{t+1})\log(1 - \tau_t) + \beta\theta_{t+1}\log r_{t+1} + \beta\theta_{t+1}\log \frac{\beta}{1 + \beta\theta_{t+1}}, \quad (14)$$

which depends on the current and future tax policies because the equilibrium real interest rate follows (11). On the other hand, from (2) and (10), the old household's indirect utility is

$$v_t^o = \log \frac{R_{t-1}B_{t-1}}{\theta_t N_{t-1} P_t}, \quad (15)$$

which depends only on the current price level and the government debt outstanding at the beginning of period  $t$  since their savings consist of nominal assets.

What is important for our analysis developed later is that the current price level will adjust to equate the real value of the assets with the discounted sum of the current and future fiscal surpluses, as we can confirm from (12). Accordingly, (15) is reduced in equilibrium to

$$v_t^o = \log \frac{n_t}{\theta_t} \left[ \tau_t + \sum_{k=t+1}^{\infty} \tau_k \left( \prod_{h=t+1}^k \frac{r_h}{n_h} \right)^{-1} \right], \quad (16)$$

which is independent of the government debt outstanding accumulated from the past but depends only on the current and future tax policies, as does the young household's indirect utility.

Recall now that (9), or equivalently (12), does not constrain the government's choice of fiscal policy but serves as valuation equation for its outstanding nominal debt that effectively determines

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<sup>10</sup>The deflationary effect of a higher survival probability partly depends on our assumption that fiscal surpluses in each period do not change in response to the population of the older generation. We will generalize this restrictive assumption in the more general model developed in the next section.

the current price level. By combining this with the fact that the indirect utilities of those living in period  $t$  are not affected by the government debt accumulated from the past, we have an interesting implication on the long-running controversy on the burden of national debt. Even in the absence of altruistic bequests, the burden of government debt cannot be shifted to future generations in the world of fiscal dominance, but must be paid by the current generation. If the government lowers the tax rate and accordingly increases bond issues in period  $t$ , the current price level rises and the real interest rate decreases as long as the future taxes are kept constant, as we can see from (11) and (13); there is no impact on the utilities of households born in period  $t + 1$  or later. Note that this implication does not depend on our specification of logarithmic utilities at all and contrasts sharply with the findings obtained from the same OLG framework used by Bowen *et al.* (1960) and Barro (1974) among others.<sup>11</sup>

Our argument is based on the issuance of nominal government bonds and fiscal dominance. In the case of real government bonds, the accumulation of deficits forces future governments to increase their taxes and thus reduce the welfare of future generations. However, in the case of nominal governments bonds, adjustments in the current price level make such future fiscal adjustments unnecessary. To our knowledge, this argument is yet to be discussed in the literature.

## 2.2 Politico-Economic Equilibrium

Thus far, we treated fiscal variables as exogenously fixed. However, it is not realistic to assume that they are independent of changes in the demographic structure. From this viewpoint, we consider the politico-economic interactions in which the choices of policy variables respond to a change in current and future demographic parameters.

To endogenize government policies and characterize the politico-economic equilibrium, we assume that tax rates and bond issues are chosen by a succession of short-lived governments, following the formulation of Song, Storesletten, and Zilibotti (2012), among others.

Each government remains in power just for one period and chooses policies to maximize the weighted average of the young and old households living in that period, taking into account their general equilibrium repercussions with the real interest rates and the price level. The government

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<sup>11</sup>As is well known, Bowen *et al.* (1960) argue that the young can avoid these burdens by selling their bonds to the next generation. Barro (1974) argues that the young generation would neutralize the burden of increased debt by increasing bequests by the same amount to keep the next generation's welfare unchanged.

in period  $t$ , which we call government  $t$  hereafter, takes over the nominal government debt  $B_{t-1}$  from government  $t - 1$  and chooses a tax rate  $\tau_t$  and the outstanding government debt  $B_t$ . Following the spirit of the probabilistic voting model, its objective function is formulated as a weighted average of constituents in period  $t$ ,

$$W_t = \gamma_t v_t^o + v_t^y, \quad (17)$$

where  $\gamma_t$  represents the political bias toward the well-being of the old generation.<sup>12</sup> In choosing  $\tau_t$  and  $B_t$ , government  $t$  has to take into account the valuation equation (9), or equivalently (12), and the bond-market clearing condition (10), which determines  $P_t$  and  $r_{t+1}$  through (11) and (13), affecting the young and old households' utility. Further, it has to anticipate the policy response by government  $t + 1$  because it affects the real interest rate,  $r_{t+1}$ , as we saw in (11). A novelty in our model is that each government takes care of income redistribution by bringing about changes in the price level when choosing fiscal policies.

The policy decisions of successive governments are usually considered intertemporally linked by the inheritance of outstanding debts. This is typical to the literature of strategic debt creation pioneered by Persson and Svensson (1989) and Tabellini and Alesina (1990), in which each short-lived government attempts to manipulate its successor's unfavorable policy-making by strategically accumulating public debt. Behind the presence of intertemporal linkage of government behavior is the assumption that the government bonds are real and each government has to honor their repayment.<sup>13</sup>

However, in our model, since the governments issue nominal bonds and are in the situation of fiscal dominance, the price level adjusts to eliminate the linkage and each government can pursue its own optimal policy irrespective of how much nominal debt it inherits from its predecessor. This point is in sharp contrast to the previous studies assuming real government bonds, such as the studies cited above. Since condition (9) does not constrain government  $t$ 's policy decision any more but serves as a valuation equation for government bonds to determine the current price level, invoking (13), (14), and (15) makes government  $t$ 's policy decision choose  $\tau_t$  to maximize

$$\gamma_t \log(\tau_t + \beta\theta_{t+1}) + (1 + \beta\theta_{t+1}) \log(1 - \tau_t) + \beta\theta_{t+1} \log r_{t+1} \quad (18)$$

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<sup>12</sup>For the probabilistic voting model, see Persson and Tabellini (2001, Ch.3) among others.

