Sraffa's System and Alternative Standards

Takashi YAGI

No. 9907

20 December 1999

Sraffa's System and Alternative Standards

Takashi YAGI

Associate Professor

Faculty of Social and Information Studies, Gunma University (Japan)

Visiting Scholar

Wolfson College and The Faculty of Economics and Politics, University of Cambridge

ADDRESS in U.K.: Takashi YAGI, Wolfson College, Cambridge, CB3 9BB (until March 2000)

ADDRESS in JAPAN: Takashi YAGI, 298 Kamitajima-cho, Ohta-shi, Gunma, 373-0044

TEL&FAX: +81-276-31-5250 **E.mail**: yagi@sta.att.ne.jp yagi@si.gunma-u.ac.jp

Introduction

Although much attention has been paid to the properties of the solution of it, Sraffa's System still appears ambiguous and also there exist various interpretations of it. In order to interpret *Part I* of *Production of Commodities by means of Commodities* (in short *PCMC*) written by Piero Sraffa, we will compare seven *Systems* in this paper. In Leontief 's system, the term 'system' is commonly used as 'quantity system' or 'price system', but, in this paper, we will use the term *System* in italics specifically for the set of the standard, the wage, the price equation, the relationship between the wage and the rate of profit. We will also examine the cinsistency and implication of each *System* and then compare the *Sysems*.

Seven *Systems* will be characterized in this paper by the chosen standard: the *Commodity Numéraire System*, the *Actual Income System*, the *Standard Income System*, the *Wage Numéraire System*, the *Commanded Labour System*, the *Embodied Labour System* and the *Value System*. The Embodied Labour System is a possible and interesting interpretation of the idea of the *Reduction to the dated quantity of labour* by Sraffa. From the viewpoint of the labour standard, the Commanded Labour System and the Embodied Labour System will provide a remarkable result that, in Sraffa's System, there exist two types of quantity of labour which can be considered as a unit: One is the quantity of labour commanded by the Standard net product and the other is the quantity of labour embodied in the Standard net product. These standards are not additional ones. They are the essencial standards which were obtained in the course of the search for a genuine unit in the real world. We will emphasize in this paper that, in Sraffa's System, the problem of choice of standard is much more important than that of price determination.

The choice of standard becomes a problem when the economy produces a surplus. Sraffa analyzes this problem after *Ch. II*. In § 12, he chooses the actual national income as the unit and maintains this condition until § 22. From § 23 to § 28 of *Ch. III*, the *Standard commodity* is introduced and explained. In § 26, Sraffa defines the *Standard net product* or *Standard national income*, and alters the standard condition from the actual national income to the Standard national income in § 34.

However, the discussions on the choice of standard do not cease at this stage. Although the Standard commodity is well known, its introduction to the analysis is half way through *Part*

I of *PCMC*. In the latter part of § 43 of *Ch. V*, the standard for prices is altered again to the quantity of labour which can be purchased by the Standard national income. Then, in § 45-47 of *Ch. VI*, the price equation is transformed into the *Reduction equation*, for which Sraffa does not refer to the standard, but we can show that the quantity of labour embodied in the Standard national income can be considered as the unit for the Reduction equation.

These conditions of standard is accompanied by the assumption of the quantity of labour. In § 10, Sraffa assumes that the actual total labour is equal to unity. In § 26, the quantity of labour of the Standard system is assumed to be equal to that of the actual quantity system. In order to interpret the section from § 43 to § 49 of *PCMC*, it is essential to understand the relation between the Standard national income and the actual total labour. This paper aims, by examining one by one the relations of the chosen standard to the wage, the price equation and the *w-r* relation, to make clear the structure and the purpose of the theory of Sraffa.

1 Actual National Income as a Standard

1) Actual Quantity System and Viable Price System

We will confine ourselves, in the following part, to the simple Sraffa's System, the case of single product industries. We will express this system with matrix algebra. There is no joint production. Land and fixed capital are excluded. Each industry produces a single commodity by using a certain quantity of labour and certain quantities of commodities as a means of production. Let n be the number of industries and thus the number of products.

If we denote the quantity vector by $\mathbf{x}$, Leontief 's input matrix by $\mathbf{A}$, and the actual net product vector by $\mathbf{y}$, then the quantity equation can be represented by

$$\mathbf{x} = \mathbf{y} + \mathbf{xA} \tag{1}$$

This implies that the quantity produced is divided into two parts; the surplus produced in the economy and the replacement of commodities which have been used up in the production processes. $\mathbf{y}$ denotes the surplus, and $\mathbf{xA}$ denotes the commodities which might be replaced. For simplicity, we will use the notation

$$\mathbf{k} = \mathbf{xA} \tag{2}$$

then (1) can be rewritten as

$$\mathbf{x} = \mathbf{y} + \mathbf{k} \tag{3}$$

From (1), we also obtain

$$\mathbf{y} = \mathbf{x}(\mathbf{I} - \mathbf{A}) \tag{4}$$

where **I** is a unit matrix. This gives the definition of the vector of the actual net product which makes up the actual national income. And the j th component of **y** represents the net product of industry j. If each industry produces a surplus, **y** will be strictly positive: $\mathbf{y} > 0$, and if there is no surplus in some industries, **y** will be semi-positive: $\mathbf{y} \geq 0$.

[Assumption 1] $\quad \mathbf{x} > 0, \quad \mathbf{y} \geq 0, \quad \mathbf{A} \geq 0$

And we assume that **x**, **y**, **A** are given exogenously.

We assume, for simplicity, that all products are basic commodities, and therefore the non-basic commodity does not exist. Thus, **A** is a indecomposable matrix.

[Assumption 2] Matrix **A** is indecomposable

Then we turn to the labour vector. We denote the labour input coefficient vector of Leontief type by $\mathbf{l}_L$. And we assume $\mathbf{l}_L > 0$. then the actual total labour will be represented as $\mathbf{x}\mathbf{l}_L$. And we assume that $\mathbf{l}_L$ is also given exogenously.

Now we proceed to define the price equation. We call the exchange ratios the prices of commodities or simply the prices, which enable the system to be viable. Let us denote the price vector by **p**. And let us denote the rate of profits by r, which is assumed to be uniform all over the economic system. Similarly, a uniform rate of wage is assumed to be prevailing in the economy, and it is indicated by ω. Thus the price equation might be written as

$$\mathbf{p} = (1 + r)\mathbf{A}\mathbf{p} + \omega\,\mathbf{l}_L \tag{5}$$

In (5), there are $(n + 2)$ unknowns and n equations, so that we have two degree of freedom. If we choose commodity j as the unit and set the price of commodity j equal to unity,

$$p_j = 1 \tag{6}$$

then we will have one degree of freedom in (5). If we make an additional assumption that one of the distributive variables is exogenous, then the degree of freedom will vanish and the equation (5) will become determinate.

