Role of Expectations in Commodity Futures Markets

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ABSTRACT

This paper considers the simultaneous determination of cash and futures prices of a commodity. The model assumes both hedging and speculative behavior in the demand for and the supply of a futures commodity. Current consumption demand is formulated to depend on the expected income which is assumed to be formed adaptively. However, the demand for and the supply of futures trading are assumed to depend on price expectations that are formed rationally within our model. From the explicit solutions of the model, the role played by rational expectations for cash and futures prices is examined in detail.

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I. Introduction*

The existence of a commodity futures market has attracted much attention because of several interrelated questions which are interesting from both the theoretical and policy-makers' standpoints. They are briefly summarized in four major issues (Stein, (1986)): (1) the intertemporal allocation of resources, (2) the informational issue concerning future demand and supply, (3) the risk premium, and (4) the welfare consideration.[1]

This paper focusses on the second and the third problems, and presents a theoretical model, paying special attention to an *endogenous* expectations mechanism, which is used to determine the cash and the futures prices simultaneously. The commodity under consideration is assumed to be storable. Because my focal point is the simultaneous determination of the cash and the futures prices, and not the causal relationship between them, some simplifying assumptions are imposed on similar traditional models such as those constructed by Peston and Yamey (1960), Goss and Giles (1986), Goss, Chan, and Avsar (1992), or Goss and Avsar (1992). Assuming familiar rational expectations in this field of research, both the cash and the futures prices are solved explicitly and expressed by the state variables and the underlying parameters in the model. Making use of these expressions, some comparative statics exercise, with respect to the expectations parameters, is developed.

The structure of the paper is as follows. In the next section II the

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[1] For the problem of intertemporal resource allocation, see Britto (1984), Diamond (1980), Feder, Just, and Schmitz (1980), and Giles, Goss and Chin (1985). For the informational issue, see McKinnon (1967), Danthine (1978), Bigman, Goldfarb and Schechtman (1983), Kaminsky and Kumar (1990). For the issue of the risk premium, see, e.g. Bray (1985), Kawai (1983), and Goss (1992). Diamond (1980), Britto (1984), and Akiba, Britto and Watanabe (1996 and 1998) considered the welfare implication of futures market.

model is presented and derived to the semi-reduced forms. Section III imposes rational expectations to close the model, and derives the true reduced forms for the cash and the futures prices. In order to emphasize our main focal point, i.e., the effects of expectations, the comparative statics exercise of the expectations parameters is developed in section IV. The last section V concludes the paper.

II. The Model

This section constructs a model of a commodity market for simultaneous determination of the spot and the futures prices with endogenous expectations. The model is, however, solved for the semi-reduced forms in the sense that, the expectations appearing in the structural equations are regarded as the exogenous variables in this section. The full reduced forms with endogenous expectations are obtained in the next section III.

The model developed in this section has its theoretical foundations in Peston and Yamey (1960), whose model has four categories of agents; (1) short hedgers, (2) long speculators in futures, (3) holders of unhedged inventories, and (4) consumers. But the Peston and Yamey (1960) model does not include any explicit representation of expectations. The model also draws on the approaches of Goss and Giles (1986), Goss, Chan, and Avsar (1992), and Goss and Avsar (1992), whose models contain behavioral relationships for long and short hedgers, and for long and short speculators in futures trading. The model presented here therefore comprises separate functional relationships for four groups of agents. Market participants under consideration in this model are called hedgers and speculators when they take either long or short positions in futures markets for hedging and speculative purposes, respectively. They are called holders of unhedged inventories when they purchase stocks spot with the expectation of a rise in the spot price. They are also called consumers when they purchase stocks for the use of current consumption.

Our stochastic version of a typical commodity market with futures is contained in equations (1) - (6). The model is adopted, with necessary modification, from a linearized model employed by Goss, Chan, and Avsar (1992) and Goss and Avsar (1992).

$$HSS_t = a_1 + a_2 P_t + a_3 P^*_{t+1} + a_4 NK^*_t + a_5 Z_t + e_{1t} \quad (1)$$

$$HSL_t = a_6 + a_7 (P_t - A_t) + a_8 (P_{t+1} - A_{t+1})^* + a_9 X_t + a_{18} C_t + e_{2t} \quad (2)$$

$$U_t = a_{10} + a_{11} A_t + a_{12} A^*_{t+1} + a_{13} Z_t + e_{3t} \quad (3)$$

$$C_t = a_{14} + a_{15} A_t + a_{16} A_{t-1} + a_{17} Y_t + a_{19} C_{t-1} + e_{4t} \quad (4)$$

$$HSS_t = HSL_t \quad (5)$$

$$K_t = U_t + HSL_t \quad (6)$$

where HSS_t = the supply of futures contracts for both hedging and speculative purposes, HSL_t = the demand for futures contracts for both hedging and speculative purposes, U_t = the demand for unhedged inventories, C_t = the demand for current (flow) consumption, P_t = the current futures prices of the commodity, A_t = the current cash (spot) price of the commodity, X_t = a measure of exogenous factor (e.g., planned export by hedgers at time t+1 (known and realized at time t, for simplicity)), K_t =

the total stock of the commodity, NK_t = part of K_t eligible for hedging, [2] Y_t = the real personal income, Z_t = either the marginal cost of storage (m), or the marginal opportunity cost of storage (r). Suffixes t denote the time period, and asterisks (*) denote expected values where:

$$Z^*_{t+j} = \underset{t-1}{E}[Z_{t+j} \mid \Omega_{t-1}] \quad j \geqq 0, \; Z = P, A \tag{7}$$