<sup>13</sup>Aghion and Bolton (1990) point out that the strategic bias of debt accumulation is eliminated if governments default on their debt.

subject to (11). From this, it follows that government  $t - 1$ 's decision has no influence on government  $t$ 's policy choice. Thus, it is natural to assume that government  $t$ 's policy-making does not affect government  $t + 1$ 's decisions if we confine our attention to a Markov perfect equilibrium of this dynamic game. Supposing that  $\partial\tau_{t+1}/\partial\tau_t = 0$ , it is straightforward to solve the equilibrium tax rate; now, government  $t$  chooses a tax rate satisfying

$$\tau_t = \frac{\gamma_t - \beta\theta_{t+1}}{1 + \gamma_t} \quad (19)$$

in the unique Markov perfect equilibrium, where we assume that  $\gamma_t > \beta\theta_{t+1}$  for internal solution. Further, plugging this into (11) and (13) yields respectively the real interest rate and current price level in the equilibrium as

$$r_{t+1} = \frac{n_{t+1}(1 + \gamma_t)}{\beta\theta_{t+1}} \frac{\gamma_{t+1}}{1 + \gamma_{t+1}} \quad (20)$$

and

$$P_t = \left( \frac{R_{t-1}B_{t-1}}{N_{t-1}} \right) \frac{1 + \gamma_t}{n_t\gamma_t}. \quad (21)$$

From (19), (20), and (21), comparative statics with respect to  $\gamma_t$  demonstrates that given the population structure, a greater political bias toward the older generation increases the tax rate and real interest rate but decreases the price level realized in the Markov perfect equilibrium. From these results, greater political influence of the older generation through a relatively high voter turnout rate, as shown in Figure 5, is one explanation for the recent concurrent deflation and aging in Japan. That is, the government, led by the older generation's strengthened political influence, attempts to suppress inflation and increase the real value of the government bonds the older generation holds.

However, the political bias may interact with the population structure. Aging may increase the bias toward the older generation. To analyze this, let us suppose that the political bias is proportional to the population ratio between the older and younger generations; that is,

$$\gamma_t = \frac{\omega\theta_t}{n_t} \quad (22)$$

where  $\omega > 0$ . Further, to focus on the impacts of simultaneous change in the current and future demographic structure, which seems to be the case according to Figures 3 and 4, we assume that  $\theta_t = \theta_{t+1} = \theta$  and  $n_t = n_{t+1} = n$ . Then, the equilibrium tax rate, real interest rate, and price

level are reduced respectively to

$$\tau_t = 1 - \frac{1 + \beta\theta}{1 + \omega\theta/n} \quad (23)$$

$$r_{t+1} = \frac{\omega}{\beta} \quad (24)$$

and

$$P_t = \left( \frac{R_{t-1}B_{t-1}}{N_{t-1}} \right) \left( \frac{1}{\omega\theta} + \frac{1}{n} \right). \quad (25)$$

From these expressions, we obtain the comparative statics results summarized in Table 1: The equilibrium tax rate increases as the survival probability becomes higher and also as the birth rate becomes lower; the equilibrium real interest rate is independent of changes in the demographic structure; and the equilibrium price level decreases as the survival probability becomes higher but increases as the birth rate becomes lower.

The price level response to demographic aging turns out to depend on its causes. If the birth rate declines, a contraction in the tax base will reduce the fiscal surplus and the government will be induced to generate inflation in order to maintain solvency at the cost of the older generation's well-being and impose higher taxes on the younger generation to mitigate the extent of inflation, based on the strengthened political influence of the older generation. In contrast, in case of an unexpected increase in life expectancy and the old survive longer than expected, the government will be inclined to favor them because of their shortage in savings for their retirement period and their strengthened political power relative to the younger generation. In such a situation, the government will opt to suppress inflation and support the older generation's well-being with deflation.

Our results are consistent with Figure 4, which illustrates repeated unexpected increases in longevity in Japan. As mentioned in the introduction, Japan experienced mild deflation and population aging during its so-called lost decades. To our understanding, this puzzling phenomenon occurred from a combination of the political and economic effects of population aging stemming from the unexpected increases in longevity and growing political influence of the older generation. In particular, the repeated upward revisions of forecasts on population aging are considered to have yielded continuous downward pressures on the price level.

### 3 Quantitative Investigations Using a Generalized Model

In this section, we quantitatively examine whether and how much the population aging in Japan accounts for the country's recent deflation through the mechanism we described in the previous section by constructing a more general model. We extend the model in three major directions. First, we consider a production economy with variable labor supply. Second, we generalize the utility functions and employ the constant relative risk aversion (CRRA) class. Third, we introduce two types of government expenditure, government consumption and transfer to the older generation. Within this framework, we derive a politico-economic equilibrium and conduct numerical simulation.

