The Generalized Ozaki-McFadden Cost Function
and Its Application to Panel Data

by

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Abstract

We introduce a globally concave flexible cost function and apply it to a panel of firms in the Japanese paper & pulp industry. This cost function (GOM) is a mixture of the *Generalized McFadden* form with the *Generalized Ozaki* form due to Nakamura (1990). We use a generalized index of technical change in place of the standard quadratic form of time-trend. The estimated GOM form satisfied global concavity, and the symmetry condition was not rejected. Homotheticity was strongly rejected.

1 Introduction

Homotheticity is an extremely useful property that greatly simplifies (and makes highly operational) the analysis of producer behavior. It is too well known that homotheticity is fundamental to the consistency of aggregation over producers and of multi-stage optimization procedures (see Lau (1982) and Blackorby, Primont and Russell (1978), among others). When it comes to empirical evidence obtained form micro data, however, the literature abounds in results rejecting homotheticity (see Baltagi and Griffin (1987), Atkinson and Cornwell (1994a,b), and Norsworthy and Jang (1992), among others).

This paper adds a nonhomothetic functional form to the already crowded literature on functional specifications (see Chung 199? for a recent survey). Nakamura (1990) introduced a nonhomothetic flexible cost function, the generalized Ozaki (GO) function, and showed an empirical example where it was found superior to the well known translog (TL) and generalized Leontief (GL) functions. A distinguishing feature of the GO is that it includes the nonlinear Leontief fixed coefficients model considered by Komiya (1962) and Ozaki (1969), among others, as a special case .

Most flexible cost functions including the TL and GL cannot satisfy global concavity without losing flexibility in the price space. This applies to the GO as well, since it is a nonhomothetic extension of the GL. In order to render global concavity to the GO without losing flexibility, we need to replace its price substitution term by that of a globally concave flexible function.

We derive a globally concave version of the GO by replacing its price substitution term with that of the generalized McFadden (GM) cost function that can be both globally concave and flexible (Diewert

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and Wales 1987) at the same time. The resulting cost function, the generalized Ozaki-McFadden form (*GOM* for short), can be globally concave and flexible in the price space, while keeping its original nonlinear nonhomothetic form.

For an empirical illustration, I apply the *GOM* to a panel of twenty-six firms in the Japanese paper & pulp industry for the period of 1976-87, and compare its results with a translog form. It is standard in the literature to specify technical change as a quadratic function of time trend (see Jorgenson (1986)). However standard this practice is, it is a mere reflection of our ignorance. Instead of following this practice, I use the alternative proposed by Baltagi and Griffin (1987) of directly estimating a general index of technical change using time dummies and a panel data set.[1]

2 The Model

2.1 The *GOM* cost function

Our starting point is the *GO* cost function introduced by Nakamura (1990):

$$c(p, y) = \left(\sum_{j \neq j} b_{ij} \sqrt{p_i p_j} + \sum_i b_{ii} p_i y^{\beta_i} \right) h(y) \tag{1}$$

where p is a $m \times 1$ vector of input prices, and y is a scalar output [2]. Except for the nonlinear-nonhomothetic term, this is simply the *GL* due to Diewert (1971). Unfortunately, the *GL* has a disadvantage that it cannot be globally concave unless all inputs are mutually substitutable. The same applies to the *GO* as well. In order to render global concavity to the *GO* without losing flexibility we have to replace its *GL* term by that of a globally concave flexible function.

Fortunately, the set of cost functions with these properties is not empty; the generalized McFadden cost function *GM* (Diewert and Wales 1987) is such a function. The *GM* has a disadvantage that it treats one of the inputs that is used as a normalizer differently from the remaining $n-1$ inputs [3]. The symmetric version of *GM* (Diewert and Wales 1987) is free of this disadvantage, and is given by:

$$c(p, y) = \left(\frac{1}{2} p^\top S p / \theta^\top p + \sum_i b_i p_i \right) h(y) \tag{2}$$

where θ is a given $m \times 1$ vector of non-negative constants, not all equal to zero, and $S = [s_{ij}]$ is a $m \times m$ symmetric negative semidefinite matrix with: [4]

$$\sum_j s_{ij} = 0, \forall i. \tag{3}$$

The negativity condition of S can be imposed upon it by representing it as $S = -AA^\top$, with $A^\top$ being an upper triangular matrix (Diewert and Wales 1987, Theorem 9). This reparametrization does not reduce the number of parameters, and preserves flexibility.

[1] Kumbhaker and Heshmati (1995) is a recent example which uses the general index of technical change.

[2] y can be an aggregate of multiple outputs. In that case we implicitly assume separability of outputs from inputs.

[3] Nakamura (1995) reports an empirical example where the use of different inputs as the normalizer yields different results for curvature conditions.

[4] Price flexibility of the *GOM* in the sense of Diewert (1974) holds only at the price point p^* which satisfies $Sp^* = 0$ (Diewert and Wales (1987) p.54). This is equivalent to (3) when p^* is a unit vector which corresponds to the normalizing point of p.

Replacing the GL term of (1) by that of (2) and introducing the general index of technical change A (Baltagi and Griffin 1987) to account for disembodied technical change, we obtain the following generalized Ozaki McFadden, GOM for short, cost function:

$$c = \left[\frac{1}{2}p^\top Sp/\theta^\top p + \sum_i b_i p_i y^{\beta_i} e^{\gamma_i A(t)}\right] y^\beta e^{A(t)} \tag{4}$$

The corresponding demand function for the $i^{th}, i = 1, \cdots, m$ input per unit of output, a_i, is then given by

$$\begin{aligned} a_i &:= \frac{\partial c}{\partial p_i} = \frac{x_i}{y} \\ &= \left[S_i p/\theta^\top p - \frac{\theta_i}{2}p^\top Sp/(\theta^\top p)^2 + b_i y^{\beta_i} e^{\gamma_i A(t)}\right] y^\beta e^{A(t)} \end{aligned} \tag{5}$$

where S_i refers to the i^{th} row of S. Estimation of (5) would be a simple matter if $A(t)$ were observable. Following Baltagi and Griffin (1987), however, we can estimate (5) utilizing dummy variables and a pooled data set as

$$a_i = \left[S_i p/\theta^\top p - \frac{\theta_i}{2}p^\top Sp/(\theta^\top p)^2 + b_i y^{\beta_i} e^{\sum_t \gamma_{it}^* D_t}\right] y^\beta e^{\sum_t \gamma_t^* D_t} \tag{6}$$

where D_t is a time specific dummy ($t = 2, \cdots, T$) and D_k is a firm specific dummy ($k = 2, \cdots, N$). We take the initial year as the base year for $A(t)$ and set $A(1) = 0$. (6) is identical to (5) iff

$$\gamma_{it}^* = \gamma_i A(t) \tag{7}$$

$$\gamma_t * = A(t) \tag{8}$$

which implies

$$\gamma_{it}^* = \gamma_i \gamma_t^* \tag{9}$$

With (9) imposed, (6) becomes

$$\frac{x_i}{y} = \left[S_i p/\theta^\top p - \frac{\theta_i}{2}p^\top Sp/(\theta^\top p)^2 + b_i y^{\beta_i} e^{\gamma_i \sum_t \gamma_t^* D_t}\right] y^\beta e^{\sum_t \gamma_t^* D_t} \tag{10}$$

Estimation of the system of equations (10) with a panel data set will enable us to identify all the parameters of the GOM.