Ω_{t-1} denotes a set of observations on variables dated t-1 and earlier. For simplicity's sake, e_{it} (i=1,...,4) are assumed to be serially uncorrelated and independently distributed random error terms obeying:

$$\underset{t-1}{E}[e_{jt+j} \mid \Omega_{t-1}] = 0, \qquad i = 1,\ldots,4, \; j \geqq 0 \tag{8}$$

Equation (1) is the supply function of futures contracts, reflecting sales of futures by both short hedgers and short speculators. The former is assumed to be an increasing function of the forward premium, thus increasing in P_t. The latter is assumed to vary directly with P_t and inversely with P_{t+1}^* and Z_t.[3] Sales of futures are also assumed to depend positively on NK_t, and negatively on Z_t. Thus, the assumed signs of the coefficients are a_1 constant, a_2 , a_4 > 0, and a_3 , a_6 < 0. The error term e_{1t}, which is assumed to incorporate supply shocks for both hedgers and speculators, makes the supply schedule of futures contracts shift randomly.

Equation (2) is the demand function for futures contracts, reflecting purchases of futures by both long hedgers and long speculators. Their demand for futures is assumed to vary inversely with the current forward

[2] If K_t-NK_t > 0, this difference can possibly be taken by government agencies. Goss and Avsar (1992) pointed out the case of the wool market in Australia where a government agency holds supply stocks in order to administer a price support scheme.

[3] This assumes risk aversion.

premium $(P_{t+1} - A_{t+1})^*$.[4] X_t represents the commitments of long hedgers, and it is assumed that current planned exports are realized one period later. It is assumed that HSL_t varies directly with X_t and C_t. Thus, the assumed signs of the coefficients are, a_6 constant, $a_7 < 0$, and a_8, a_9, and $a_{18} > 0$. The error term e_{2t}, which is also assumed to incorporate unexpected demand shocks for both hedgers and speculators, makes the demand schedule of futures contracts shift randomly.

Equation (3) is the demand function for unhedged storage. These inventories are assumed to be held by those who expect the spot price to rise in the next period.[5] Thus, this relationship exhibits the characteristics of a speculative demand function. Specifically, U_t is assumed to vary directly to the expected spot price, and inversely to the current spot price, the marginal net cost of storage (m_t) and the marginal opportunity cost (r_t) (Dewbre, (1981)). The assumed signs of the coefficients are thus a_{10} constant, a_{11}, $a_{13} < 0$, and $a_{12} > 0$. The error term e_{3t}, which is also assumed to represent unexpected demand shocks, makes the demand schedule U_t of unhedged storage shift randomly.

Equation (4) represents the current consumption of the commodity. For commodities such as oats (Goss, Chan, & Avsar, (1992)) or wool (Goss & Avsar, (1992)), the current consumption is one of intermediate inputs for the final products.[6] Then, C_t exhibits the characteristics of the derived demand function which is assumed to have a form of:

$$C_t = a'_{14} + a'_{15}A_t + a'_{16}Y^*_{t+1} \tag{9a}$$

[4] Alternatively, purchasers of futures by long-speculators could be specified to depend negatively on $(P_t - P_t^*)$ and positively on $(P_{t+1} - P_{t+1}^*)^*$, where P^* denotes the futures price of the closely-related commodity (substitute). See, e.g., Goss, Chan and Avsar (1992), pp. 136-7.

[5] It can also be thought that these inventories include those held by a government agency that sells the commodity at times of buoyant demand and buys it at times of deficient demand. See, e.g., Goss and Avsar (1992), p. 194.

[6] If there are more than two intermediate inputs for the final output, the spot price of either complementary or substituting input is an additional independent variable in (9a) (see Goss, Chan and Avsar (1992), p. 138). Such a possibility is precluded in the present investigation for ease of calculation.

where a_{14}' is a constant, and the sign of parameter a_{15}' is assumed to be negative, but that of a_{16}' is positive. Because Y is an aggregate real variable, its expectation is assumed to be formed rather gradually. To be more specific, the adaptive expectations hypothesis of Nerlove type is assumed here:

$$\Delta Y^*_{t+1} = Y^*_{t+1} - Y^*_t = \alpha\,(Y_t - Y_t^*) \quad (1>\alpha>0) \tag{9b}$$

and the solution is given by:

$$Y^*_{t+1} = (1-\beta)\,Y_t/(1-\beta L) \tag{9c}$$

where $\beta = 1-\alpha$, and L is the lag operator. Substituting (9c) into (9a) yields:

$$C_t = a_{14}' + a_{15}' A_t - a_{15}'\beta A_{t-1} + a_{16}'(1-\beta)\,Y_t + \beta\, C_{t-1} \tag{9d}$$

Redefining the coefficients in (9d) and adding a similar stochastic error term to it yields equation (4). Thus, the assumed signs of the coefficients in (4) are a_{14} constant, $a_{15} < 0$, $a_{16}, a_{17} > 0$ and $1>a_{19}>0$.