#### 3.1 The Model

A young household in period  $t$  supplies labor  $\ell_t$  and receives before-tax labor income  $\ell_t$ , where we assume a linear production technology and thus the real wage is normalized to unity. An old household does not supply labor and its consumption relies entirely on annuity income and the per-capita government transfer  $g^T > 0$ .

The household chooses  $c_t^y$ ,  $c_{t+1}^o$ , and  $\ell_t$  to maximize its utility,  $u^y(c_t^y, \ell_t) + \beta\theta_{t+1}u^o(c_{t+1}^o)$ , subject to the budget constraints

$$c_t^y + a_t = (1 - \tau_t)w_t\ell_t, \quad (26)$$

and

$$c_{t+1}^o = \frac{r_{t+1}a_t}{\theta_{t+1}} + g^T, \quad (27)$$

where  $a_t \equiv A_t/P_t$ . The consumption, labor supply, and demand functions for annuities are determined to satisfy the first-order conditions.

A short-lived government remains in power for just one period. The law of motion for government debt is

$$R_{t-1}B_{t-1} + P_tN_tg_t = B_t + P_tN_t\tau_t\ell_t. \quad (28)$$

The government spending per young household,  $g_t$ , consists of its own consumption,  $g^C$ , and its transfer to the older generation,  $g^T$ , and is thus defined by  $N_tg_t \equiv (N_t + \theta_tN_{t-1})g^C + \theta_tN_{t-1}g^T$ . The tax rate,  $\tau_t$ , is now proportional to the labor income. If we rewrite government debt (28) in real terms, we have

$$r_t b_{t-1} = n_t b_t + n_t \tau_t \ell_t - (n_t + \theta_t) g^C - \theta_t g^T, \quad (29)$$



where  $b_t \equiv B_t/N_t$ . By expanding (29) forward to infinity, we obtain a valuation function similar to (12),

$$\left(\frac{R_{t-1}B_{t-1}}{N_{t-1}}\right) \frac{1}{P_t} = n_t \left[ s_t + \sum_{k=t+1}^{\infty} s_k \left( \prod_{h=t+1}^k \frac{r_h}{n_h} \right)^{-1} \right], \quad (30)$$

where  $s_h \equiv \tau_h \ell_h - (n_h + \theta_h) g^C - \theta_h g^T$  is a fiscal surplus in period  $h$ . Note that  $P_t$  is the only variable determined by the right-hand side of this expression at the beginning of period  $t$ .

The general equilibrium system is as follows. The market clearing conditions for goods and bonds are respectively

$$n_t c_t^y + \theta_t c_t^o + (n_t + \theta_t) g^C = n_t \ell_t. \quad (31)$$

and

$$a_t = b_t, \quad (32)$$

which are not independent owing to Walrus' law whenever (29) holds.

Now, consider the politico-economic equilibrium of the model. The objective function of each short-lived government is given by

$$W_t = \gamma_t u^o \left( \frac{n_t b_t + n_t \tau_t \ell_t - (n_t + \theta_t) g^C}{\theta_t} \right) + u^y((1 - \tau_t) \ell_t - b_t, \ell_t) \\ + \beta \theta u^o \left( \frac{n_{t+1} b_{t+1} + n_{t+1} \tau_{t+1} \ell_{t+1} - (n_{t+1} + \theta_{t+1}) g^C}{\theta_{t+1}} \right), \quad (33)$$

for the derivation of which we use (26), (27), (29), and (32) and eliminate  $r_t$  from the expression of the older generation's consumption. As in the previous analysis,  $b_{t-1}$  does not appear in government  $t$ 's objective function. Thus, as far as a Markov perfect equilibrium is concerned, government  $t + 1$ 's choice of  $b_{t+1}$  and  $\tau_{t+1}$  is independent of government  $t$ 's choice of  $b_t$ ; their strategic interactions disappear owing to the adjustments in the price level. Then, formally it is only the bond market clearing condition (32) that constrains government  $t$ 's policy choice. Needless to say, behind the market clearing condition, there are the first-order household conditions for utility maximization, and the demand for government bonds  $a_t$  depends on the tax rate  $\tau_t$  and real interest rate  $r_{t+1}$ . In models with real government bonds, if government  $t$ 's policy choice induces a change in the real interest rate, it would affect government  $t + 1$ 's policy choice through an impact on its budget constraint. This intertemporal linkage between the policy choices makes it rather difficult to solve even the steady state of the model. In our model, however, thanks to the equilibrium price adjustment, we need not bother about such a linkage. Specifically, the value of  $r_{t+1} b_t$  is kept constant irrespective of government  $t$ 's choices, and so  $r_{t+1}$  decreases proportionally

in response to an increase in  $b_t$ . This property facilitates a numerical solution for the steady state of the model.<sup>14</sup>

## 3.2 Simulation

In this subsection, we conduct numerical simulations. We first calibrate the model to fit Japanese data and then quantitatively investigate the effects of demographic and political changes on the Japanese economy. We also conduct comparative statics and compute the transition paths.