2) Assumption on the Actual Total Labour in § 10

Let us proceed to consider the assumption of § 10 about the actual total labour. Sraffa assumes in this section that the actual total labour is equal to unity. Although Leontief 's labour coefficient vector is a common tool to represent Sraffa's System, it is incompatible with the assumption of § 10, because, by Leontief 's vector, the actual total labour might be represented as $\mathbf{xl}_L$, which may not be necessarily equal to unity. Hence we introduce into our analysis the Sraffa type labour coefficient vector. In order to distinguish it from the Leontief type labour coefficient vector, we denote it by $\mathbf{l}_S$, and define as

$$\mathbf{l}_S = (1 / \mathbf{xl}_L)\mathbf{l}_L \tag{7}$$

We will call this vector the *Standard labour coefficient vector*. From (7), the actual total labour can be represented by definition as

$$\mathbf{xl}_S = 1 \tag{8}$$

Since $\mathbf{l}_L > 0$, the Standard labour coefficient vector becomes strictly positive. Therefore,

[Assumption 3] $\mathbf{l}_S > 0$

Since we altered the labour coefficient vector from $\mathbf{l}_L$ to $\mathbf{l}_S$, we must alter the notation of the wage variable. In place of ω corresponding to $\mathbf{xl}_L$, we denote the wage corresponding to (8) by w.

Let us make clear here the implication of w corresponding to (8). Whereas we have called ω the wage rate, w means the total wage. Since we have two types of labour coefficient vector, i.e. $\mathbf{l}_L$ and $\mathbf{l}_S$, the total wage will be represented either by $\omega\,\mathbf{xl}_L$ or by $w\mathbf{xl}_S$. While $\omega \neq w$ and $\mathbf{xl}_L \neq \mathbf{xl}_S$, $\omega\,\mathbf{xl}_L$ is equal to $w\mathbf{xl}_S$ in the aggregate. Let w^T be the total wage, so that

$$w^T = \omega\,\mathbf{xl}_L = w\mathbf{xl}_S \tag{9}$$

This shows the relation of ω and w. Substituting (8) into (9), we have

$$w^T = w \tag{10}$$

This implies that w means the total wage. If we denote the wage per unit of labour by w^L, then it can be represented by

$$w^L = w^T / \mathbf{xl}_s \tag{11}$$

From (8) (10) (11), it can be written as

$$w^L = w \tag{12}$$

Thus, w has two implications under the assumption (8): the total wage and the wage per unit of labour. However, it is important to bear in mind that both w^T and w^L do not represent an actual bundle of heterogeneous commodities for the wage or the evaluated value of such wage goods, while they only represent the value of the claims for some products which should be distributed as the wage.

Then, with the vector $\mathbf{l}_s$ and the wage variable w, the equation (5) can be rewritten as

$$\mathbf{p} = (1+r)\mathbf{Ap} + w\mathbf{l}_s \tag{13}$$

Comparing (5) and (13), the variable for the wage is altered from ω to w, but price vector $\mathbf{p}$ of (13) is the same as the equation (5). *Assumptions 1-3* assure that the equation (13) may have a positive solution of $\mathbf{p}$ for all r $(0 \leqq r \leqq R)$. In the following part, we will regard the equation (13) as that of Sraffa's System.

3) Standard Condition in § 12 : Actual National Income as a Standard

In § 12, Sraffa chooses the actual national income as a unit. If we denote the price vector in terms of the actual national income by $\mathbf{p}_y$, it will be represented as

$$\mathbf{p}_y = \mathbf{p} / \mathbf{yp} \tag{14}$$

If the actual national income is considered as the standard, $\mathbf{yp}$ will be set equal to unity

$$\mathbf{yp} = 1 \tag{15}$$

then the degree of freedom of (13) will become unity. The equation (15) represents the condition in § 12.

In order to aggregate the actual net product itself, we should multiply the actual net product vector by the price vector. Thus the product of $\mathbf{y}$ and $\mathbf{p}_y$ implies the value of actual national income. Multiplying $\mathbf{y}$ by $\mathbf{p}_y$, it can be represented as

$$\mathbf{yp}_y = 1 \tag{16}$$

This means that a bundle of heterogeneous commodities or a composite commodity constituting the actual national income is considered as the unit to measure the prices.

Similarly, the total wage in terms of the actual national income

$$w_y{}^T = w\,\mathbf{xl}_s\ /\ \mathbf{yp} = w \tag{17}$$

and the wage per unit of labour in terms of the actual national income

$$w_y{}^L = w_y{}^T\ /\ \mathbf{xl}_s = w \tag{18}$$

is also measured by, as a unit, a bundle of commodities or a composite commodities which constitutes the actual national income.

However, there can be another interpretation of w under the condition (15). If we consider w as the ratio of the wage to the actual national income (wage-income ratio), which is a proportion between two aggregates, then it will not be necessary to consider that the wage is expressed in terms of a bundle of heterogeneous commodities, because the wage-income ratio is a pure number which is independent of any unit to measure. If we denote the wage-income ratio by $w_y{}^R$, it can be written as

$$w_y{}^R = w\mathbf{xl}_s\ /\ \mathbf{yp} \tag{19}$$

Substituting (8) (15) into (19), we have the following equation

$$w_y{}^R = w \tag{20}$$

It should be noticed that the pair of the assumption (8) and the condition (15), which are introduced in § 10 and § 12 in *Ch. II*, enable us to consider w of the price equation (13) as the wage-income ratio. Under the condition (15), w will be real numbers from 1 to 0. Sraffa states that the wage will take successive values ranging from 1 to 0 in § 13 of *Ch. III*. Thus *Ch. II* is a preliminary part for the analysis in *Ch. III*.

4) Actual National Income and Distribution

In *Ch. III*, Sraffa does not discuss the distributive relation between the wage and the rate of profits. He refers only to the wage reduction and states in § 16, 'Nothing is assumed at the moment as to what rate of profits corresponds to what wage reduction' Let us consider here the relation between the wage and the rate of profits.

From (13), we have

$$(\mathbf{I} - \mathbf{A})\ \mathbf{p} = r\,\mathbf{Ap} + w\mathbf{l}_s \tag{21}$$

Pre-multiplying (21) by the vector **x**, we obtain

$$\mathbf{x}\,(\mathbf{I} - \mathbf{A})\ \mathbf{p} = r\,\mathbf{xAp}_j + w\mathbf{xl}_s \tag{22}$$

Substituting (2) and (4) into (22), we obtain

$$\mathbf{yp} = r\,\mathbf{kp} + w\mathbf{xl}_s \tag{23}$$

This implies that the actual national income is divided into two parts, which are distributed to the profit and the wage. From (8), we can rewrite (23) as

$$r = (1/\mathbf{kp})(\mathbf{yp} - w) \tag{24}$$

where $0 \leqq w \leqq 1$.

If we choose commodity j as a *numéraire*, then the *System* in terms of commodity j can be represented for $0 \leqq w \leqq 1$ as follows:

[System 1]

$$\begin{aligned} p_j &= 1 \\ w_j &= w / p_j \\ \mathbf{p}_j &= (1+r)\mathbf{Ap}_j + w_j \mathbf{l}_s \\ r &= (1/\mathbf{kp}_j)(\mathbf{yp}_j - w_j) \end{aligned}$$

We will call *System 1* the *Commodity Numéraire System*.