Equations (5) and (6) are the market clearing (equilibrium) conditions for both flows and stocks of the commodity, respectively. The former signifies that, for simplicity's sake, the flow supply of futures contracts (HSS_t) must always be equal to the flow demand for them (HSL_t).[7] The latter means that the given endowment stocks of the commodity (K_t) in each period are comprised of unhedged stocks (U_t) and hedged stocks (HSL_t).

Equations (1) - (6) determine the equilibrium values of P_t, A_t, HSS_t, HSL_t, U_t, and C_t as a function of the time paths of exogenous and predetermined variables, the random errors (e_{it}), and expectations. To

[7] An alternative specification that makes distinction between hedging and speculation is possible. See, e.g., Giles, Goss and Chin (1985) or Goss, Chan and Avsar (1992).

close the model, rationality is imposed, so that the expectations appearing in the system of equations (1)-(6) are linear least squares forecasts conditional on Ω_{t-1}.[8]

To determine the equilibrium paths of endogenous variables, solve (1) - (6) for the values of P_t and A_t that clear the flow and the stock markets of the commodity simultaneously. These semi-reduced forms are:[9,10]

$$P_t = \alpha_0 + \alpha_1 P^*_{t+1} + \alpha_2 A^*_{t+1} + \alpha_3 K_t + \alpha_4 Z_t + \alpha_5 X_t + \alpha_6 e_{1t} + \alpha_7 e_{2t} + \alpha_8 e_{3t} \qquad (10)$$

where:
$\alpha_0 = [(a_6-a_1)\alpha_7+(a_1+a_{10})\alpha_8]$ $\qquad \alpha_1 = (a_8\alpha_7+a_3\alpha_6)$
$\alpha_2 = (a_{12}\alpha_8-a_8\alpha_7)$ $\qquad \alpha_3 = -\alpha_8+\delta\cdot a_4\alpha_6$
$\alpha_4 = a_{13}\alpha_8+a_5\alpha_6$ $\qquad \alpha_5 = a_{10}\alpha_7$
$\alpha_6 = \alpha_8-\alpha_7$ $\qquad \alpha_7 = a_{11}/H$
$\alpha_8 = a_7/H$ $\qquad H = (a_2-a_7)(a_{11}-a_7)-a_7^2$

$$A_t = \beta_0 + \beta_1 P^*_{t+1} + \beta_2 A^*_{t+1} + \beta_3 K_t + \beta_4 Z_t + \beta_5 X_t + \beta_6 e_{1t} + \beta_7 e_{2t} + \beta_8 e_{3t} \qquad (11)$$

where:
$\beta_0 = (a_1+a_{10})\beta_6+(a_6+a_{10})\beta_7$ $\qquad \beta_1 = (a_8\beta_7+a_3\beta_6)$
$\beta_2 = a_{12}\beta_6+(a_{12}-a_8)\beta_7$ $\qquad \beta_3 = -\beta_7-(1-\delta a_4)\beta_6$
$\beta_4 = (a_5+a_{13})\beta_8+a_{13}\beta_7$ $\qquad \beta_5 = a_{10}\beta_7$
$\beta_6 = a_7/H$ $\qquad \beta_7 = -a_2/H$
$\beta_8 = \beta_6+\beta_7$

To determine (10) and (11), NK_t is replaced by δK_t $(1 \geq \delta \geq 0)$ by

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[8] For notational simplicity, we use the notation $E[Y_{t+j}]$, $j \geq 0$ to refer to $E[Y_{t+j} \mid \Omega_{t-1}]$.

[9] These are semi-reduced forms because they contain expectations yet to be endogenously determined in the model.

[10] In order to avoid unnecessary complexity and to make calculation easy and tractable, C_t in equation (2) and C_{t-1} in equation (4) are assumed to be hidden in e_{2t} and e_{4t}, respectively. This actually means $a_{16} = a_{15} = 0$ in the following calculation.

assuming that the part of K_t eligible for hedging is a positive fraction of the total stocks.

As discussed earlier, the signs of a_7 and a_{11} are both negative. It is assumed here that the absolute magnitude of $a_7 \equiv \partial HSL_t / \partial P_t$ is greater than that of $a_{11} \equiv \partial U_t / \partial A_t$, implying that the demand schedule for futures contracts is steeper than that for spot unhedged storage. This assumption seems to be legitimate in view of the accumulated empirical evidence; e.g. Goss, Chan, and Avsar (1992, Table 6.3, p.145), or Goss and Avsar (1992, Table 8.1, p.198). Then, $H \equiv (a_2 - a_7)(a_{11} - a_7) - a_7^2$ is unambiguously signed as negative, because $a_{11} - a_7 < 0$.[11]

Taking expectation of the semi-reduced form (10), and successive substitution yields, given A_{t+1+i}^*:

$$P_t^* = [\alpha_0/(1-\alpha_1)] + \sum_{i=0}^{\infty} \alpha_1^{i} M_{t+i}^* + \alpha_2 \sum_{i=0}^{\infty} \alpha_1^{i} A_{t+i+1}^* \qquad (10a)$$

where $M_t \equiv \alpha_3 K_t + \alpha_4 Z_t + \alpha_5 X_t$, the aggregate exogenous variable at time t. Applying a similar procedure to (11), A_t^* is solved, given P_{t+1+i}^*:

$$A_t^* = [\beta_0/(1-\beta_1)] + \sum_{i=0}^{\infty} \beta_2^{i} (M')_{t+i}^* + \beta_1 \sum_{i=0}^{\infty} \beta_2^{i} P_{t+i+1}^* \qquad (11a)$$

where $M'_t \equiv \beta_3 K_t + \beta_4 Z_t + \beta_5 X_t$, another aggregate exogenous variable at time t.