2.2 Technical Change, Scale, and Substitution Effects

We now turn to economic implications of the GOM form, and start from technical change. The dual rate of technical change (the growth rate of adjusted TFP) is given by

$$\begin{aligned} \dot{T} &= -\frac{\partial \ln c}{\partial A}\frac{dA}{dt} \\ &\approx -\left\{1 + \frac{\sum_i \gamma_i b_i p_i y^{\beta_i} e^{\gamma_i A}}{\Gamma}\right\} \Delta A \end{aligned} \tag{11}$$

where Γ refers to the expression inside the square brackets of (4) and Δ to the first order difference operator. (11) shows decomposition of technical change into two components consisting of the term referring to pure technical change (ΔA) and of that referring to the effects of nonneutral technical change and scale augmentation.

When time trend is used, the unit cost is given by (see Nakamura 1990)

$$c = \left[\frac{1}{2}p^{\top} Sp/\theta^{\top} p + \sum_i b_i p_i y^{\beta_i} e^{\gamma_i t}\right] y^{\beta} e^{\gamma t + 1/2\delta t^2} \tag{12}$$

where γ and δ are unknown parameters and t is the time trend. The rate of technical change then becomes

$$\dot{T} = -\left\{\gamma + \delta t + \frac{\sum_i \gamma_i b_i p_i y^{\beta_i} e^{\gamma_i t}}{\bar{\Gamma}}\right\} \tag{13}$$

where $\bar{\Gamma}$ refers to the expression inside the square brackets of (12). Thus, with time trend the rate of technical change can never be zero unless all the parameters referring to technical change are zero, which corresponds to the case of no technical change at all. In contrast to this, $\dot{T}$ in (11) can be zero when there is no change in A.

If the technology was subject to constant returns to scale (CRS), the rate of technical change would be identical to the traditional TFP measure given by

$$T\dot{F}P = -\dot{c} + \sum_i w_i \dot{p} \tag{14}$$

where $\dot{}$ refers to the growth rate and w_i to the share of the i^{th} input in the total cost of production with $\sum w_i = 1$. Otherwise, the following would hold (Baltagi and Griffin 1988, p.25)

$$T\dot{F}P = \dot{T} - \epsilon_{cy}\dot{y} \tag{15}$$

where ϵ_{cy} is the elasticity of unit cost with respect to output, that is given by

$$\epsilon_{cy} = \beta + \frac{\sum_i \beta_i b_i p_i y^{\beta_i} e^{\gamma_i A}}{\Gamma} \tag{16}$$

Partial derivatives of (11) with respect to input prices gives the bias of technical change

$$\frac{\partial \dot{T}}{\partial \ln p_i} = -\frac{\Delta A}{\Gamma}\left[\gamma_i b_i p_i y^{\beta_i} e^{\gamma_i A} - \left(\sum_j \gamma_j b_j p_j y^{\beta_j} e^{\gamma_j A}\right) w_i\right]. \tag{17}$$

When this takes a negative value, technical change is said to be the i-th input using; since an increase in p_i decreases x_i, the rate of technical change is positively associated with x_i, hence is increasing in x_i[5]. Similarly, when the value is positive, technical change is said to be the i-th input saving. From the definition it follows that:

$$\sum \frac{\partial \dot{T}}{\partial \ln p_i} = 0 \tag{18}$$

Similarly, the effect of output augmentation on the growth of TFP is given by

$$\frac{\partial \dot{T}}{\partial \ln y} = -\frac{\Delta A}{\Gamma}\left[\sum_i \beta_i \gamma_i b_i p_i y^{\beta_i} e^{\gamma_i A(t)} - \left(\sum_j \gamma_j b_j p_j y^{\beta_j} e^{\gamma_j A}\right) \epsilon_{cy}\right]. \tag{19}$$

We next turn to issues of scale effects. Under nonhomotheticity factor proportions change with a change in the size of production. In other words, a given change in the size of production can have

[5]This terminology is used by Jorgenson and Fraumeni (1980).

different effects on different inputs. The partial input elasticity of output gives input specific effects of the size of output:

$$\frac{\partial \ln a_i}{\partial \ln y} = \beta + \frac{\beta_i b_i y^{\beta_i} e^{\gamma_i A}}{\Gamma_i} \tag{20}$$

where Γ_i refers to the the expression inside the square brackets of (5). In the special case of factor limitationality ($S = [0]$), this simplifies to

$$\frac{\partial \ln a_i}{\partial y} = \beta + \beta_i \tag{21}$$

The partial input elasticity of output (20) is related to ϵ_{cy} given by (16) via

$$\epsilon_{cy} = \sum \frac{\partial \ln a_i}{\partial \ln y} w_i. \tag{22}$$

ϵ_{cy} is thus the input shares weighted mean of partial input elasticities of output. On the other hand, the overall scale elasticity, es, is given by

$$es = (1 + \epsilon_{cy})^{-1} \tag{23}$$

Substituting from (22), we obtain the following representation of es :

$$es = \left(1 + \sum \frac{\partial \ln a_i}{\partial \ln y} w_i\right)^{-1}. \tag{24}$$

The presence of nonhomotheticity itself does not imply economies of scale; share weighted partial elasticities of different signs can cancel each other resulting in es close to unity.

Finally, we turn to issues of substitution among inputs. The Allen Uzawa partial elasticity of substitution between inputs i and j, σ_{ij}, is given by

$$\begin{aligned} \sigma_{ij} &= c\left(\frac{\partial c}{\partial p_i}\frac{\partial c}{\partial p_j}\right)^{-1} \frac{\partial^2 c}{\partial p_i \partial p_j} \\ &= \frac{\Gamma}{\Gamma_i \Gamma_j}\left[S_{ij}(\theta^\top p) - \theta_i S_i p - \theta_j S_j p + \frac{\theta_i \theta_j p^\top S p}{\theta^\top p}\right] \frac{1}{(\theta^\top p)^2}. \end{aligned}$$

3 Stochastic Specification and Estimation

We apply the *GOM* to a panel of *KLM* (capital, labor and materials) data set on 26 firms in the Japanese paper & pulp industry for the period of 1976 to 87.Data Appendix gives details of the data.