Then if we choose the actual national income as a unit, then (24) can be written as

$$r = (1/\mathbf{kp})(1 - w_y) \tag{25}$$

where $0 \leqq w_y \leqq 1$. In this case, we can consider that w_y is the wage-income ratio which will take real numbers from 1 to 0. The *w-r* relation of (25) is much simpler than that of (24). However, since **kp** may change as w changes, the *w-r* relation is still complicated. We can represent the *System* in terms of the actual national income as follows

[System 2]

$$\begin{aligned} \mathbf{yp} &= 1 \\ w_y &= w / \mathbf{yp} \\ \mathbf{p}_y &= (1+r)\ \mathbf{Ap}_y + w_y \mathbf{l}_s \\ r &= (1/\mathbf{kp}_y)(1 - w_y) \qquad 0 \leqq w_y \leqq 1 \end{aligned}$$

We call *System 2* the *Actual Income System*, which provides the basis for the analysis of wage reduction and price changes in § § 13-22 of *Ch. III*.

2 Standard National Income and Actual Total Labour

1) Definition of Standard National Income in § 26

Standard commodity is well known as an invariable standard. We proceed to the discussions of the Standard commodity by Sraffa, which is introduced in *Ch. IV*.

The Standard system is defined as a virtual system whose quantity vector corresponds to the eigenvector of the input coefficient matrix A and which has a uniform rate of surplus in physical terms throughout industries. If we denote the physical rate of surplus by Π, and the quantity vector of the Standard system by **h**, it can be represented as

$$\mathbf{h} = (1 + \Pi)\mathbf{hA} \tag{26}$$

If we denote the Standard commodity vector by **u**, it can be defined from (26) as

$$\mathbf{u} = \mathbf{h}(\mathbf{I} - \mathbf{A}) = \Pi\mathbf{hA} \tag{27}$$

The components of this vector correspond to the commodities constituting the Standard commodity.

It is indeed true that Sraffa introduced the Standard commodity for the analysis of price changes and uses the term frequently, but not the Standard commodity in general but the Standard national income was adopted for Sraffa's analysis. The Standard national income was defined as the Standard commodity with an additional assumption introduced in § 26: that is, the total labour of the Standard system is equal to the total labour of the actual system. If we denote the vector of the total quantities produced in the Standard system corresponding to the Standard national income by **q** in place of **h**, the relation between **q** and **h** will be represented by

$$\mathbf{q} = t\,\mathbf{h} \tag{28}$$

where t is a positive scalar. Thus the vector **q** is defined as a vector which can be represented, like (26), as

$$\mathbf{q} = (1+\Pi)\mathbf{qA} \tag{29}$$

and, in addition, satisfies the following assumption:

[Assumption 4] $\mathbf{ql}_s = \mathbf{xl}_s$

This represents the assumption of § 26.

From (28), the Standard national income can be defined, like (27), as follows

$$\mathbf{s} = \mathbf{q}(\mathbf{I} - \mathbf{A}) = \Pi\, \mathbf{qA} \tag{30}$$

This is called the *Standard net product* or *Standard national income* in § 26. The vector $\mathbf{s}$ is uniquely determined if the matrix $\mathbf{A}$ and the total labour $\mathbf{xl}_s$ are given exogenously.

Now if we denote by R the maximum rate of profits with $w = 0$, the price equation (13) will become

$$\mathbf{p} = (1 + R)\mathbf{Ap} \tag{31}$$

This implies that $\mathbf{p}$ is the right-hand eigenvector of the matrix $\mathbf{A}$ and $1/(1 + R)$ is its eigenvalue. On the other hand, in equation (29), the vector $\mathbf{q}$ is the left-hand eigenvector of the matrix $\mathbf{A}$ and $1/(1+\Pi)$ is its eigenvalue. Hence, from the *Perron-Frobenius theorem*, we have

$$\Pi = R \tag{32}$$

Substituting (32) into (30), we obtain

$$\mathbf{s} = \mathbf{q}(\mathbf{I} - \mathbf{A}) = R\, \mathbf{qA} \tag{33}$$

This is a more useful expression of the Standard national income.

2) Alteration of Standard in § 34: Standard National Income as a Standard

In § 34 of *Ch.IV*, Sraffa considers the Standard national income as a unit, in place of the actual national income. The condition to consider the Standard national income as a unit is represented by

$$\mathbf{sp} = 1 \tag{34}$$

From (34), the degree of freedom of the price equation (13) becomes unity.

Let us consider the results stemming from the condition (34). First, the price vector in terms of the Standard national income becomes

$$\mathbf{p}_s = \mathbf{p} / \mathbf{sp} \tag{35}$$

Aggregating $\mathbf{s}$ by $\mathbf{p}_s$, we obtain

$$\mathbf{sp}_s = 1 \tag{36}$$

This implies that the value of Standard national income is measured by, as a unit, a bundle of heterogeneous commodities, which make up the Standard net product.

Secondly, the wage in terms of the Standard national income

$$w_s = w / \mathbf{sp} \tag{37}$$

will also be measured by the unit of a composite commodity of the Standard net product. In this case, w_s has three different implications: the total wage in terms of Standard national income

$$w_s^{T} = w\mathbf{xl}_s / \mathbf{sp} = w_s \tag{38}$$

and the wage per unit of labour

$$w_s^{L} = w_s^{T} / \mathbf{xl}_s = w_s \tag{39}$$

and then, the wage-income ratio in terms of the Standard national income,

$$w_s^{R} = w_s^{T} / \mathbf{sp} = w_s \tag{40}$$

Under the condition (34), w_s will take real numbers from 1 to 0. And w_s^R is a pure number and is therefore independent of a particular unit.

Finally, let us consider the *w-r* relation under the condition (34). Sraffa analyzes this in § § 29-32. Pre-multiplying (21) by **q**, we obtain

$$\mathbf{q}(\mathbf{I} - \mathbf{A})\mathbf{p} = (r / R)R\mathbf{qAp} + w\mathbf{ql}_s \tag{41}$$

Substituting (32) and *Assumption 4* into (41), we obtain

$$(1 - r / R)\mathbf{sp} = w\mathbf{xl}_s \tag{42}$$

From (8), we obtain

$$(1 - r / R)\mathbf{sp} = w \tag{43}$$

From (33), we obtain

$$r = R(1 - w) \tag{44}$$

where $0 < w \leqq 1$ or alternatively $0 \leqq r < R$. We have got, at this stage, to the well known *w-r* relation of (44). Only in this case, the rate of profits is independent of the price changes. From the above arguments, we can indicate the following system

$$\text{[System 3]} \quad \begin{aligned} &\mathbf{sp} = 1 \\ &w_s = w / \mathbf{sp} = R / (\mathrm{R} - \mathrm{r}) \qquad 0 \leqq r < R \\ &\mathbf{p}_s = (1 + r)\mathbf{Ap}_s + w\mathbf{l}_s \end{aligned}$$

$$r = R(1 - w_s) \qquad\qquad 0 < w_s \leqq 1$$

We call this the *Standard Income System*. It might be commonly held that *System 3* is an important contribution by Sraffa. But Sraffa's arguments about the standards does not end up with *System 3*. In § 43, a new device for the Standard is introduced. In § § 45-47, the price equation called *Reduction to dated quantities of labour* will be more interesting in interpreting the *Production of Commodities* by Sraffa.

