Exchange Rate Dynamics of Portfolio Balance Model by Stochastic Difference Equations

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First draft: October 1998
This draft: June 1999

JEL Classification Number:

F31 Foreign Exchange

F41 Open Economy Macroeconomics

Keywords: Exchange Rate Dynamics, Portfolio Balance Model, Stochastic Difference Equations,

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ABSTRACT

This paper focuses on one central question of an appropriate and consistent theoretical as well as empirical model of a reduced form of the rational expectations version of the asset market approach to the exchange rate determination. Using the portfolio balance model, and formulating the model by the stochastic difference equations system, the explicit solutions are obtained as functions of forcing variables extending to both infinite future as well as past dates. This characteristic of the solution is interpreted as reflecting both of the "forward-looking" nature of the exchange rate and the "backward-looking" nature of the interest rate.

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I. Introduction

This paper focuses on one central question of an appropriate and consistent theoretical as well as empirical model of a reduced form of the rational expectations version of the asset market approach to the exchange rate determination. There has existed a clear inconsistency between models in the asset market approach to the exchange rate determination in the literature. For example, according to the rational expectations version of the flexible-price monetary model, the reduced form of the exchange rate is theoretically shown to depend on the discounted present values of the vector of the expected forcing variables in *infinite future* (e.g., Isard (1995), chapter 7). However, the empirical versions of the model have usually been formulated by an AR or a VAR representation using only the *past* (or predetermined) forcing variables, without discussing any explicit rationale for using them instead of the *future* expected forcing variables (e.g., Isard (1995), chapter 8, van de Gucht, Dekimpe, and Kwok (1996)). Those empirical researches seem to be motivated by actual observation. "Nominal exchange rates ... showed considerable variation. Much of this variation took the form of protracted swings, swings that ... appeared to bear little or no, and at times even a perverse, relationship to movements in fundamental economic variables" (Lothian (1997), p.21).

This inconsistency motivated me to reconsider rational expectations versions of the asset market approach to the exchange rate determination. In order to pursue my motivation, I took up the so-called "sticky-price" monetary model (which has also been known as the "overshooting" model in the exchange rate literature) because of the following two reasons. First of all, this essentially dynamic model is a simultaneous equations system with the interest rate and the exchange rate as the endogenous

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* This is a companion piece to Akiba (1998) which is an outgrowth of a graduate independent seminar on Advanced Macroeconomics at Waseda University in the spring semester of 1997. I have benefited from stimulating discussions with and constructing comments by the participating graduate students, especially Kenshi Takeda, Yoshiaki Nagai, and Takayuki Tsuruga. The usual disclaimer applies with respect to all remaining errors. This research was financed in part by the Waseda University Grant for Special Research Projects in 1998.

variables. Thus, once the original equilibrium point is disturbed by some shock, the time paths of adjustment towards the new equilibrium can be easily traced dynamically. Secondly, if the rate of interest is slow to adjust, then the simultaneous difference equations system is an alternative way for formulation to reconsider the problem at hand.

It will be shown that the "one-sidedness" (or only "forward-looking") of the theoretical solution to the flexible-price monetary model (i.e., only the expected *future* forcing variables matter) is not replicated here,[1] but the "two-sidedness" is derived from our solutions (i.e., both *past and future* forcing variables matter for exchange rate determination). This characteristic of the solution definitely alleviates empirical studies, simply because our solution validates the empirical models formulated to estimate the exchange rate with an AR or a VAR model using only the exogenous and predetermined variables.

II. The Model

In order to emphasize our purpose of this paper mentioned at the outset with greater clarification, let me begin with the so-called portfolio-balance model of exchange rate determination.[2] The model was extended by Branson (1979), and Dooley and Isard (1983). The model adopted here, however, resembles more closely to Akiba's (1993) version, suppressing the expected rate of inflation from the model.[3] A small open economy is assumed to consist of two broadly defined markets, one is the asset market and the other is the good market. However, following Isard (1995), the latter is

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[1] This one-sidedness, requiring observations of the expected future forcing variables, has been particularly troublesome for empirical studies, simply because expectations are unobservable. Thus, arbitrariness comes in when the asset market models are estimated by replacing those expectations with observed variables.