Table 1 shows the variance of the price ratio of capital to labor and of the size of output over firms for each year (each normalized by the corresponding cross-sectional mean). Of the factors occurring in the demand equations (10), the size of production is the one that mostly differentiates individual firms[6]. Allowing for nonhomotheticity, we take account of this most differentiating factor. Still, it is likely that there are elements of firm specific effects which cannot be accounted for by nonhomotheticity alone.

A simple and widely used way to take account of firm specific effects is to introduce additive fixed effects via dummy variables (see Hsiao (1986), among others). In our setting, however, the usefulness of this standard specification is rather limited. First, since the model is nonlinear we cannot sweep out the firm-specific effects by using the covariance transformation.

[6]By construction, the price index of materials has only a negligible variance.

Table 1: Variance of factor price ratio and output:normalized at the mean

year	P_K/P_L	output
1977	0.07803	1.20079
1978	0.07803	1.20079
1978	0.17005	1.15142
1979	0.10785	1.17696
1980	0.09397	1.24132
1981	0.07255	1.16879
1982	0.06586	1.15803
1983	0.02868	1.16420
1984	0.04592	1.22765
1985	0.06122	1.21804
1986	0.05811	1.22834
1987	0.05675	1.25755
1988	0.09082	1.26063

Secondly, additive fixed effects are difficult to justify on economic grounds. Suppose that we introduce additive firm specific (but not input specific) dummy variables, say α_k with k referring to individual firms, into each of the three factor demand equations. Since the factor demand equations are related to the cost function via Shephard's lemma and the function is homogeneous of degree one in factor prices, the cost function of the k^{th} firm should be augmented by an additive term:

$$c_k = \alpha_k \sum_{i=K,L,M} p_i + c(p, y, A) \tag{25}$$

Since units of measurements are different among inputs, it is difficult to give reasonable economic interpretation to this term unless it is made input specific. Doing so, however, will drastically increase the number of estimating parameters.

An alternative to the fixed effects specification is the random effects specification.However, it is hard to accept that the twenty-six firms in our sample, after having controlled for explanatory variables, are different in a purely random manner. The assumption that the random disturbance are uncorrelated with prices and output may be unrealistic[7]. We therefore do not consider random effects specification of firm specific effects. Furthermore, the GLS estimation with a random effect specification could be quite cumbersome within the current nonlinear multivariate framework. [8]

Based on above considerations, we choose to introduce fixed firm specific effects in a multiplicative and input neutral manner as follows:

$$c \;=\; e^{\sum_k \alpha_k D_k} \left[\frac{1}{2} p^\top Sp/\theta^\top p + \sum_i b_i p_i y^{\beta_i} e^{\gamma_i A(t)} \right] y^{\beta} e^{A(t)} \tag{26}$$

where α_k are constant parameters referring to firm specific effects. This specification of firm specific efficiency corresponds to input technical efficiency, which refers to an over-utilization of inputs given output and the input mix (see Atkinson and Cornwell (1994a,b)).

[7] See Atkinson and Cornwell (1995, p.1)

[8] One could obtain consistent estimates of variance components following Srivastava and Giles (1987, p.272).

The estimating system of demand equations is then given by

$$\begin{aligned}
a_K &= e^{\sum_k \alpha_k D_k}\left[\frac{1}{\theta^\top p}\left(s_{KK}(p_K-p_M)+s_{KL}(p_L-p_M)\right)-\frac{\theta_K}{2}p^\top Sp/(\theta^\top p)^2\right.\\
&\left.+b_K y^{\beta_K}\exp\left(\gamma_K\sum_t \gamma_t^* D_t\right)\right] y^\beta \exp\left(\sum_t \gamma_t^* D_t\right)+u_K \qquad (27)\\
a_L &= e^{\sum_k \alpha_k D_k}\left[\frac{1}{\theta^\top p}\left(s_{KL}(p_K-p_M)+s_{LL}(p_L-p_M)\right)-\frac{\theta_L}{2}p^\top Sp/(\theta^\top p)^2\right.\\
&\left.+b_L y^{\beta_L}\exp\left(\gamma_L\sum_t \gamma_t^* D_t\right)\right] y^\beta \exp\left(\sum_t \gamma_t^* D_t\right)+u_L \qquad (28)\\
a_M &= e^{\sum_k \alpha_k D_k}\left[\frac{1}{\theta^\top p}\left((s_{KK}+s_{KL})(p_M-p_K)+(s_{KL}+s_{LL})(p_M-p_L)\right)\right.\\
&\left.-\frac{\theta_M}{2}p^\top Sp/(\theta^\top p)^2+b_M y^{\beta_M}\exp\left(\gamma_M\sum_t \gamma_t^* D_t\right)\right] y^\beta \exp\left(\sum_t \gamma_t^* D_t\right)+u_M \qquad (29)
\end{aligned}$$

where the quadratic form $p^\top Sp$ takes the form

$$p^\top Sp = s_{KK}(p_K-p_M)^2+s_{LL}(p_L-p_M)^2+2s_{KL}(p_K-p_M)(p_L-p_M), \qquad (30)$$

θ is set equal to the sample means of a_K, a_L, a_M, and u_i, $i=K,L,M$, isa $TN\times 1$ vector of error terms.

Let $u_{i,t,s}$ be the element of u_i referring to time t, $t=1,2,\cdots,T$ and firm s, $s=1,2,\cdots,N$, and $u_{t,s}$ be the 3×1 vector obtained by stacking $u_{i,t,s}$ over three inputs. We assume that the contemporaneous error terms are identically distributed over time and firm:

$$E\left(u_{t,s}u_{t,s}^\top\right)=\Sigma,\ \forall t,s \qquad (31)$$

where Σ is positive definite. Note that our dependent variable is the input per unit output and not the input level itself. To the extent that the variance of the latter is proportional to the level of output, our assumption of homoscedasticity will be a reasonable one. We further assume that the errors of different firms are mutually uncorrelated and that there is no serial correlation. Write u for the $3TN\times 1$ vector obtained by stacking $u_{t,s}$ first over time and then over firms. Then

$$E\left(uu^\top\right)=\Sigma\otimes I_{TN} \qquad (32)$$

4 Empirical Results

4.1 Estimation Results and Hypothesis Testing

We estimated the system of 3 equations (27), (28), and (29) for the pooled data of 312 observations by using the iterative *SUR* [9]. Provided the error terms are uncorrelated with prices and output, the iterative *SUR* estimators of the parameters of the *GOM* are consistent and asymptotically normally distributed when $N\to\infty$ and $T\to\infty$ (Gallant 1975; pp. 45-48).[10]

The second column of Table 2 shows the estimates of parameters. Prior to making inferences, I tested the null of homoscedasticity by use of a Lagrange multiplier test. In particular, I regressed the square

[9] I used RATS version 4.2.

[10] We need both T and N to go to ∞ because α_i are estimated using the time series variation alone and γ_i by the cross section variation alone.