3) Standard Conditions in § 43

The Standard national income gives a clear insight to the *w-r* relation as (44). However, it is still a composite commodity composed of heterogeneous products, which are measured by the different units of each product. It is this problem that Sraffa discusses in § 43 of *Ch. V*. He sweeps away the heterogeneous commodity bundle as a unit from the analysis, and suggests that it can be replaced by the following methods, preserving the properties as an invariable standard.

In the former paragraphs of § 43 (paragraph 1-5), Sraffa suggests that it is possible to replace the Standard net product by (44), because, from (43), the following equivalence holds

$$sp = 1 \qquad \Leftrightarrow \qquad r = R\ (1 - w) \tag{45}$$

where $0 \leqq r < R$. From (45), the condition (44) comes to have an implication that the Standard national income is considered as a unit, even if we do not indicate the composition of the Standard national income and thus it remains anonymous.

However, in the latter paragraphs of § 43, Sraffa introduced a more interesting standard, which works as a standard and is measured by a genuine unit which we can find out in the actual system. It is 'the quantity of labour that can be purchased by the Standard net product' The standard condition (44) is an easy device but ambiguous, whereas the quantity of labour is a unit which exists in the actual system and is measured by its physical unit. Thus Sraffa find much significance in the quantity of labour.

The quantity of labour that can be purchased by a commodity can be obtained by dividing the price of a commodity by the wage. Before we proceed to consider the quantity of labour purchased by the standard national income, we will represent the relative prices to the wage or the prices in terms of wage. If we denote it by $\mathbf{p}_w$, it can be represented by

$$\mathbf{p}_w = \mathbf{p} / w \tag{46}$$

This implies that the prices are expressed in terms of the quantity of labour commanded, what is called by Classical economists, by a commodity. If the wage is set equal to unity,

$$w = 1 \tag{47}$$

then the wage w is considered as a unit. The condition (47) makes it possible that the degree of freedom of (9) becomes unity. But w is a pure number because w is interpreted as the wage-income ratio. As has been mentioned already, w represents the value of the claims for some products which should be distributed as the wage, and not the value of the wage goods. Hence, in order to interpret w as a unit which exists in the actual system, the wage goods must be specified in physical terms.

If we denote the wage commodity vector by $\mathbf{b}$ ($\mathbf{b} > 0$), then the value of wage goods will be represented by $\mathbf{bp}$ in the aggregate. Since the value of the wage is represented both as w or as $\mathbf{bp}$, the following equation can be obtained

$$w = \mathbf{bp} \tag{48}$$

The value of the wage commodity bundle in terms of the quantity of labour commanded can be represented as

$$\mathbf{bp}_w = \mathbf{bp} / w = 1 \tag{49}$$

Alternatively, if we denote the wage in terms of wage goods bundle by w_b, then, from (48), we have

$$w_b = w / \mathbf{bp} = 1 \tag{50}$$

In this case, the standard is in the end the wage commodity bundle $\mathbf{bp}$, so that

$$\mathbf{bp} = 1 \tag{51}$$

From above arguments, we have *System 4* in terms of the wage commodity as

[System 4]

$$\begin{aligned} &\mathbf{bp}_w = 1 \\ &w = 1 \\ &\mathbf{p}_w = (1+r)\mathbf{A}\mathbf{p}_w + \mathbf{l}_s \qquad 0 \leqq r < R \\ &r = (1/\mathbf{kp}_w)(\mathbf{yp}_w - 1) \qquad 0 \leqq r < R \end{aligned}$$

We call *System 4* the *Wage Numeraire System*. In *System 4*, the condition (49) or alternatively (51) can be considered as the standard condition, but, it may be more important that even if we eliminate the condition (49) or (51) from *System 4*, the situation will be the same. Hence, *System 4* stands up only by the condition (47), even if the wage commodities are not specified and remain anonymous.

To be precise, we should make an additional assumption on the vector **b** of $\mathbf{b} \neq t\,\mathbf{s}$, ($t$: positive scalar), because if $\mathbf{b} = t\,\mathbf{s}$, some fraction of the Standard national income will be considered as the wage goods bundle. In this case, the results will be something like *System 3*. If $\mathbf{b} = t\,\mathbf{y}$, some fraction of the actual national income will be considered as the wage goods bundle, Let $\mathbf{e}^{j}=(0,\cdots,1,\cdots,0)$ be the unit vector whose components are zero except the jth one. Then if $\mathbf{b} = \mathbf{e}^{j}$, one unit of commodity j will be considered as the wage.

There will be many scholars who may refer to *System 4* in case that the prices are represented in terms of wage. The assumption of exogenous wage might be what the Classical economists conceived for their analysis. But it may be too difficult to specify the exact wage goods. Sraffa introduced, in the latter part (from paragraph 6) of § 43, a standard much clearer than that of *System 4*. The standard is the quantity of labour that can be purchased or commanded by the standard national income. It will be obtained by dividing the standard national income by w,

$$\mathbf{sp}_{w} = \mathbf{sp} / w \qquad (52)$$

This is the quantity of labour purchased by the standard national income. And if $\mathbf{sp} = 1$, the wage will be represented from (44) by

$$w = 1 - r / R \qquad (53)$$

From (53), (52) can be rewritten as

$$\mathbf{sp}_{w} = \frac{1}{w} = \frac{R}{R - r} \qquad (54)$$

In this equation, the value of the Standard national income is measured by the quantity of labour commanded by the Standard national income. It should be noticed that (54) depends upon the condition (34) implicitly.

It should be noted that, in § 43 of *Ch. V*, Sraffa represents the standard condition (54), but not the price equation itself, while, in § § 45- 47 of *Ch. VI*, he represents the price equation in the form of the *Reduction equation*, but he does not refer to the standard for it. Here we will indicate the price equation as a function of the rate of profits (Reduction equation). Since $[\mathbf{I} - (1 + r)\mathbf{A}]$ is a non-singular matrix for $0 \leqq r < R$, from (13) we can rewrite the price equation (13) as

$$\mathbf{p}_w = \mathbf{p} / w = [\mathbf{I} - (1 + r)\mathbf{A}]^{-1} \mathbf{l}_s \tag{55}$$

where $0 \leqq r < R$. Thus we have *System 6* as follows

$$\text{[System 5]} \quad \begin{array}{ll} \mathbf{s}\mathbf{p}_w = R / (R - r) & 0 \leqq r < R \\ w = 1 & \\ \mathbf{p}_w = [\mathbf{I} - (1 + r)\mathbf{A}]^{-1} \mathbf{l}_s & 0 \leqq r < R \\ r = R(1 - w_s) & 0 < w_s \leqq 1 \end{array}$$

We can call *System 5* the *Commanded Labour System*. It should be noticed that both *System 4* and *System 5* have the same condition of $w = 1$, but the latter system makes things clearer than the former, in the points of the standard condition and the *w-r* relation.