The previous expressions describe some important characteristics of our model. First, P_t and A_t are jointly determined variables depending on the same set of underlying variables, although their dynamic paths need not be identical. Second, both P_t and A_t depend on the time paths, current and

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[11] It follows from (10) that the signs of the coefficients are $\alpha_1, \alpha_2, \alpha_7, \alpha_8 > 0$, $\alpha_3, \alpha_4 < 0$, and the rest are ambiguous. The signs of the coefficients in (11) are $\beta_1, \beta_2, \beta_7, \beta_8 > 0$, $\beta_3, \beta_4 < 0$, and the rest are indeterminate.

expected, of exogenous variables. Thus, our model of a commodity market with futures transaction embodies the main features of the asset market approach for financial assets.

Furthermore, for both (10a) and (11a) to be dynamically stable, $|\alpha_1| < 1$ and $|\beta_2| < 1$ must be satisfied. These conditions are easily shown to reduce to the following sufficient conditions:

$$a_2 + a_3 > 0 \text{ and } a_7 + a_8 < 0 \tag{12}$$

In the following analysis these conditions are assumed to be satisfied.[12] This completes the description of our model. In the next section rational expectations are assumed to close the model.[13]

III. Solutions with Endogenous Expectations

To obtain an explicit solution for P_t and A_t, it is necessary to specify the process generating the exogenous variables in the model, K_t, Y_t, X_t, and Z_t.[14] In order to avoid unnecessary complexity and to ease calculation we consider a simple case where all exogenous variables are independent processes. In other words it is assumed that they are generated by the following simple independent processes:

$$\Delta K_t = K_t - K_{t-1} = K + u_{1t} \tag{13a}$$

$$\Delta X_t = X_t - X_{t-1} = X + u_{2t} \tag{13b}$$

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[12] These sufficient conditions mean that $a_2 > |a_3|$ and $|a_7| > a_8$. They are equivalent to: $\partial HSS_t/\partial P_t > |\partial HSS_t/\partial P^*_{t+1}|$ and $|\partial HSL_t/\partial P_t| > \partial HSL_t/\partial P^*_{t+1}$. These are plausible, and their meanings are easy to understand: The impact effect of a change in the current futures price on the current supply or demand is stronger than that of a change in the expected and unobservable futures price in the next period. Empirical evidence also supports this assumption; see Goss and Avsar (1992), Table 8.1, p.198.

[13] The semi-reduced form for C_t is easily obtained as:

$$C_t = \gamma_0 + \gamma_1 P^*_{t+1} + \gamma_2 A^*_{t+1} + \gamma_3 K_t + \gamma_4 Z_t + \gamma_5 X_t + \gamma_6 e_{1t} + \gamma_7 e_{2t} + \gamma_8 e_{3t} + e_{4t} + \gamma_9 A_{t-1} + \gamma_{10} Y_t$$

where $\gamma_0 = a_{14} + a_{15}\beta_0$, $\gamma_i = a_{15}\beta_i$ $(i=1,\cdots,8)$, $\gamma_9 = a_{16}$ and $\gamma_{10} = a_{17}$.

[14] X_t should be denoted as $X_{t,t+1}$, meaning that it is realized in the next period. However, for simplicity's sake, it is abbreviated as X_t.

$$\Delta Z_t = Z_t - Z_{t-1} = Z + u_{3t} \tag{13c}$$

where K, X, and Z are all non-negative constants (possibly zero), and the u_{it} (i=1, 2, 3) are the unavoidable random shocks pertaining to the i-th stochastic process. These shocks are assumed to be serially uncorrelated and independently distributed random process with zero mean and finite variance. Each exogenous variable is thus assumed to follow a random walk process with a non-negative drift term, but hereafter the drift term is set to zero for simplicity.[16]

To solve the model we use a familiar method of undetermined coefficients, with the details relegated to the Appendix section. Briefly speaking, P and A can be written as time invariant functions with respect to the underlying state variables.[16] The market clearing semi-reduced solutions (10) and (11) thus yield the cross equation restrictions implied by rationality. Neglecting the constant terms, the explicit rational expectations solutions for the futures price and the spot (cash) price are:

$$P_t = Q_1 K_{t-1} + Q_2 X_{t-1} +\text{-} Q_3 Z_{t-1} + [(a_7 - a_{11})/H] e_{1t} + (a_{11}/H) e_{2t} + (a_7/H) e_{3t}$$

$$+ Q_4 u_{1t} + Q_5 u_{2t} + Q_6 u_{3t} \tag{14}$$

where $Q_1 = [(a_7 + a_8)(1-\delta a_4) + \delta a_4 (a_{11} + a_{12})]/E$

$Q_2 = -a_9 (a_{11} + a_{12})/E$

$Q_3 = [a_5 (a_{11} + a_{12}) - (a_7 + a_8)(a_5 + a_{13})]/E$

$Q_4 = [-a_7 + \delta a_4 (a_{11} - a_7)]/H$

$Q_5 = a_9 a_{11}/H$

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15 Because C_t and C_{t-1} in equation (2) and (4) were assumed to be hidden in e_{2t} and e_{4t}, Y is no longer a state variable for P_t and A_t.