Table 2: Estimates of Parameters: *GOM*

	unrestriced			restrictied[a]		
parameter	estimate	t-value[b]	P-value	parameter	t-value	P-value
s_{KL}	-2.13391	-1.30	0.1947	-1.37174	-2.60	0.0092
s_{KK}	-2.38903	-1.26	0.2089	-1.51666	-2.49	0.0129
s_{LL}	-9.23643	-1.40	0.1623	-4.92625	-4.15	0.0000
β_{KK}	0.04159	2.10	0.0355	0.03773	2.56	0.0104
β_{LL}	4.41637	2.71	0.0068	5.38520	3.73	0.0002
β_{MM}	3.91665	3.25	0.0012	3.68448	4.84	0.0000
β	-0.37884	-5.73	0.0000	-0.32543	-14.84	0.0000
β_K	0.46215	7.11	0.0000	0.41579	14.27	0.0000
β_L	0.07098	1.02	0.3067	0.00000	na	na
β_M	0.25145	4.69	0.0000	0.20319	11.26	0.0000
γ_2	-0.00490	-0.19	0.8479	0.00000	na	na
γ_3	-0.10672	-1.05	0.2944	-0.09951	-1.04	0.3002
γ_4	-0.08061	-1.00	0.3188	-0.07288	-1.00	0.3164
γ_5	0.13434	1.07	0.2833	0.12926	1.05	0.2919
γ_6	0.03594	0.78	0.4349	0.03159	0.84	0.4029
γ_7	0.00004	0.00	0.9988	0.00000	na	na
γ_8	-0.03914	-0.83	0.4045	-0.04172	-1.01	0.3126
γ_9	-0.00870	-0.27	0.7864	-0.01231	-0.63	0.5316
γ_{10}	0.00870	0.19	0.8527	0.00000	na	na
γ_{11}	-0.11507	-1.10	0.2718	-0.11498	-1.11	0.2663
γ_{12}	-0.07884	-1.03	0.3020	-0.08038	-1.09	0.2753
γ_K	-1.49094	-3.24	0.0012	-1.66039	-2.82	0.0047
γ_L	-1.70344	-3.03	0.0025	-1.65649	-2.92	0.0035
γ_M	-0.54392	-1.49	0.1371	-0.52095	-1.31	0.1910
α_2	-0.09250	-4.21	0.0000	-0.08929	-5.76	0.0000
α_3	-0.00002	0.00	0.9974	0.00000	na	na
α_4	-0.01950	-4.99	0.0000	-0.02015	-5.15	0.0000
α_5	-0.03705	-8.35	0.0000	-0.03838	-10.01	0.0000
α_6	-0.06375	-7.20	0.0000	-0.06248	-8.66	0.0000
α_7	-0.17568	-8.20	0.0000	-0.17320	-10.42	0.0000
α_8	-0.26510	-4.49	0.0000	-0.25877	-5.67	0.0000
α_9	-0.23955	-5.51	0.0000	-0.23389	-7.82	0.0000
α_{10}	-0.25997	-5.94	0.0000	-0.25406	-8.36	0.0000
α_{11}	-0.08583	-10.59	0.0000	-0.08613	-10.55	0.0000
α_{12}	-0.36788	-5.44	0.0000	-0.36065	-7.92	0.0000
α_{13}	-0.20672	-6.43	0.0000	-0.20140	-8.80	0.0000
α_{14}	-0.22834	-4.79	0.0000	-0.22276	-6.77	0.0000
α_{15}	-0.48424	-7.50	0.0000	-0.47757	-10.71	0.0000
α_{16}	-0.31820	-7.27	0.0000	-0.31234	-10.15	0.0000
α_{17}	-0.38835	-8.55	0.0000	-0.38255	-10.93	0.0000
α_{18}	-0.24321	-4.66	0.0000	-0.23656	-6.04	0.0000
α_{19}	-0.30963	-4.33	0.0000	-0.30107	-6.19	0.0000
α_{20}	-0.52829	-5.84	0.0000	-0.52134	-8.58	0.0000
α_{21}	-0.54481	-5.95	0.0000	-0.53700	-8.66	0.0000
α_{22}	-0.12496	-11.00	0.0000	-0.12355	-12.15	0.0000
α_{23}	-0.27249	-5.99	0.0000	-0.26689	-8.35	0.0000
α_{24}	-0.49848	-6.65	0.0000	-0.49138	-9.63	0.0000
α_{25}	-0.36427	-5.80	0.0000	-0.35635	-8.28	0.0000
α_{26}	-0.27613	-6.95	0.0000	-0.27066	-9.64	0.0000

[a] $\beta_L, \gamma_{2,7}, \alpha_2$ set equal to zero.
[b] Heteroscedasticity robust t -values

and cross products of estimated residuals on a constant, time as well as firm dummies, price of capital- and labor services, and output , and tested the null of zero slope coefficients (Davidson and MacKinnon 1993, p.560). It turned out that the null on the auxiliary equations is decisively rejected. The third column of Table 2 shows heteroscedasticity consistent t-values obtained by using the *robusterrors* option of *RATS*.

The estimates of s_{ij}, while insignificant, indicate that the estimated cost function is globally concave.[11] In obtaining the estimates I imposed the symmetry of s_{ij}. This is a testable hypothesis. The symmetry condition is usually formulated as a cross equation restriction. In the *GM* case, however, the situation is a bit different, because the full set of s_{ij} parameters appear in each of the demand equations. Accordingly, we will have to test for equality of all s_{ij} parameters across the three demand equations and not simply for the cross equation equality of $s_{ij},\ i \neq j$. The *Wald* test statistic of the null of symmetry was 3.22 with six degrees of freedom the P-value of which is 0.78. The estimated *GOM* demand functions thus satisfied the integrability conditions.[12]

For a comparison, I also estimated the *TL* form used by Baltagi and Griffin (1987):

$$\begin{aligned} \ln c &= \alpha + \sum_k \alpha_k D_k + \sum_i \beta_i \ln p_i + \beta_y \ln y + \sum_t \gamma_t D_t + 1/2 \sum_{i,j;i \neq j} \beta_{ij} \ln p_i \ln p_j \\ &\quad + 1/2 \beta_{yy} y^2 + \sum_i \beta_{iy} \ln p_i \ln y + \sum_t \gamma_t D_t \sum_i \beta_{iA} \ln p_i + \beta_{yA} \sum_t \gamma_t D_t \ln y. \end{aligned} \tag{33}$$

Table 3 shows iterative *SUR* estimates of the parameters obtained by estimating (33) together with the corresponding two share equations. With the estimates of β_{KK} and β_{LL} both being positive, the estimated translog cost function does not satisfy the global concavity condition.

I checked for the local curvature condition and found that it was violated at 25 of 312 observation points. Although the proportion of data points with improper curvature could be regarded as minor, this result is still in sharp contrast to that obtained for the *GOM* where the curvature condition was globally satisfied. The fact that the *TL* and *GOM* yielded remarkably different results for the curvature condition will be attributable to that their regularity regions have mutually exclusive areas (see Barnett and Lee 1984).