[2] For references to the portfilio-balance model, see, e.g. Isard (1995) and Taylor (1995a,b).

[3] For an extensive analysis of a portfilio-balance model of an open economy, see, e.g. Allen and Kenen (1980).

assumed to be imn equilibrium. The former, asset market, is represented by the following four equations:

$$M/P = m(r,\ r^*,\ Q)W \qquad (1)$$

$$B/P = b(r,\ r^*,\ Q)W \qquad (2)$$

$$sF/P = f(r,\ r^*,\ Q)W \qquad (3)$$

$$W = (M + B + sF)/P \qquad (4)$$

where M= nominal stock of money supply, B=nominal stock of domestic bonds, F=nominal stock of foreign bonds held by domestic residents, P=the national price level, s=the spot exchange rate (defined as number of units of domestic currency needed in order to purchase one unit of foreign currency), r=the nominal rate of interest (* means the foreign counterpart), Y=real output, and W=the real wealth. Q denotes the vector of other variables that are relevant to home residents.

The composition of private portfolio in an small open economy is assumed to depend on the own-currency rates of return on domestic and foreign bonds, together with a vector of other relevant variables, which conventionally includes a variable measuring the scale of monetary transactions within the country. The rates of return reflect the own-rates of interest on home and foreign bonds, respectively denoted by r and r^*.

The following signs are assumed with respect to the partial derivatives for the three demand functions, m, b, and f (a suffix means a oartial derivative):

$$\begin{aligned} &m_r < 0, \qquad m_{r^*} < 0 \\ &b_r > 0, \qquad b_{r^*} < 0 \\ &f_r < 0, \qquad f_{r^*} > 0 \end{aligned} \qquad (5)$$

Using equation (4), the equilibrium conditions for the money, domestic and foreign bond markets are summarized in a single, well-known expression:

$$m + b + f = 1 \qquad (6)$$

Partial differentiation of (6) with respect to r and r^*, and the condition (5) yield:

$$m_r + f_r = - b_r < 0 \qquad (7.1)$$

$$m_{r^*} + b_{r^*} = - f_{r^*} < 0 \qquad (7.2)$$

The characteristics of this portfolio-balance model have been well-known (e.g., Isard (1995) chapter 6), and no further explanation may be necessary. It has only to be noted that the expected rate of domestic inflation is assumed to be zero, and the real capital stock is assumed to remain constant. Because of the balance sheet restriction (6), one of the equilibrium conditions (1), (2), and (3) is not independent of the others.

The short-run equilibrium for given stocks (M, B, and F), and the effects of their change are considered. We first solve two equations from (1)-(3) for r and s as a function of given stocks. The solution is given by r=r(M.B.F;Q) and s=s(M.B.F;Q). Differentiation of the solution r with respect to M, B, and F yields:

$$\partial r/\partial M = -bF/D < 0, \quad \partial r/\partial B = mF/D > 0, \quad \partial r/\partial F = 0$$

where $D=WF[mb_r - bm_r] > 0$.

Likewise, differentiation of the solution s with respect to the same variables yields:

$$\partial s/\partial M = -W[fm_r + bf_r + ff_r]/D > 0$$

$$\partial s/\partial B = mfW[\zeta_{mr} - \zeta_{fr}]/(rD) \gtrless 0$$

$$\partial s/\partial F = sW[(1-f)m_r + mf_r]/D < 0$$

where ζ_{mr} (ζ_{fr}) is the interest elasticity of demand for money (foreign bond).

Next, following Torre (1977), Scinasi (1981,1982), and Akiba (1993), suppose an adjustment process in disequilibrium of our model is described by the following two differential equations:

$$dr/dt = \alpha[B_t - b(r_t, r_t^*, Q)W_t] + \eta_{rt} \qquad (8.1)$$

$$ds/dt = \beta[f(r_t, r_t^*, Q)W_t - s_tF_t] + \eta_{st} \qquad (8.2)$$

This short-run dynamic adjustment system for r and s represents disequilibrium in the domestic and foreign asset markets. The adjustment coefficients, α and β, are both assumed to be positive. η_{rt} and η_{st} are iid stochastic disturbances for sr/dt and ds/st equation, respectively.