16 The reason for it lies in the fact that the structure of the model is not expected to change over time.

$$Q_6 = [a_7 a_{13} + a_5(a_7 - a_{11})]/H$$

$$E = (a_7 + a_8)(a_2 + a_3 + a_{11} + a_{12}) - (a_2 + a_3)(a_{11} + a_{12}) > 0$$

$$A_t = R_1 K_{t-1} + R_2 X_{t-1} + R_3 Z_{t-1} + (a_7/H) e_{1t} - (a_2/H) e_{2t} + [(a_7 - a_2)/H] e_{3t}$$

$$+ R_4 u_{1t} + R_5 u_{2t} + R_6 u_{3t} \qquad (15)$$

where

$$R_1 = -[(a_2 + a_3) - (1 - \delta a_4)(a_7 + a_8)]/E$$

$$R_2 = a_9(a_2 + a_3)/E$$

$$R_3 = [a_{13}(a_2 + a_3) - (a_7 + a_8)(a_5 + a_{13})]/E$$

$$R_4 = [a_2 - a_7(1 - \delta a_4)]/H$$

$$R_5 = -a_2 a_9/H$$

$$R_6 = [a_7(a_5 + a_{13}) - a_2 a_{13})]/H$$

To facilitate our theoretical analysis of the effects of endogenous price expectations below, empirical evidence to date is invoked to determine some of the sign of Q_i and R_i. According to Goss, Chan and Avsar (1992, Table 6.3), a_{11} is significantly negative, and dominates the positive estimated sign of a_{12} in Goss and Avsar (1992, Table 8.1). Thus, it can be assumed that $a_{11}+a_{12} < 0$. Furthermore, the negativity of $a_{11}+a_{12}$ dominates the positive estimate of the sum of the positive a_2 and a_3 in Goss and Avsar (1992, Table 8.1). Therefore, it can also be assumed that $(a_2+a_3)+(a_{11}+a_{12}) < 0$.[17] Thus, our assumptions fed back from empirical evidence, enable us to sign $E > 0$. Then, we can further determine the sign of coefficients in (14) and (15), namely, Q_2, Q_5, $R_2 > 0$ and Q_1, Q_4, R_1, R_3, R_4, $R_6 < 0$ (the rest is undetermined).

These signs are easy to understand once we recognize the simultaneous nature of the cash and futures prices determination process. For example,

[17] It should be pointed out here that, although our model is formulated under rationality, the model of Goss, Chan and Avsar assumed an adaptive expectations mechanism. The estimates of Goss and Avsar were based on rationality, and consistent with our model.

an exogenous increase in K(X) causes an incipient rightward shift of the HSS (HSL) schedule, which has downward (upward) pressure on P_t in the futures market. Thus, Q_1 is negative, while Q_2 is positive. In the spot market the increase in K also causes a rightward shift of K-HSL, because only a part of the increase is absorbed in the futures market. This shift has a downward pressure on A_t and, therefore, R_1 is negative. On the other hand, the increase in X, given the fixed quantity of K, causes a leftward shift of the K-HSL schedule in the spot market. This shift has an upward pressure on A_t and, therefore, R_2 is positive.

The effect of a change in Z (either m or r) is a little more complicated, as it affects two schedules, U+C in the spot market and HSS in the futures market simultaneously. For example, an increase in Z brings about a leftward shift of the U+C and the HSS schedules. The former shift has a downward pressure on A_t and, therefore, R_3 is negative. At the same time, the fall in A_t also causes a leftward shift of the HSL schedule through a fall in U, which has a downward pressure on P_t. But this is not the end of the story, because a fall in A_t causes a leftward shift of the HSL schedule simultaneously. Thus, the sign of Q_3 turns out to be ambiguous (see Figure 1).

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Insert Figure 1 here

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This completes the description of true (explicit) reduced forms of P_t and A_t with our assumption of rationality. Before leaving this section the following fact should be noted: A brief glance and simple comparison between the semi-reduced forms and the true reduced forms, i.e., (10) and (14), and (11) and (15), reveal that their coefficients of exogenous variables (K, X and Z) have exactly the same sign pattern. Thus, it is difficult, if not impossible, to measure the effects of the existence of futures trading by simply observing the signs and the magnitudes of empirical estimates. In the next section the role played by expectations in a typical commodity market with futures trading is theoretically assessed in some detail.

IV The Effects of Expectations

There are three expectations variables, P_{t+1}^*, $(P_{t+1}-A_{t+1})^*$, and A_{t+1}^*, in our structural model. Their effects within our commodity market are reflected by their coefficients, a_3, a_8 and a_{12}, respectively. Thus, if expectations are formed rationally, they will manifest themselves in the coefficients of the exogenous variables, K, X, and Z in the true reduced forms (14) and (15). This section examines the marginal effects of these expectations on P_t and A_t through those exogenous variables.