The *GOM* and *TL* are nonnested to each other. From the point of view of consistency with economic theory, the *GOM* preformed better than the *TL* due to its automatic satisfaction of global concavity. To obtain some rough idea about the statistical goodness of fit, the *Akaike Information Criterion* (*AIC*) could be used. *AIC* was 0.44282 for the *GOM* and 0.44366 for the *TL*. The two forms thus seem to have almost the same goodness of fit.

I now turn to hypothesis tests. Test results are also shown for the *TL* for reference purposes. Table 4 shows test results of the null of homotheticity, factor limitationality (no price induced substitution effects), and price independence (neutrality) of the rate of technical change. Homotheticity is decisively rejected by the *GOM* as well as the *TL*. Factor limitationality is rejected by the *TL*, but not by the *GOM*. Finally, the *GOM* decisively rejects neutrality of technical change whereas the *TL* does not. Our finding of nonhomotheticity is robust to alternative specifications of the cost function, whereas the results of factor limitationality and neutral technical change are mixed.[13]

[11] For reference purposes, the right half side of the table shows estimation results obtained by imposing zero restrictions on some parameters the estimates of which were insignificant statistically and numerically as well (these consist of β_L, $\gamma_{2,7,10}$, and α_3). With the restriction imposed, the estimates of s_{ij} become statistically significant at 2 percent.

[12] When tested for the restricted estimates of Table 2, the P-value turned out to be 0.69. The test result of symmetry was thus robust to the imposition of zero-restrictions.

[13] For the restricted *GOM* the test statistics were 213.14, 17.2 and 88.7, respectively. Each of the three hypothesis was decisively rejected including that of factor limitationality.

Table 3: Estimates of Parameters: TL

parameter	estimate	t-value*	P-value	parameter	estimate	t-value*	P-value
β_K	-0.16558	-7.29	0.00	α_{14}	-0.21439	-5.27	0.00
β_L	0.37469	12.79	0.00	α_{15}	-0.42842	-8.53	0.00
β_Y	0.13987	6.01	0.00	α_{16}	-0.29682	-7.84	0.00
β_{KK}	0.01293	1.36	0.1732	α_{17}	-0.36547	-10.25	0.00
β_{LL}	0.0394	2.83	0.0047	α_{18}	-0.20234	-4.92	0.00
β_{YY}	-0.02221	-5.94	0.00	α_{19}	-0.24166	-4.5	0.00
β_{KL}	-0.06801	-8.2	0.00	α_{20}	-0.39743	-6.3	0.00
β_{KY}	0.02176	11.48	0.00	α_{21}	-0.40655	-6.48	0.00
β_{LY}	-0.02246	-9.33	0.00	α_{22}	-0.13715	-8.43	0.00
β_{KT}	-0.80263	-0.72	0.4744	α_{23}	-0.28828	-7.32	0.00
β_{LT}	-0.53269	-0.7	0.4869	α_{24}	-0.41247	-7.35	0.00
β_{YT}	0.0019	0.01	0.9881	α_{25}	-0.3161	-6.4	0.00
α_2	-0.14409	-4.06	0.00	α_{26}	-0.27861	-7.87	0.00
α_3	0.01042	0.72	0.47	γ_2	-0.00076	-0.15	0.8791
α_4	-0.03103	-2.26	0.0236	γ_3	-0.02171	-0.73	0.4672
α_5	-0.05307	-3.76	0.0002	γ_4	-0.01369	-0.72	0.4734
α_6	-0.11506	-6.06	0.00	γ_5	0.02117	0.72	0.4732
α_7	-0.17105	-7.6	0.00	γ_6	-0.00304	-0.44	0.6568
α_8	-0.23058	-5.08	0.00	γ_7	-0.00675	-0.64	0.5198
α_9	-0.21743	-5.68	0.00	γ_8	-0.01901	-0.73	0.467
α_{10}	-0.23411	-6.12	0.00	γ_9	-0.01064	-0.7	0.483
α_{11}	-0.0749	-5.44	0.00	γ_{10}	-0.01013	-0.7	0.4848
α_{12}	-0.30641	-5.81	0.00	γ_{11}	-0.03274	-0.73	0.4628
α_{13}	-0.19016	-6.22	0.00	γ_{12}	-0.02493	-0.73	0.4642

* Heteroscedasticity robust t -values

Table 4: Test Results of Hypothesis

model	statistics	d.f.	P-value
homotheticity			
GOM	244.97	3	0.000
TL	213.14	2	0.000
factor limitationality			
GOM	1.98	3	0.576
TL	136.44	2	0.000
neutral technical change			
GOM	83.49	3	0.000
TL	0.5119	2	0.774

4.2 Elasticities

Table 5 shows the output weighted means of elasticities . We find that this industry is characterized by significant economies of scale with *es* around 1.13. The estimates of partial scale elasticities indicate that with an increase in output the demand for capital increases more than proportionally, while the demand for materials and labor increase less than proportionally with the latter to the least extent. Consequently, with other things being equal, the ratio of capital to labor increases with output. Estimates of β_K and β_L in Table 3 indicate that the TL model also shares this nonhomothetic feature. Over time, the elasticity was increasing for capital whereas for labor and materials it remained relatively stable.

To convey a cross sectional picture, Figure 1 shows the scale elasticities (overall minus unity and partial) of individual firms for 1985. At the aggregated level it was found that the ratio of capital to labor increases with output. We find that this holds at the level of individual firms as well. The partial scale elasticity for capital tends to increase with the size of output. Since the output size was growing over time, this correlation with the output size explains the increase over time of the partial scale elasticity for capital.

From the estimates of *Allen Uzawa elasticity of substitution*($AUES$) we find that $K - L$ are complements, $L - M$ are fairly substitutable, and $K - M$ are also substitutable but to a much smaller extent. It thus appears that not the relative factor prices but the nonhomothetic scale effect was the major determinant of the capital/labor ratio.

We proceed further with the analysis of aggregated elasticities, and next turn to those related to technical change and TFP growth shown in the lower half panel of Table 5. The rate of technical change $\dot{T}$ in the second column is highly volatile with its value fluctuating from .001 to -0.063. This is a common feature of the generalized technology index (see Baltagi and Griffin (1987) and Kumbhaker and Heshmati (1995)). The sharp and large drop in 1980 corresponds to the second oil crisis. The 'standard' trend model (see (13)), by definition, cannot share this feature. This is demonstrated in Figure 2.[14].

Figure 3 shows the corresponding picture for the TFP growth rate . By construction, the TFP growth rate can fluctuate even for the trend model reflecting fluctuations in the growth rate of output (see (15)). Compared to the general index, however, the extent of fluctuation is still limited to a much smaller range.