### 3.2.1 Calibration

We specify the utility functions as

$$u^y(c_t, \ell_t) = \frac{(c_t)^{1-\sigma}}{1-\sigma} - \chi \frac{(\ell_t)^{1+1/v}}{1+1/v}, \quad (34)$$

and

$$u^o(c_t) = \frac{(c_t)^{1-\sigma}}{1-\sigma},$$

where  $\sigma$ ,  $\chi$ , and  $v$  represent the inverse of the elasticity of consumption, a utility weight, and the Frisch elasticity of labor supply, respectively. We set  $\sigma = 1$  and  $v = 0.5$ ,<sup>15</sup> and calibrate  $\chi$  to have labor supply close to unity in the equilibrium. We assume that a period consists of 40 years and that  $\beta$  is equal to 0.99<sup>40</sup> (1% annually). Government spending is set to satisfy  $g^C = g^T = 0.01$ .

The variables associated with demographic structure and political influence are calibrated as follows. We simulate an initial steady state as a benchmark by using data for the year 1976, which is about one period prior to the current period in our model. We derive the demographic parameters  $\theta$  and  $n$  from the forecasts of the National Institute of Population and Social Security Research (IPSS) released in 1976. We assume that the economy converges consistently with long-term forecasts and hence use the forecasts for the year 2050. We set the survival probability  $\theta$  equal to 0.482, following the survival rate of people until the age of 77.5 in 2050. The birth rate  $n$  is set to 1.058, which is the growth rate of the population aged from 20 to 24 in 2050. To measure the older generation's political influence  $\omega$ , we use the data on voter turnout rates shown

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<sup>14</sup>A more detailed mathematical explanation of this point can be obtained from the authors upon request.

<sup>15</sup>See Imrohorglu and Kitao (2009) and Trabandt and Uhlig (2011) for studies pointing out the importance of incorporating variable labor supply into the model.

in Figure 5. Specifically, we choose election No. 34 held in 1976 and set  $\omega$  equal to 0.969, which is the ratio of the voter turnout rate of the 20–59 age group to that of the 60–79 age group.

As parameters for a new steady state, we use the long-term forecasts for 2060 published in 2012. The survival rate is now 0.781, which is significantly higher than the forecast published in 1976. The birth rate, on the other hand, declines to 0.629. As for the measure of the older generation’s political influence, we use the result of election No. 46 held in 2012. This allows us to set  $\omega$  equal to 1.230, which is also higher compared to that in 1976.

### 3.2.2 Comparative Statics

We first calculate and compare the tax as well as real interest rates (in annual terms) realized in the steady-state politico-economic equilibria by changing one of the parameter values of  $\theta$ ,  $n$ ,  $\omega$ ,  $v$ ,  $g^C$ , and  $g^T$ , keeping the other parameters fixed at their 1976 levels. Figure 6 gives the results. The blue lines with circles represent the tax rates (the vertical axis on the left-hand side) and the green lines represent the real interest rates (the vertical axis on the right-hand side).

The figure shows that the results in this generalized model are not qualitatively different from those of our analysis developed in the previous section. The tax rate increases in response to an increase in life expectancy  $\theta$ , a decline in birth rate  $n$ , and an increase in the older generation’s political influence  $\omega$ . It also increases with government expenditure irrespective of  $g^C$  and  $g^T$ . The real interest rate is, on the other hand, almost unaffected by changes in demographic parameters  $\theta$  and  $n$ . Only an increase in the older generation’s political power  $\omega$  can increase the real interest rate.

### 3.2.3 Transition Path

We next calculate the changes in tax rate and real interest rate when the economy experiences unexpected simultaneous changes in the demographic parameters  $\theta$  and  $n$  with an associated change in  $\omega$ . For this, we assume that the economy is initially in the steady state with the 1976 parameters and that unexpected demographic changes occur to the 2012 levels at the beginning of the next period and persist forever thereafter.

Figure 7 illustrates the transition paths of the tax rate  $\tau$ , the annualized real interest rate  $r$ , the real value of outstanding government debt per young household  $b_t$ , and the annualized

inflation rate  $\pi_t$ .<sup>16</sup> The figure shows that the inflation rate falls by about 0.6 percentage points annually. Considering the finding in the previous section that an increase in  $\theta$  and  $\omega$  leads to deflation while an decrease in  $n$  leads to inflation, this result suggests that the effects of the increases in  $\theta$  and  $\omega$  dominate those of the decrease in  $n$ . Given that the Japanese economy did not experience a deflationary spiral, the deflation in this exercise is moderate but sensible. figure also shows that the tax rate rises from 10% to 40%, which is consistent with the result in the previous section. Finally, the real interest rate rises by about 0.6 percentage points annually. Note that in the long run, that is, from  $t = 2$  onward, the real interest rate increases even more. Our findings in the previous section and in Figure 6 suggest that such a rise is brought about by an increase in  $\omega$ .

## 4 Concluding Remarks

In this study, we considered the effects of population aging on fiscal balances and general prices by embedding the FTPL within a simple OLG model in which fiscal policy decisions are endogenized in the spirit of a probabilistic voting model. A key feature of our model is that the government uses the fiscal impact on general prices as a means to redistribute income between the current younger and older generations, and because of this, short-lived governments cannot pass on fiscal burdens to the future generations by issuing government bonds.

