Rank-based mobility measurement*

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Extended Abstract. The measurement of income mobility is an increasingly important area within the analysis of income distributions. The fundamental problem is to design measures that reflect the extent to which members of a society can move within the income distribution from one period to the next. A crucial aspect that distinguishes mobility from most other criteria that are used to assess the performance of a society (such as inequality or poverty) is that mobility is difficult—if not impossible—to define without any reference to intertemporal movements. Of course, intertemporal approaches to the measurement of inequality, poverty and other social phenomena have been explored but they can also be defined without any difficulties in a single-period setting; in contrast, there is no mobility without movement. Thus, the objective is to find a way of assessing the extent to which individuals move within an income distribution between one time period and the one that follows. As a consequence, the arguments of a mobility measure are pairs of income distributions–one distribution of each of the two periods under consideration.

Fields and Ok (1996) characterize a measure that consists of the sum (over all individuals in a society) of the absolute values of the differences in the individual incomes in the two periods under consideration. Mitra and Ok (1998) provide a characterization of a more general class of the measure due to Fields and Ok (1996). Furthermore, dominance criteria that bear a family resemblance to the Lorenz criterion known from income inequality measurement are discussed in their contribution. Other important contributions include (but are not restricted to) those by Shorrocks (1978), Chakravarty, Dutta and Weymark (1985), Dardanoni (1993), Fields and Ok (1996) and Mitra and Ok (1998), to name but a few. Excellent surveys and guides to the literature are provided by Fields and Ok (1999) and Jäntti and Jenkins (forthcoming).

In this paper, we propose to measure mobility exclusively in terms of the movements in ranks that the members of a society experience in the transition from the previous period to the current period. This choice of approach immediately suggests to make use of the literature on measuring the distance between orderings—in this specific case, the rankings of individuals in the income distributions before and after the intertemporal move. The most prominent measure of distance between orderings is the Kemeny distance; see, for instance, Kendall (1938), Kemeny (1959) and Kemeny and Snell (1962). This distance function is characterized in Kemeny and Snell (1962) but, as pointed out in a remarkable contribution by Can and Storcken (2013), one of the axioms employed in the original characterization is redundant. As a consequence, Can and Storcken (2013) succeed in obtaining a considerable strengthening of the result due to Kemeny and Snell (1962). The axiom in question is a reducibility condition—the only property used by Kemeny and Snell (1962) that (at least implicitly) links distances between orderings involving different numbers of objects to be ranked. Can and Storcken's (2013) observation that the Kemeny distance can be characterized without this axiom represents, in our opinion, a very fundamental and important contribution to this literature.

Returning to the issue of income mobility, the Kemeny distance (or, more precisely, a variant that explicitly takes into consideration how pairs of distributions involving different population sizes are to be ranked) emerges as a natural candidate for a rank-based measure. We make use of the results established by Can and Storcken (2013) to obtain a characterization of a mobility measure that is a multiple of the Kemeny distance, applied to pairs of income distributions. The multiplicative factor is determined by a population-replication property familiar from the theory of economic index numbers. To the best of our knowledge, this property has not appeared in the literature on measuring the distance between orderings. This seems likely to be the case because replication invariance is a natural property in the context of measurement issues involving the ranking of individuals in income distributions but is not of immediate appeal in the more abstract setting of measuring the distance between orderings.

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