The last four columns of the second half panel of Table 5 refer to bias of technical change. We find that technical change is capital and labor saving and materials using. With other things being equal, an increase in the prices of capital and labor increases the rate of technical change, whereas an increase in the price of materials works in the opposite direction. Furthermore, technical change is increasing in the size of output.

4.3 Efficiency

Finally, we turn to the analysis of measures of firm specific efficiency. Table 6 shows the correlation with output size (normalized by the cross sectional mean) of several variables related to efficiency for each year. As a supplement, Figure 4 shows the picture for individual firms for 1985 (the cross sectional pattern remained stable over time). We note that, the presence of significant economies of scale notwithstanding, the unit cost is not significantly negative correlated with the output size. This implies that firm specific efficiency and/or factor prices must be distributed across cross section in such a way that mitigates effects of economies of scale.

[14]The trend model was also estimated by the iterative SUR. Unfortunately, no conversion was achieved in the iteration without imposing zero restrictions on the firm specific parameters. Figure 2 uses the restricted estimates.

Table 5: Aggregated estimates of elasticities

year	es	$\partial \ln a_K/\partial \ln y$	$\partial \ln a_L/\partial \ln y$	$\partial \ln a_M/\partial \ln y$	σ_{KL}	σ_{KM}	σ_{LM}
1976	1.1364	0.0515	-0.3196	-0.1177	-1.9607	0.93	1.4104
197	1.1425	0.0367	-0.3196	-0.1164	-1.9046	0.93	1.4307
1978	1.1399	0.0588	-0.3148	-0.1206	-1.5712	0.74	1.2376
1979	1.1410	0.0545	-0.3146	-0.1206	-1.5594	0.69	1.2773
1980	1.1409	0.0182	-0.3258	-0.1128	-2.3390	1.09	1.6629
1981	1.1342	0.0635	-0.3176	-0.1202	-2.1154	0.87	1.4607
1982	1.1363	0.0698	-0.3141	-0.1225	-1.9711	0.76	1.4041
1983	1.1342	0.0885	-0.3074	-0.1272	-1.8161	0.59	1.3113
1984	1.1376	0.0821	-0.3058	-0.1271	-1.9245	0.60	1.3874
1985	1.1397	0.0840	-0.3024	-0.1284	-2.0297	0.57	1.4368
1986	1.1385	0.0966	-0.2982	-0.1321	-1.5040	0.40	1.1601
1987	1.1400	0.0945	-0.2957	-0.1323	-1.6155	0.40	1.2321

year	$\dot{T}$	$T\dot{F}P$	$\partial \dot{T}/\partial \ln p_K$	$\partial \dot{T}/\partial \ln p_L$	$\partial \dot{T}/\partial \ln p_M$	$\partial \dot{T}/\partial \ln y$
1977	0.001	-0.0014	-8.6E-05	-6.73E-05	-5.32E-05	0.00135
1978	0.024	0.0335	7.5E-03	0.00889	-0.01633	0.02936
1979	-0.006	0.0170	1.8E-03	0.00225	-0.00407	0.00744
1980	-0.063	-0.0797	1.1E-02	0.01202	-0.02275	0.05684
1981	0.025	0.0297	7.8E-03	0.00675	-0.01449	0.02803
1982	0.009	0.0133	2.9E-03	0.00283	-0.00568	0.01031
1983	0.009	0.0198	3.6E-03	0.00357	-0.00714	0.01164
1984	-0.007	-0.0049	2.5E-03	0.00288	-0.00539	0.00887
1985	-0.004	-0.0047	1.4E-03	0.00175	-0.00314	0.00505
1986	0.024	0.0265	1.1E-02	0.01479	-0.02607	0.03773
1987	-0.007	0.0000	3.1E-03	0.00438	-0.00752	0.01091

In fact, the third and fifth columns indicate that the firm specific input efficiency is negatively, and the aggregated factor price level (the aggregation was facilitated by using the estimated cost function with the level of output evaluated at the sample mean) is positively correlated with the size of output. Furthermore, scale effects diminish with an increase in the output size; at the output size of the largest firms the scale effects are almost nonexistent.

Table 6: Correlation of efficiency measures with the output size

year	unit cost	efficiency	scale effects	factor prices
1976	0.0165	-0.8000	-0.8337	0.6516
1977	-0.0343	-0.8031	-0.8374	0.6525
1978	-0.0849	-0.7997	-0.8458	0.3741
1979	-0.0520	-0.7953	-0.8358	0.5704
1980	-0.1761	-0.7991	-0.8311	0.4884
1981	-0.0758	-0.7949	-0.8306	0.4362
1982	-0.0831	-0.7981	-0.8320	0.5497
1983	-0.0836	-0.7893	-0.8396	0.5208
1984	-0.1029	-0.7903	-0.8385	0.5275
1985	-0.1668	-0.7890	-0.8413	0.5328
1986	-0.1438	-0.7818	-0.8350	0.4662
1987	-0.1551	-0.7828	-0.8413	0.4972

unit cost: fitted values of unit cost

efficiency: unit cost with the factor prices at unity and at the mean output level

scale effects: unit cost with no fixed effects and the factor prices at unity

factor prices: unit cost with the actual factor prices, no fixed effects and at the mean output level

scale elasticity: *es*

It appears that, other things being equal including the size of output, smaller firms tend to have higher efficiency than larger ones. However, talking about 'smaller' firms when the size of output itself is assumed constant across firms may not make much sense. This will in particular be the case when the sample contains firms with significantly different output size as in the present one where the output size of the smallest firm is less than 2 percent of the largest ones. Given the huge difference in the output size among firms, we should not strictly interpret firm specific fixed effects as a measure of efficiency, but loosely as a conglomerate of numerous omitted factors including differences in the product mix.

5 Concluding Remarks

We introduced a globally concave flexible cost function, the *Generalized Ozaki McFadden* form (*GOM*), and applied it to a panel of firms in the Japanese paper & pulp industry. The form has a distinguishing feature that it contains the nonlinear version of Leontief fixed coefficients model (Komiya 1962, Ozaki 1969) as a special case(see Nakamura 1990). Our specification is also characterized by the use of the generalized index of technical change in place of the standard quadratic form of time-trend.

The estimated *GOM* form satisfied global concavity automatically. Furthermore, the symmetry condition of the Slutsky matrix was not rejected; the estimated *GOM* demand functions satisfied the integrability conditions. We estimated the translog form (*TL*) as well, and found that it violated the concavity condition at eight percent of the observation points. The regular region of the *GOM* appears to give a better covering of our sample than the *TL*.

The hypothesis of homotheticity was strongly rejected by the *GOM* and *TL* as well. In particular, there was a strong indication that the capital to labor ratio increases with the output size. As for the hypothesis of factor limitationality, the test result was mixed .

The model in this paper is static, and assumes instantaneous adjustments for each of the factors of production. This is a testable assumption. An important future direction for research will therefore be to relax this stringent assumption, and to examine if our finding of nonhomotheticity is robust to general dynamic specifications [15].