Linear approximation of equation (8) yields the following system:

$$\begin{pmatrix} dr/dt \\ ds/dt \end{pmatrix} = \begin{pmatrix} -\alpha b_r W & 0 \\ \beta f_r W & -\beta F \end{pmatrix} \begin{pmatrix} r_t - r_0 \\ s_t - s_0 \end{pmatrix} + \begin{pmatrix} Q_r \\ Q_s \end{pmatrix} \tag{9}$$

where r_0 and s_0 are the equilibrium values. Q_i is defined by $R_i + \eta_i$ (i=r,s) where R_i is the remainder of linear approximation of di/dt. Denoting the determinant of the Jacobian of (9) by H, it is easy to verify that:

$$\text{trace (H)} = -[\alpha b_r W + \beta F] < 0, \text{ and } \quad |H| = \alpha\beta b_r WF > 0$$

Thus, the equilibrium point is "asymptotically globally stable", as proved in, e.g. Takayama (1994)(theorem 7.4, p.406).[4]

The exchange rate is a "jump variable" (Taylor (1995a,b)) of the model that compensates for stickiness in other variables, including another endogenous variable, the interest rate in our model. Thus, exchange rate expectations are related to expectations about other forcing variables in a forward-looking manner, while the interest rate behaves sticky in a backward-looking manner (Isard (1995), Taylor (1995a,b), MacDonald and Taylor (1992)). These two endogenous variables of the model are considered to be different in nature.

Therefore, it is not necessarily considered to be an appropriate way to formulate the model by a differential equations system (5). An alternative way for formulating the model is by a difference equations system. Although it could be argued that the exchange rate behaves rather smoothly compared to the interest rate, so that the discrete-time framework also has some drawbacks, an attempt to do so has been made at least for the exchange rate (Isard (1995) chapters 5,7, and 8). In fact, it has been well known that the rational expectations solution of the discrete-time flexible price monetary

[4] The two eigenvalues are shown to be real. Because the determinant is positive, but the trace is negative, the equilibrium point is a node. See Takayama (1994), p.407.

model is expressed as the discounted present value of the vector of the expected forcing variables in infinite future (Isard (1995), p,127; Taylor (1995a), p.22; MacDonald and Taylor (1992), p.5). However, if the exchange rate behavior is formulated with the interest rate behavior in a simultaneous equation system, this "one-sidedness" of expectations seems inappropriate. The reason for it lies in a simple observed fact that the interest rate behaves quite sticky in the short and medium runs, compared with the exchange rate.

If equation (8) is approximated by the difference equations with transformation by $dx/dt = x_{t+1} - x_t$ for r and s, we obtain:

$$\begin{pmatrix} r_{t+1} \\ s_{t+1} \end{pmatrix} = \begin{pmatrix} 1-\alpha b_r W_t & 0 \\ \beta f_r W_t & 1-\beta F_t \end{pmatrix} \begin{pmatrix} r_t \\ s_t \end{pmatrix} + \begin{pmatrix} Q_{r,\,t+1} \\ Q_{s,\,t+1} \end{pmatrix} \tag{10}$$

which is compactly denoted by vectors and matrices as:

$$x_{t+1} = Ax_t + \varepsilon_{t+1}, \qquad \text{or} \qquad x_t = Ax_{t-1} + \varepsilon_t \tag{11}$$

where $Q_{r.t+1} = R_{r.t+1} + \eta_{r.t+1}$ and $Q_{s.t+1} = R_{s.t+1} + \eta_{s.t+1}$.