Now let us proceed to consider the remarks of Sraffa in the last paragraph of § 43. The wage in terms of a commodity is explained in that paragraph. Although we have already introduced the relative wage in terms of commodity *j*, as appears in *System 1*, the explanation carried out by Sraffa is based on the price equation of *System 5* (or *System 4*), and not of *System 1*. The wage in terms of commodity *j* is obtained by taking the reciprocal of the price of commodity *j*. If we denote the relative price of commodity *j* to the wage by

$$p_w{}^j = p^j / w \tag{56}$$

then, the wage in terms of commodity *j* can be represented by

$$w_j = 1 / p_w{}^j \tag{57}$$

It should be noted that, in *System 1*, the wage in terms of commodity *j* is represented only by $w_j = w / p^j$, whereas, as we can see in the above discussions, w_j is derived as the reciprocal of the relative price of any commodity to the wage, which appeared in *System 4* or *System 5*.

4) Reduction to Dated Quantity of Labour in § § 45-47

Now let us proceed to the examination of the price equation which is called the equation of *Reduction to the dated quantities of labour* or in short the *Reduction equation.* Sraffa introduces this equation in § § 46-47 of *Ch. VI*, but does not indicates the standard for it. We will show that the actual quantity of labour can be considered as the standard for the Reduction equation.

The Reduction equation is a function of the rate of profits, and like (55), is represented by

$$\mathbf{p} = w\,[\mathbf{I} - (1 - r)\mathbf{A}]^{-1}\,\mathbf{l}_s \tag{58}$$

or in the form of a power series expansion,

$$\mathbf{p} = w\mathbf{l}_s + w\,(1 + r)\mathbf{A}\mathbf{l}_s + w(1 + r)^2\mathbf{A}^2\mathbf{l}_s + \cdots\cdots \tag{59}$$

From (44) (58) (59), we have

$$\begin{aligned}\mathbf{p} &= (1 - r / R)\,[\mathbf{I} - (1 - r)\mathbf{A}]^{-1}\,\mathbf{l}_s \\ &= (1 - r / R)\mathbf{l}_s + (1 - r / R)\cdot(1 + r)\mathbf{A}\mathbf{l}_s + (1 - r / R)\cdot(1 + r)^2\mathbf{A}^2\mathbf{l}_s + \cdots\cdots\end{aligned} \tag{60}$$

One of the members of the right-hand side of (60) is called in § 47 the dated quantity of labour. The Reduction equation (60) assumes the condition (44). Hence we can consider that the Standard national income is chosen as a unit. However, there will be another interpretation. We can obtain the following equation, not from (43) but from (42) and (44)

$$\mathbf{sp} = \mathbf{xl}_s \tag{61}$$

This implies that the Standard national income is equal to the actual total labour.

Now let us consider this equality in income and labour. For this purpose, let us denote the value of the Standard national income per unit of labour by v_L, then it can be defined as

$$v_L = \mathbf{sp} / \mathbf{xl}_s \tag{62}$$

where v_L is a scalar. In other words, v_L is the value of actual total labour embodied in the Standard national income. If the condition of $r = R(1 - w)$ is satisfied, from (61) (62), we can obtain the following equation, for all r ($0 \leqq r < R$),

$$v_L = 1 \tag{63}$$

This implies that the value of labour embodied in the Standard national income is equal to unity.

On the other hand, from (62),we have

$$\mathbf{sp} = v_L\,\mathbf{xl}_s \tag{64}$$

If we define the price vector under the condition (63) by

$$\mathbf{p}_v = \mathbf{p} / v_L \qquad (65)$$

This implies that the prices is expressed in terms of the quantity of labour embodied in the Standard national income. Hence, under the condition of $r = R(1 - w)$, from (63)-(65), we have

$$\mathbf{s}\mathbf{p}_v = \mathbf{x}\mathbf{l}_s \qquad (66)$$

This implies that the aggregate value of the standard net product obtained by post-multiplying the vector $\mathbf{s}$ by (65) is equal to the actual total labour. This is a possible, and even natural, interpretation of (61). This should cause no surprise, because Sraffa considers that the members of the Reduction equation (60) are expressed in terms of the quantity of labour. It follows that the prices of the Reduction equation are also expressed in terms of the quantity of labour. Therefore it is important to note that, in (66), both the standard national income and the actual total labour are measured in terms of the quantity of labour.

From the normalization condition (8) and the equation (66), we have

$$\mathbf{s}\mathbf{p}_v = 1 \qquad (67)$$

This implies that the aggregate value of the Standard net product by (65) is equal to unity. In (36), we have already seen the aggregate value of the Standard net product by post-multiplying s by $\mathbf{p}_s$. the equation (36) is an interpretation of (34), but now we have obtained an alternative interpretation in the form of (67). The prices in terms of the Standard national income in *System 3* are measured by the unit of a heterogeneous commodity bundle. Moreover, the standard national income is a virtual and hypothetical construction. On the other hand, it should be emphasized that, as is seen in (66) (67), the prices in terms of the actual total labour embodied in the standard national income are measured by the unit of the quantity of labour, which exists in the actual system.

Now let us represent an interesting System as follows

$$\text{[System 6]} \quad \begin{aligned} \mathbf{s}\mathbf{p} &= \mathbf{x}\mathbf{l}_s & & \\ w_v &= 1 - r/R & & 0 \leqq r < R \\ \mathbf{p}_v &= (1 - r/R)[\mathbf{I} - (1+r)\mathbf{A}]^{-1}\,\mathbf{l}_s & & 0 \leqq r < R \\ r &= R(1 - w_v) & & 0 \leqq r < R \end{aligned}$$

We call *System 6* the *Embodied Labour System*. The standard conditions (63) (66) and (67) may be alternative. It should be noticed that *System 6* is based on the following Theorem,

[Theorem] Let $\mathbf{p} = (1 + r)\mathbf{Ap} + w\mathbf{l}_s$ be the price equation system under *Assumption 1-4*.

Then the Standard national income is equal to the actual total labour if and only if

$$r = R(1\text{-}w)$$

where $0 \leqq r < R$.

[Proof] As we have seen in (41), from *Assumption 4*, the price equation can be transformed into

$$(1\text{-}\ r\ /\ R)\mathbf{sp} = w\mathbf{xl}_s$$

then, for $0 \leqq r < R$, we have

$$\mathbf{sp} = \mathbf{xl}_s \quad \Leftrightarrow \quad r = R(1 - w)$$

From this, the theorem is verified. Q.E.D.

I call this *Sraffa's Theorem*. This theorem implies that the Standard national income is equal to the actual total labour. In other words, the equality between the flow of the aggregate value of the Standard net product and the flow of actual total labour is established if and only if $r = R(1\text{-}w)$ for $0 \leqq r < R$. It provides the basis of *System 6* and, with the assumption (8), it also becomes the basis of *System 3* and *System 5*.