Table 1 Marginal Effects of Expectations

	Pt (14)			At (15)		
	Q1	Q2	Q3	R1	R2	R3
a3	?	−	+	+	−	−
a8	−	+	?	+	−	?
a12	−	+	−	−	+	−

Table 1 summarizes the marginal effects of expectations on P_t and A_t. For example, the (1, 1) cell of the table means that the marginal effect of P_{t+1}^* on P_t (i.e., a_3) through K_{t-1} (i.e., Q_1) at the initial equilibrium evaluated at a_3=0 is ambiguous, i.e.

$$\left.\frac{\partial Q_1}{\partial a_3}\right|_{P_t,\ a_3=0} \gtrless 0 \qquad (16)$$

Similarly the (1,3) cell means that

$$\frac{\partial R_1}{\partial a_3}\Bigg|_{\substack{A_t \\ a_3=0}} > 0 \qquad (17)$$

and so forth. Thus, although the effect of a marginal increase in P_{t+1}^* on P_t through K_{t-1} is ambiguous, it will definitely weaken the negative effect of R_1 on A_t.

An intuitive explanation behind Table 1 runs as follows: When a_3 is marginally increased, the HSS schedule shifts leftward in the short-run. But because $a_2+a_3 > 0$ is assumed, the HSS schedule shifts back in the long-run. Thus, there is a downward pressure on P_t from the increase in a_3 in the futures market. If K_t increases marginally, the K-HSL schedule in the spot market also shifts rightward. However, because the HSS schedule shifts rightward in the futures market in the long-run, and because HSS=HSL in equilibrium, there also exists a pressure to make the K-HSL schedule shift leftward. According to (17) the latter is stronger than the former, which causes R_1 to increase. As the result of smaller transactions in the spot market, those in the futures market must increase, implying that the HSL is increased. This increase has an offsetting effect on the initial downward pressure on P_t, and the final outcome is ambiguous, as shown in (16). A similar interpretation is possible for the rest of the cell in Table 1.

One remarkable result from Table 1 is the long-run effect of expectations of the futures spot price (A_{t+1}^*) on P_t and A_t, a_{12}. The direction of changes in parameters in both equations (P_t and A_t) is exactly the same. This contrasts keenly with changes in a_3 or a_8, where the signs do not follow any systematic pattern. For example, if it is certain that the future spot price is expected to increase in the market, then we can safely predict that, as long as X stays constant, any marginal exogenous change in either K or Z will make both P and A decrease in the long-run.

When expectations originate either from P_{t+1}^* or $(P_{t+1}-A_{t+1})^*$, the effects are somewhat equivocal. The same sign pattern is observed only in one case where a marginal change in a_3 has a negative impact on both P_t and A_t through X_t. But it has an opposite impact on P_t and A_t through Z_t. It is also observed that a_8 has an opposite impact on P_t and A_t through K_t and X_t.

Next, a familiar result of a "normal backwardation" (Keynes, (1930), Hicks, (1946), and Kawai, (1983)) is examined within our framework. It means that the expected gain from hedging a unit forward contract, $E[A_{t+1}] - P_t > 0$ exists. It is this expected gain that induces speculators to take a long position in the futures market. Subtracting P_t (eq. (14)) from a one-period updated and expected A_{t+1}, i.e. $E[A_{t+1}]$ yields:

$$\overset{E}{t}[A_{t+1}] - P_t = \sum_{i=1}^{3} (\varepsilon_i D_i - Q_{i+3} u_{it}) - [(a_7 - a_{11}) e_{1t} + a_{11} e_{2t} + a_7 e_{3t}]/H \tag{18}$$

where $\varepsilon_i \equiv R_i - Q_i$ $(i=1,2,3)$ and $D \equiv (K, X, Z)$. Simple calculation yields $\varepsilon_1 = -[a_2+a_3+\delta a_4(a_{11}+a_{12})]/E$, $\varepsilon_2 = a_9[a_2+a_3+a_{11}+a_{12}]/E$, and $\varepsilon_3 = [a_{13}(a_2+a_3) - a_5(a_{11}+a_{12})]/E$. According to our assumptions on some of the parameters a_i, it is confirmed that ε_2 and ε_3 are negative, although ε_1 is ambiguous in sign. Thus, it is shown that, starting from the initial equilibrium point of $E[A_{t+1}] = P_t$, a ceteris paribus increase in X makes $E[A_{t+1}] - P_t$ go towards a "contango", while a ceteris paribus decrease in Z makes it go towards a "normal backwardation". The effect of a change in K is ambiguous, as the sign of ε_1 is indeterminate.

Equation (18) also suggests that a case of contango is likely when an unanticipated exogenous increase in X, or an unanticipated shift in the HSL or the U schedule occurs in the market. But an unanticipated exogenous increase in K, or an unanticipated shift in the HSS schedule is likely to bring about a normal backwardation. Furthermore, we can also investigate the effects of marginal change in expectations on ε_i, and hence on

$E[A_{t+1}]-P_t$. Simple computation clarifies these effects which are summarized in Table 2.