Our main findings are as follows. First, population aging stemming from an increase in longevity leads to deflation by increasing the political influence of the older generation, as does an increase in the election turnout rate among older voters. This happens because, to appease older voters, the government increases its income tax rates (which harms the young) to an avoid increase in prices (which would harm the older generation). Second, population aging stemming from a decline in birth rate leads to inflation by shrinking the tax base and raising fiscal expenditure. These findings revealed that the effects of population aging on general prices depend on the cause of the aging. Numerical simulation showed that aging in Japan over the past 40 years brought about an annual deflation of about 0.6 percentage points.

Our study represents a first step toward embedding the FTPL in a politico-economic framework. This approach presents two main challenges. The first is to address Japan's accumulation

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<sup>16</sup>The annualized inflation rate is calculated from the path of real interest rates by assuming that the nominal interest rates remain constant at the level of the real interest rate at  $t = 0$ .

of government bonds over the past 40 years. Our finding that aging improves fiscal balances by increasing the tax rate imposed on the young seems questionable in reality. Second, it would be interesting to introduce foreign investors buying government bonds into the model. It is well known that around 90% of Japanese government bonds are held by domestic investors. This fact justifies the use of a closed economy model in which only domestic investors are present. However, if the government cares less about foreign investors than domestic ones, it might devalue its bonds if the proportion of bonds held by foreign investors were to increase.

## References

- [1] Aghion, Philippe, and Patrick Bolton, Government Domestic Debt and the Risk of Default: A Political-Economic Model of the Strategic Role of Debt, in *Public Debt Management: Theory and History*, edited by Rudiger Dornbush and Mario Draghi, 1990, pp.315–345, Cambridge: Cambridge University Press.
- [2] Bassetto, Marco, A Game-Theoretic View of the Fiscal Theory of the Price Level, *Econometrica*, 70, 2002, pp. 2167–2195.
- [3] Barro, Robert, Are Government Bonds Net Wealth?, *Journal of Political Economy*, 82, 1974, pp.1095–1117.
- [4] Bianchi, Francesco and Cosmin Ilut, Monetary/Fiscal Policy Mix and Agents’ Beliefs, NBER Working Paper No. 20194, 2014.
- [5] Bowen, William G., Richard G. Davis, and David H. Kopf, The Public Debt: A Burden on Future Generations?, *American Economic Review*, 50, 1960, pp. 701–706.
- [6] Braun, Anton R. and Tomoyuki Nakajima, Why Prices Don’t Respond Sooner to a Prospective Sovereign Debt Crisis?, Bank of Japan IMES Discussion Paper Series, 2012-E-2, 2012.
- [7] Buiter, Willem H., The Fiscal Theory of the Price Level: A Critique, *Economic Journal*, 112, 2002, pp. 459–480.
- [8] Bullard, James, Carlos Garriga, and Christopher Waller, Demographies, Redistribution, and Optimal Inflation, *Federal Reserve Bank of St. Louis Review*, 94(6), 2012, pp. 419 –440.
- [9] Canzoneri, M.B., Cumby, R.E., and B.T. Diba, Is the Price Level Determined by the Needs of Fiscal Solvency?, *American Economic Review*, 91, 2001, pp. 1221–1238.
- [10] Christiano, L.J. and T. Fitzgerald, Understanding the Fiscal Theory of the Price Level, Federal Reserve Bank of Cleveland Economic Review, 36, 2000, pp. 1–37.
- [11] Cochrane, John H., Money as Stock, *Journal of Monetary Economics* 52(3), 2005, pp. 501–528.
- [12] Cushing, Matthew J., The Indeterminacy of Prices under Interest Rate Pegging: The Non-Ricardian Case, *Journal of Monetary Economics*, 44, 1999, pp. 131–148.