If the remainders are disregarded. Q_r and Q_s consist of unanticipated shocks, η_r and η_s, so that $\varepsilon_{t+1} = (Q_{r.t+1} \;\; Q_{s.t+1})' = (\eta_{r.t+1} \;\; \eta_{s.t+1})'$. It is assumed here that ε_{t+1} is a vector of white noise with mean zero and contemporaneous covariance matrix $E[Q_tQ_t'] \equiv V$, a 2x2 matrix.[5] It is also assumed that $E[Q_tQ_{t-k}] = 0$ for all $k \neq 0$.

Applying the lag operator to equation (8) yields:

$$(I - AL)x_t = C(L)x_t = \varepsilon_t \tag{12}$$

where I is a (2x2) identity matrix and $C(L) \equiv (I-AL)$ is given by:

[5] V is given by:

$$V = \begin{pmatrix} \sigma_r^2 & \sigma_{rs} \\ \sigma_{rs} & \sigma_s^2 \end{pmatrix}$$

where σ_r and σ_s are the standard deviations of r and s, and σ_{rs} is the covariance between them. They are assumed to be constant.

$$C(L) = \begin{pmatrix} 1-(1-\alpha b_r W_t)L & 0 \\ -\beta f_r W_t L & 1-(1-\beta F_t)L \end{pmatrix} \quad (10)$$

The matrix C(L) is assumed to have an inverse under convolution $C(L)^{-1} \equiv B(L)$, where $C(L)^{-1}$ is defined as the matrix that satisfies $C(L)^{-1}C(L) = I_{(2x2)}$. $I_{(2x2)}$ means the (2x2) identity matrix. If it exists as assumed, $C(L)^{-1}$ can be determined in the following way (Protter and Morrey (1972), chapter 10, Sargent (1987), chapter 11):

First evaluating the z-transformed matrix C(L) at $z=e^{-i\omega}$ yields $C(e^{-i\omega})$. Then, inverting $C(e^{-i\omega})$ frequency by frequency yields $C(e^{-i\omega})^{-1}$. Finally, the matrix coefficients $C(L)^{-1} = B(L) = \sum_{j=0}^{\infty} B_j L^j$, B_j being a 2x2 matrix, can be found from the inversion formula of the Fourier transform using a Lebesgue integral:

$$B_j = \frac{1}{2\pi}\int_{-\pi}^{\pi} C(e^{-i\omega})^{-1} e^{i\omega j} d\omega \quad (14)$$

where by integrating a matrix, element-by-element integration is calculated.

Because in our exchange rate dynamics C(z) is given by:

$$C(z) = \begin{pmatrix} 1-(1-\alpha b_r W_0)z & 0 \\ -\beta f_r W_0 z & 1-(1-\beta F_0)z \end{pmatrix}$$

where W_0 and F_0 are the initial values. Thus, $C(e^{-i\omega})$ is given by:

$$C(e^{-i\omega}) = \begin{pmatrix} 1-(1-\alpha b_r W_0)e^{-i\omega} & 0 \\ -\beta f_r W_0 e^{-i\omega} & 1-(1-\beta F_0)e^{-i\omega} \end{pmatrix} \quad (15)$$

Then, the inverse of (15) is calculated as:

$$C(e^{-i\omega})^{-1} = \frac{1}{D}\begin{pmatrix} 1-(1-\beta F_0)e^{-i\omega} & 0 \\ \beta f_r W_0 e^{-i\omega} & 1-(1-\alpha b_r W_0)e^{-i\omega} \end{pmatrix} \quad (16)$$

where D is the determinant of (15), given by $D \equiv [1-(1-\beta F_0)e^{-i\omega}][1-(1-\alpha b_r W_0)e^{-i\omega}]$, which is assumed not to be vanished.