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[15]Nonseparability of multiple outputs from inputs does not produce seemingly nonhomothetic relationships of otherwise homothetic relationships. See Färe and Mitchell (1993)

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A Data

The set of data used in this study is based on annual financial reports of corporations taken from the NEEDs tape, and was kindly provided to me by Professor Ichiro Tokutsu, Kobe University. In the following, we give a brief summary of how the data set was constructed. Tokutsu (1994) gives a full detail of the data set.

A.1 Price indices of output and materials

In order to obtain real values of output from nominal values in the financial report, we used the price index at the industry level that was obtained from national income accounts supplemented by the wholesale price index published by the Bank of Japan. This implies the absence of any cross-sectional difference in the price of output.

To obtain real values of intermediate inputs, we also used the price indices of materials and energy at the industry level. These indices were obtained as weighted averages of the price indices of output mentioned above and the price indices of imports with weights being given by input coefficients from input-output tables. These price indices at the industry level were further Divisia aggregated to obtain the price index of intermediate inputs, using value shares of materials and energy of individual firms.

Accordingly, the aggregate price index of intermediate inputs can be different over firms reflecting the difference in value shares among them.

A.2 Labor input

Labor is measured by the number of employees times hours of work. The wage rate was obtained by dividing the nominal labor compensation by labor input. We implicitly assume homogeneity of labor both over time and across cross section.

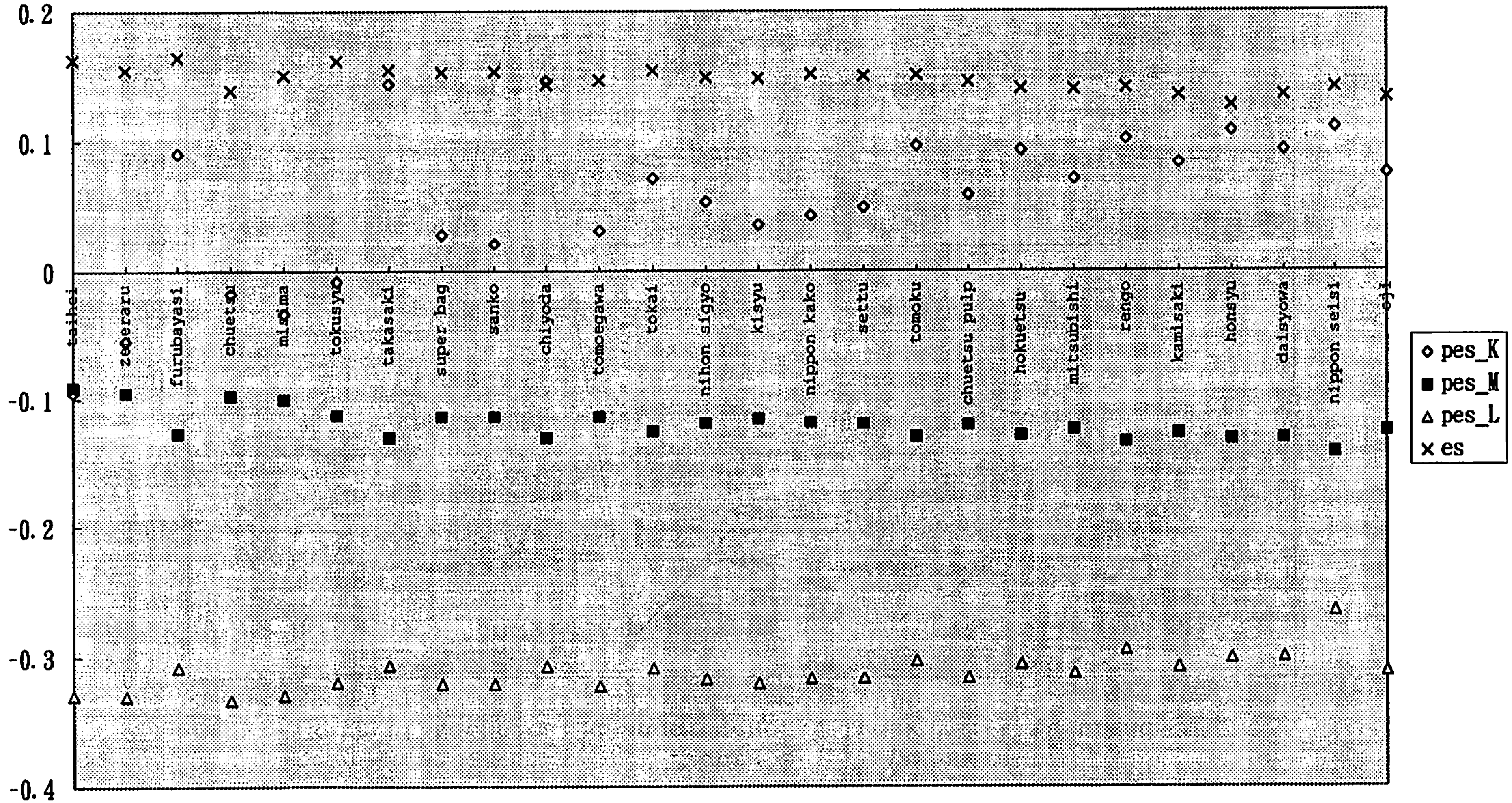
A.3 Capital input

The time series of real capital stock in 1980 prices was obtained by applying the perpetual inventory method to the benchmark value of real capital stock for 1965 and a fixed rate of depreciation, δ, given by the mean of annual rates of depreciation. The user cost of capital p_K was obtained for each firm from

$$p_K = q(r + \delta) - \dot{q}$$

where q is the price index of investment goods, r is the mean interest rate of firm's loans.