Now let us consider the case that the rate of profits is zero, i.e. $r = 0$. In this case, the wage becomes equal to unity, i.e. $w = 1$, and the prices are proportional to the values which can be defined as

$$\mathbf{v} = [\mathbf{I} - \mathbf{A}]^{-1}\mathbf{l}_s \tag{68}$$

In (68), $\mathbf{v}$ implies the quantity of labour used directly and indirectly in the economic system to obtain one physical unit of each commodity as a final commodity. We can consider that $\mathbf{v}$ is expressed in terms of the quantity of labour. Thus we have a *System* for the case of $r = 0$, or $w = 1$, as follows

[System 7]

$$\mathbf{sv} = \mathbf{xl}_s$$
$$w = 1$$
$$\mathbf{v} = [\mathbf{I} - \mathbf{A}]^{-1}\mathbf{l}_s$$
$$r = 0$$

Sraffa refers to this case in § 14. We call *System 6* the *Value System*. By *Assumption 2*, we excluded the trivial case that the proportion of labour to the means of production is uniform throughout industries. Thus *System 7* is valid only in the case of $w = 1$. It should be noticed that Sraffa uses the term 'value' only in the case that the rate of profits is zero. This is the case of *System 7*. And he uses the term 'prices' for the exchange ratios in the case that the rate of profits is positive. Our interpretation as (66) or (67) makes it possible to consider that the prices of (60) are expressed in terms of labour for all r ranging from 0 to R. This may bring a consistent interpretation between (60) and (68), and between (55) and (60), in which the prices are considered to be expressed in term of the quantity of labour.

By comparing *Systems*, we have remarkable results. *System 7* is a special case of *System 6*, and it is also a special case of *System 5*. It may be much more interesting that the unit of labour is a commesurable unit for *System 5-7*. In *System 3*, the unit to measure the prices is the unit of the Standard national income, which is a virtually constructed heterogeneous commodity bundle. On the other hand, the standard of *System 6* is the quantity of labour embodied in the standard national income, or in the end, it is the actual total labour which can be measured in terms of a physical unit in the actual world. Therefore, we can consider that *System 6* is much more firmly based on the actual system.

4 Distribution and Price Changes

1) Choice of Exogenous Distributive Variable in § 44

We have seen the features of *Systems 1- 7*. However, *Systems 1-6* remain indeterminate, because we still have one degree of freedom. In order to make the *Systems* determinate, we must make an additional assumption that one of the distributive variables should be given exogenously. If the actual national income is regarded as the unit of the prices and the wage, the w-r relation can be written, as is already seen in *System 2*, by

$$r = (1 / \mathbf{kp}_r)(1 - w_r) \tag{69}$$

On the other hand, as has already been seen in *System 3*, after the standard national income is chosen as unit, the w-r relation becomes,

$$r = R(1 - w_s) \tag{70}$$

Sraffa assumes in § 13 that w_y is the exogenous distributive variable, whereas in § 44, it is assumed that the rate of profit is exogenously given by the level of money rates of interest. Let us consider here the problem of the choice of exogenous distributive variable.

Let us first consider the case that the wage might be determined exogenously. In this case, in order to measure the true value of w before we start into the analysis, the wage commodity bundle must be specified beforehand. It is necessary to know the composition of the wage bundle, even in the case that w is considered as the wage-income ratio. It is indeed true that the wage-income ratio itself is a pure number, which is independent of any unit to measure, but, in order to measure its value in advance, we need to know two aggregates of heterogeneous goods: one is the aggregate value of the wage commodities and the other is the aggregate value of the net product constituting the national income. Let **b** be the vector of the total wage, then the wage-income ratio can be represented as

$$w_y{}^R = \mathbf{bp} / \mathbf{yp} \tag{71}$$

or the wage-income ratio in terms of the standard national income can be represented as

$$w_s{}^R = \mathbf{bp} / \mathbf{sp} \tag{72}$$

Substituting (71) into (69) or (72) into (70), r can be determined. However, to measure $w_y{}^R$ of (71) or $w_s{}^R$ of (72) in advance, the wage commodity bundle **b** and also the proper price vector **p** must be known beforehand.

Let us turn to the case of the exogenous rate of profits. In this case, r in (69) or (70) is considered as exogenous. The rate of profits is also a pure number, but in order to measure it in advance, we must know two aggregates of heterogeneous commodities; one is the aggregate value the products used as the means of production and the other is the aggregate value of goods distributed to the profit. If we denote the total profit vector in physical terms by **d**, then r will be represented as

$$r = \mathbf{dp} / \mathbf{kp} \tag{73}$$

Substituting (73) into (69) or (70), we can determine the wage. However, to know the value of r in advance, **d**, **k** and **p** must be known beforehand. And in order to know **p** in advance, we must know the value of r. The problem of this circular argument is described in § 4. And the difficulties in specifying **b** or **d** are described in § 44.

In § 44, Sraffa assumes simply that the rate of profits is given, from outside the system of production, by the level of the money rates of interest. Let i be the monetary rate of interest, then the assumption can be represented as

$$r = i \tag{74}$$

This assumption appears an easy byway to avoid the above mentioned puzzles. However, this makes *System 1-6* determinate. And it is essentially important for *System 5* and *System 6*, rather than for *System 3* and *System 4*, to make the system determinate by a exogenous variable which can be measured by a genuine unit, the quantity of money. Thus, we can consider that, in *System 5* and *System 6*, the quantity of labour as a genuine unit, which can be found inside the system of production, and the monetary rate of interest, which is given outside the system, make the system determinate.

2) Analysis of Wage Reduction and Price Changes in § § 13-22

System 2 provides a basic framework for the analysis in § 13-22 of *Ch.III*, in which the actual national income is considered as the standard. As has been seen, w can be interpreted as the wage-income ratio. In *Ch.III*, Sraffa discusses not only the problem of price changes, but also the properties of the methods of production, by examining, first, the effects of wage-reduction to a 'high-proportion industry'or a 'low-proportion industry' under the constant prices (§ § 16-17), second, the relation between wage-reduction and price-changes(§ § 18-20), third, the properties of the 'critical proportion' or 'balancing proportion'of the methods of production(§ § 17,21-22).

Let us take two industries from the system and denote them by a and b. And let $\mathbf{x}^a$ be the unit vector whose components are zero except the component corresponding to the quantity produced in industry a. Then $\mathbf{x}^a$ is defined as

$$\mathbf{x}^a = (0, \cdots, x^a ,\cdots,0) \tag{75}$$

Similarly, for industry b, $\mathbf{x}^b$ is defined as

$$\mathbf{x}^b = (0, \cdots, x^b ,\cdots,0) \tag{76}$$

Let $\mathbf{k}^a$ be the vector of products used as the means of production, which are necessary to produce the net product of industry a. Similarly, let $\mathbf{k}^b$ be the vector of products used as the means of production, which are necessary to produce the net product of industry b. Then , from

(75) (76), we have

$$\mathbf{k}^a = \mathbf{x}^a \mathbf{A} \tag{77}$$

$$\mathbf{k}^b = \mathbf{x}^b \mathbf{A} \tag{78}$$

Let p_y^a, p_y^b the prices of products in terms of the actual national income in each industry, and l_s^a, l_s^b be the quantity of labour used directly in each industry, Then, pre-multiplying (13) by (75) and (76), we have the following equations

$$x^a p_y^a = w x^a l_s^a + r \mathbf{k}^a \mathbf{p}_y + \mathbf{k}^a \mathbf{p}_y \tag{79}$$

$$x^b p_y^b = w x^b l_s^b + r \mathbf{k}^b \mathbf{p}_y + \mathbf{k}^b \mathbf{p}_y \tag{80}$$

In (79) and (80), $x^a p_y^a$ and $x^b p_y^b$ imply the value of total quantity produced in each industry, $w x^a l_s^a$ and $w x^b l_s^b$ imply the wage cost in each industry, $r \mathbf{k}^a \mathbf{p}_y$ and $r \mathbf{k}^b \mathbf{p}_y$ imply the profit in each industry, and $\mathbf{k}^a \mathbf{p}_y$ and $\mathbf{k}^b \mathbf{p}_y$ imply the aggregate value of the means of production in each industry.