Since we have assumed that $E[\varepsilon_t \varepsilon_{t-k}] = 0$ for all $k \neq 0$, so that the random process ε_t is said to be *covariance stationary* (or, *wide-sense stationary*), our solution given by $x_t = C(L)^{-1}\varepsilon_t = B(L)\varepsilon_t$ is in effect to define a new random process x_t by ε_t through filtering. In other words, our solution $B(L)x_t$ is to decompose the variance of x_t by frequency of the spectrum. Thus, for $B(e^{-i\omega})=(B_{ij})$ $(i,j=1,2)$ in equation (16), suppose, for example,

$$B_{11}(e^{-i\omega}) = \begin{cases} 1/2 & \text{for } \omega \in [a, b] \cup [-b, -a], \quad 0 < a < b < \pi \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

which means that the random process ε_t is transformed into another random process x_t by selecting a filter (17). A filter obeying (17) shuts off all the spectral power for frequencies outside the union of the closed region $[a,b]$ and $[-b,-a]$.

$B_{11}(e^{-i\omega})$ can be easily determined by equation (14) as follows:

$$B_{11h} = \frac{1}{2\pi}\left(\frac{\sin(bh)-\sin(ah)}{h}\right) \quad \text{for all integer } h$$

As shown in the Appendix section, the other B's are also inverted based on equation (17) as follows:

$$B_{12h} = 0$$

$$B_{21h} = \frac{1}{2\pi}\left(\frac{\beta f_r W_0}{(1-\alpha b_r W_0)+(1-\beta F_0)}\right)\left(\frac{\sin(bh)-\sin(ah)}{h}\right)$$

$$B_{22h} = \frac{1}{2\pi}\left(\frac{\sin(bh)-\sin(ah)}{h}\right) \quad \text{for all integer } h$$

Thus, the final form of our solution is described as:

$$\begin{pmatrix} r_t \\ s_t \end{pmatrix} = \sum_{h=-\infty}^{\infty}\left(\frac{1}{2\pi}\right)\left(\frac{\sin(bh)-\sin(ah)}{h}\right)\begin{pmatrix} 1 & 0 \\ \dfrac{\beta f_r W_0}{(1-\alpha b_r W_0)+(1-\beta F_0)} & 1 \end{pmatrix}\begin{pmatrix} \varepsilon_{r,t-h} \\ \varepsilon_{s,t-h} \end{pmatrix} \tag{18}$$

for all integer h. Thus, r_t and s_t are expressed in explicit forms as:

$$r_t = \frac{1}{2\pi}\sum_{h=-\infty}^{\infty}\left(\frac{\sin(bh)-\sin(ah)}{h}\right)\varepsilon_{r,t-h} \tag{19.1}$$

$$s_t = \frac{1}{2\pi}\sum_{h=-\infty}^{\infty}\left(\frac{\sin(bh)-\sin(ah)}{h}\right)\left\{\left[\frac{\beta f_r W_0}{(1-\alpha b_r W_0)+(1-\beta F_0)}\right]\varepsilon_{r,t-h} - \varepsilon_{s,t-h}\right\} \tag{19.2}$$

Some important implications are derived from this results (18), or (19), compared with those from the differential equation version. First of all, r_t and s_t are jointly determined variables depending on the same sets of underlying variables, although their dynamic time paths are apparently different. Secondly, suppose $m_t = m_0 + u_{mt}$ where m_0 is a constant, but u_{mt} is an i.i.d. disturbance term, representing unanticipated part of monetary policy of the economy. By the assumption that the inverse relationship exists between the demand for money and the interest rate, it can be assumed simply that $\varepsilon_{r,t} = \equiv \eta_{r,t} = u_{mt}$. Then, substituting $\varepsilon_{r,t} = - u_{mt}$ into (18) yields:

$$\begin{pmatrix} r_t \\ s_t \end{pmatrix} = \sum_{h=-\infty}^{\infty}\left(\frac{1}{2\pi}\right)\left(\frac{\sin(bh)-\sin(ah)}{h}\right)\begin{pmatrix} 1 & 0 \\ \frac{\beta f_r W_0}{(1-\alpha b_r W_0)+(1-\beta F_0)} & 1 \end{pmatrix}\begin{pmatrix} -1 \\ -1 \end{pmatrix} u_{mt-h} \tag{20}$$

for all integer h. Note that the sign of the product of the matrix and the vector in (20) is negative for r_t but ambiguous for s_t. However, whether the overall effect of u_{mt-h} on r_t and s_t is plus or negative depends entirely on the sign of the first summation term of circular functions in (20), which is of either sign.[6] Thus, we can *no longer* predict the sign of the effect of u_{mt-h} on s_t within a portfolio-balance model. But, it can be observed that, in the long-run as $h \to \infty$, the effect of u_{mt-h} on logs of the exchange rate (s_t) and the interest rate (r_t) is one-to-one, as Dornbusch (1976) predicted for a sticky-price monetary model.