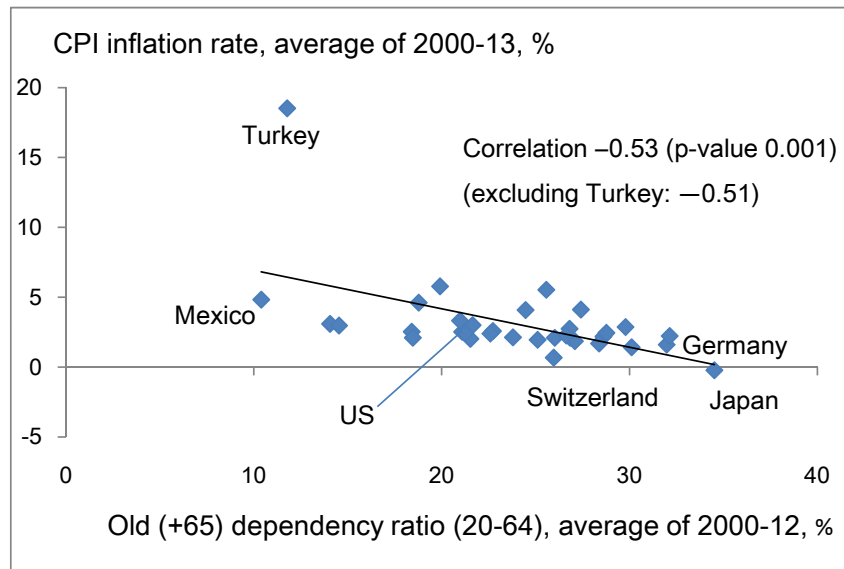
- [13] Davig, Troy, Eric M. Leeper, and Todd B. Walker, Inflation and the Fiscal Limit, *European Economic Review*, 55, 2011, pp. 31–47.
- [14] Fujiwara, Ippei and Yuki Teranishi A Dynamic New Keynesian Life-cycle Model: Societal Aging, Demographics, and Monetary Policy, *Journal of Economic Dynamics and Control*, 32, 2008, pp. 2398–2427.
- [15] Hall, George, J. and Thomas J. Sargent, Interest Rate Risk and Other Determinants of Post-WWII US Government Debt/GDP Dynamics, *American Economic Journal: Macroeconomics*, 3, 2011, pp. 192–214.
- [16] Heer, Burkhard, Alfred Maussner, and Paul D. McNelis, The Money-Age Distribution: Empirical Facts and the Limits of Three Monetary Models, *Journal of Macroeconomics*, 2011, pp. 390–405.
- [17] Imrohorglu, Selo and Sagiri Kitao, Labor Supply Elasticity and Social Security Reform, *Journal of Public Economics*, 93, 2009, pp. 867–878.
- [18] Leeper, Eric M., Equilibria Under “Active” and “Passive” Monetary and Fiscal Policies, *Journal of Monetary Economics*, 27, 1991, pp. 129–147.
- [19] Leeper, Eric M. and Todd B. Walker, Fiscal Limits in Advanced Economies, *Economic Papers*, 30, 2011, pp. 33–47.
- [20] Nishimura, Kiyohiko G., Ageing, Finance and Regulations, Keynote Address at the Joint Forum Meeting held in Tokyo, Nov 14, 2012.
- [21] Persson, Torsten and Lars E. O. Svensson, Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences, *Quarterly Journal of Economics* 104, 1989, pp. 325–345.
- [22] Persson, Torsten and Tabellini, Guido Enrico *Political Economics: Explaining Economic Policy*, 2000, MIT Press, Cambridge, MA.
- [23] Reinhart, Carmen M. and Kenneth Rogoff, *This Time Is Different: Eight Centuries of Financial Folly*, 2011, Princeton University Press, Princeton, NJ.

- [24] Sargent, Thomas J. and Neil Wallace, Some Unpleasant Monetarist Arithmetic, *Federal Reserve Bank of Minneapolis Quarterly Review*, 5, 1981, pp. 1–17.
- [25] Shirakawa, Masaaki, Demographic Changes and Macroeconomic Performance: Japanese Experiences, Opening Remark at 2012 BOJ-IMES Conference hosted by the Institute for Monetary and Economic Studies, The Bank of Japan, 2012.
- [26] Sims, Christopher A., A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy, *Economic Theory*, 4, 1994, pp. 381–399.
- [27] Sims, Christopher A., Paper Money, *American Economic Review*, 103(2), 2013, pp. 563–584.
- [28] Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti, Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt, *Econometrica*, 80, 2012, pp. 2785–2803.
- [29] Tabellini, Guido Enrico and Robert Alesina, Voting on the Budget Deficits, *American Economic Review*, 80, 1990, pp. 37–49.
- [30] Trabandt, M. and H. Uhlig, The Laffer Curve Revisited, *Journal of Monetary Economics*, 58, 2011, pp. 305–327.
- [31] Woodford, Michael, Price-Level Determinacy Without Control of a Monetary Aggregate, *Carnegie-Rochester Conference Series on Public Policy*, 43, 1995, pp. 1–46.



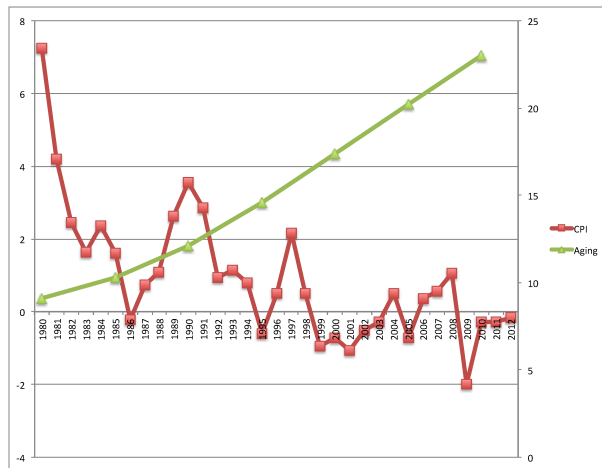
Table 1: Effects of Aging

	Tax ( $\tau_t$ )	Bond ( $b_t$ )	Price ( $P_t$ )	Real interest rate ( $r_{t+1}$ )
Increase in the older generation's political power ( $\omega_t \uparrow$ )	+	-	-	+
Decrease in birth rate ( $n_t \downarrow$ )	+	-	+	0
Increase in life expectancy ( $\theta_t \uparrow$ )	+	+	-	0



Source: OECD Databases.