Let us consider the proportion of labour to the value of the means of production. The case that the proportion in industry a is larger than that of industry b can be represented, from (79) (80), as

$$x^a l_s^a / \mathbf{k}^a \mathbf{p}_y > x^b l_s^b / \mathbf{k}^b \mathbf{p}_y \tag{81}$$

In this case, industry a is called a 'high-proportion industry', whereas industry b is called a 'low-proportion industry'.

If the inequality in the proportions of two industries is given as (81), the prices of each industry will change as w is reduced. Let us assume that p^a, p_y^b, $\mathbf{p}_y$ remain constant when the wage is reduced. Then industry a enjoys a surplus, on the other hand, industry b falls into a deficit, because the reduction of the wage cost in industry a (i.e. $wx^a l_s^a$) is larger than that of industry b (i.e. $wx^b l_s^b$), whereas the increase of the cost for the profit in industry a (i.e.$r \mathbf{k}^a \mathbf{p}_y$) is smaller than that in industry b (i.e. $r \mathbf{k}^b \mathbf{p}_y$). Thus, industry a is called a 'surplus' industry, whereas industry b is called a 'deficit' industry.

Sraffa proceeds to the analysis of the case that the prices may change when the wage is reduced. In this case, the price changes are not predictable. p_y^a may increase or decrease or even alternate as the wage is reduced. and p_y^b may also increase or decrease or even alternate, The reason is that the aggregate value of the means of production in each industry is not predictable, because the prices of commodities used as means of production may increase or decrease or even alternate as the wage is reduced.

The proportion of the industry in which the prices do not change when the wage is reduced is called the 'critical proportion' or 'balancing proportion' The proportion can be found in the case that the wage is zero. In this case, the equation (79) (80) will be rewritten as

$$x^a p_r^a = R\mathbf{k}^a \mathbf{p}_r + \mathbf{k}^a \mathbf{p}_r \tag{82}$$

$$x^b p_r^b = R\mathbf{k}^b \mathbf{p}_r + \mathbf{k}^b \mathbf{p}_r \tag{83}$$

where R is the maximum rate of profits. The net product might be distributed only to the profit, so that , if y^a, y^b denote the net product in each industry, we have

$$y^a p_r^a = R\mathbf{k}^a \mathbf{p}_r \tag{84}$$

$$y^b p_r^b = R\mathbf{k}^b \mathbf{p}_r \tag{85}$$

From (84) (85), we can find out the critical proportion as follows

$$R = y^a p_r^a / \mathbf{k}^a \mathbf{p}_r = y^b p_r^b / \mathbf{k}^b \mathbf{p}_r \tag{86}$$

The equation (86) is explained in the last section of *Ch.III*. The analysis of the 'critical proportion' is succeeded by the discussions of *Ch.IV*. On the other hand, The analysis of price-changes is succeeded by the discussions of § 48 of *Ch. VI*.

3) Analysis of the Relationship between the Profit Rate Change and Price-Changes in § 48

System 3 or alternatively *System 6* will provide the basic framework for the analysis of price changes in § 48 of *Ch. VI*. The analysis in *Ch.III* is a preliminary survey for the analysis in § 48, and the exogenous distributive variable is considered not as w_r but as r. The clarity is much better in § 48 (*System 3* or *System 6*) than in *Ch.III* (*System2*).

In § 48, Sraffa does not see the relative price like

$$p_b^a = p^a / p^b \tag{87}$$

Even if each price is expressed in terms of the Standard national income, the relative price is, in the end, expressed in terms of a commodity. Let ps^a and ps^b be the prices in terms of the Standard national income, and then the relative price of ps^a and ps^b will be represented as

$$ps^a / ps^b = (p^a / \mathbf{sp}) \cdot (\mathbf{sp} / p^b) = p_b^a \tag{88}$$

Thus, $ps^a / ps^b = p_b^a$ is the price in terms of commodity b. Hence Sraffa does see not the relative price, but the difference of two prices in terms of standard national income. Let d_p be the difference, then d_p is defined as

$$d_p = ps^a - ps^b \tag{89}$$

The difference d_p will change as the rate of profit changes, but the value of the standard is independent of the changes in the rate of profits or the price-changes. The values of d_p may be positive, negative and zero (see *graph 3* of § 48). From this, Sraffa denies that there is a general rule for the price changes of a commodity.

Although the relation between the wage-reduction and the price-changes is analyzed in *System 2* in *Ch. III*, it will be analyzed more clearly on *System 3* or *System 6*.

4) Analysis of the Simultaneous Changes of Three Variables in § 49

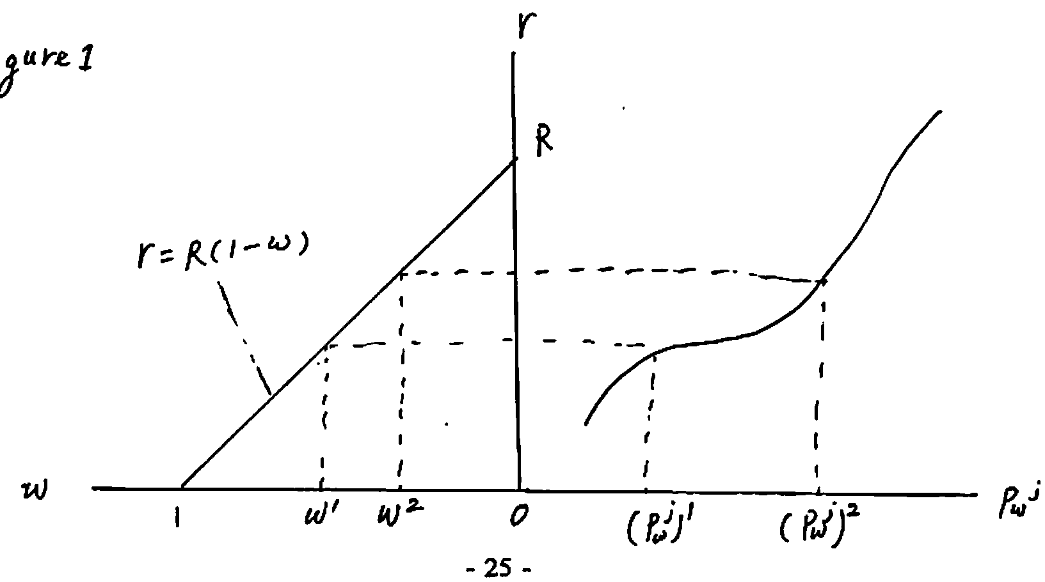
System 5 provides the basic framework for the analysis in § 49, where Sraffa analyzes the simultaneous changes of the rate of profits, the price, and the wage, and points out that, although the conclusion in § 48 appears destructive, there is a rough rule in the movements of these three variables. Sraffa states, in the first paragraph of § 49, the last section of *Part I* of *PCMC*, 'There is however a restriction to the movement of the price of any product: if as a result of a rise in the rate of profits the price falls, its rate of fall cannot exceed the rate of fall of the wage.'