[6] For example, if $b=\pi/2$ and $a=\pi/6$, then, for h=1,...,10, we have the value of [sin(bh)-sin(ah)]/h=(1/2, -$\sqrt{3}/4$, -2/3, -$\sqrt{3}/8$, -1/10, 0, -1/14, $\sqrt{3}/16$, 2/9, $\sqrt{3}/20$). The simple summation turns out to be negative in this case.

Thirdly, there is a basic difference in stability property. Although the differential equation version of the model (9) is *not* asymptotically stable, but only saddle point stable, the difference equation version (10) is covariance stationary because Q_t is covariance stationary (or wide-sense stationary).

Finally, and the most important for our present purposes, the rational expectations solutions for r_t and s_t depend on both (1) all the past movements, as well as (2) all the future movements, of shocks, as clearly described in equation (20). As mentioned earlier, the rational expectations solution for the flexible-price monetary model depends on the discounted values of the vectors of the forcing variables in all future periods only. [7] Contrary to it, our solutions for the sticky-price monetary model were shown to depend on the shocks in infinite past, as well as those in infinite future.

The last point has an empirical implication for exchange rate economics. Although sin(bh) and sin(ah) has the period of $(2\pi/b)$ and $(2\pi/a)$ with $2\pi/a > 2\pi/b$, the amplitude is (1/h) for both sin (bh) and sin(ah). Because both ε_{rt} and ε_{st} are assumed to be covariance stationary, and their coefficients in (20) are constant, then their effects are fading away as $h \rightarrow \pm\infty$. Thus, the time-varying amplitude in this case has a similar effect as the discounted factor does in the flexible-price monetary model (MacDonald and Taylor (1992), p.5). In other words, sample observations of ε_{rt-h} and ε_{st-h} need not be infinite $(\pm\infty)$.

Before leaving this section, one more comment is in order. Obstfeld and Rogoff (1995) showed that, making use of their two-country dynamic model based on the intertemporal approach and the sticky-price Keynesian approach, the exchange rate "jumps" immediately to its long-run level when prices are unable to adjust in the short-

[7] Rational bubbles are assumed away here. For rational bubbles and its related topics, see Taylor (1995a, p.22; 1995b pp.38-9), and MacDonald and Taylor (1992, pp.13-5).

run.[8] However, it can easily be observed from our solution (20) that the spot exchange rate is still shown to *fluctuate*, depending on circular function, despite the inability of prices to adjust in the short-run.

According to Obstfeld (1997), "real and nominal exchange rate movements are nearly perfectly correlated in the short-run" (p.3), and we can observe "a persistent effect of nominal shocks on real exchange rates" or "the persistence of monetary effects on real exchange rates" (p.8). Thus, our stochastic difference equations model predicts more plausible short-run behavior of the exchange rate than that of Obstfeld and Rogoff (1995).

III. Concluding Remarks

This article considers an appropriate and consistent model of rational expectations version of asset market approach to the exchange rate determination for the both purposes of theoretical and empirical analysis. Using the portfolio-balance model, it was shown that both the interest rate and the exchange rate are a function of a set of forcing variables that extends from present to both directions of infinite future and infinite past. At first glance, this characteristic of the solution seems to be more restrictive than the one-sidedness required for the solution of the flexible-price monetary model.