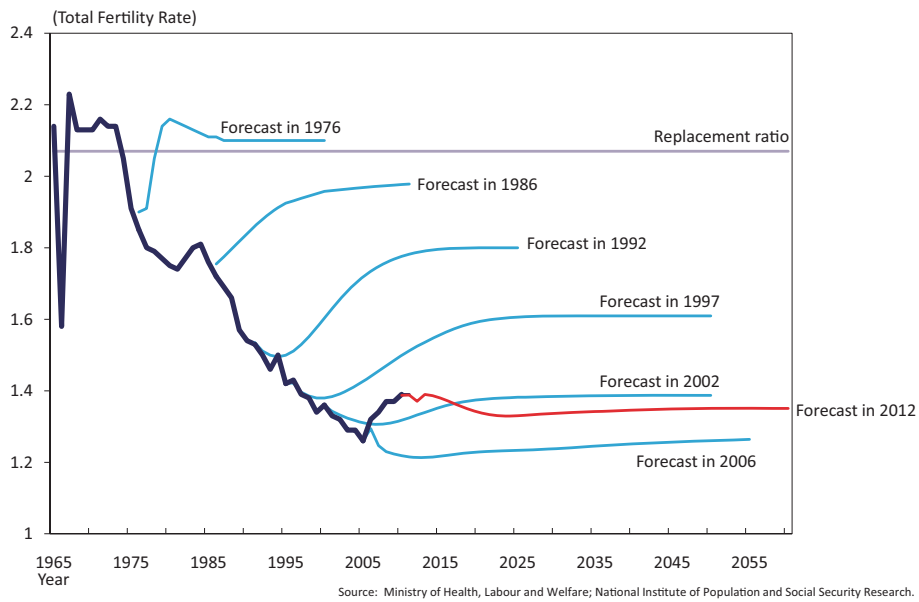
Figure 1: Aging and Deflation in OECD Countries



Note: The red line with squares represents the annual rate of change in consumer price index (measured in % on the left vertical axis), and the green line with triangles records the change in share of the population aged 65 or above (measured in % on the right vertical axis).

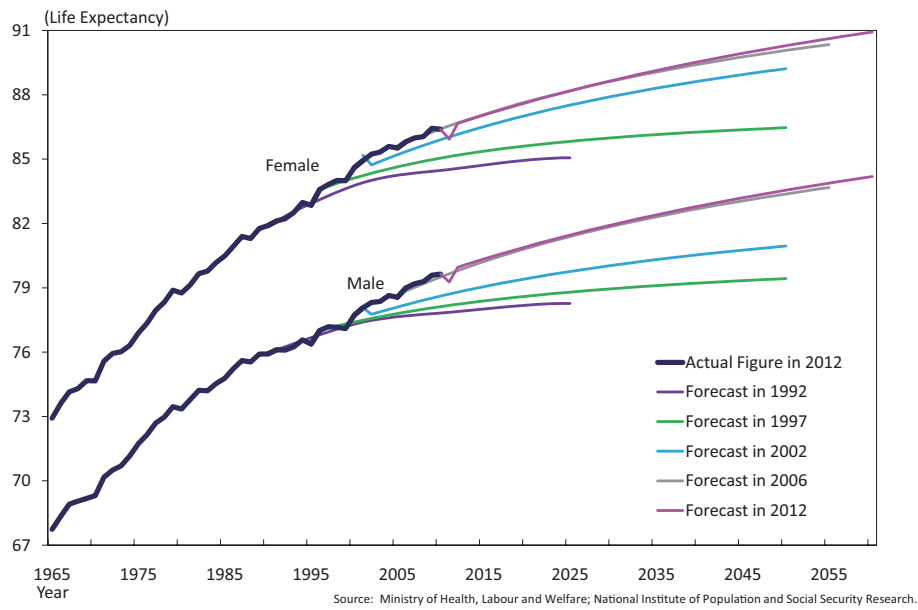
Source: IMF World Economic Outlook Databases, 2012.

Figure 2: Recent Aging and Deflation in Japan



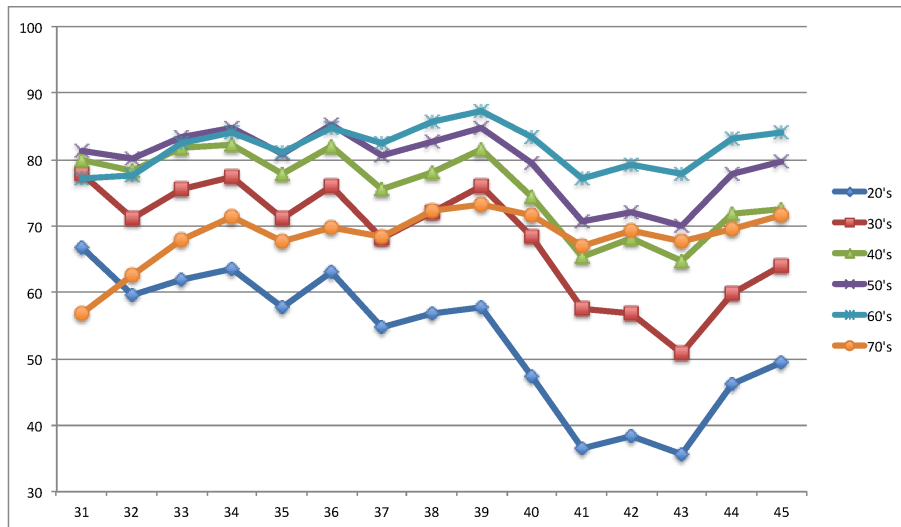
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Figure 3: Revisions to Japan's Total Fertility Rate Forecasts



Source: [http://www.boj.or.jp/en/announcements/press/koen\\_2012/ko121114a.htm/](http://www.boj.or.jp/en/announcements/press/koen_2012/ko121114a.htm/)