Figure 1
Partial and overall elasticities of scale: 1985



Firms in the ascending order of the size of production

Figure 2
Rate of technical change: general index vs. time trend

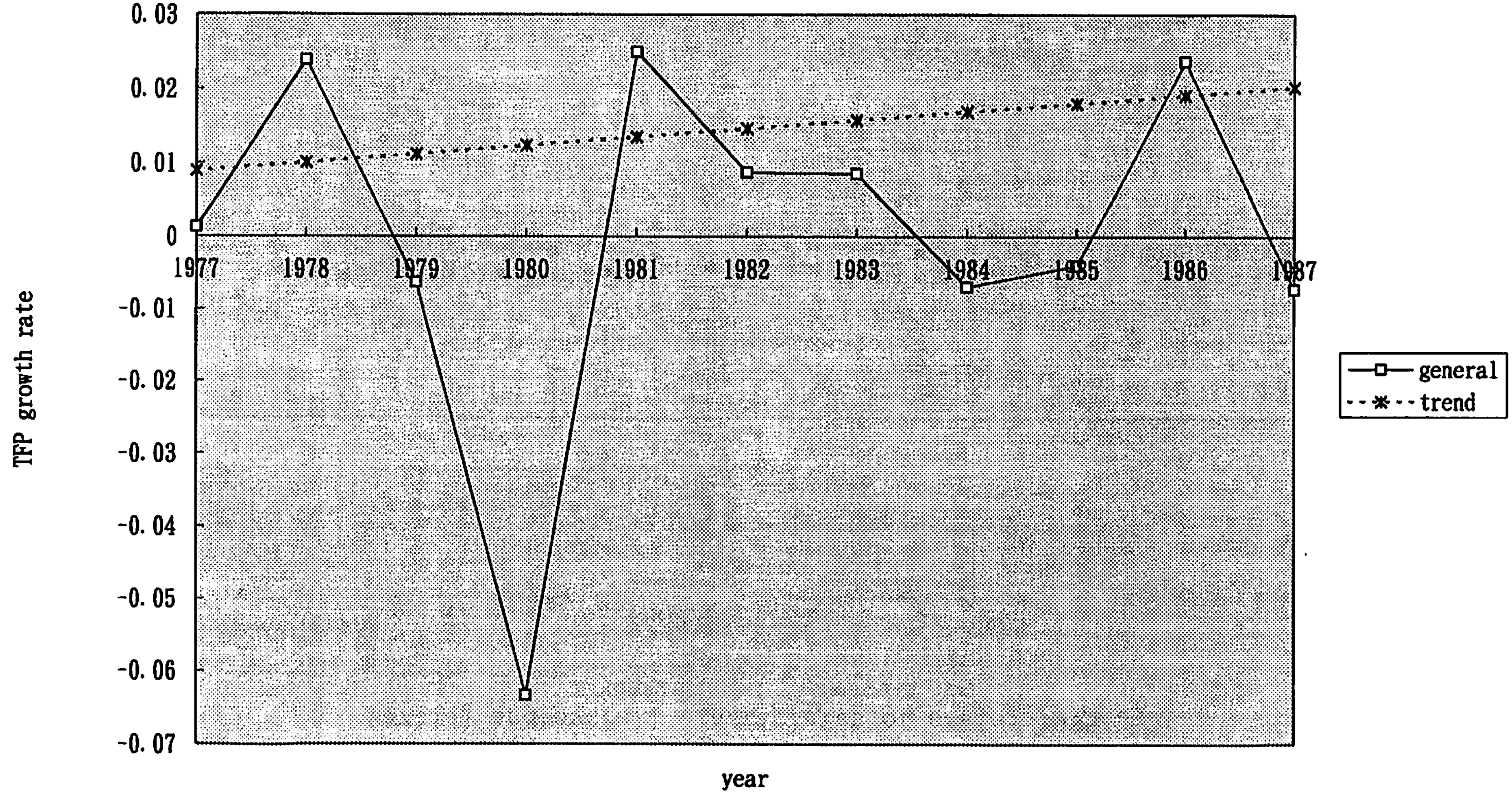


Figure 3
TFP growth rate: general index vs. time trend

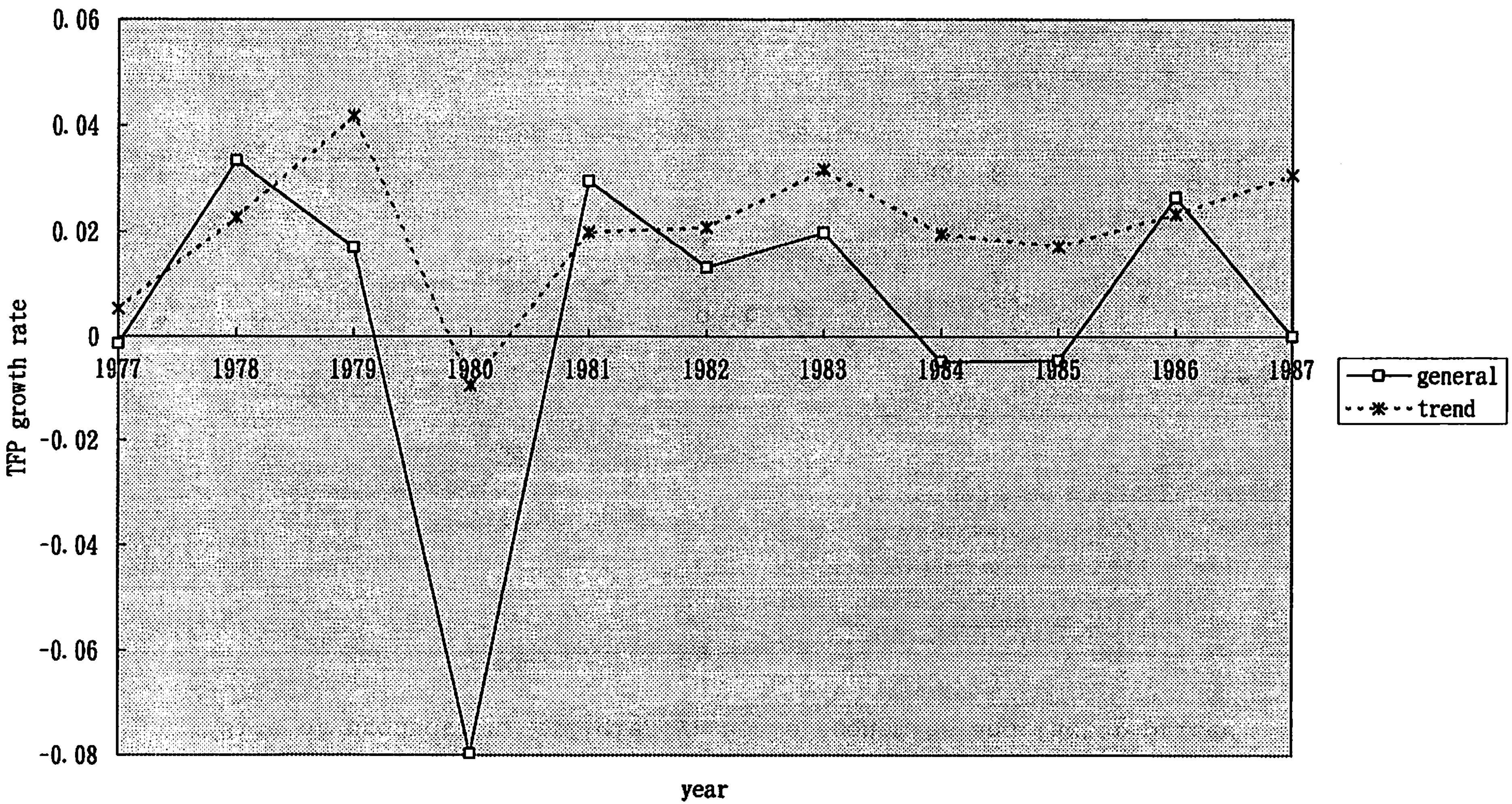


Figure 4

Size of production, efficiency, scale economies and factor prices