However, as is clear from the solution (20), the cyclical function part of the solution tends to be negligibly small as time h tends to be away from present for both directions. Thus, the solution (20) can be thought as a generalization of those for the flexible-price

[8] However, Obstfeld and Rogoff (1995, p.644) wrote that "...the lack of empirical support for the overshooting hypothesis...". They also wrote in another place that "the evidence in support of overshooting is thin indeed" (Obstfeld and Rogoff (1996), p.678). For an explanation for it, see Akiba (1996).

monetary model. The reason for it lies in a simple fact that, in the modern theory of asset market approach to the exchange rate determination, the exchange rate is considered as a "jump" variable (Isard (1995, p.118), Taylor (1995a, p.230; 1995b, p.39) in one direction, or a "forward-looking" variable, while the "other variables" (Taylor (1995a, p.23)) are not jump but "backward-looking" variables (Kawai and Murase (1990, p.57), Stansfield and Sutherland (1995, p.221)). As Obstfeld and Rogoff (1996, p.525) write, "adaptive" manner is an example of backward-looking behavior. Thus, our reduced forms for the exchange rate and the interest rate are interpreted to represent these two characteristics in a simple but a plausible model. It seems to the author that this characteristics also reflect the basic idea of rational expectations underlying the modern asset view of the exchange rate determination.

APPENDIX

This section develops and briefly sketches some of the algebra underlying the results of B_{ijh} (i,j=1,2) in section II.

If $B_{11}(e^{-i\omega}) = 1/2$ for $\omega \in [a,b] \cup [-b,-a]$, $0<a<b<\pi$, then, $[1-(1-\beta F_0)\, e^{-i\omega}]/D = 1/2$ means that, after some rearrangement,

$$e^{-i\omega} = -1/(1-\beta b_r W_0) \tag{A-1}$$

On the other hand, the inversion formula means that

$$B_{11h} = \frac{1}{2\pi}\int_{-\pi}^{\pi} B_{11}(e^{-i\omega})e^{i\omega h}d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2}e^{i\omega h}d\omega = \frac{1}{2\pi}\int_{-b}^{-a}\frac{1}{2}e^{i\omega h}d\omega + \frac{1}{2\pi}\int_{a}^{b}\frac{1}{2}e^{i\omega h}d\omega$$

$$= \frac{1}{4\pi}\int_{a}^{b}(e^{i\omega h} + e^{-i\omega h})d\omega = \frac{1}{4\pi}\int_{a}^{b}2\cos(\omega h)d\omega = \frac{1}{2\pi}\int_{a}^{b}\cos(\omega h)d\omega$$

$$= \frac{1}{2\pi}\left[\frac{\sin(\omega h)}{h}\right]_a^b = \frac{1}{2\pi}\left[\frac{\sin(bh)-\sin(ah)}{h}\right] \quad \text{for all integer } h \tag{A-2}$$

Because $B_{12}(e^{-i\omega}) = 0$, it is trivial that $B_{12h} = 0$. Next, because $B_{11}(e^{-i\omega}) = 1/2$ for $\omega \in [a,b] \cup [-b,-a]$, $0<a<b<\pi$, $B_{21}(e^{-i\omega}) = -\beta f_r W_0\, e^{-i\omega}/D = \beta f_r W_0 /2[(1-\alpha b_r W_0)+(1-\beta F_0)]$. Thus, using (A-1), the inversion formula for B_{21h} is given by:

$$B_{21h} = \frac{1}{2\pi}\int_{-\pi}^{\pi} B_{21}(e^{-i\omega})e^{i\omega h}d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\beta f_r W_0}{2[(1-\alpha b_r W_0)+(1-\beta F_0)]} e^{i\omega h}d\omega$$

$$= \frac{1}{2\pi} \frac{\beta f_r W_0}{[(1-\alpha b_r W_0)+(1-\beta F_0)]} [\frac{\sin(bh)-\sin(ah)}{h}] \quad \text{for all integer } h \qquad \text{(A-3)}$$

Likewise, we have $B_{22h}(e^{-i\omega})= 1/2$, so that $B_{22h}(e^{-i\omega})= B_{11h}(e^{-i\omega})$, and the inverted B_{22h} is the same as (A-2).

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