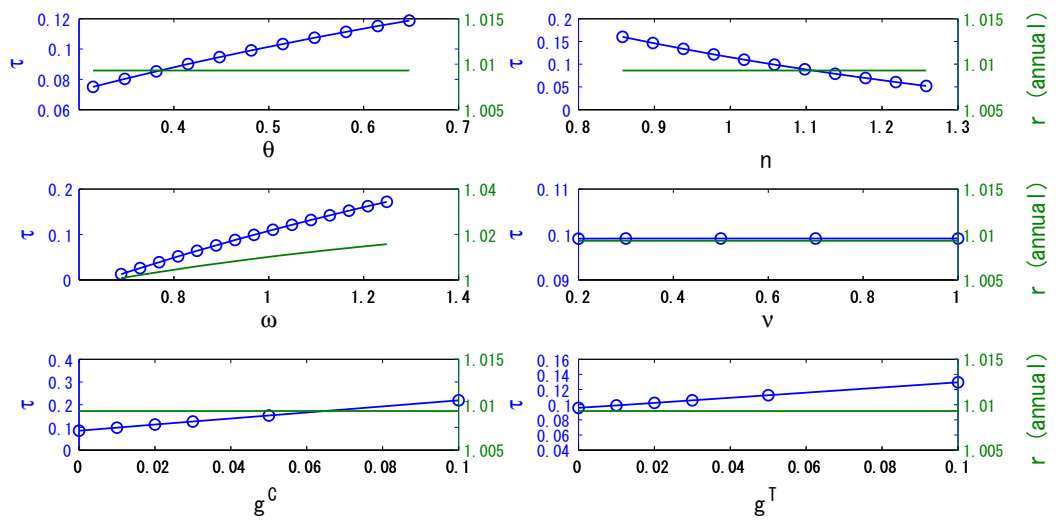
Figure 4: Revisions to Japan's Life Expectancy Forecasts



Note: The voter turnout rate is the fraction of eligible voters who cast a ballot in an election. The figure illustrates the voter turnout by age in the Japanese lower house election Nos. 31–45 (from 1967 to 2009). Specifically, the lines with diamonds, squares, triangles, Xs, asterisks, and circles, respectively, represent the turnout rates of people in their 20s, 30s, 40s, 50s, 60s, and 70s.

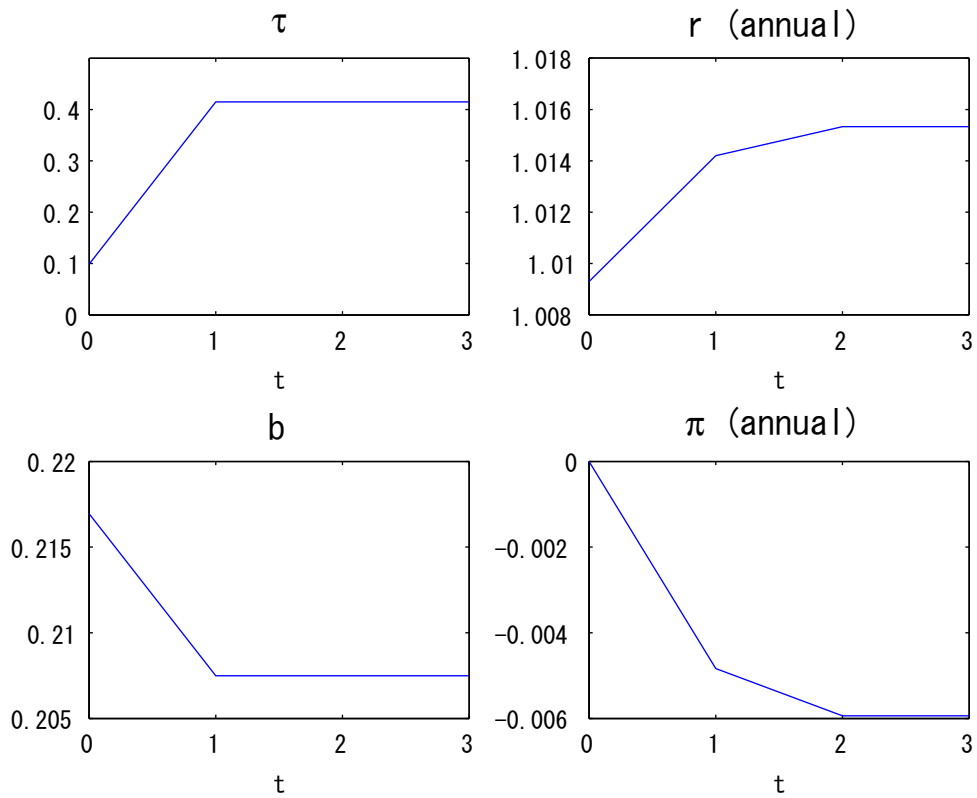
Source: The Association For Promoting Fair Elections.

Figure 5: Japan's Voter Turnout Rates by Age



Note: The blue lines with circles represent the tax rate  $\tau$  (left axis) and the green lines the annualized real interest rate  $r$  (right axis).

Figure 6: Comparative Statics for the Tax Rate and Real Interest Rate



Note:  $t = 0$  corresponds to 1974. At  $t = 1$ , unexpected demographic and political changes occur, which corresponds to the year 2012. From  $t = 2$  onward, demographic and political parameters are unchanged. The inflation rate at  $t = 0$  is set 0.

Figure 7: Transition Path under Japan's Aging Population