Table 2

Effects of Marginal Change in Expectations on $E[A_{t+1}]-P_t$

	ε1	ε2	ε3
a3	+	+	?
a8	?	−	−
a12	−	+	−

Remark: +(−, ?) = $\partial \varepsilon_i / \partial a_j$ is positive (negative, ambiguous)

Thus, out of the total of seven unambiguously signed cases, five cases (a marginal increase in a_3 through K on ε_1, those in a_3 and a_{12} through X on ε_2, a marginal decrease in a_8 and a_{12} through Z on ε_3) are shown to have a tendency towards normal backwardation. Two cases (a decrease in a_{12} through K on ε_1 and that in a_8 through X) unambiguously make $E[A_{t+1}]-P_t$ towards contango.

Although somewhat equivocal results are obtained from the effects of a marginal change in expectations for a_3 and a_8, those for a_{12} $(A_{t+1}{}^*)$ are, as in Table 1, unambiguously signed. For an increase in a_3 through X, both P and E[A] will increase in our model. But a greater increase in $E[A_t]$ than in P_t brings about normal backwardation.[18] On the contrary an increase in a_{12} through K causes both $E[A_t]$ and P_t to decrease, but the former decrease is greater in absolute value than the latter, causing a tendency towards contango. A similar interpretation applies for an increase in Z.

Thus, when an increase in expectations occurs from either $a_8[(P_{t+1}-A_{t+1})^*]$ or $a_{12}(A_{t+1}{}^*)$, a fall in Z makes $E[A_{t+1}]-P_t$ positive, inducing

[18] Note that ε_2 is negative.

speculators to hold a long position of futures contracts. In other words, the commodity futures market in our model provides a mechanism by which risk averse producers are insured by risk averse speculators. In this sense, a positive risk premium $E[A_{t+1}]-P_t$ (> 0) plays the role of the insurance price paid by producers to speculators.[19]

Another implication derived from Table 2 is familiar in these types of models. A quick glance at Table 2 and equation (18) reveals that P_t, the current futures price, is not an unbiased predictor of the future cash price, $E[A_{t+1}]$. As is evident from our discussion so far, this failure of the unbiasedness property is not necessarily evidence of market inefficiency in our model, because expectations are formed rationally using all available information within the mode.[20]

V Concluding Remarks

This paper constructed a simple but typical model of a commodity market with futures trading. Parametrizing expectations of the cash and the futures prices in the next period, these prices were endogenized, and the true reduced forms were derived by using the semi-reduced forms.

The marginal effects of expectations on the cash and the futures prices were examined in some detail, and summarized in Table 1. The marginal effects of expectations parameters on the risk premium were also examined and summarized in Table 2. As observed in these tables, our analyses, which parametrized expectations, made the effects of the resulting changes on the interrelationship between the cash and the futures prices both clearer and tractable. Our Table 2 also explains why, under

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[19] The negative sign of the effect of an increase in a_{12} on ε_1 in Table 2 corresponds to the analysis presented in Figure 2 (p. 930) of Dewbre (1981). Also, the fact that the effect of a decrease in either a_8 or a_{12} on ε_3 is negative in Table 2 corresponds to that of his Figure 3 (p. 931).

[20] See, e.g., Britto (1984), Bigman, et al. (1983) or Kawai (1983).

uncertainty, the futures price is not an unbiased predictor of the cash price in the next period when some of the expectations parameters undergoes a marginal change.

Although our analysis is in line with many preceding works devoted to the explanation of the effects of futures trading on spot price fluctuations, the welfare implication of the existence of the futures trading is totally neglected from the present investigation. I have argued the issue in a different model without rationality elsewhere (Akiba, Britto & Watanabe, (1996) and (1998)). The welfare effects of futures trading with rational expectations seems to be somewhat unresolved and left unanswered, simply because it was presumed that an increase in the spot price fluctuations would be detrimental to welfare. Unfortunately, this presumption was rarely made explicit, and is therefore a future research agenda.

APPENDIX

This appendix develops and briefly sketches some of the algebra underlying the results in section III. To obtain an explicit solution for P_t and A_t in terms of exogenous and predetermined variables, we consider writing P_t and A_t as unknown functions of state variables:

$$P_t = \Pi_{10} + \Pi_{11}K_{t-1} + \Pi_{12}X_{t-1} + \Pi_{13}Z_{t-1}$$

$$+\Pi_{14}e_{1t} + \Pi_{15}e_{2t} + \Pi_{16}e_{3t} + \Pi_{17}u_{1t} + \Pi_{18}u_{2t} + \Pi_{19}u_{3t} \qquad (A\text{-}1)$$

$$A_t = \Pi_{20} + \Pi_{21}K_{t-1} + \Pi_{22}X_{t-1} + \Pi_{23}Z_{t-1}$$

$$+\Pi_{24}e_{1t} + \Pi_{25}e_{2t} + \Pi_{26}e_{3t} + \Pi_{27}u_{1t} + \Pi_{28}u_{2t} + \Pi_{29}u_{3t} \qquad (A\text{-}2)$$