Let us denote the unit vector whose components are zero except the j th one, by $\mathbf{e}^j = (0,\cdots,1,\cdots,0)$, then, from (54), we obtain

$$p_w^j = \mathbf{e}^j[\mathbf{I} - (1+r)\mathbf{A}]^{-1}\ \mathbf{l}_s \tag{90}$$

This is a strictly monotonic increasing function of the rate of profits. The graphs of (44) and (90) are shown in *Figure 1*.

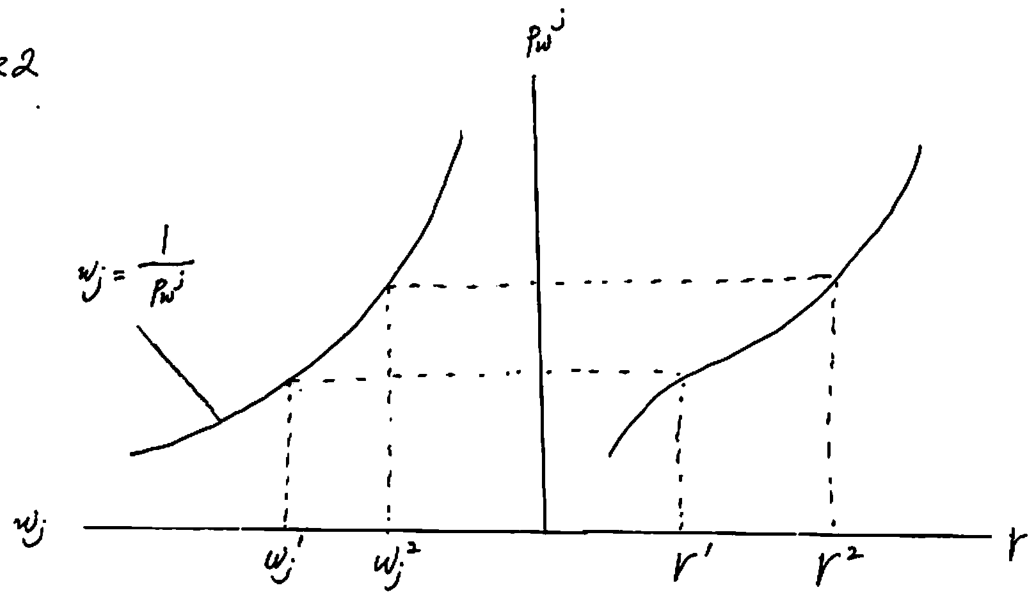
Figure 1



By *Figure 1*, if the rate of profits increases with the fall of the wage, the price in terms of the wage will increase. In other words, if the wage decreases, the rate of profits will increase. And since p_w^j is a strictly increasing function of the rate of profits, as the rate of profits increases, the price in terms of in the wage increases. This means that the rate of fall of the wage cannot exceed the rate of fall of the price of a commodity.

Sraffa gives another explanation in the fifth paragraph of § 49. He states there, 'if the wage is cut in terms of any commodity (no matter whether it is one that will consequently rise or fall relatively to the Standard) the rate of profits will rise.' We can explain this by the graphs of (56) and (90), which are shown in *Figure 2*.

Figure2



By *Figure 2*, if w_j decreases, p_w^j will increase, and if p_w^j increases, the rate of profits will increase. Thus, if the wage in terms of commodity *j* decreases, the rate of profits will increase.

The simultaneous changes of the rate of profits, the wage, the prices can be analyzed, as has been seen in the above discussions, by *System 5* clearly.

5 Conclusion

The most significant features of Sraffa's System, in comparison with Leontief 's System, lie in the relations of the chosen standard, the wage, the prices of commodities, and the *w-r* relation. We have examined it in seven *Systems* in this paper.

If we confine ourselves to the problem of exchange or the problem of price determination, then it will be analyzed by *System 1*. If we proceed to the analysis of distribution and price changes, then *System 2* or *System 3* will provide the basic framework for its analysis, where the standard is a heterogeneous composite commodity. And *System 3* will provides a more clear insight than *System 2*, because *System 3* make it possible to express the w-r relation in the well-known simple form. If we want to eliminate a heterogeneous commodity bundle as a standard from the analysis, then we can do this by considering $r = R(1-w)$ as the standard condition.

However, it should be emphasized that we have two important units as standard: one is the quantity of labour commanded by the Standard national income(*System 5*), the other is the quantity of labour embodied in the Standard national income(*System 6*). Thus we can conclude that the units to make Sraffa's System determinate can be considered in the end to be the unit of the quantity of labour inside the system of production, and the unit of the quantity of money outside the system. The quantity of labour embodied in the Standard national income is measured as the actual total labour, and it can express the prices of commodities. The quantity of money is the unit to determine the level of money rates of interest. Both the quantity of labour and the quantity of money are measured by its pure and genuine unit which we can find out in the real world.

Sraffa states in the *Preface* of *PCMC* as follows

> This Standpoint, which is that of the old classical economists from Adam Smith to Ricardo, has been submerged and forgotten since the advent of the 'marginal' method.

and in the *Appendix D 2*, he also states as follows

> The conception of a standard measure of value as a medium between two extremes (§ 17) also belongs to Ricardo and it is surprising that the Standard commodity which has been evolved from it here should be found to be equivalent to something very close to the standard suggested by Adam Smith, namely 'labour commanded' (§ 43), to which Ricardo himself was so decidedly opposed.

These remarks indicate that Sraffa does not confine himself to the problem of distribution in Ricardian perspectives, which may be seen in *System 3*. The Smithian perspectives may exist in Sraffa's System. We might have had a glimpse of this, in our *System 5-7*.

Notes

1) Although there are many studies on *PCMC*(1960), we will refer only to *Part I* of *PCMC*. Sraffa's analysis is given by elementary algebra, but we will use Matrix algebra. Therefore, refer only to Pasinetti[1977]. My ideas related to this subjects can be seen in Yagi[2000a]. My extension of Sraffa's model can be found in Yagi[1998][2000b].

2) See Pasinetti[1977]p.112.

3) See Pasinetti[1977]pp.117-119.

4) See Pasinetti[1977]pp.99-101,pp.130-134.

[Reference]

Pasinetti,L.L.[1977] *Lectures on the Theory of Production*, Columbia University Press

Sraffa,P.[1960] *Production of Commodities by means of Commodities*, Cambridge University Press, Cambridge.

Yagi,T.[1998] "Alternative Theories on the Evaluation of Real National Income(in Japanese)," in *Keizaigaku No Shosou: Essays in honor of Professor Toshinosuke Kashiwazaki*, Egawa,M., Tomono, N., Uemura,T., and T.Yagi (eds),Gakubunsha, Tokyo, March.

Yagi,T.[2000a] "The Structure of *Production of Commodities*(in Japanese)," in *Gendai Keizai Ronsou*, Kataoka,H. and M.Matusmoto(eds) Gakubunsha, Tokyo, March.

Yagi,T.[2000b] "Productivity Index and Capital Theory(in Japanese)," in *Frontiers in International Political Economy*, Kuniharu Date(ed), Chuokeizaisha, Tokyo, (forthcoming).