This trial solution makes use of the fact that we assume that the linear structure of the model and the stochastic specification (13a), (13b) and (13c) are not expected to change over time. Given (A-1) and (A-2) it is a simple matter to calculate the expressions for $E[P_{t+1}]$ and $E[A_{t+1}]$ which appear in the market clearing conditions (10) and (11). If the trial solution in (A-1) and (A-2) is in equilibrium, then substituting from (A-1) and (A-2) for expressions into (10) and (11) must yield identities in the state variables. The resulting identities are (neglecting constants):

$$\begin{aligned}
&\Pi_{11} = \alpha_1\Pi_{11} + \alpha_2\Pi_{21} + \alpha_3 && \Pi_{21} = \beta_1\Pi_{11} + \beta_2\Pi_{21} + \beta_3 \\
&\Pi_{12} = \alpha_1\Pi_{12} + \alpha_2\Pi_{22} + \alpha_5 && \Pi_{22} = \beta_1\Pi_{12} + \beta_2\Pi_{22} + \beta_5 \\
&\Pi_{13} = \alpha_1\Pi_{13} + \alpha_2\Pi_{23} + \alpha_4 && \Pi_{23} = \beta_1\Pi_{13} + \beta_2\Pi_{23} + \beta_4 \\
&\Pi_{14} = \alpha_6 && \Pi_{24} = \beta_6 \\
&\Pi_{15} = \alpha_7 && \Pi_{25} = \beta_7 \qquad\qquad (A\text{-}3) \\
&\Pi_{16} = \alpha_8 && \Pi_{26} = \beta_8 \\
&\Pi_{17} = \alpha_3 && \Pi_{27} = \beta_3 \\
&\Pi_{18} = \alpha_5 && \Pi_{28} = \beta_5 \\
&\Pi_{19} = \alpha_4 && \Pi_{29} = \beta_4
\end{aligned}$$

Solving (A-3) for Π_{ij} $(i=1,2; j=1,\cdots,9)$ yields:

$$\Pi_{11} = [(a_7+a_8)(1-\delta a_4) + \delta a_4(a_{11}+a_{12})]/E$$

$$\Pi_{21} = [(a_7+a_8)(1-\delta a_4) - (a_2+a_3)]/E$$

$$\Pi_{12} = -a_9(a_{11}+a_{12})/E \qquad \Pi_{22} = a_9(a_2+a_3)/E$$

$$\Pi_{13} = [a_5(a_{11}+a_{12}) - (a_7+a_8)(a_5+a_{13})]/E$$

$$\Pi_{23} = [a_{13}(a_2+a_3) - (a_7+a_8)(a_5+a_{13})]/E$$

The remaining coefficients are as in (A-3). Substituting back into (A-1) and (A-2) yields the expressions for P_t and A_t appearing in equations (14) and (15).

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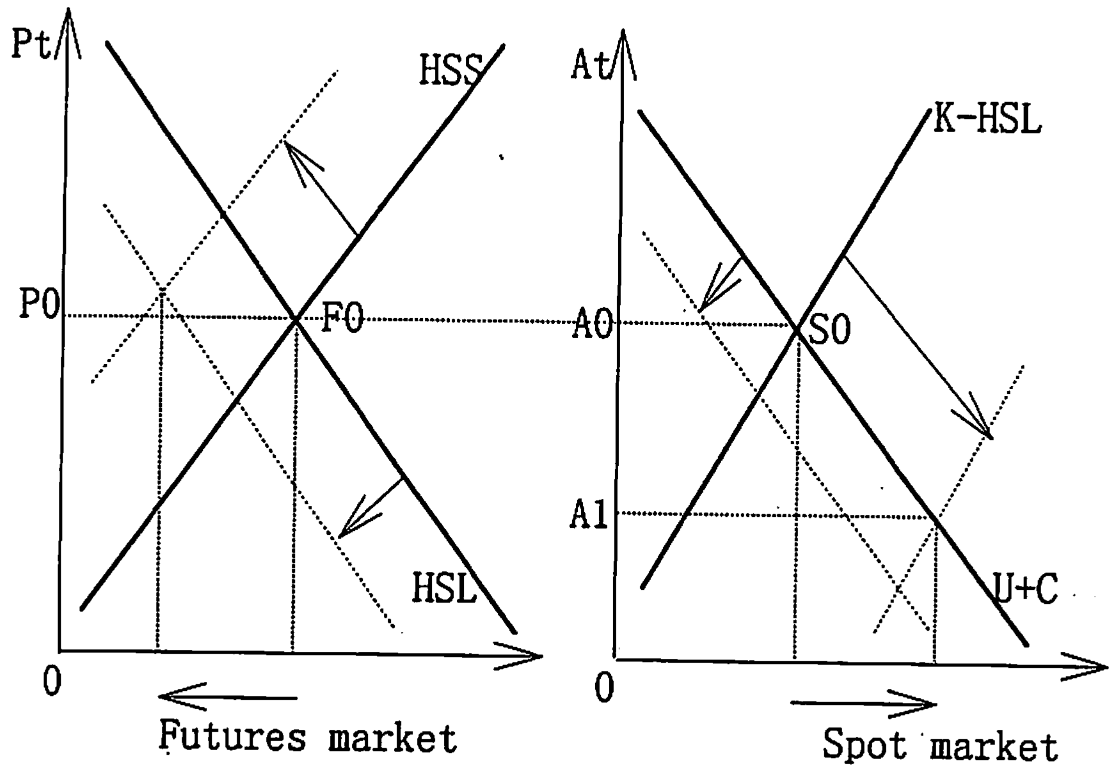

Figure 1 The effect of an increase in Z